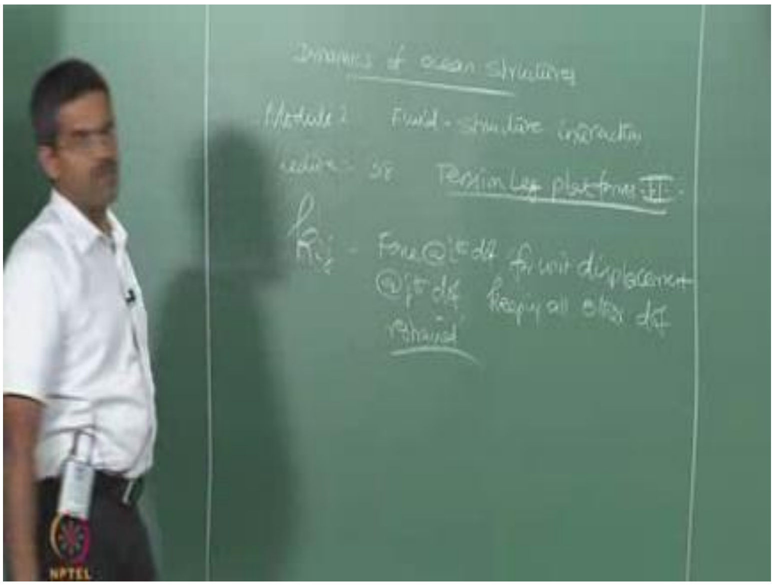
Dynamics of Ocean Structures
Prof. Srinivasan Chandrasekaran
Department of Ocean Engineering
Indian Institute of Technology, Madras

Lecture – 38
Tension Leg Platforms II

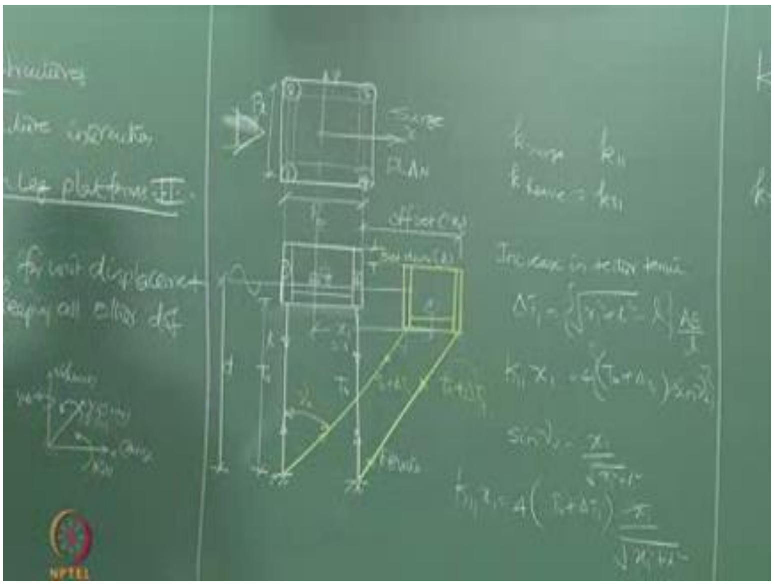
In the last lecture we discussed about requirements of deriving a stiffness matrix from a fundamental principle.

(Refer Slide Time: 00:22)



We already said k ij being the element member of a stiffness matrix is nothing but the force ith degree by giving unit displacement, for unit displacement at the jth degree keeping all other degrees of freedom restrained. So, we picked up (Refer Time: 01:00) element we showed you how one can derive the first column of this.

(Refer Slide Time: 01:07)



So, now we will pick up with the same problem in the TLP. So, we know that TLP comprises of contour in column members I will pick up as square plan shaped TLP where all the corners are having column members of circular in shape and all these are contour members and the typical elevation of this looks like this - is the top side, multi level deck. Let us say these are my column members, these are my tethers, this (Refer Time: 01:53) is my main sea level. Let me give some nomenclature and designation to this.

This is of course the plan, this is the elevation. Let us assume that the TLP is subject to dual directional wave in this axis, this is the wave propagating direction I call these members as 1, this column member as 2, clockwise; this column member as 2, this is 3 and this is 4 and assume a symmetric geometry let us say this becomes my x axis, this becomes my y axis in plan of course, this becomes my z axis in the elevation. The dimension of TLP centered central column along the wave propagation is taken as P b, b stands for the breath of the platform p is the plan let us say. And dimension normal to P b that is normal to the wave propagation is taken as P suffix L, L stands for length of the platform and P is the again the plan.

So, this becomes (Refer Time: 03:15) axis I am subjecting it to uni-directional way. So, when you drawn an elevation this column becomes 1, 3, 2 is hidden, this column becomes 4 and 3 is hidden and this becomes (Refer Time: 03:29) that is my contour number let us say this is my c g somewhere here let us see. So, I give unit displacement in the x axis. The cables are in extensible, they do not get plugged off from the location and we all know in classical terms this might be called as offset, this is what we call as set down which is indicated as delta in the nomenclature. An offset in my case is the degree of freedom x 1; let us say to this c g to this c g this is x 1. So, for derivation or definition this should be unity. Initially there were some tension in the cable which were nothing but T naught that T naught is a tension per cable or per tether.

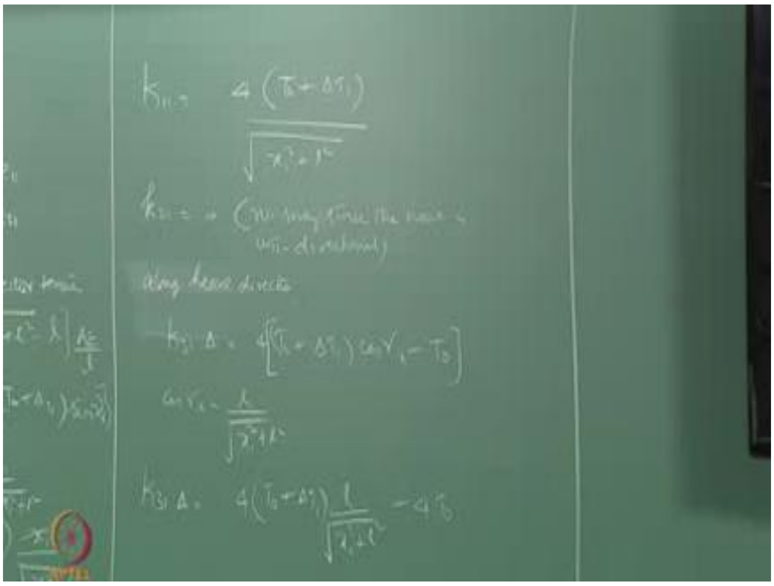
Now, since the length of the cable or the tether is inextensible; obviously, this tension now will be different I call this tension as T naught plus delta T 1. Delta T is an increment with respect to T naught and 1 stands for a surge degree, I call this angle. Now from the physical observation we can understand that there will be forces in surge direction because it has to reach to, there will be a force in heave direction because there is a set down happening which is coupled to offset; let us try to find out these 2 first which I will call as k 11 which I will call k 31.

The second substitute one stands for the first column of the matrix which also tells me that I am giving unit displacements about the first degree of freedom and the first subscript of 1 3 etcetera are respective degrees of freedom to area of measuring the values. To be very clear we know this that this what we are measuring this is x, let us say this is y and this is z, this is surge, this is sway, this is heave and this is roll put a thumb towards this forefingers it will point out this - this is my pitch and this is my m. These are the degrees of freedom which we are actually measuring and the point we are measuring is nothing but the c g of the platform c g we are trying to measure.

So, one can ask me why you are not bothered about the tethers, we are not talking about tether displacement, we are not talking about the hull separately, we are talking this as the rigid body motion. The contours and the columns and the deck move together it is a rigid body, they do not move separately. So, let us quickly find out the increase in tether tension delta T 1 is nothing but the change in length with a e by l that is actual stiffness with the member change in length from here can be easily found out as x 1 square plus l square root of that minus l that is the change in length multiplied by the axial stiffness will give him a delta T 1.

Now, this tension has gone in horizontal component, vertical component. Horizontal component is the restoration inside the axis vertical component is the restoration in the heave axis. So, let us talk about the horizontal component. So, that component is going to be the force in x axis multiplied by the displacement that gives me the force now because stiffness newton per meter which will give me the force now, which should be actually equal to T naught plus delta T 1. If this angle is gamma x this is also gamma x wave symmetry, so I should say sin gamma x now I am saying T naught and delta T 1 is the axial tension in each tether, I should multiply with 4 sets of such value because there are 4 such sets and from the geometry sin gamma x is nothing but x 1 value or let us say l we will replace l with root of x 1 square plus l square - that is the new length.

(Refer Slide Time: 09:34)

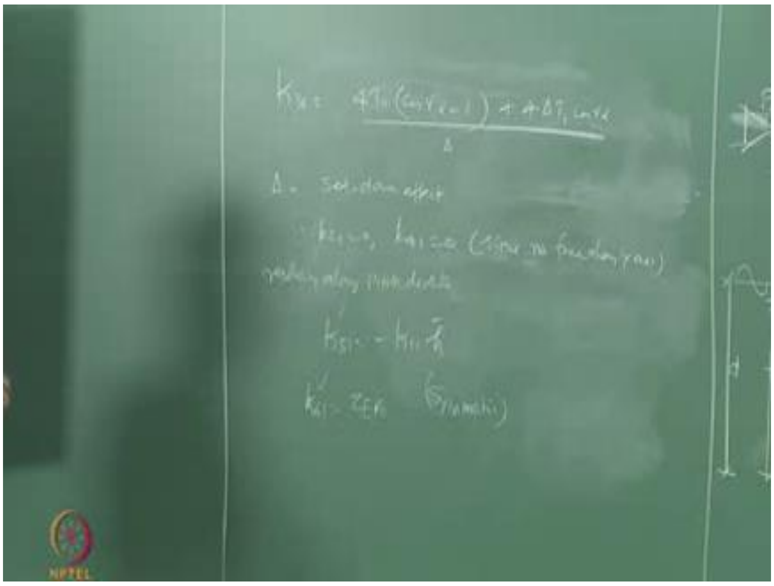


So, now I can easily get k 11 as 4 p naught delta T 1 x 1 (Refer Time: 09:27) plus l square, so k 11 4 times of T 0 plus delta T 1 by root of x 1 square plus l square. So, l is the length of the cable or the tether. So, fill up of this figure the platform does not move normal to the board, so there is no save movement. So, to get to k 21 is going to be 0 because there is no sway moment, since the wave is uni-directional - is an assumption that the wave is no hitting the platform only along x, there is no angle of attack. Now let us talk about the force in the vertical degree that is heave, k 31 should be the force multiplied by the value in terms of displacement of heave. So, I should say in heave axis let us say along heave direction k 31 multiplied by delta because that is the set down effect we have here that is the displacement.

Now, this should have a component of the vertical direction, the component resolved in the vertical axis which is going to be the cause of this value. So, T 1 or the T naught plus delta T 1 of cos gamma x of 4 times minus T naught because I am looking only for the increment initially it was in pre tension so I do not have to use T naught as such because T naught was not contributed because of the movement of x 1 because when the x 1 was not happening platform was stable and T naught was imposed already in the platform. So, T naught is not a component of x 1 or T naught does not arise because of x 1 the one is causing or influenced by x 1 is delta T 1. So, I have to subtract them.

So, cos gamma x again is l by root of x l square plus l square. So, substitute back here. So, I will get k 31 delta as 4 p naught delta factor one of l by root of x 1 square plus l square minus 4 T naught.

(Refer Slide Time: 12:31)



So, I can even do this as k 31 is 4 T 0 minus 1 then you will say 4 T naught cos gamma x minus one I am just replacing this value back as cos gamma x for simplicity, cos gamma x minus 1 plus 4 delta T 1 cos gamma x 4 divided by delta - where delta is what is called set down effect.

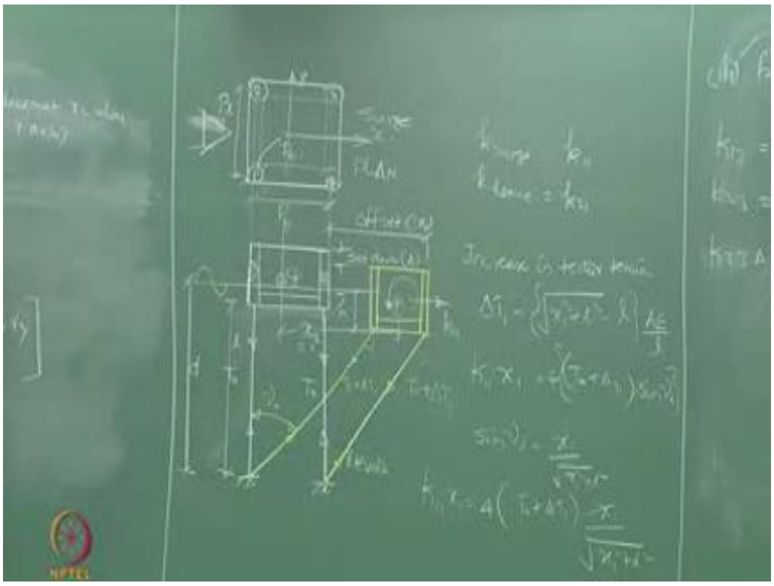
So, we have k 11, we have k 21, we have k 31, since k 21 is 0; k 4 – 1. You must understand that there is no force in the y axis therefore, there is no moment about x axis so roll will be 0. Since there is no force along y axis there is no moment about x axis, since there is force along x axis there will be movement about y axis there is pitch, resolving above pitch axis. So, k 11 is going to act this way, one can ask me a question how I have marked this direction - force is always happening only in the displacement of the cos T axis for example, take a beam, you give a displacement this way, so you have a displacement this way, the force always the direction as that of the displacement we cannot always have a force upper displacement at least in the first value. And we also know that k 11 being the diagonally direct member in the matrix it has got to be positive, it cannot be negative.

So, k 11 is marked this way if I call the distance of k 11 action which is going to happen at c g with respect to keel in a new position as h bar, which is also same earlier and k 51 is the restoring component then look at the original degree of freedom k 51 is restoring therefore, k 51 is going to be minus k 11 of h bar. So, k 51 I have k 61 is going to be 0 because the platform is symmetric.

On the other hand look at the geometry 1 2 3 4 all contour are equally spaced from the c g being a square platform. I have taken a dimension of P L and p b knowingly, but let us assume it as square platform they will have a symmetry. Even if it is rectangle they will be symmetrically placed otherwise also the diameter are same contour diameters are same therefore, the absolute the symmetry in geometry the platform will not (Refer Time: 15:49). So, k x 1 is 0 for a given x 1 displacement. So, we have all the six values available in the first column of the matrix 11 21 31 41 51 61, now we can enter them and we get the first column of the matrix.

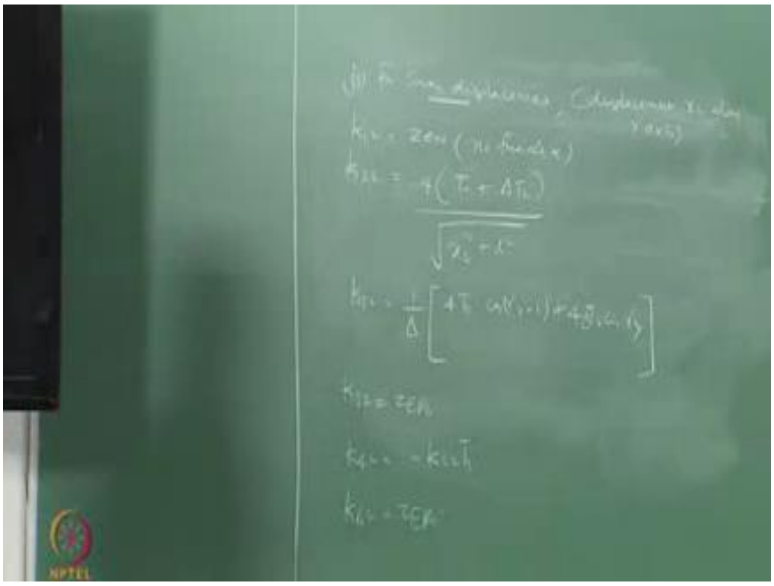
Similarly, if you want to give or derive the second part of the matrix in the sway direction instead of giving displacement along this I must give displacement along y.

(Refer Slide Time: 16:16)



So, if I give displacement along y then I have to draw a view from this direction. So, the right hand side will be to, I am drawing from here - left hand side is going to be 1, right hand is going to be simply 2 and I can say this is x 2, I get the same equation back again only thing is I will change the nomenclature slightly for a sway motion which I am doing it here.

(Refer Slide Time: 16:40)

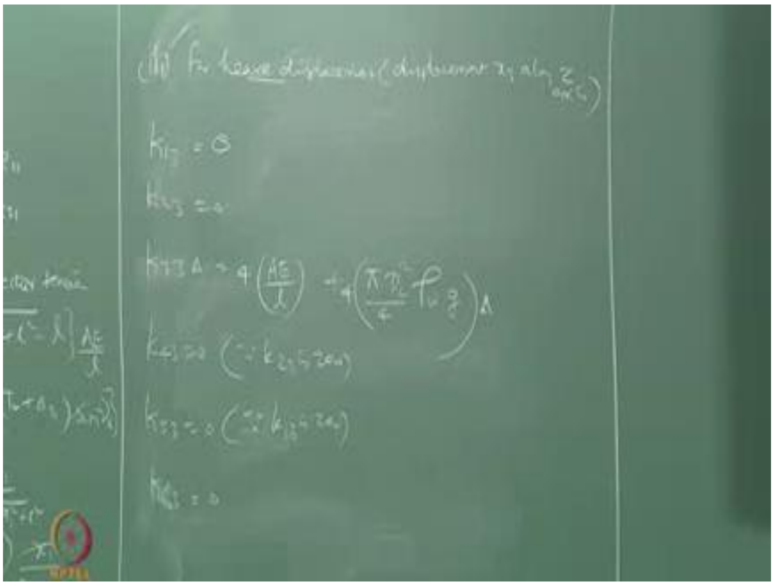


So, for sway displacement - that is displacement along y axis. So, same figure holds good except that I will change the nomenclature slightly I can rewrite the equations readily. So, I will get k 22, k 12 will be 0 because no force along x because of displacement caused along y. Force is already acting along x there is no movement along x because of displacement caused along y because we have to keep all other degrees restrained that is the derivation fundamental in stiffness matrix.

So, k 12 is going to be 0, we can always see that the symmetric k 21 was 0 and I am getting k 12 back as 0. So, that is called symmetry of the matrix. K 22 is going to be similar to that of 11 except the nomenclature will change, so I will say simply values are same nomenclature there 4 T naught plus delta T 2 because I am giving along the y axis the second degree root of instead of x 1 I will say x 2 square plus l square because x 2 is the displacement I have given along y and so on. Similarly, when I move along y there will be offset along y there is going to be set down along y also. So, there is going to be an influence of this y moment along the z axis which is in the heave direction therefore, there will be k 32, this is as same as k 31 which is going to be 1 by delta of and then 4 T naught cos gamma y minus 1 plus 4 T 2 cos gamma. The angle is now going to be gamma y then I am going to give displacement along y axis going to be delta T is it not, maybe delta t.

Now, I have no force along x therefore, I will have no moment about y therefore, k 52 will be 0 no pitch, and k 42 will be as similar to that of the pitch moment earlier which is going to be minus of k 22 of h bar same as that of the earlier case and k 62 is going to be 0, the platform is symmetric converted way actually there is no moment. Now I have all the columns, now, the second column 12 22 32 42 52 and 62 as the second column of the stiffness matrix now for the TLP.

(Refer Slide Time: 19:51)



For heave displacement that is displacement along displacement x 3 along z axis, all these displacement along the positive axis respectively.

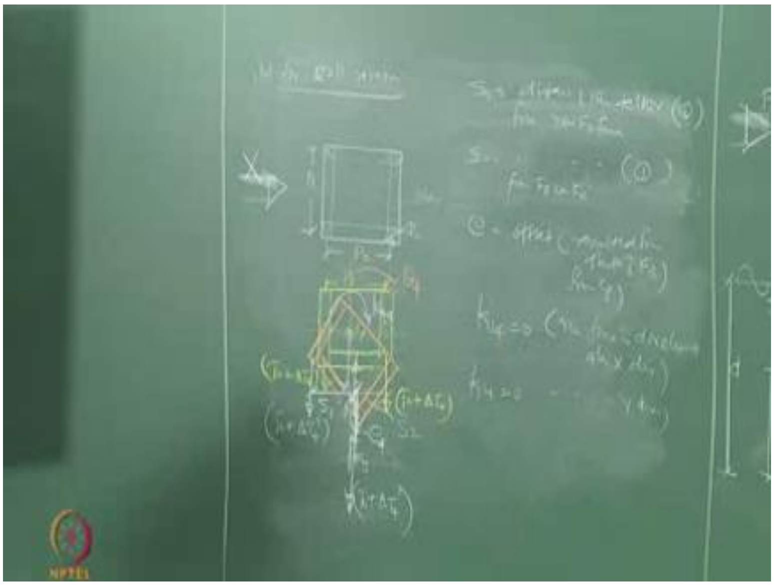
If I purely give displacement only along the z axis let us say only along the z axis, assuming that the platform hypothetically does not move along x and along y. So, the platform will remain like this not like the other one, when the platform moves only up and down there is going to be draft variation alone which will affect the submerged volume of the contours and the columns. So, the respiration force in the heave direction is going to come only by the change in displaced volume of the submerged members of 1 and 2 or 1 and 4 whatever may be the direction we will look at. All the 4 will become effective only because the platform is ideally moving only along the z axis. Therefore, k 13 there is no contribution in source surge direction because of the movement in the heave direction 0. Similarly k 23 there is no sway force because of key movement 0.

K 33 delta should be now counteracted by the initial tension in the tethers which is a e by l of 4 tab 4 tethers plus the submerged volume where d c is the diameter of the column, c stands for the column. Contour side will be completely submerged they will have no variable y and c only the column variation will be the variation nothing but delta that is the degree we have given. But delta is unity in this case by derivation, because I have to give a unit displacement to get the force in d. Just pushing the platform only in the positive heave axis ideally speaking it does not move along x and y.

Obviously, since k 13 is 0 k 43 is 0 since k 23 is 0 because roll and sway are associated. This is sway, this is roll, if sway is 0, roll is 0. K 53 is 0 since k 13 is 0 because pitch and size are associated when you give surge the cause the moment about y axis which is nothing but the pitch because of symmetry the platform does not (Refer Time: 23:32) you have the symmetry, symmetry is at 0. So, I have got the third column of the stiffness symmetry now they are only 33 present.

Let us talk about the roll displacement any doubt till this point. Now we are talking about rotational displacement I have given the transitional displacement along x along y and along z respectively. I have got the coordinates or I have got the members of the stiffness matrix for the first three columns. Now I am going to give the roll displacement or roll rotation.

(Refer Slide Time: 24:00)



For roll let us say rotation. Let us draw the diagram now, just say this is 1, this is 2, this is 3, this is 4 and this is p b and this is P L, this is minus wave axis, this is x, this is my plan, this is d c diameter of the column, this is diameter of the contour. Ideally speaking an alt TLP is usually they have kept same because they will have fabrication difficulty at these junctions they have taken almost same in all the cases usually. Now I am giving at give unit roll, roll means it has to be rotate about this axis. So, I have to draw the view from this direction now right hand side column will be one and left will be 2 let me draw that here.

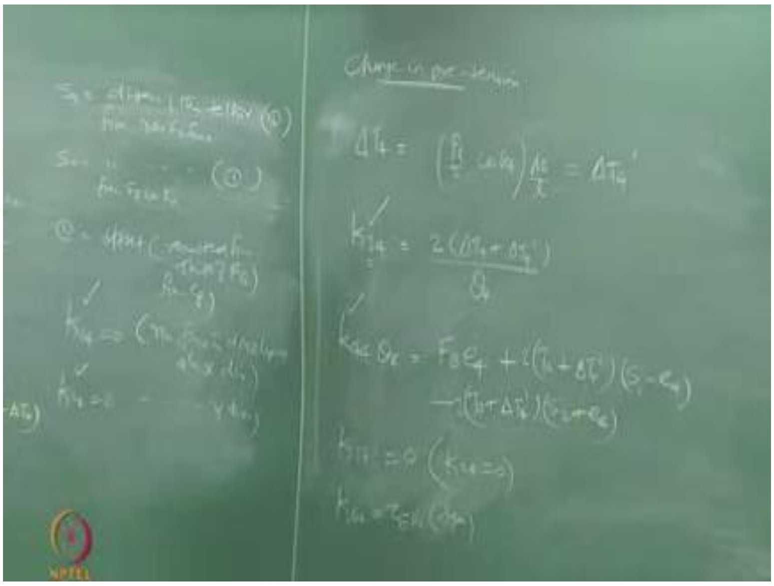
Strictly speaking, the central centre of this will be actually P L; I am looking from this direction I am looking for roll. This is pitch the moment about this axis is pitch, I am looking for this axis, I have to draw this blue. So, I can call this member as 1 and this member as 2. So, now, it is rolls, let us now mark the rolled value. This is the contour, let us say have the c g at this point, now I am drawing new configuration this is 1, this is 2.

Well I say this is my c g, my c g I call this angle as theta 4. So, k 44 will be responsible as is theta 4. Now obviously, you see the column one is submerged more compared to that of column 2 now. So, buoyancy force is nothing but an uproar force acting on the submerged value of the member the moment the member submerged more compare to the other one initially the buoyancy was co planar and co linear with center of gravity for static stability it has now it will now move towards 1. Because one is submerged more compared to 2, 2 is lifted up as per this configuration. So, I mark the new value of f b let us try to extend this slightly, extend it like - let me mark the new value f b here this will be new value f b which was the original value here I called offset of these as e suffix 4 - 4 stands for degree of freedom, 4 which is roll, e is the extent c.

Now, how this has been caused, how you are able to give this rotation it is because when you give this rotation; obviously, this cable what we have here will be pulled off more; let us mark this, will be pulled off more which will be T naught plus delta T 4 dash I will come to what is dash means and this cable which was here which is T naught plus delta T 4 dash. Initially they had only, T naught plus delta T 4 and T naught plus delta T 4. Now the difference between the tethers and variation with and without roll is the delta T 4 dash only that is what it has caused the rotation to theta 4.

Now, from the c g I call this force as S 1, the distance this as S 1. I hope the drawing is clear from the c g I call this as S 1 not from the buoyancy center, from the new buoyancy center I call this as S 1, this as S 1, of course, this is e which we already have here and the remaining distance for the new one is what we call as S 2. So, S 1 is the distance tether from the new buoyancy for center, S 2 is the distance of the tether you can say attached to column 1, this is column 2, this is attached to column one from the buoyancy center. E is off set which is created or resulted from shift advance from c g. So, this is not developing any force along x axis therefore, I should say k 14 is set to 0 because no force is developed along x direction, k 24 is also 0 because there is no force developed along y direction.

(Refer Slide Time: 32:24)



Now, there see what is the change in pre tension, the change in pre tension delta T 4. I am talking about delta T 4 which is change in pre tension caused because of the roll this should be actually equal to half of this dimension, half cos theta 4 there is going to be the x component like this cos theta 4 of course, multiplied by the initial axial tension available in the cable.

Now, one can ask me a question what is the difference between delta T 4 and delta T 4 prime. Delta T 4 is constant and equally gets involved in all the legs because of the roll movement, delta T 4 dash is only on those farthest tethers. So, delta T 4 ideally speaking is taken same as that of T 4. So, we assume that the tension change in the farthest leg is as same as that of the nearer leg also because the platform is symmetric that is why I am using P L by 2, had these been a triangular one then I would see that tension in the nearer leg will be different than that of the further one or if it is P L and P b and they are rectangular if you look at this dimension will be different, this dimension will be different.

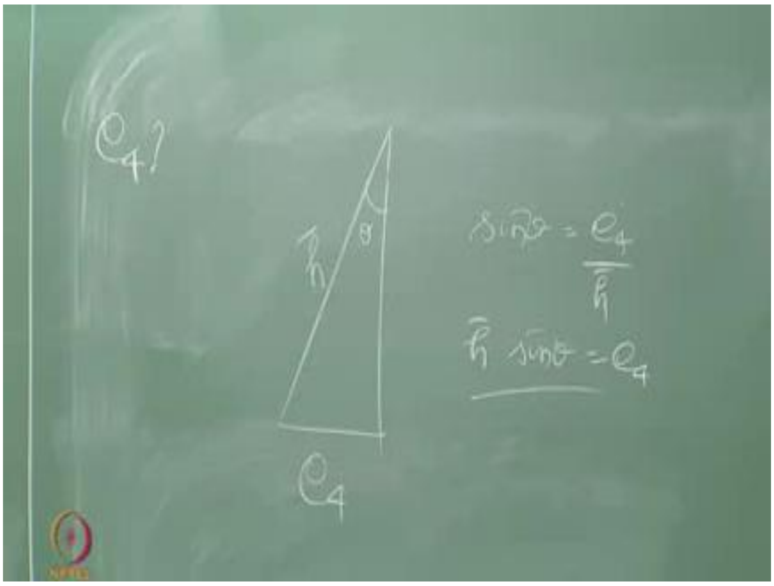
So, we are assuming that the platform is symmetric initially at P L by 2, since I am using P L by 2 I assume that the change in pin in the nearest leg and the farthest leg which are delta T 4 and delta T naught T 4 prime are same in all the legs; that is why I am multiplying this by 4. So, I should now get k 34 that is the force in the heave degree because of the roll given in the 4th degree of freedom which is going to be delta T 4 less delta T 4 dash, because this is going to be only nothing but the change in tension that is all because the change in tension alone is actually rolling the platform. Since I have got 2 near tension and 2 farther tensions, I will multiply this by 2 and I will divide this by theta 4 for dimension symmetry.

Now, let us understand how to compute k 44, how to compute k 44. Now there are 2 components involved in this - one is the set of these 2 legs, what are these 2 legs? For example, 2 and 1, 2 and 3 I am looking this platform from this direction, 2 and 3 are overlaid there can be tension at 2, there can be tension at 3 also. They will have moment about this f b of the distance S 1 and tethers attached to column 1 and 4 which are overlaid will have moment about f b again which is distanced at S 2, and S 1 and S 2 are not equal because they are offset by e. Initially they were equal which is nothing but p v by 2, which were equal to p v by 2 because all tension was same. Now it is not there because there is an (Refer Time: 36:03) because this column is getting submerged more compared to this column. On the other hand 1 and 4 are getting submerged more compared to 2 and 3 therefore, there is going to be change in tension marginally between these legs of tension to that of these legs therefore, there is an offset of f b, f b initially shifted from here towards this and that (Refer Time: 36:24) what e.

Now, I am trying to work out the moment why I am working out the moment because k 44 is a restoring moment in roll degree. So, to k 44 of theta 4 should be equal to this is anticlockwise you can see this is clockwise will be equal to now, this is tension in the cable and this is the point where f b is acting, f b is acting this way this is my buoyancy force. So, this is clockwise f b is having (Refer Time: 37:07) with c g which is e 4. So, f b of e 4 plus T 0 plus delta T 4 dash twice I am talking about this leg, this leg into S 1 minus c 4 I am talking about this leg which is counteractive to k 44 like f b minus now I am talking about this leg this one. So, I should say T 0 plus delta T 4 prime of S 2 plus e 4 of twice I will get k 44, k 14 was 0, k 24 was 0, k 34 is obtained k 44; k 54 and 64 were 0 due to y k 54 is 0, pitch was 0 because there was no sway. Similarly k 64 is 0 because it is symmetric.

Now in this equation you see f b is known to me that is a buoyancy force which is known to me for a given system of displacement, T 0 is of course, initial edition given to the system. S 1 and S 2 are geometric values, if you know the plot you can easily find out vectorially what is the shift of S 1 and S 2 because you have to find out the e 4 values, if you know e 4 there, now how to compute e 4?

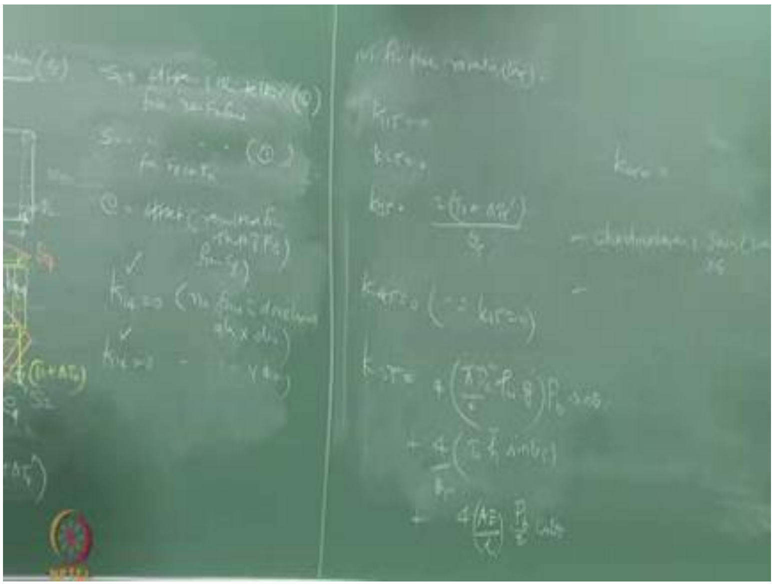
(Refer Slide Time: 39:17)



Now, how to compute e 4 the angle is subtended from the center from this center to this center look at the half of the portion. So, I have to plot it here. If this is e 4 this is the (Refer Time: 39:38) this is my h bar, this is my theta. This is rotating about the center half of that. So, therefore, sin theta e 4 by h bar, so h bar sin theta is e 4. So, e 4 has now become a function of displacement because theta is a displacement now, theta is actually a displacement it has become a function of displacement that is the beauty of the whole problem. All stiffness coefficients rather in all the cases they will all become function of displacements now, all stiffness coefficient will become function of displacements now that is the beauty of the whole problem.

Similarly I can extend the same discussion for pitch. So, I have to draw the figure from this view now because pitch is moment about this axis. So, 1 and 4 I will draw here 2 and 3 will be overlaid behind 4 and 1 respectively, I will give again theta 5, I will write directly the equations for the fifth column of the stiffness matrix - the same manner.

(Refer Slide Time: 41:01)

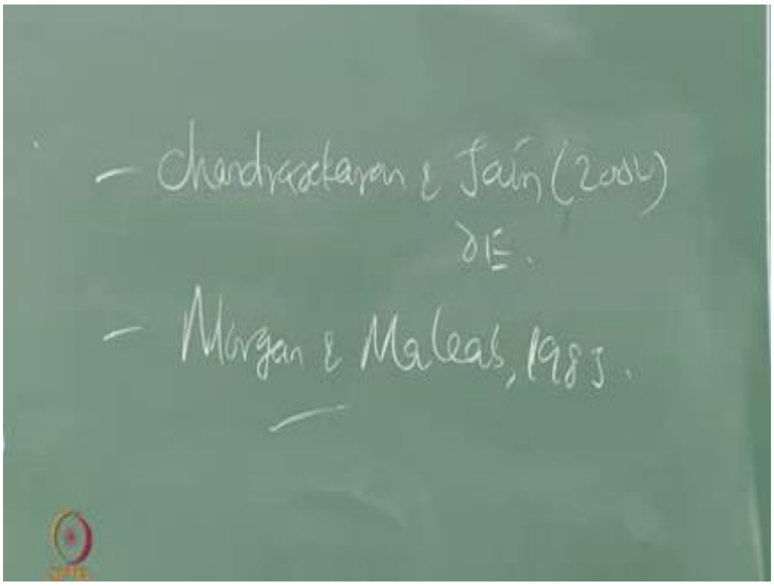


So, for pitch rotation which I call as theta 5, if by derivation all these are unity that is why we get the forces. So, k 15 remains 0, 25 remains 0, k 35 will be T 0 plus delta T 5 dash twice of 5, k 45 will remain 0 because k 15 is 0, k 55. Similar value how we wrote for k 44. The classical derivation given in the equation shows like this phi d c square by 4 root w g 4 times; that is nothing but the immersed volume of p b of sin theta 5. You can also write the similar way for theta 4 also which is roll plus 4 times of T 0 h bar sin theta 5 by theta 5 because this is coming exactly from e 5 plus 4 times of a e by l of that is change in tension which is p by p b by 2 cos theta 5. You can write either k 44 or 55 in the form earlier shown to you or in this new form, one - if you substitute back they will be exactly same.

So, I have a now k 65 is 0 due to symmetry. We have the sixth column which is (Refer Time: 43:13), we will do that in the next lecture. Then we will talk about some important characteristics of the stiffness matrix, we will talk about the mass matrix derivation then, we will move on to the damping matrix of TLP then we will talk about the solution procedure for TLP. So, any doubt here we have derived the members or elements of stiffness matrix step by step from the first principles like giving unit displacement respective degree of freedom and we got the forces in the respective degrees keeping all other degrees constraint.

Then there are some very special characteristics of the stiffness matrix members which I will discuss in the next lecture which is very important, any doubt? So, please look at the references available in the NPTEL I have got a classical paper there are 2 issues – one, I have my own paper with Jain, Chandrasekaran and Jain 2002, Ocean Engineering journal. Please see this reference the whole derivation is available there. The other one is Morgan and Meleab, 1983, again the reference is available in NPTEL please see that.

(Refer Slide Time: 44:21)



So, these are the 2 key papers based on which this derivation was transcribed actually. So, please see this papers you will get more, one is the conference there is ONE Houston. This is journal paper, ocean engineering journal paper. This is of course, ONE conference. The reference is available in NPTEL site you please see that, you get the full reference, locate the paper and try to read them. So, you will have the full derivation and explanation of what has been discussed in the blackboard here for more clarity in terms of understanding this. So, please look at this we will continue in next lecture.