Dynamics of Ocean Structures

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Lecture – 35
Response control of complaint structures (MLATs)

In the last lecture we started discussing about the advantages of articulated towers where they have been applied. So, we already said that they can be used for 3 main usages - 1 is can be used as anchor platform for transporting ships or vessels, you can use as a temporary base in naval application, you can use as a pipe line supporting system (Refer Time: 00:36) support system which is commonly used.

However, the tower being supported or the deck being supported on a single tower it has got lot of action in terms of rotation which was undesirable. So, people extended this by having multi legs which are parallel we already saw the advantages they design concepts evolved by MLAT. Now to write the equation of motion to MLAT or to solve let us try to see and understand very clearly that researchers have showed in the recent pass that response control of MLAT is mandatory because they give lot of excitation to the top side. And if you are able to control the response to some extent then one can improve the usage or the functionality or the operability of these towers not only we are having multi legs, but also we are having response control systems.

Before we move on to multi leg (Refer Time: 01:29) let us quickly talk about multi hinge and try to see how I can derive the equation of motion then, how I can solve just an introduction to that for few minutes. Then we will move on to MLAT where I focus on response control algorithm directly and we will see how we can do that.

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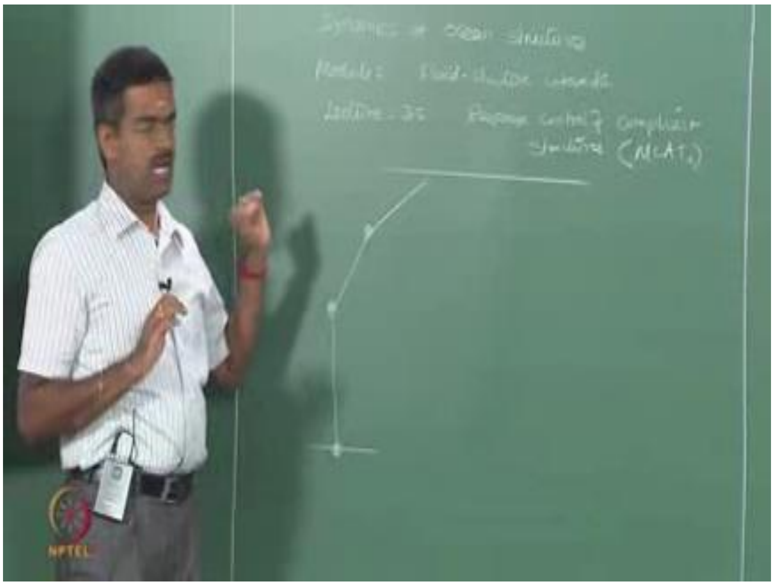

So, now to understand multi hinge we already said that I have a hinge at a bottom which can be identified as a universal joint which have a spring which is wide spring because it is having a rotational stiffness m theta, if it is a single tower it is a t, if it is multi tower then it is hinged and so on. So, these are the points where the hinges are located and so on and so forth. So, it is I say inverted pendulum actually. So, let us take up an example of an inverted pendulum of 2 degree quickly. Try to write equation of motion for that and assimilate that with this easily, it is easy for us to do that.

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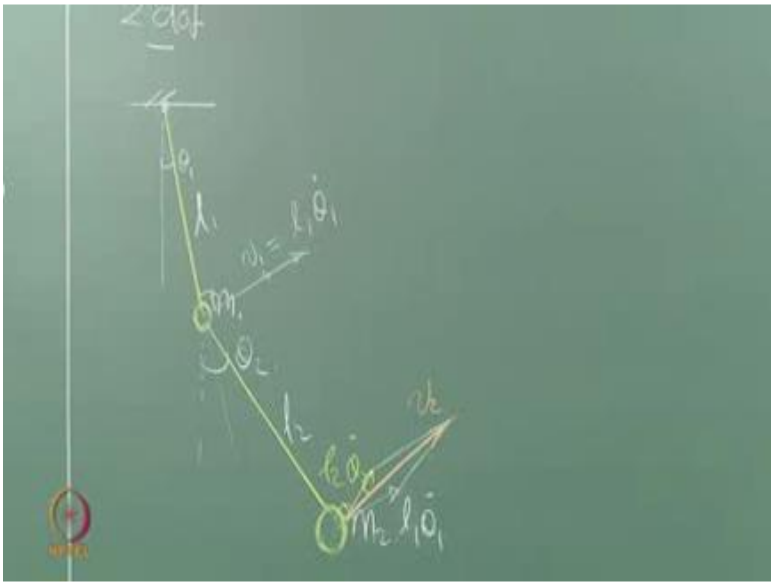

Let us take up a 2 degree freedom system a simple pendulum which is suspended like this, let us say hinge. We say the mass is lumped here, mass, let us call this as theta 1 and this as theta 2 this of course, as m 1 and this of course, as m 2 and l 1 and l 2 and so on. We know that the velocity vector of this which I can call as v 1 which is simple equal to l 1 theta 1 dot. I can also write down the velocity for this in a simple vectorial notation like this, see how we are doing it I draw a line parallel because I understand you know that this going to be normal.

So, I draw a line parallel to this here; obviously, this will not be normal to this because they are not parallel. So, I draw a line parallel to this here I presume that they are parallel, I mark v 1 here as a velocity of the first particle. I draw another line which is normal to this from this point, these 2 are normal. So, I rewrite this slightly in a different manner this going to be l 1 theta 1 dot and this can be l 2 theta 2 dot. I complete this parallelogram and get me resultant vector which is actually may be 2, this v 2. the angle between them will be theta 2 minus theta 1 that is a relative angle between the initial and the final position, this angle. The angle between these 2 will be theta 2 and theta, theta 2 minus theta 1; theta 2 is larger than theta 1. I can apply a simple cosine rule; I write down this v 2 let us do that.

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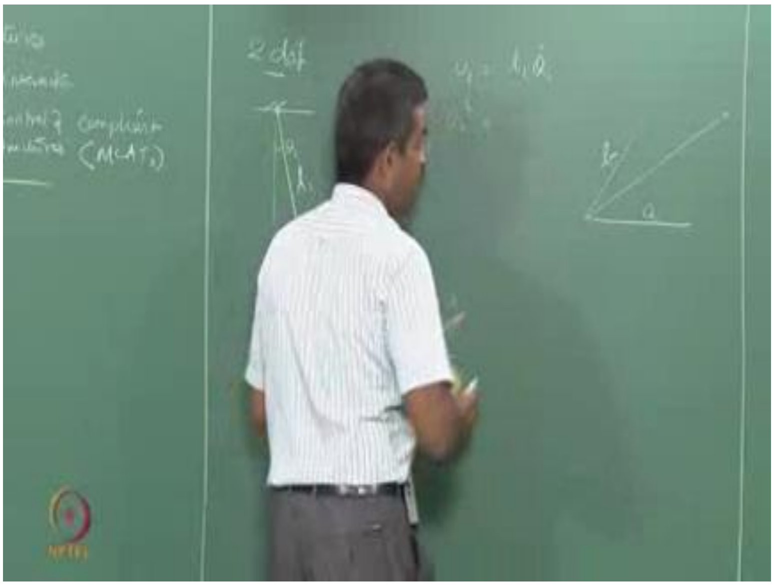

So, v 1 is going to be l 1 theta 1 dot where as v 2 square will be using a cosine rule, we already know a cosine rule. If I have let us say this is a and this is b I should say a square plus b square minus 2 a b c cos invert angle between a and b that will give me resultant of this, is it not.

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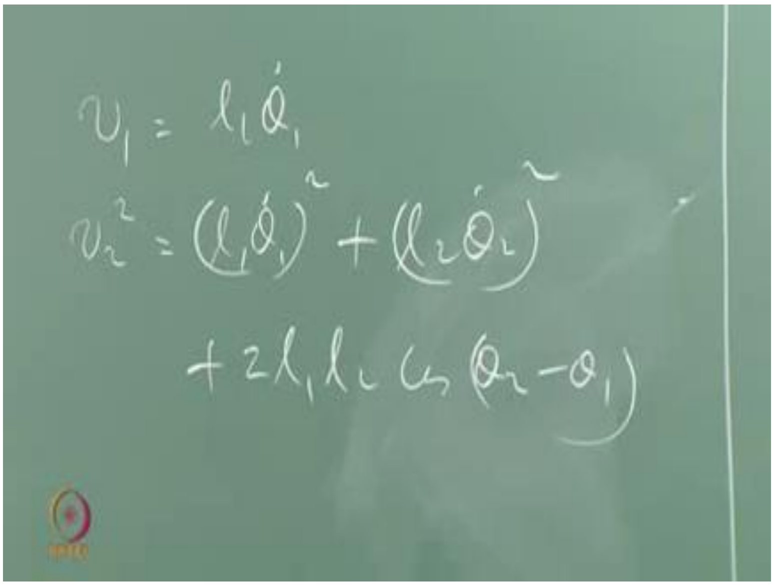

Similarly, I do it here l 1 theta 1 dot square plus l 2 theta 2 dot square plus 2 l 1 l 2 cos theta 2 minus theta 1, simple cosine rule.

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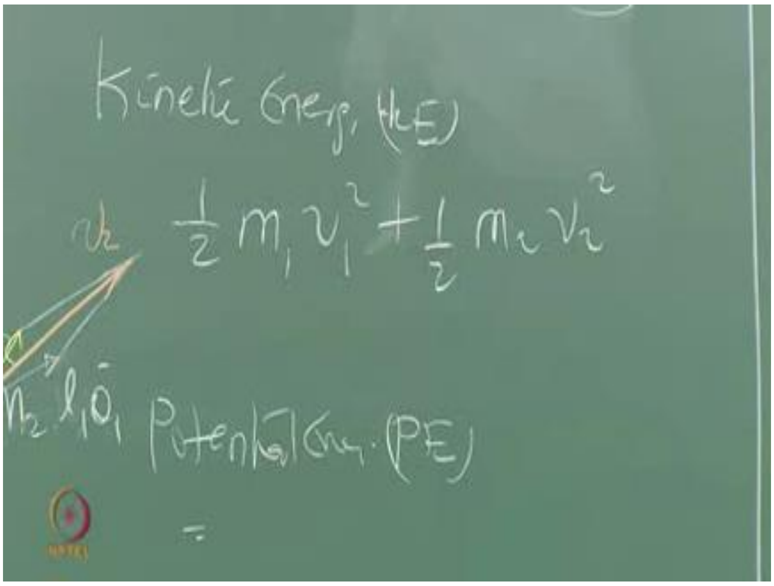

Now, we know for the system the kinetic energy is half m 1 v 1 square that is the mass m 1 plus half m 2 v 2 square and the potential energy of this system, let us say PE, let us say KE is m 1 g l 1 1 minus cos theta 1; we all know that because this is going to be l 1 this is l 1 cos theta 1 the difference of these 2, is it not.

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This difference l 1 minus l 1 cos theta 1 plus m 2 g of l 1 1 minus cos theta 1 l 2 1 minus cos theta, this is my potential energy, this is my kinetic energy. I will call this as let us say equation 1 a, equation 1 b, equation 2, equation 3.

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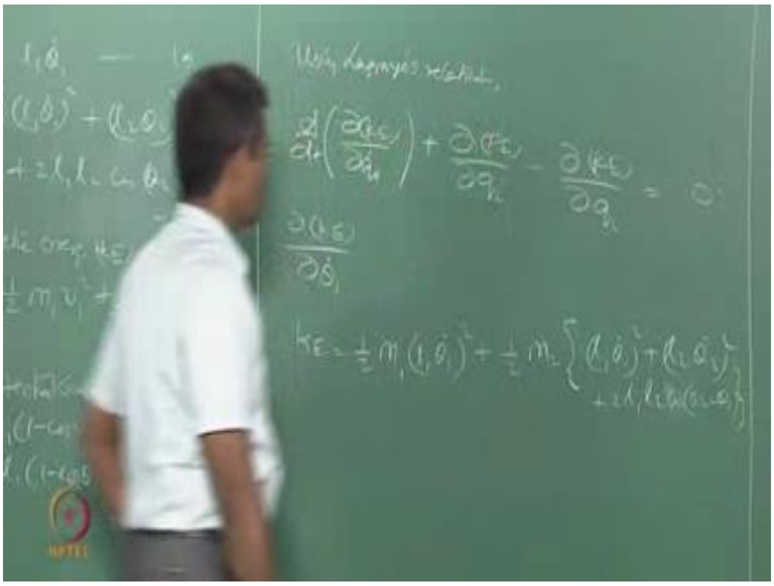

Now, we will be using Lagrange relationship. I can write the equation of motion energy method, it says that d by d t of dou kinetic energy of dou q I dot the partial derivative of potential energy of q i minus partial derivative of kinetic energy of q i should be set to 0. So, let us try to find out the partial derivative of kinetic energy with respect to q is the degree of freedom I should say this as theta 1 dot the 2 degrees of freedom, theta 1 and theta 2 by the displacement theta 1 velocity is theta 1 dot and acceleration theta 1 double dot.

So, let us write down the equation of kinetic energy first which is half m 1 l 1 theta 1 dot square plus half m 2 l 1 theta 1 dot square plus l 2 theta 2 dot square plus 2 l 1 l 2 theta 1 cos theta 2 minus theta 1, is already squared. So, need not to do square again. But this was not squared this v 1 only, so I squared it here.

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So, let us expand this. So, let kinetic energy be expanded as half m 1, let say l 1 square theta 1 dot square half m 2 l 1 square theta 1 dot square l 2 square theta 2 dot square plus 2 l 1 l 2 cos theta 2 and theta 1. Now, I have to partially differentiate this with respect to theta 1 dot that is what I have to do, which I say half m 1 l 1 square 2 of theta 1 dot plus half m 2 l 1 square 2 theta 1 dot plus l 2 square 2 is going to be 0 anyway and partial derivative plus 2 l 1 l 2. Please make a change because this is l 1 theta 1 dot this l 1 theta 1 dot this l 2 theta 2 dot, this square plus this square plus twice of a b cos. So, I am missing here theta 1 dot and theta 2 dot, I am sorry I am missing that here I think we have to make those everywhere theta 1 dot and theta 2 dot.

So, here theta dot will stay cos theta 2 minus theta because the derivatives is respect to theta 1 dot. So, simplify and give me what is dou kinetic energy by theta 1 dot. So, m 1 l 1 square theta 1 dot plus m 2 l 1 square theta 1 dot plus l 1 l 2 theta 2 dot, I can say this set to 1 because actually theta 2 and theta 1 is very less and practically that cosine angle will be practically equal to 1. Now, I want to find the differential of this respect to time because that is see equation required in Lagrange, when you do that it will become m 1 l 1 square theta 1 double dot that is a derivative (Refer Time: 12:11) plus m 2 l 1 square theta 2 1 double dot there is m 2 missing here, this m 2 is the multiplier here plus m 2 l 1 l 2 theta 2 double dot. Then similarly we can find the partial derivative of potential energy respect to theta 1 partial derivative of potential energy respect to theta 1 can give me this value because potential energy equation is already here, so to theta 1.

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So, m 1 plus m 2 g l 1 sin theta 1. So, I can write the first equation of motion by substituting in a Lagrange's, can you give me the first equation of motion, substitute that and give me the first equation of motion - m 1 plus m 2 l 1 square theta 1 double dot that is a first term you will get plus m 2 l 1 l 2 theta 1 double dot, it is here theta 2 double dot that 2 double dot; theta 1 goes away, plus of course the value here which is m 1 plus m 2 g l 1 sin theta 1. For lower value of theta 1 I can simple say this as theta 1 itself set to 0. Let us just look at this equation, now I write a similar equation for a 3 hinged MLAT. Let us look at this equation and try to generate the parallel mass matrix for a 3 hinged tower.

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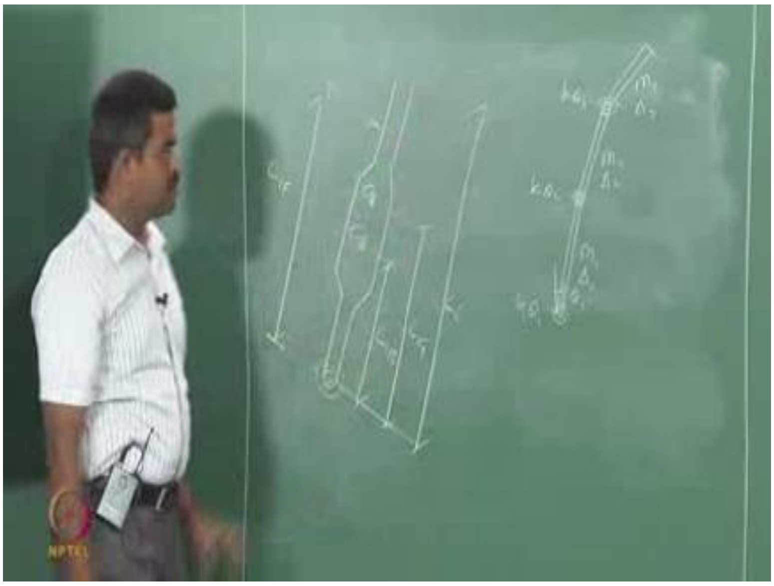

Like this, they are not parallel they hinge. So, there are some spring restrains here which are those universal joints I call this as m 1 delta 1 displacement m 3 delta 3 and the rotational stiffness I call this as k theta 1, this as k theta 2, this as k theta 3, where theta 1 of course, is a (Refer Time: 15:25) vertical to this centre this is theta 1 and so on and so forth.

If we enlarge only 1 element and try to draw it here we know an element will have a buoyancy chamber which is slightly larger, let us say this is my hinged point, universal joint. Let us say I have a central buoyancy here and mass centre here this is central buoyancy, this is central gravity of the mass centre we already know for stability this is important. I call this distance from the hinge as l 1 b l stands for the length 1 stands for the number of the member and b stands for the buoyancy centre. So, similarly I would call this as l 1 g. If I really wanted to find a force at any point on this member where the force is acting as f 1, I would call this distance as l f 1 because I want this for a moment because I want this for a moment on the right hand side of equation of motion and of course, I will call this entire length as typically l i in this case l 1; can be even l 1 f for maintaining the similarity l 1 b, l 1 g, l 1 f, and l 1, so typical configuration or geometric configuration of a member.

Similarly, for member 1, member 2, member 3, all will be same accept that the suffix 1 will all replace by 2 and 3 respectively. Now, I will not write a mass matrix copying from their directly without deriving it. So, we all know being a 3 degree because there are theta 1, theta 2, theta 3, independent notations they are not depended on each other.

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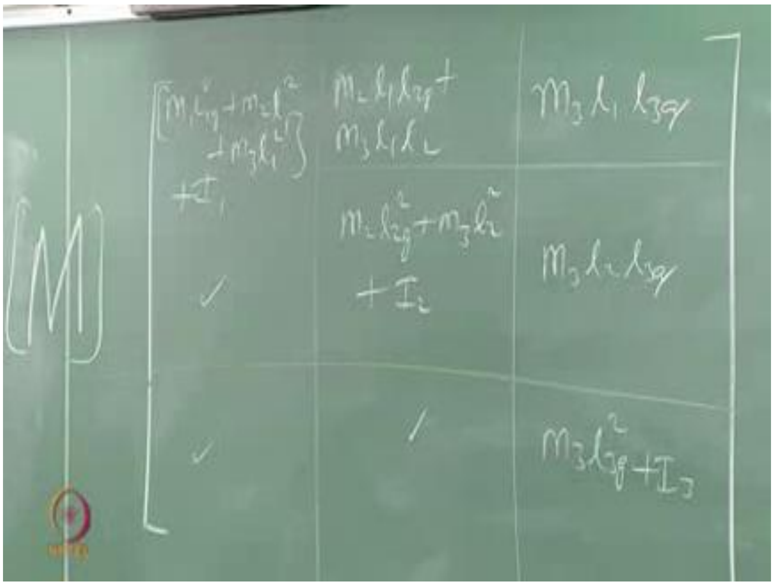

So, the mass matrix will be 3 by 3, I can write here if there are 2 members m 1 m 2 there are 2 here, I can write here m 1 plus m 2 plus m 3 of l 1 square. To make it very precise let is expand this, this is what I must get similarity from here, but I will not expand this slightly. Let us rewrite this equation slightly in a different manner like here. It will be m 1 l 1 g square for m 1 it is l 1 g actually, for m 2 and m 3 it is only simple l 1 and l 2. So, it is going to be m 2 l 1 square and m 3 l 1 square, because we borrow the same term from here, sum up all the mass that is what I will get here. In addition to this I will also have the moment of inertia of this system itself alone, in the earlier case of neglected because spring mass system having a neglected mass of the self body, but here this is present therefore, I should say plus I 1, where I 1 is a moment of this system itself because this is now substantially high we cannot neglect this. In the pendulum problem we neglected; it we neglected it, but here we cannot neglect.

Similarly, can I fill up this? So, I should say m 2 l 2 g square plus m 3 l 2 square plus I 2 where is a I 2 is a moment of the inertia of the second member. If I want to do this, this will be m 3 l 3 g square plus I 3. I have borrowed this exactly from this. Now let us look at this term, the second term which is related to theta 2 whereas there are 2 only, but it will be theta 2 and theta 3, both theta 2 double dot, theta 3 double dot - second term.

Now if you have only m 1 m 2 mass, m 1 is not appearing the second term m 2 is appearing whereas a plot this l 1 l 2. In this case is going to be m 2 and m 3, let us expand this slightly. Now let us rewrite this, this is going to be m 2 l 1 l 2 g because I am talking about m 2 m 1 l 2, l 1 l 2. So, l 2 g plus for m 3 this is going to be only m 1 l 2 there is no g, the same term here, same term here. If I write this second equation you will know it is going to be symmetric.

The third one will be, can you give me the third one, can you write this as m 3 l 1 l 3 g, same here. Now I can quickly fill up this, can you give me this value? Where the mass matrix is ready, we can derive the mass matrix for a multi hinged tower like this.

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So, I took up this example only for you to understand how the term related to theta 1 double dot and theta 2 double dot will appear. Of course, this term will appear in the stiffness matrix is it not because that is related to theta 1 displacement, I will give you a very simple method to derive the stiffness matrix, give (Refer Time: 21:45) displacement here get alpha 11, 21 and 31 and invert it to get k and give unit displacement here and try to find influence coefficient the single manner, in the earlier manner what we explained you will get the alpha matrix which is 3 by 3, invert that you will get a k matrix which I am not writing it here there are (Refer Time: 22:09) refer back in the presentation, you can see that. You will exactly have the same mass matrix and same k matrix available in paper.

We will move on now to, now once I have k and m matrix c is going to be 2 zeta omega m the classical damping matrix I have the equation of motion now set from a problem, I can solve this using any iterative schema which I have not so far discussed. In the last case we had a similar problem we are not discussed they are holding a (Refer Time: 22:37) scheme later we will discuss all of them at the end. So, now, equation motion is now complete I can solve them if you know the iterative scheme. Let us hold it here, come back to the response control of the system, any doubt here in simulating how we are getting the mass matrix, how we will get the (Refer Time: 22:57).

I am living it for the homework to you. You should be able to do this because we are able to derive the first column of the influence matrix or the whole matrix we can easily invert it and get the k matrix which will be exactly similar to what we have done in theta 1 theta 2 and theta 3 times which you will appreciate. If you have still there to compare you can always look at the paper which will refer back now, you can look at the papers of course, the derivation is not shown in the paper you will have only the mass and k matrix given in the paper you can compare and find out. But the derivation comes from here like this. So, instead of writing this matrix directly from a 3 hinged tower I have picked up a 2 pendulum system problem and I derive that value for you then, I just picked up the same idea and extended it for a 3 way 3 n by n system. It is very easy to understand, that is a reason why we have picked up that. So, you hold it here we get back to the response control, any questions here, any questions?

So, in the last lecture we appreciated that for deep waters from shallow to medium depth offshore platform designs have been formulated to have a compliance introduce in the design. The compliance introduction will also cause increase flexibility therefore, the system should have a recentering capability, when the system is pushed by lateral force there should be a recentering element in the system which will try to bring it back. There are 2 elements present in an MLAT – one element is the spring or the universal joint which has got m phi characteristics, for any m given to you it will bring it back in opposite direction.

The second will be the alteration the floating chamber which is nothing, but the buoyancy chamber because the variable (Refer Time: 24:41) effect the buoyancy chamber will also try to bring back the system, But however, it has been noticed with the researchers as we present in the last slide or last lecture that the response of the top deck is sufficiently large therefore, these platforms cannot be used for production facilities. So, that has got to be recommendations made by the researchers saying that let us try to attempt response control mechanisms on this. Now let us look at the real structure engineering where response control or where are they applied, what are the different concepts available. Out of the concepts available what I will pick up for this problem. Then I will show a video that how this works out then, I will again derive the equation of motion for an MLAT with tuned mass damper then, I will solve this here and the lead equation of motion show with the results directly - that is the idea of this lecture.

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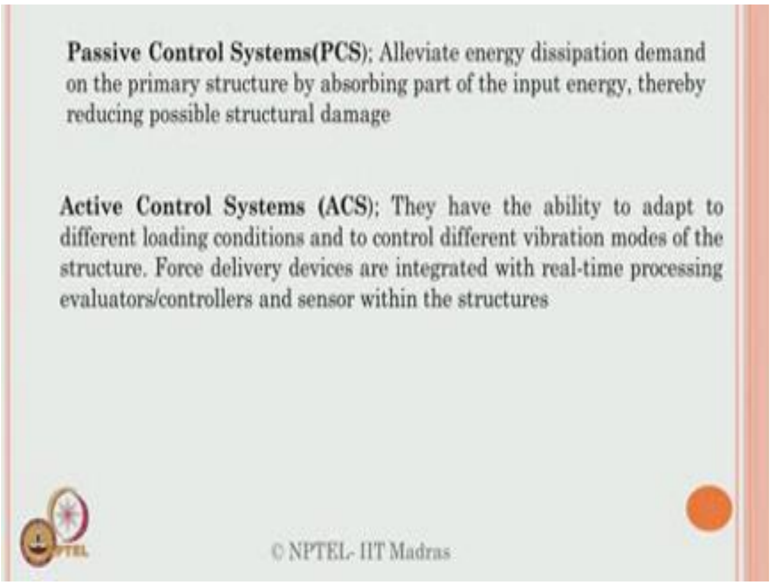

Let us see now, what are the difference response control mechanisms we have in this system in general. Let us talk about response control of multi leg articulated towers. There are different systems available in general - passive control, active control. Passive control actually alleviates energy dissipation demand on the primary structure by absorbing the input energy; therefore reducing the possible structural damage is called passive control. Active control is you have to give a control mechanism from the system where the control mechanism also receives external forces. Passive control does not demand external input to the system, whereas active control demands an external input to the system you have got to. For example, if I want to push a larger mass I have to full back by an external with supply which will again needs a tremendous amount of energy.

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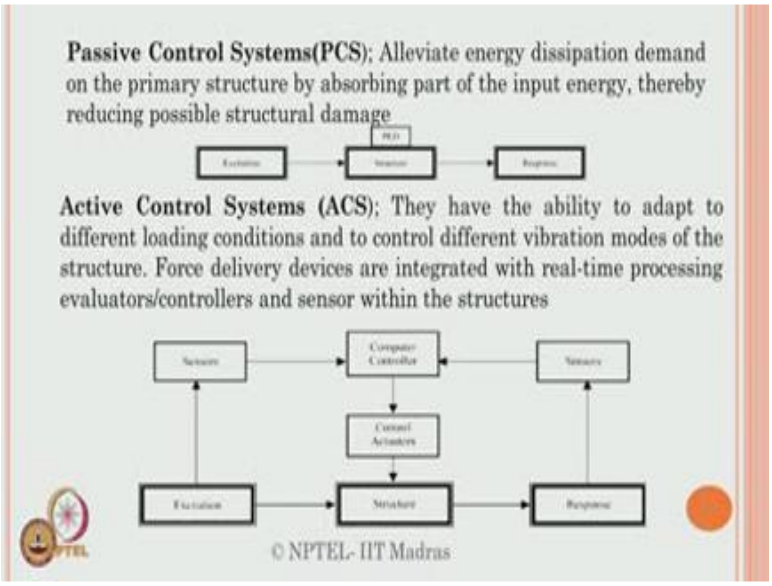

So, you need excitation force, you have a structure, you have a response, whereas you have a control mechanisms there attach to the structure which can be either passive or active. The active mechanism comes like this because there are senses available, excitation is given to the structure, structure response - the response is sensed by the sensor, the computer control receives the sensor, activates a controller which gives an opposite force to the structure to stop the structure that is called active control mechanism.

So, in this loop you will see that the control sensors or the control actuators require lot of energy to dissipate the response of the structure in an opposite direction in case of (Refer Time: 27:07) failure, let us say in case of sensor fail to organise a response, in case of there is a phase lag of the sensor response recorded compare to that of a structure the active control may not work efficiently.

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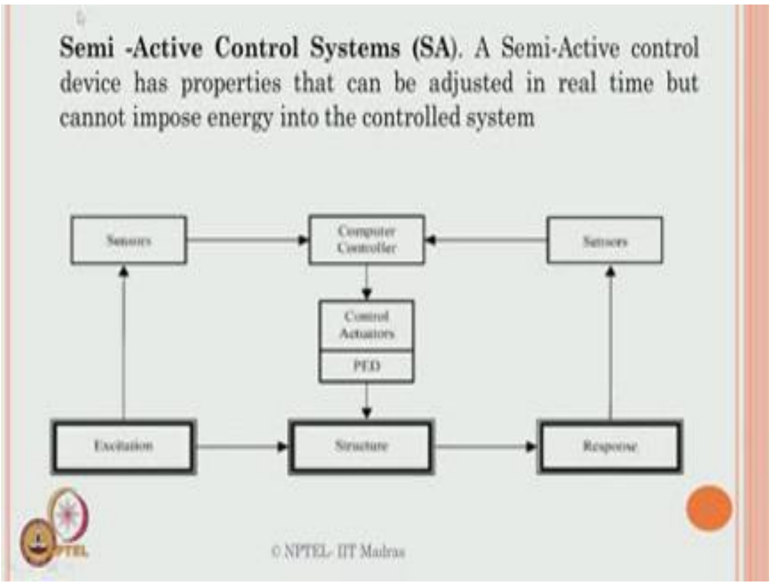

The something on semi active, this is an elevation of these two. So, you have a passive control mechanism also in place partly and partly you have got actuators also. So, it is called semi active control system. These are all very commonly used in land based structures especially to elevate forces either from wind or from earthquakes, but the application of this is never being recommended and used in offshore structures at all for 2 reasons – one, people never felt in offshore structure require as a response control, one can ask me a question counter how people felt response requirement to be control in land based structures. For example, I think you will know hotels in abroad let us say especially in European countries as you go higher and higher, may be 20th storey 40th storey they rent is half of the ground floor because it requires lot of uncomfortability, as you go higher and higher to stay the rent is particular half.

So, people feel that the response on the structure under unexpected forces can caused undesirable modes number 1, and number 2 many structures failed because of earthquake or lateral forces occurring from wind when there was no response control mechanism introduces in the system. Therefore, international codes came out with the string and regulation that mostly all public buildings like schools, hospitals, etcetera should have some isolation control mechanism in the system which can elevate not 100 percent, but at least partially the unexpected design force on the system may be caused by earthquake or by wind - earthquake means massive structures, wind means tall structures. Because tall structures will have active wind force on system.

But this idea was not let us say applied upon or taught of in offshore structures because people never taught that offshore structures will also have uncomfortability in terms of production facilities. Now this was in provides in MLATs because at 80s or early 90s people taught that MLATs are very successive adaption of a complain system for deep waters, but it has got some demerit it is having lot of (Refer Time: 29:35) displacement I want to control.

Now, the displacements of 2 order here - one the deck and more in a heap direction which is vertical on the vertical plane people are (Refer Time: 29:45) bother about this because the topside load is about 25,000 tons is very very high. So, it cannot be felt right. But people are really bothered about the surge and sway motion or roll and pitch motion. So, roll and pitch motion will not occur because they are almost stiff degrees surge and sway are almost complain degrees. So, the response compared (Refer Time: 30:07) will be larger with that of roll and pitch. Therefore, people entirely focused on controlling surge or sway motion, surge will be for unidirectional surge and sway will be for multi directional waves we already known this is not. So, people taught about the response control (Refer Time: 30:24) specific degree of freedom in offshore structures.

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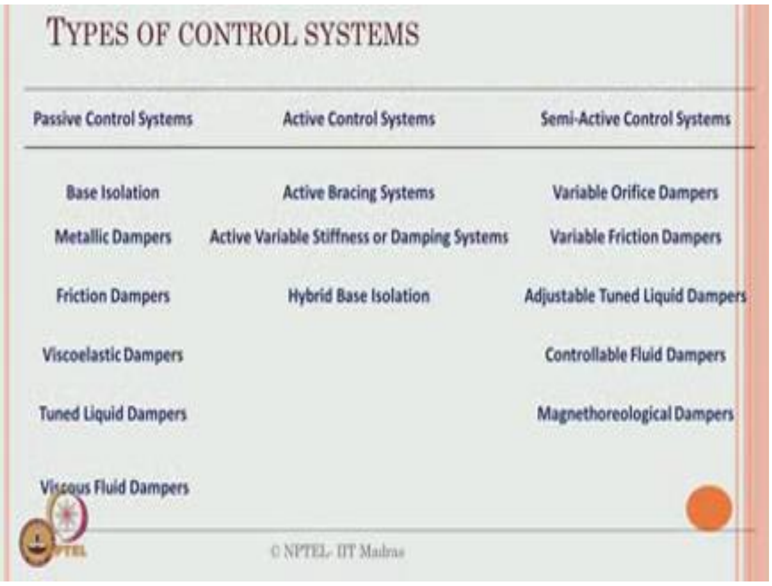

So, let us see the type of control system here, passive control are listed as base isolation metallic dampers friction dampers, viscoelastic dampers, tuned liquid dampers called as TLDs and viscous fluid dampers. Active control systems are active bracing systems, variable stiffness and hybrid base isolation systems. Semi-active are variable orifice devices, Magnethoreological Dampers, MR dampers etcetera all these are comprehensively available discussed apply and you will ultimately see all of this kind of systems are available in all structural systems of mechanical, arrow space, etcetera. For example, (Refer Time: 30:59) door closer in the door has one of the mechanism systems here which is actually a spring mass system which is nothing, but a passive system. My door closer operates in a passive control system here, when I open the door the spring applies the forces opposite way the door closes.

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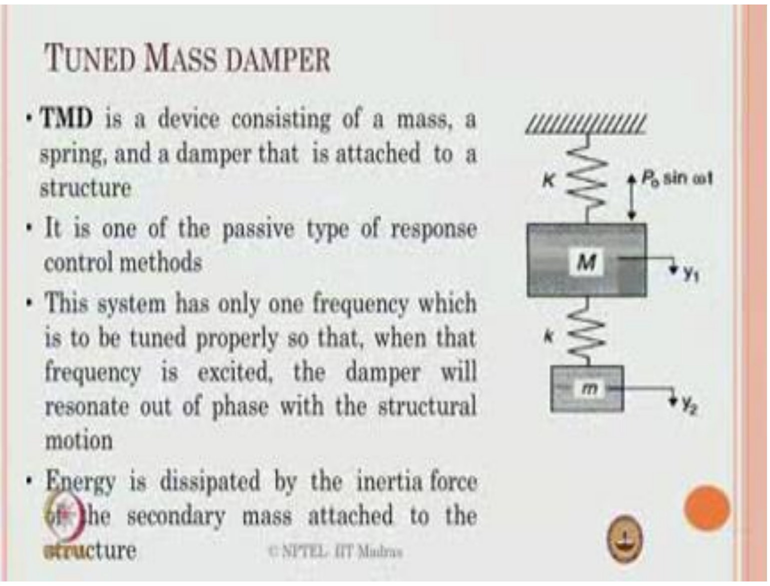

So, all systems in reality if you see mechanical system do have one of the control mechanisms implemented in them, whereas structural system have to be implemented by force. So, offshore structures had an idea from implementation of passive control mechanism which I will discuss here. So, tuned mass dampers as I picked up from here which is one of the, one can have a liquid damper, one can have a mass damper also. So, TMD is one of the passive types of response control.

So, it is very simple I attach a spring mass system a spring mass system to the primary system. So, primary system m is capital M, the secondary mass is small m capital K is for the primary system and small k is for the secondary system. Now obviously, the primary system will or will not have a damper. The secondary system may not have a damper at all it will be simply spring mass system. So, now, I can write an equation of motion of this system easily because I got 2 degrees of freedom y 1 and y 2, I can easily write equation of motion of this we can solve this equation of motion and see what is the complexity to tune this. Now very interestingly where you are going to tune this, it is very important the primarily system will have the maximum response only when (Refer Time: 32:29).

So, the excitation force is p 0 sin omega t for example, or EA omega t may be sin and cos sin components both are present in system for given amplitude of p naught. Now when the excitation frequency matches with the fundamental frequency of the primary system the system will excite to its maximum that is always we know that. So, the idea is to control that maximum excitation. So, we generally take care of dampers in the system right let us see 2 zeta omega m we introduce damping the system 1 percent, 2 percent, 5 percent, etcetera, which are fortunately derive hydro dynamic damping in offshore structures models or one can also have external dampers which I am going to apply in this system here. So, the spring mass system of k m which I call a secondary mass system is actually nothing but an additional damper to the primary system. So, there is no requirement of an additional damper to the secondary mass system. So, I do not have c 2 here I have only k 1 m 1 and c 1, but I have only k 2 and m 2 I do not have to c 2 here because k 2 m 2 itself is an integral c 2 or damper to the primary system.

So, I have 2 degree of freedom system model here, can easily write the equation of motion and solve. The difficult is when you are got to have the tuning of the secondary mass system to that of the primary (Refer Time: 33:46) system as in excitation frequency this cannot be analytically studied because you know at analytical equations when this equation of omega equals omega bar you will see it becomes unbounded. That is a primary mass system does not have additional damper, solution cannot be obtain. So, you have to tune this. Using experimental methodology by which what m 2 verses m 1 ratio what omega 2 verses omega 1 ratio can be adopted for the given system therefore, the system of the response of the primary system can be minimised.

So, we too conduct an experiments and obtain that parameter substitute that parameter and equation of motion and solve, because you cannot directly solve this an equation of motion for omega equal to omega bar if that is a case. Even if you say omega 2 is equal t omega 1 even then it cannot be solved because they will resign it. Most interestingly one can physically ask me a question – sir, when you attach a secondary mass system the primary mass system how actually the response will get controlled, you have to tune the mass secondary mass in such a manner that they should be out of face.

On the other hand if the primary mass moves to the right the secondary mass should move to the left. So, it should drag the primary mass to the left on the other hand when the primary mass moves to the left because the wave can act in both direction, the prime secondary mass should move opposite way and drag it back to the normal (Refer Time: 35:13). So, m 2 and m 1 or omega 2 and omega 1 should be design in such a manner they should remain out of face for the entire period of vibration.

So, it is very interesting problem, physically you can see this (Refer Time: 35:28) video now I show you how this can easily we see then we derive this mathematically. First let us understand the physical problem, physical problem I have a k m system, large displacement I want to control. I am controlling to putting a secondary mass system to it. Now I cannot analytically solve this problem straight away because I am going to tune it to the maximum response of the primary system which will happen only when omega equal to omega bar of the acceleration frequency. So, I have to do experimentally find out the parameter where I have to tune the secondary mass ratio with respect to the primary mass such a manner that they not only remain out of face, but also I will be able to get the response experimentally and therefore, analytically.

What are those parameters, how this equation becomes complicated and so on so forth we will just now see, but I will I want to run a video and this moment and see physically how this happens. Now you can see easily the leg is oscillating is put to the hinged point you can see the moment of the deck and unidirectional way is actually surge action. Now let us see more deliberately for a very low frequency it has got very beautiful motion in terms of pendulum action and now I want to stop this because this is very large as for a researcher concern, I want to stop this.

Now I put a secondary mass system suspended from the primary mass and see what happens. Secondary mass suspended now, wave is applied unidirectional scale model 1 is to 100. Look carefully when the system moves to the left pendulum to the right you can see that, I show you very interesting the top that does not move at all only the pendulum is oscillating you see, there is a (Refer Time: 37:33) control obtain in this. So, very simple mechanism by which I can tune the secondary mass to that of the primary mass so that is out of face and I can control the response of the deck now. Physically this is what it is, but the complication comes when you really want to derive the equation of motion for this to solve.

The system as only one unique frequency which is to be tuned properly so that the frequency when it is excited the damper will resign it out of phase and the response is achieved. Energy dissipated by inertia force secondary mass attach to the structure, these are examples of the problem.

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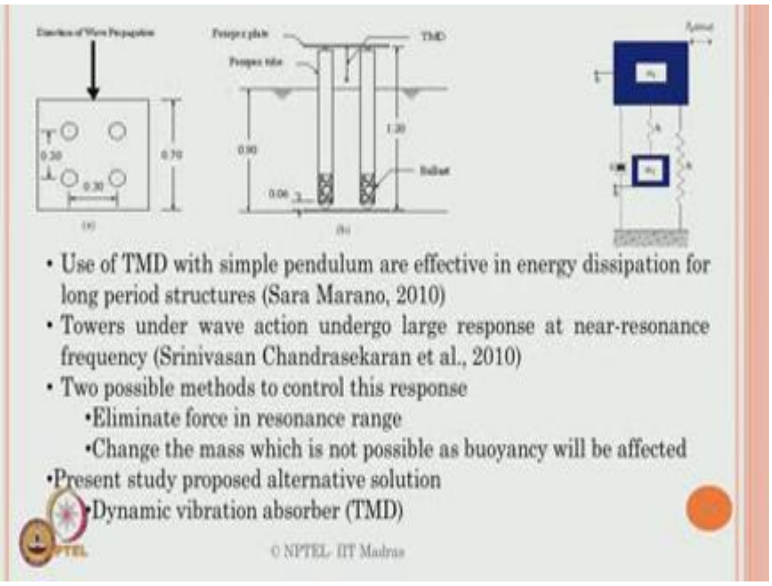

This is actually the physical model which has been done at IIT, Madras. These are the dimensional of the scaled model that is we say system which is shown on the right hand side and this got very good references of Sara Marano which I showed in the last class also. We have also published 1 paper on the same idea on 2010 which I am referring from results. So, the present study actually proposes alternate solution in terms of tuned mass dampers.

So, I will stop the lecture here then I will continue this lecture in next class just to make you to understand how I do equation of motion for the system available here, and then how do we solve, what are parameters, what are governing curves I get and how do I actually tune to get me out of this relationship between m 2 and m 1.