Dynamics of Ocean Structures
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Lecture – 33
Articulated Towers

So, in the last lecture we discussed about the earth quake forces acting on the given system. And, we also decided that why an earthquake analysis should be done for a fixed as well as floating (Refer Time: 00:27) structures.

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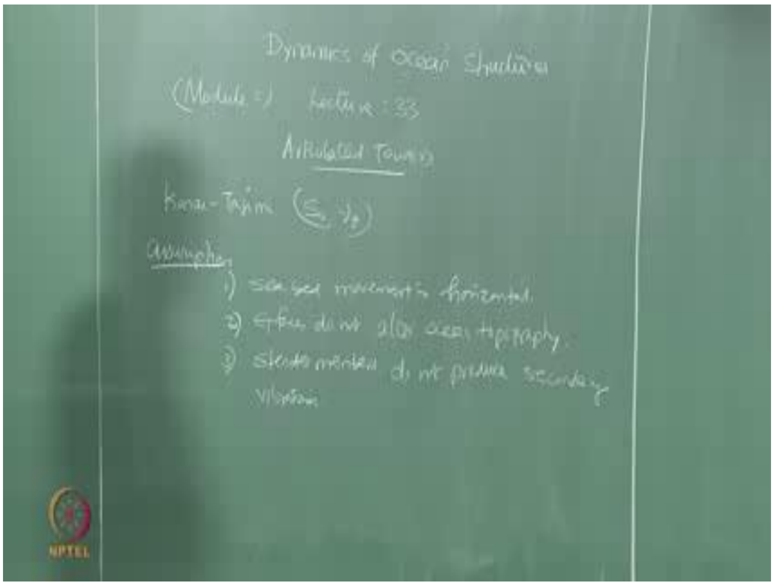

So, we have given you an equation which is the governing equation for calculating the spectral forces which is given by Kanai-Tajimi power shift in this function, which is given by Kanai Tajimi's equation, which has got side specific parameters which is S naught and nu g and so on.

There are some interesting studies which has prompted that the earthquake analysis should be done for complain structures. I have already given you a take home assignment to find out what kind of structure is having in close epicenter to the top in recently recorded earthquake in the Gulf and Mexico. So, it is very important. We will talk about that later when we do real time analysis and show you how the earthquake process can be super imposed on analysis of compliance structures.

We will continue with the discussion here that let us say there are some assumptions what earthquake analysis in offshore structures need to be the foremost assumptions are the following. The sea bed moment is always considered to be horizontal. The sea bed movement or the earthquake forces do not alter ocean topography. Slender members do not produce secondary vibrations; need not be considered in the analysis. These are the basic assumptions.

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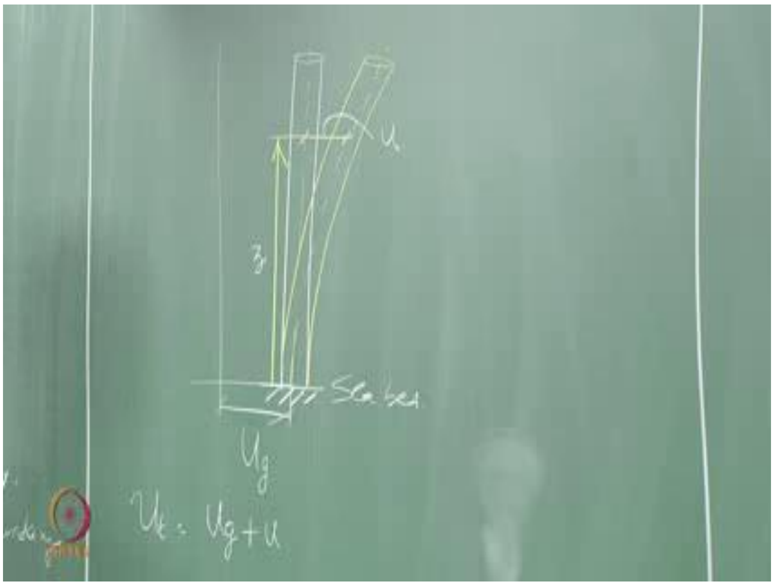

So, if you have a cylinder whose initial position is vertical, which is fixed to the sea bed and hollow cylinder, when the displacement for the ground or the sea bed is given, the cylinder being a cantilever stack will bend.

So, at any point on the CG at any distance, that is, a z from here, we call this displacement as u. And, of course this is a displaced position. Therefore, from the origin let us say this displacement has a value of u double dot g. Or, let us say u g for the time being; let us say ground displacement. We will talk about acceleration slightly later. So, now the u total displacement of the structure will be the sum of the ground displacement and the displacement of the structure; because of the vibration at any instance z or any distance z from the sea bed. This is sea bed.

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Now, interestingly let us look at some terminology; let us say K u. Let say C u dot. Now, u being the displacement of the structure, K u will be the restoring force which is stiffness proportional of the structure - u naught b, the damping force which arise from the structure, so structural damping, not the hydro dynamic part; because u dot has been taken, where u is the displacement of the structure.

So, now we can write the equation of motion for the system M u total because system has two displacements. One is the ground and the system displacement plus C u dot plus K u is let us say F z comma t. We know that water particle kinematics varies spatially along xy; where x and y are the orthogonal directions are accessed. Depending upon the spacing, value keeps on changing. Of course, in a two dimensional plane y does not make any role, but it varies with the depth of water, z and t. That is why we said z. And, of course varies with time and so on. Please note that the damping and the stiffness terms are associated only with the structural velocity and structural displacement, not with the total of the ground. It has nothing to do with the ground at all because I am analyzing the structure after the structure is displaced.

Now, that is why it is, whereas in the case of u double dot, you must include the ground acceleration. If you do not include that, then it is as good as analyzing the structure without the ground displacement at all. So, that is why it is u double dot total, which of course, F z of t.

Now, let us try to expand F z of t, which we have already written in the last class. It has got two components; inertia and the drag components, which I can write here as C m v dot minus u double dot t. There is the inertia component with the relative acceleration because v dot is acceleration of water particle plus the pressure variation, which is C A v dot; C A is rho a, we have already said in the last lecture, plus linearization of the drag which is given by C D eight by pi sigma r. I put sigma r. I will explain what sigma r is v minus u dot t. So, let us call this is equation number one; this is equation number two.

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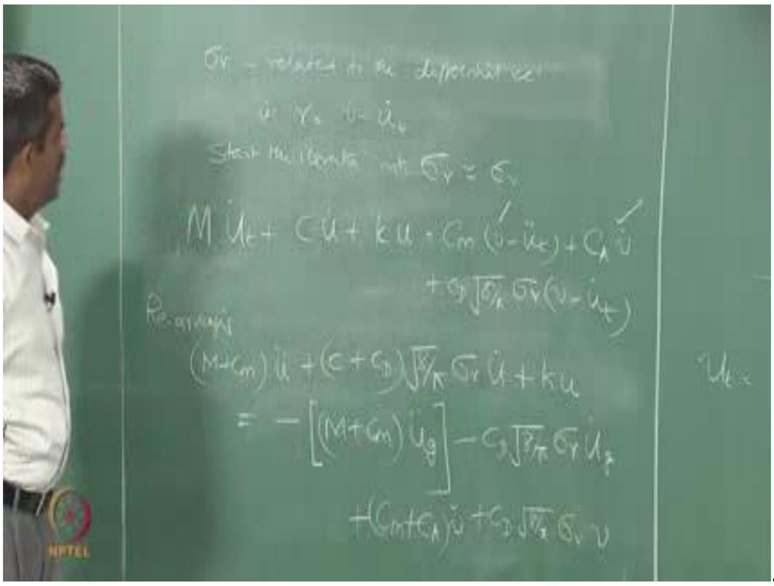.

r actually is related to the differential velocity. That is, r v minus u dot of t. Now, it is very interesting for us to ask a question that I do not know in u dot of t or in u of t, for example, I have got two components. One is the ground displacement; other is the displacement of the structure. Initially, you know displacement of the structure will be zero because this structure does not get displaced in the initial part.

So, when you put zero here, it is not differentiable. Therefore, you will not get u double dot, u dot t at all in your equation. So, you will not be able to find sigma. So, it is very interesting that one must start the iteration with sigma r being approximately equal to sigma v, which was the term what we explained in the last lecture. It is only related to water particle variations, nothing to be the relative value. To start with, you do. One may ask the question how the whole scheme of iteration is set in here.

The scheme of iteration is coming into play here because look at the equation of motion, the solution of this equation of motion should land up in actually u. You may say, “Sir, where u is available, where u double dot is there”. u double dot is there as a part of this part. u double dot g is known to me. Therefore, in the whole equation if you solve you will get the displacement of the solution for this problem.

Unfortunately, if you do not know the displacement of this problem, you cannot actually solve this equation. So, there is a coupling between the equation itself to solve, one; two, if you do not know the displacement or the value, you will not be able to find the relative velocity term in F z of t. If it is fixed as we have shown here, no problem. But, if it is coupling, then I have to have a relative velocity term also, which will not be available to me when u or u dot or u double dot is not known to me, therefore, there is a strong coupling between the right hand side motion of the equation of motion and left hand side of the equation of motion, which has dependency on one variable which is the displacement of the system, which is not known to me. Therefore, in such situations people go for numeric integration.

We will explain the scheme later. Let us understand that there is iteration necessary to solve this problem. Once I say iteration is necessary, I have to have an initial value. I say that sigma r can be as same as sigma v because sigma v is known to me. You know for a given water particle, see straight or water particle or the location of the platform for a given series of sea waves or earthquake waves, I must be able to get the variance of say standard deviation of the velocity. So, it is known to me. So to start with, I will say sigma r in that equation is same as sigma v. But sincerely speaking, sigma r should be the value related to v minus u dot of t. That is what it is.

Now once we understand this, let us rewrite these equations substituting the back. Let us say M u double dot plus C v dot, sorry, u dot plus K u is C m v dot u double dot t plus C A v plus C D root of eight by pi sigma r, v minus u total. This is u total. One can erase. Why it is u total because the drag non-linear session at the drag linear session, actually comes only at water particle velocity. If we will look at the original equation, it is nothing to do with the dependency, the velocity of the system. So, that is the u total.

Now, I have got two terms here; u double dot t here, u double dot t here, also u dot u dot here also and so on. So, I can rearrange this terms. So, rearranging; M plus C m of u double dot because I am substituting u total as u g plus u in this equation, wherever I have got total. So, I can expand M as u g plus u and so on and so forth. Then, I am rearranging the terms; C plus C D u dot. I have; also have u dot here because u total is u dot of u. So, C plus C D eight by root pi sigma r u dot, of course plus (Refer Time: 12:01) value.

Now, the left over terms, this goes this side, minus M plus C m of u double dot g. There is a u double dot t here, which is sum of u double dot g plus u double dot. I have taken this u double dot already here. So, u double g is (Refer Time: 12:24) here. I bring M here back. That is a classical way of writing equation of motion, when system subjected to earthquake forces. You must get a negative sign here. Of course, the other term because here is u total here; there is linearization happening here. So, I should say C D eight by pi sigma r. I put sigma r or sigma v, does not matter because initial iteration is going to be same. Sigma r u dot of g because I get u dot here. This is the sum of these two. So, u dot of g plus the pressure value; that is, C m plus C A of V dot. This term plus this term here and this term here, v dot. In this, the v term here, which is drag linear session which is plus C D root eight by pi sigma r. I will call this as equation number three a. I am rearranging this 3 b. So, let us look at this equation carefully and see what are the unknowns here.

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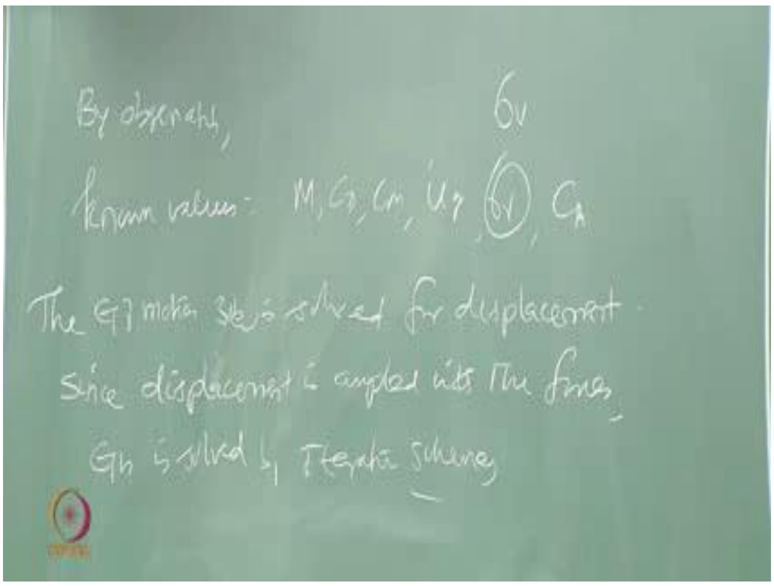

By observation, observe a equation three b or three a. They are one and the same. I have just rearranged them. Known values are the system mass. Of course C D, C m, u double dot g for a given side specific, it is known to me. Sigma r or sigma v, it is known to be A. The one which you do not know the equation is a displacement, which we can now solve.

So, the equation of motion three b is solved for displacement. Since, the equation of motion contains the displacement terms both in left hand side and its derivatives and in the right hand side and so on. There is a relative term here. This term actually is a difference of this. So, there is a term available here.

Since, the displacement is coupled with the forces; equation is solved by iteration schemes whereas, many iteration schemes are available. We will talk about that later. Now, let us come to a conclusion of understanding that, yes, we are going to use a iterating scheme.

So, this ends the discussion of how to compute or how to, let us say impose the earthquake forces in the given system, if it is relatively compliant or fixed. As we all understand, if it is fixed, this will be practically zero. So, it is one and the same actually. All these terms will be become zero because there is no movement of the system in terms of relative motion, with respect to the water particle kinematics. So, the system motion will be there, but that will be in significant. Therefore, still you have to solve. But that is not be, would not be predominantly important. Whereas in the compliant system, the relative water particle velocity becomes important. Therefore, the system becomes iterative.

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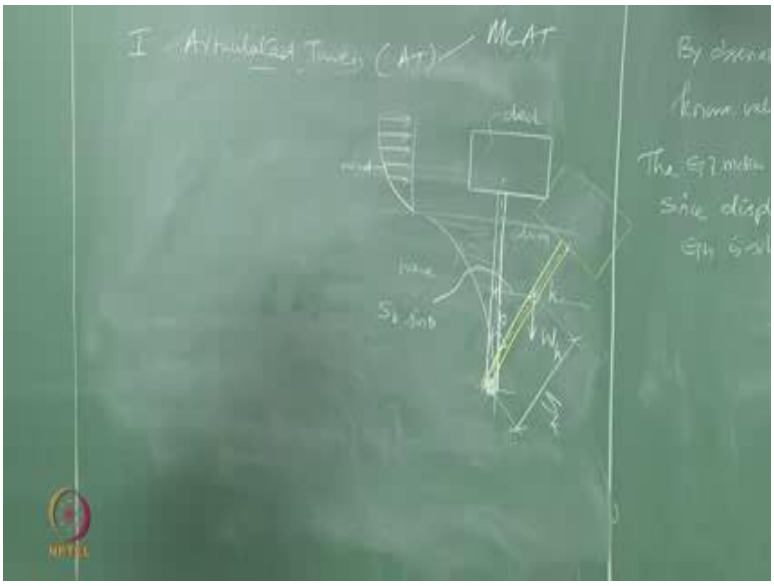

Now, let us talk about other kind of platform, which are articulated towers. We already know that they are of course abbreviated as AT s. You are also find some terminologies like MLAT; multi-legged articulated towers. Essentially, AT is actually a single tower system. It looks like this. There is a top side. We have got a single column. A single column will be hinged. Let us see, it is not fixed. You can call the whole system as top side or a deck. I can call this as column, when the wind force and the wave force act on the system as lateral loading. The system naturally has a tendency to deflect or displace from the given position. Since, it is hinged because the system design is inherent to bring it back to equilibrium, the system will have, may be a position or take a position like this. And, keep on oscillating like an inverted pendulum.

So, let me have a segment where the weight of the part is W. And, this node is the k th node, W k. And, let us say this is at an angle of theta. And, let us say that the segmental length of this k from the origin or from the hinged point is taken as S k. And, it is segmental length of the kth node. Now, this distance in the given system is nothing but S k sin theta. This is S k. So, this is S k cos theta; S k sin theta. That is the horizontal value of the kth node from the original equilibrium position of the column.

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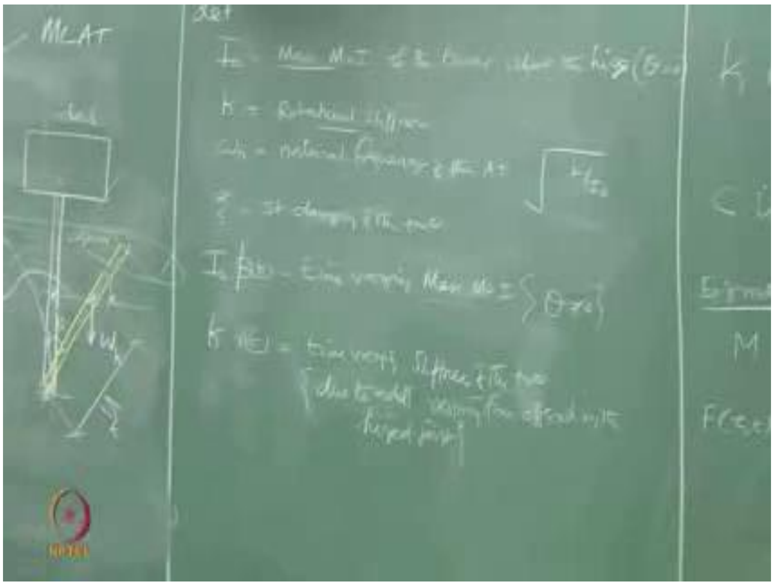

Now, let us say I naught is the mass moment of inertia of the given deck of the tower. Then, the moment I say mass moment of inertia, it is also called as second moment of area. The moment I say second moment of area; I have to identify a node with which the moment is actually taken. So, I should say about the hinge part. It is about the hinge. I can always find out this.

So, how can I do this? I can divide the column into segments. You can always find the second moment of the area of those segments or finite elements, with respect to this point keep on sending them up, I will get I naught. This is very easy. Geometrically, you can calculate this.

Of course, this worked out for an initial condition where theta remains zero. That is very important because when theta is not zero, it will become a new value. Now, I will come to that point what it is. K is the rotational stiffness. How do we get rotational stiffness? It is the force required to give unit rotation to the tower. Of course, omega is the natural frequency of the tower, of the articulated tower. Which is from this expression, you can easily understand this is mass and this is stiffness. So, can I say this has root by I naught. Zeta, the structural damping on the tower I naught, these are all let; I mean, these are all terms which is used in the derivation.

I naught beta of t is the time varying mass moment of inertia. Now, one may ask me a question how do you compute time varying mass moment of inertia, why mass moment of inertia will be time varying? Now, we all agree that mass moment of inertia will be the sum of the mass, second moment of area of the stem or the column and the tower deck area. It should be calculated generally when theta is zero. When theta is zero, the fixed positions of the stem or the tower in water and above water are known to me for calculation purposes.

The moment theta is not zero, this will happen only when the time passes. When the wave hits the structure, when the time passes, the theta will never stay zero. If it would stay at zero, it is identical design. It does not respond at all. It is good. That will happen only when it is fixed at the bottom. When it is hinged for any small literal action, we already know, we have studied in the first module that the restoration or restoring stiffness of the spring is designed in such a manner, the spring will always invoke a negative motion to the tower. So, the tower is brought back to normal sea. But, unfortunate part is that it do not come back to normal sea, it will swing. And, after some time it will come back to normal sea like a pendulum. And that is the problem with AT s. We already said that. there it is compliant in nature, it is flexible. But, this is undesirable because its movement is large enough.

So therefore, when the time passes when the lateral forces hits the structure, you will obviously see because of the trough and crest of the wave, some part of the deck will get immersed. And, you will get some variable submergence effect; are added to the system. Therefore, the mass will now change.

Once the mass changes, the second moment of the area that mass will also change about the theta. Though, I said that collects beta t. And, t refers to the time vary and content of the mass moment of inertia. And obviously, this we can even, if you want you can write when theta is not zero, otherwise it will not going to be there.

Similarly, when we talk about time variant mass moment of inertia, I must also talk about time variant stiffness. One may ask me a question, “Sir, how stiffness will vary with time? Stiffness is actually either b d q by twelve or e a by l or ae by l; bending stiffness or axial stiffness.

So, in this case it is changing because when the spring is; when the spring or the hinged joint is also altered, the hinged joint will have a rotational stiffness by its characteristic. This will be invoked like unbounding a spring, let say you have a spring in the ballpoint pen. Take out the spring, compress it, and release it. It will bounce back. It means when the spring is unbounded, the spring will restore its capacity. So, that is an added time; that will happen only when theta is not zero. When theta is zero, it will stay in its position. It will not invoke or it will not have any unbounded capacity. When it is not zero, when the platform is moved or oscillated, then this will invoke an additional stiffness that should be added here; which is time variance stiffness now. I call that value as K. I should say it is time variance stiffness of the tower. If you want, you can put it in bracket, due to the additional restoring force offered by the hinged joint.

Now, one can ask me a question how a hinged joint will offer a spring resistance. Actually, they are an unbounded springs, you know. Even for a hinged joint, there is an M theta curve which you can plot. Under given moment, the theta is not infinite. It has got a characteristic. Under given force and moment, theta is not infinity. It is not free. It will have resistance, that resistance that I am trying to capture. That will invoke only when theta is there. When theta is zero, it is zero. That is why K will not have that component to mind. Whereas when theta is not zero, this will have an additional component coming into.

And, I call that as a component or factor of K itself. Just to have identification. I am trying to group the terms. I could simply say time varying mass moment of inertia only as beta t, but will not giving any meaning because when I club it with I naught, I will know it is a mass moment of inertia. Similarly, when I club this with K, I will know it is time varying stiffness. That is the idea why we are clubbing them because in some of the papers and literature, you will see that - this term will not be called as I naught b theta. They will call this as I m, I k. They put a new subscript to this. I am not, I am avoiding this. I am saying let us club this to the initial value. So, we understand that they are part of this itself. So, it becomes easy for me. When I write the equation of motion, I can club the terms easily. That is the reason why I am doing this here.

Now, interestingly, we required to compute the following. What do we require to compute? I naught beta t, I need to compute this. I naught, I also wrote I naught, beta t, K, r of t. I need to compute this.

Now, let us see individual expressions for these variables and see how they are displacement independent. All will be displacement independent. We must know that. All will be displacement independent. One can ask me a question how mass momentum of inertia are displacement independent. It has got a component of the initial position of the system.

So, it will be displacement independent. I will show you that. Of course, beta t, we all agree it will be displacement independent theta. And, theta and displacement can be connected by this relationship because if we call this as displacement of the k th segment from the equilibrium position, this is the function of theta. So, it can be connected. K of course, we know it is going to be a by l. l is the component of displacement. The displaced l and the original l will be different. And, of course r of t is a function of time itself. Therefore, we know all these components are going to be displacement dependent. we have to now show that. We have to derive these and we have to now show.

Once I show in the equation of motion all of them are displacement dependent, write the equation of motion. Then, you converge to give an idea saying that this now will be solve in time domain using iterative frequency. I mean, using iterative schemes or numerical methods. So, we will talk about that now.

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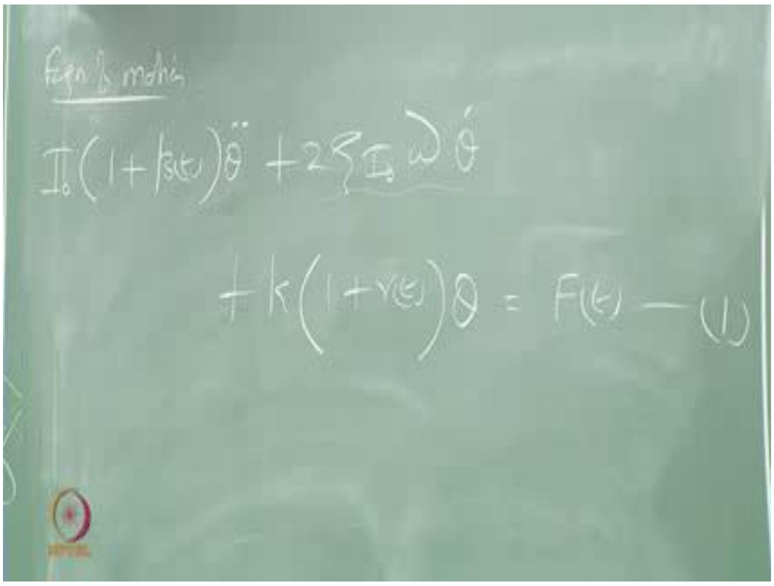

So, let us write the equation of motion. Now, the given system; combining all these terms, I should say the mass moment of inertia will have two terms; theta double dot. I am using a classical damping; not really a damping. plus, K of one plus r t of theta should be equal to; let say equation number one. That is the equation of motion now.

So, I must try to find out all the terms; I naught, beta t, zeta, of course an assumed value. It varies between two to five percent in a given steel system or two to three percent of concrete system. Omega, of course we know, if we know these two I can find out omega. Just now we wrote an equation for omega. This is a variable to be solved because if this is solved, these are all known to me because they are derivative of the time domain; K and r of t, which you have already indicated here. Ok. So, this is what I want. So, I must now have an expression to fill up all these in the equation of motion. When I have all these in equation of motion, I will put an integration scheme to this and try to find theta. The moment I know theta, if I proved that these variables are all functions of theta, I know all of them now. Ok. So, that is what we are trying to do now. Let us talk about over turning moment. Let us draw this figure.

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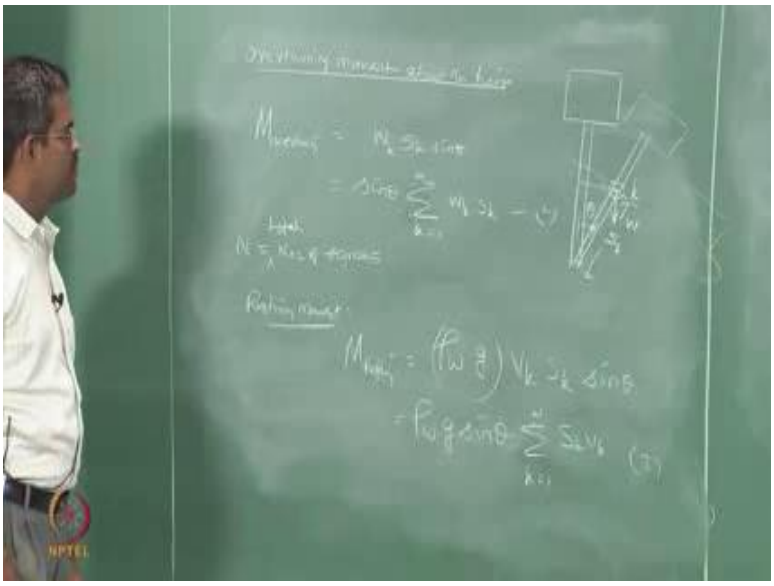

Let us say over turning moment about the hinge. Try to compute the over turning moment. The overturning moment actually, should be actually w into this distance. So, I, it practically should be w. And, this value is going to be S k sin theta. That is the overturning moment. But unfortunately, this is only for a k th segment. Is it not? I have got n number segments like that. So, I must say for all the segments theta will not of course vary; because it is going to be a unique thing. So, we can say it varies w k S k; where k varies from one to; this k is not stiffness. It is a count. Capital N, where N is the number of segments.

To be very precise, let us say total number of segments in a given system. I can divide them into any number of segments. It is the total number of segments within this. That is overturning moment I have, total number of segments. This is the one; rest is the crossing the overturning. Now, restoring moment of the tower will be given by let say rho w g of volume of S k. And, I am looking for the restoring action. So, it is going to be the horizontal component of this. So, I must say rho w g sin theta k one to n S k v k, where equation 1, equation 2, equation 3.

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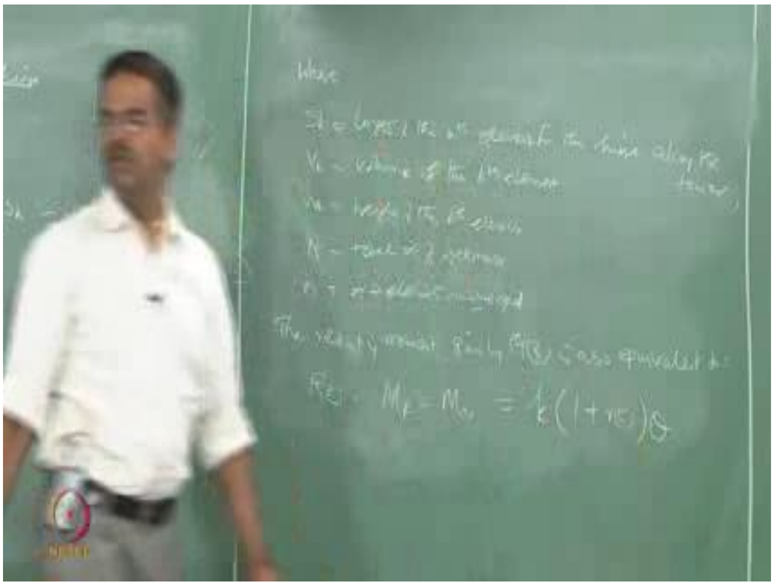

Where, S k, the length of the k th segment from the hinge - v k is the volume of the kth element; theta, of course the degree of displacement in angular section, S k is actually measured along the tower. This is S k, measured along the tower. Remaining all terms is known to us. And, w k is a weight of kth element. In fact, you should say element, not a segment. Let say by the summation here is going to be only for the small n, where small n, capital N is the total number of segments, we have already said that. Total number of segments, we have already said that. Whereas small n is the total number of segments immersed. We are looking only the submerged elements.

Now, the restoring moment, the restoring moment given by equation three is also equivalent to which is equivalent to some other value. Can you tell me what is that value? It is not restoring, resisting moment. So, the resisting moment r of t should be the restoring moment minus over turning. This should also be practically equal to one term;

should it not be equal to stiffness of theta? Because that is the restoring component in the whole system, let us equate this. So, it is called equation number 4.

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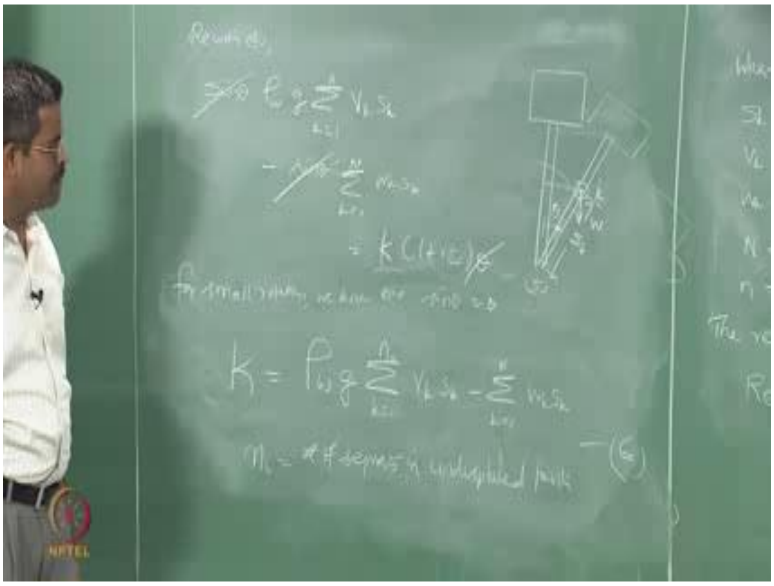

So, now rewriting four, sin theta rho w g summation of k equals one to small n of v k s k minus sin theta summation of k equals one to capital N w k S k should be equal to K of k of one plus r t of theta because there is K here also. So, I am writing K in a different manner, just to show that it is different. Now for very small rotations of theta, I call this is equation number five. For small rotations, we know that sin theta is set to theta itself. Therefore, these terms will get cancelled out automatically. I can now find K from this equation. Just check.

So, rho w g summation of k; small k is equal to one to n naught. I will come to this n naught later. v k S k minus summation of k equals one to n w k S k. Now, this term has been taken away. So, I should say n zero is a total number of segments in undisplaced position. If I say undisplaced position, r of t will set to zero automatically. And, one can ask me a question why K should be arrived at undisplaced portion. Remember, K is actually the stiffness of the system. Stiffness of the system is initially present where the system is straight or inclined. I am working at that property. I am not working at the property, when the system is inclined. Therefore, need not have to consider r of t in this equation actually; because the restoring moment should otherwise be always invoked, which is the difference of these two, which should be a function of k. And, I think I will replace this k also with this K for clarity because there is a k count running here. That is why. Ok.

So, I removed r of t. The moment I say n naught for the submerged volume of the elements, I am removing r of t component into this. Therefore, this goes away. I will get K directly from here. I will call this equation number 6.

So, out of the four constants what I want or four variables what I want in the whole equation, I have got one now, which is K. So, similarly I will get I naught, beta t and r of t, then substitute them and show all of them are function of displacement. You can see here in this case; K will be again the function of displacement because S k volumes of submergence are all dependent on the displaced value. So, K is again the function of displacement of the system. Ok. On the other hand, if the displacement is not there, K will be actually equal to the original (Refer Time: 38:42) from the system.

So, we will stop here. We will continue in the next class how we can work out the remaining terms of the given equation of motion. When we do that we substitute them back again and show you that all of them will be displacement independent. Therefore, we can go for an iterative scheme, which is given by many methods available. One of the classical method is (Refer time: 39:05) beta technique. We will talk about that later. Once we take down all the problems, try to understand all the equations formulations and show them that all of them are time dependent. And, one can be solved in the time domain. Then, we will work out the iteration scheme later.