

Dynamics of Ocean Structures
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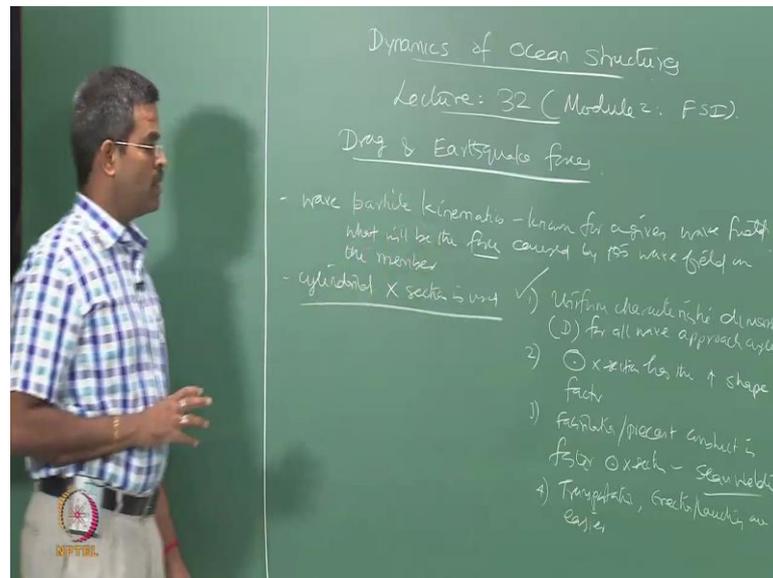
Lecture – 32
Fluid Structure interaction: Drag & Earthquake Forces

So, in the last lectures we discussed about what is the influence on water body kinematics on velocity variation and the flow field variations by interference caused by the cylinder which is blocking the flow of field wave field.

Now we have also seen what is the influence of current on the given velocity profile variation along the depth. We have also seen the secondary vibrations passed by the system or the cylinder kept horizontal, kept vertical. What we call as the characteristic dimension of the cylinder in the wave approach angle. And we understood that how the water particle kinematics is altered and subsequently we have said that how this alteration or the velocity variation can be captured to understand the force reduction on a given member by simply putting a outer perforated cover, on a inner shield of the cylinder.

So, the force on the inner cylinder is reduced which can be considered as one of the effective method of retrofitting or rehabilitation of a given inner cylinder, provided the inner cylinder is not replaced because of it is damage. But only putting encapsulating cover outside the cylinder, on a specific zone this is damaged caused by the splashing effect of the corrosion effect.

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So, now in all these cases one is actually interested to ultimately get through an important point. When, I had let us say water particle kinematics, known for a given wave field as an engineer, one is interested to know what will be the force caused by this wave field on the given member. Usually cylindrical members are used in offshore structures because of many reasons foremost reason; it has got uniform characteristic dimension which is nothing, but the diameter of the cylinder, for all wave approach angles that is the one reason foremost reason.

The second reason is if you look at the method of analysis let us say look at the plastic analysis or LRFD load resistance factor design method circle or circular shape has a decently higher number of shape factor. That is the second reason those who know the limit state design or plastic design you know what the importance of shear factor is estimated in load carrying capacity of a given section. So, it has got a higher number. Third is the fabrication and precast construction is faster with cylindrical cross section because we go for a seam welding.

So, the connections become rigid and will establish using a weld connection and 4. Of course, transportation erection launching is easier with circular cylinders. So, cylindrical cross section generally used in offshore structures because of many reasons

fundamentally this and remaining all are auxiliary reasons why they have been used. Therefore, we use cylindrical cross section, now, one is primarily interested what would be the effect of the wave field on the interfering cylindrical member in terms of it is forces; therefore, Morison equation came into play, Morison actually suggested in the empirical equation on the basis of experiments conducted way back in 1950.

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Morison, J.R., O'Brien, M.P., Johnson, J.W. and Schaaf, S.A.
 (1950) Force exerted by surface waves on piles,
 Petrol. Trans. AIME, 184.

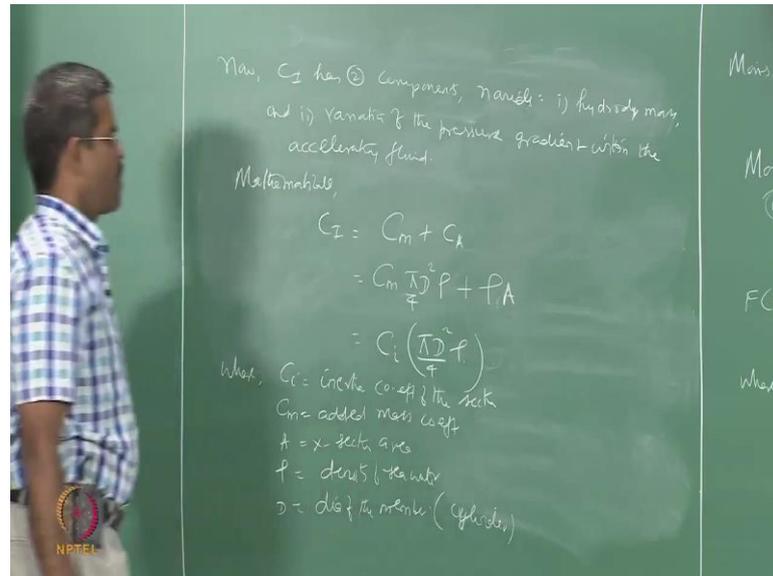
$$F(z, t) = \underbrace{C_I \dot{v}_x}_{F_I \text{ (Inertia)}} + \underbrace{C_D v_x |v_x|}_{F_D \text{ (Drag)}}$$

Where $v_x = \text{vel of the water particle}$
 $\dot{v}_x = \text{acc}$
 $C_D = \text{Drag coeff}$
 $C_I = \text{Iner coeff}$



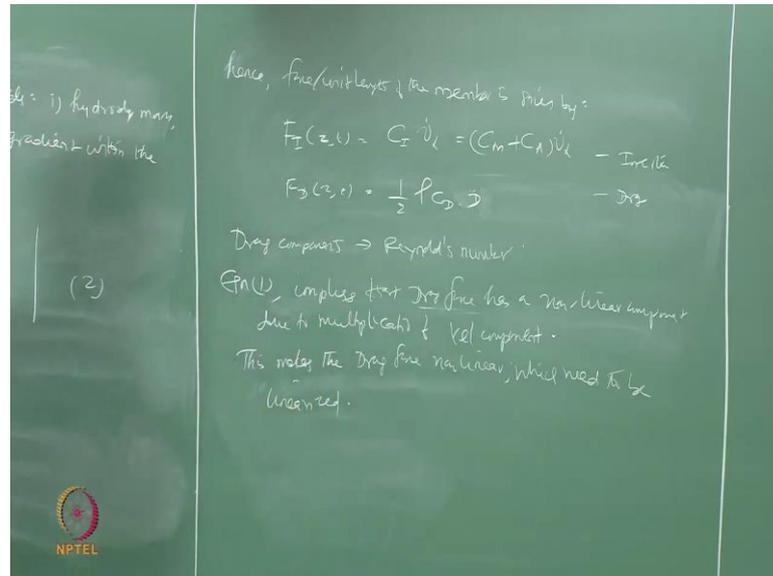
So, it is way back in 1950, which is based on some experiments conducted by a team of people on measuring the forces on the cylinder which is exerted by the wave action on the cylinder and is given in the equation, which I will right now. which infamously know, but still we will write down that the general equation suggested by Morison is like this f of z comma t is $C_I v_x \text{ dot}$ plus $C_D v_x v_x$ which can be written in 2 forms this is called F_I this is called F_D this is called drag force, this is called inertia force. Where v is the velocity in $v \text{ dot}$ is the derivate of the velocity which is acceleration both are water particles where v_x is the velocity of the water particle and $v_x \text{ dot}$ is the acceleration of the water particle and C_d and C_i are called drag coefficients and inertia coefficients respectively which, has to be estimated based on the experiments conducted by the team of people supported by Morison. If you looked at this equation carefully the component of C_I has got 2 subcomponents to it.

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Now, C_i has 2 components namely; one is arising from the hydrodynamic mass and the second is arising from the variation of the pressure gradient within the accelerating fluid within the accelerating fluid particles there are 2 components. So, mathematically C_i has got 2 components 1 is C_m plus other is C_A which can be C_m multiplied by plus ρa , which I can write as C_i C_i actually is inertia term which concerns both the pressure gradient variation where C_i is inertia coefficient of section C_m is the added mass coefficient A is the cross section area of course, ρ is density of sea water and d is the diameter of the member essentially cylindrical member calls equation number one this equation number two.

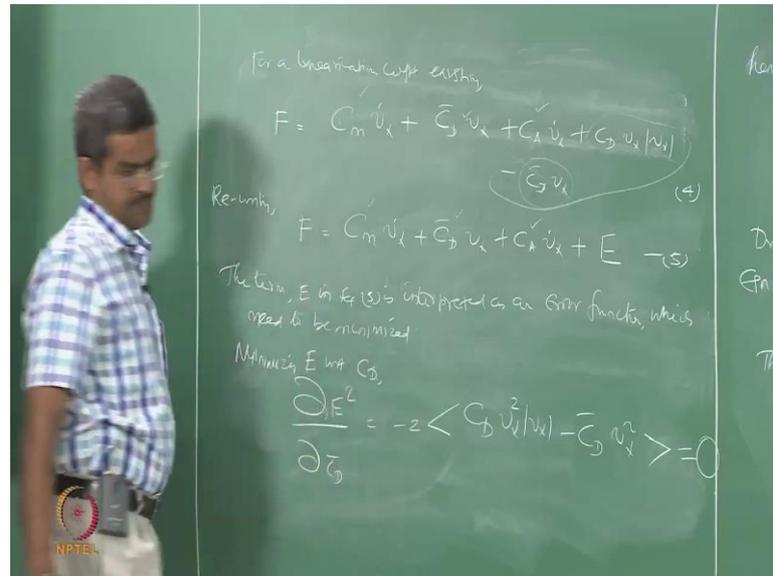
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Hence force or unit length of the member is given by F_i x of t which is C_i not x of t that is z of t z is the axis along the vertical $C_i V_x$ dot which is C_m plus $c_a V_x$ dot that is inertia force.

Now, the drag component f_d for unit length of the member the drag component is given by half ρC_d drag and of course, the drag force the drag component heavily depends on the Reynolds number now looking at equation one implies, that the drag force has a non-linear component due to the multiplication of the velocity component this has to linearize which actually makes the drag force non-linear. So, this makes the drag force non-linear which need to be linearized many researchers attempt this linearization and they have given a alternative 1 can ask the question why linearization is important I will come that points actually once we later once we understand how to linearize this.

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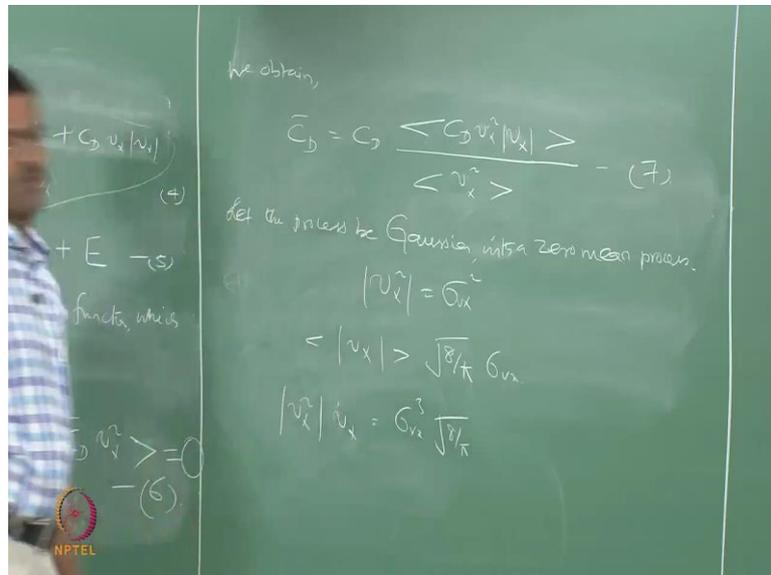
For a linearization coefficient existing now force, now will be given by a different expression which will be let us say $C_m \dot{V}_x$ which is $\bar{C}_d V_x$. There is no multiplier of V_x again here which of course, will have the pressure $c_a V_x$ bar because there are 2 components available in the inertia force plus $C_d V_x$ absolute V_x minus $\bar{C}_d v_x$. So, meaning of the equation will not change compared to equation 1 it is the same I will call this equation number this is number 3. I can call this equation number four. So, compare 4 and 1, the mean the meaning of the equation 4 and 1 does not change except that I have added and subtracted an additional term which is $\bar{C}_d V_x$ I will explain what is \bar{C}_d is because this term was any way expanded from equation 2 and this equation was reproduced from equation 4 with substituting 2 in 1. Y

ou get equation 4 like this you compare 4 and 1 it is 1 and the same except 2 components are added and subtracted to make it balanced now, I get rewrite this equation slightly in a different form rewriting f will be given by now $C_m \dot{V}_x$ plus $\bar{C}_d V_x$ because I want to avoid the non-linear part plus $c_a V_x$ bar V_x dot plus e the e is an interpreted as an error function the term e in equation 5 is interpreted as an error function which need to be minimized to make the equation accurate which need to be minimized.

Now, if we look at the error function which is component of; because term is present this term is present this term is present. So, it is a component of this which is again function of $c d$. So, I have to minimize the error function respect to Cd , So, minimizing e with respect to Cd . So, error functions. In fact, I should say $\bar{C}d$ because I am minimizing with respect to the new linearized drag term it is a linearised drag term because, there is no non-linear available here the original term is this which has got non-linearity here. So, linearized data we presume that 1 function of this order existed it is an assumption we minimize it with respect to that assumption which is given by negative 2 of there is a negative term here.

Time average of this symbol indicates time average time average of $Cd Vx^2$ minus $\bar{C}d$ let me write this minus $\bar{C}d Vx^2$ I must set this to 0 because I am minimizing the square of the inner function once you do that an perform the operation minimization respect to $\bar{C}d$ you will get.

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Let me call equation number 6 we can get $\bar{C}d$ as Cd of time weighted average of $Cd Vx^2$ by Vx^2 . I mean all are small v please all are small v whole square of course; this is other time weighted average equation 7. So, I get $\bar{C}d$ is equal to 0.2 has no meaning and I am looking at time weighted average negative will have no

consequence on that. So, try to find \bar{C}_d from the equation like this now interestingly you see if we really wanted to find out the value of \bar{C}_d which is a linear drag term I must try to find out the time weighted average of this. So, I have to compare really the velocity square from the original process of the wind velocity. So, I must know what is the nature of the wind velocity processed it itself. So, generally the wind velocity generally let the process be Gaussian, it is a Gaussian distribution generally it is Gaussian distribution, with a 0 mean process in that case you got a great advantage. The V_x square will be actually be equal to the sigma square of the process we can even call this sigma V_x square to make it very clear.

And the absolute value of V_x the mode value of V_x between the time weighted average will be an approximation of $8 \sqrt{\pi}$. Now can you tell me what is the value of $8 \sqrt{\pi}$ the square root of $8 \sqrt{\pi}$ square root of $8 \sqrt{\pi}$ please change what is the value of this.

Student: 1.5 (Refer Time: 20:22).

One pi.

Student: (Refer Time: 20:23).

So, it is linearization instead of square we are having 1.569. So, it is not actually equation to 2 now when you multiply V_x square with V_x you get sigma V_x cube root $8 \sqrt{\pi}$ now here you can ask me a question, how are we eliminating the non-linearity term by introducing a higher order of sigma V_x ? Now sigma V_x actually is a number for a given process whereas, V_x is not a number V_x is actually a velocity term which is time weighted average it is a time weighted value whereas, this is a constant value for a Gaussian process. So, cubing anything is not a problem simply number. So, simply cube this and multiply this with $8 \sqrt{\pi}$ therefore, \bar{C}_d now will become hence let me call this equation number let us say 8.

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Hence,

$$\bar{C}_D = C_D \frac{\sqrt{8/\pi} \sigma_{V_x}}{\sigma_{V_x}^2} = G \sqrt{8/\pi} \sigma_{V_x} \quad (9)$$

Hence linearized Morison Eq is given by:

$$F(z,t) = C_m \dot{v}_x + G \sqrt{8/\pi} \sigma_{V_x} v_x \quad (10)$$


\bar{C}_D now becomes substituting back C_D of let us say 8 by π $\sigma_{V_x}^3$ $\sigma_{V_x}^2$ which will give me 8 by π σ_{V_x} of C_D of course, C_D is there here call equation number 9. So, we presume that a linearization of drag term exists it will be a multiplier of some weighted factor of C_D which is given by this number where as this is not, $6 \sigma_{V_x}$ this is σ_{V_x} which is known to me for a given process being Gaussian and 0 mean we know this for the velocity and I think we all know how to compute this if you got velocity variation of water particle velocity variation horizontal, for a given wave field for a specific time domain you can take a mean average of this find the standard deviation and variance of this easily for a given statistical process.

So, it is easily obtainable for a given Gaussian process once you know this I can use this for linearization get the value of force now it will become hence linearized Morison equation sorry is given by F_z of t $C_m \dot{v}_x$ with an acceleration term which includes C_m and C_v both C_m and C_v we already know we refer to equation 3 C_m is a component of 2 things plus $C_D \sqrt{8/\pi} \sigma_{V_x} v_x$. So, 1 is the velocity term other is the acceleration term. So, inertia and drag components - it is the linearized equation there is no non-linear terms to this compared to 1. Let us call this equation number 10. So, 10 is the linearized equation which has got some approximation with respect to non-linear original drag term given by Morison, but this is necessary. Now, one can ask me a

question why the drag term got to linearized what is the problem now we have seen how it is linearized we have derived it we know the value, but let us quickly see as a self question to understand why a non-linear session cannot be handled as it is.

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Compliant member.

$$F_D = \frac{1}{2} \rho C_D D \underbrace{(v - \dot{x})}_{v_r} |v - \dot{x}|$$

Let us take for example, a compliant member let us look at the drag force alone which is half rho Cd di av minus x x dot v minus x dot where, x dot is the velocity of the structure or the member and v is the velocity of the water particle in the horizontal direction and this term can also be called as relative velocity we can call as v r this is relative velocity where system is also now compliant members is also now moving. Therefore, I had to capture a relative velocity term, if you if you expand this term and maintain the sign because of the mode on only 1 of this you will see there is a square term in the product which will not be able to their square terms available at v squares which is velocity square structural actuation square and the product of v and x. Which will become non-linear which cannot be computed because x dot is not known to me is the solution of the problem actually because, the equation of motion the equation of motion is what I have displayed as the solution. Therefore, velocity and acceleration are not known to me.

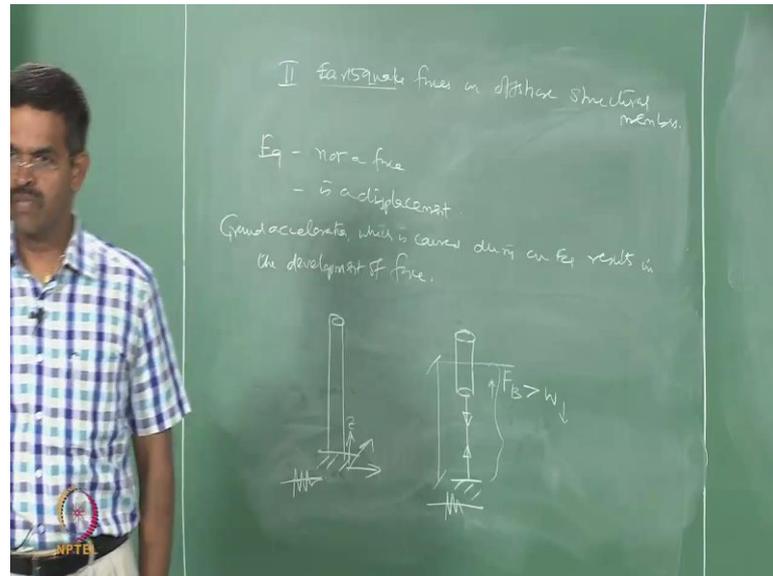
So, initially you cannot evaluate this term provided you have to assume this and keep on iterating when you iterate we got a non-linear term iteration becomes cumbersome it is

very difficult to iterate. So, I want to linearize this compared to this of power of 2 I have got 1.569 as we have pointed on the previous minutes. For example; there is an approximation available here, but this term is relatively easy here the variable is of course, velocity and acceleration, but if you solve the displacement equation of motion you will be able to get this that is the beauty in this problem I will come back to this slightly later, how the right hand side of the equation of motion is coupled with the left hand side. Therefore, if you do not know the velocity and acceleration you cannot solve the equation of motion to start with we will use Gaussian method etcetera later I will show you the scheme later, but anyway this term does not have any non-linearity in its evaluation process as you had the in terms of complaint system it becomes easy that is the reason why we linearize this.

Suppose if the force on and this about the vertical cylinder if you have got a inclined cylinder then you got to resolve the forces along and across or along and normal to the cylinder find the respective components of the force in the respective directions and try to develop a vector in all 6 degrees of freedom. See if the member is inclined with respect to the water particle approach angle then you have got to find the velocity and acceleration or the force in general one is tangential to the member and other normal to the member and you have to find the equivalent components of them in all 6 degrees in terms of relative degrees of freedom and fill up the whole vector though the force may be acting in one direction since the member is inclined. Now you will get forces on all respective activated degrees of freedom that is the idea.

The second part of the lecture we will focus now on how do we get earth quake forces on offshore structures.

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The second part now is to get earthquake forces or seismic forces on offshore structural members as we all know we talk about the force it has to generate from the velocity or the motion of the body because a force of the vibrating system is essentially from the motion displacement. It is a very clear idea even in the case of water particle or the force exerted by the waves on the member there is actually no force there is only a moment of the water particle which causes force it is actually displacement of the water particle which has got since the displacement is continuous in the time history it has got a velocity and acceleration that arises to a force which we just now calculated and computed using a Morison equation. So, displacement gives rise to the force and so and so forth. Similarly here in earth quakes also the earthquake actually is not a force actually it is a displacement which causes the force.

Now we are very clear about this idea how displacement can cause a force just now we saw in Morison equation when the water particle displaces it generates velocity and acceleration when the cylinder interferes the flow field it generates the force in the member and the cylinders vibrating it will give secondary force which we say radiation etcetera in the last lectures. We know how the displacement can give arise to a force in a fluid medium same concept in earthquake now earthquake gives forces through it is displacement now the question is which is getting displaced in the earlier case it was

water particle or the member in this case it is neither of them, but the ground the ground is getting displaced. So, the ground acceleration which is caused during an earthquake results in the development of force.

So, one can ask a very simple question and try to understand how serious it is let say I have one cylinder which is fixed another cylinder which is anchored using a cable; obviously, system is designed. So, far that the buoyancy force will exceed the weight of the system therefore, we will try to push the system up because the buoyancy will act upward weight, of course will act downwards we know that. So, when the buoyancy of the system exceeds the weight of the system or basically there is always an upward motion given to the body when this is putting water may be it was partially immersed of course, the buoyancy force depend upon the circumferential area of immersion and the depth of the immersion what we call as design draft it immerses.

Therefore, it pushes the body because the weight is very low right since the weight is very low and the body is being pushed up all the time we always need a cable which is in tension to pull this body down otherwise the body cannot be held in position the body will get up float body will be thrown out from the media because, the buoyancy force is externally high compared to w that is the design concept one may ask me a question, why this concept is involved what was the reason the moment you keep this design concept on commissioning installation towing transportation everything is very, very easy because weight is compromised on buoyancy force simply float it you drag it will drag it would have been other way it will be very, very difficult to install this that is for installation purposes.

Then one can ask me as counter question, why for installation purposes designed get evolved that is the beauty of the offshore structure because an offshore structure schemes installation commissioning decommissioning is about 80 percent or 70 percent of the cost of the structure it itself it is very, very expensive.

Therefore, we should have a mechanism there are very special equipments cranes available there is weather window within which you have got to operate etcetera are all consequence are there we already vaguely we already know this at least what are the

forces coming on. Therefore, we must facilitate easier erection and commissioning during a launching we must have a system which is having excessive buoyancy by design number 1, number 2 this will also enable flexibility to the system which is required in the design for deep water systems, but this is a fixed system.

So, rigid flexibility or what you call compliant systems in both the cases you applied the earthquake force what will be the consequences now since the cylinder or the member is fixed to the bottom; obviously, when the sea when the sea bed in this case moves which is displacement which causes an acceleration in all the 3 degrees x, y and z all the 3 because earthquake forces will apply or the displacements will apply in all the 3 axis. So, let us say it is happening in all the 3 since this member is rigidly fixed to this. Whatever forces offered by the system will be transferred as stress to the member material if the member is able to take the stress the member will not fail otherwise the member will fail.

So, it is actually a design concept in terms of adding this as an additional force to the member. Whereas in this case it is different you see what happens here when an earthquake force is applied in this case since the member is not resting on the sea bed practically speaking this member should not be influenced at all by an earthquake happening at the bottom because the depth what you are talking about is approximately about thousand meters.

So, we talk about acceleration happening at the bottom kilometer bellow we should not rather bother about this because the member should not be influenced because 2 factors 1 it is happening at a very far epicenter from the bottom number 2 there a good amount of hydrodynamic damping available the water body which will try to pressurize this and control and compromise this or disperse this force which will not reach remember this is the idea originally till seventies people thought this will not be a force acting on the member because the member is far away from this.

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$$F_B < W$$
$$F_0 = W + T_0$$
$$\left. \begin{matrix} [k] \\ [m] \end{matrix} \right\} f_n(T_0)$$

Now, there another interestingly concept here the interesting concept is since F_B is less than w . So, F_B should be equal to w plus something to equalize this that something was nothing, but the tension in the cable.

Now, the stiffness matrix the mass matrix or the variation submergence because of the added mass in the mass matrix are all unfortunately functions of t naught which will derive later are functions of t naught. Now let us see for what happens to this idea in this problem here when the earthquake is appeared in the bottom since the theta is connected one end to the ground which is moving the tension in the theta is changing because theta is actually inaccessible or in extensible. So, it is in extensible cable strictly speaking.

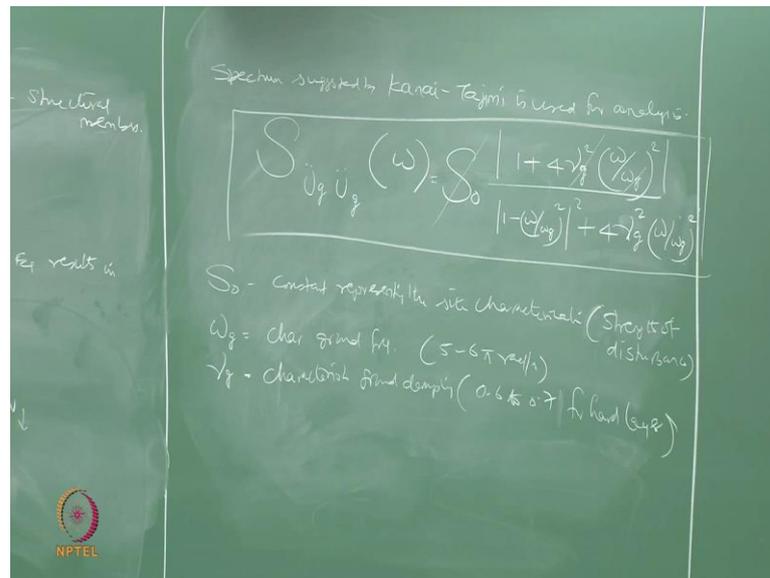
So, when it moves because the length variation that is caused by the moment the tension has got to change because tension is $a e$ by l when the length changes $a e$ by l will change because k will change when the tension changes your stiffness matrix will change when the stiffness matrix changes the frequency of vibration of the whole system will change when the frequency of vibration of whole system changes it will get into a mode of resonance with the wave therefore, theta tension variation is indirectly an accelerator problem because, of earthquake was happening at the bottom though it will not affect the super structure directly as in this case.

So, this is most serious one, reason to the second reason is when, this extension displacement is very large there is a possibility that the cable get pulled off. When the cable get pulled off the floating member will have it is stability problem because this equation will no more be met there will be no balance that cable is pulled off there is not t, t_0 . When there is no t_0 system is having a over buoyancy design the member will be flown off thrown off from the sea bed automatically for stability will be a problem, when it is thrown off or blew off from the position the top facility which is holding this will all crash. So, earthquake moment will cause indirect effect on the top side of the member though there is no direct connectivity between the member and the ground is it clear.

Where as in this case there is direct connectivity now in this case the advantage is since the system is not designed for over buoyancy the weight will always compromise this effect therefore, the effect of the ground activity of the ground motion on the super structure is insignificant mostly whereas, in this case this is significant because of 2 reasons 1 the system it itself is very complaint in nature second it is affected directly by the t_0 variation which is affected subsequently by the displacement of the sea bed therefore, in both the cases displacement give rise to the force.

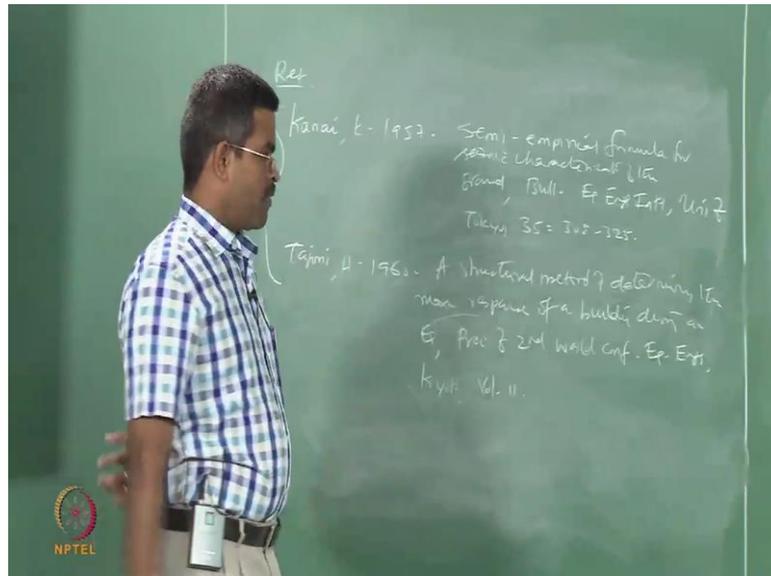
So, as we saw in the previous case in wave we need to have a spectrum because there we are spectrum ISCC spectrum modified 2, parameter p_m spectrum which we have seen all equation in the previous module in the wave forces. So, similarly for earthquakes also we have seen the spectrum which is given by Kanai Tajimi of course, Knai Tajimi spectrum is not directly influencing up 1 for anlaysis of offshore structures Kanai Tajimi is actually a spectrum generated for studying earthquake motion on land based structures, but here the extension is done.

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So, spectrum suggested by Kanai Tajimi is used for analysis. So, let us see what is the spectrum $\ddot{u}_g \ddot{u}_g$ this is a classical way of writing a spectrum because even in power spectral density function or even in wave spectrum we write omega. In this case omega is given by this is the spectrum this is given by Kanai Tajimi by working out the forces because S_0 is called a constant representing the site characterization indicating the strength of disturbance ω_g is called characteristic ground frequency which is usually 5 to 6 pi radians per second γ_g is called the damping term which is the characteristic ground damping which is usually 0.6 to 0.7 per hard line. So, this constant is known this ratio is also known ω_g is of course, known. So, the entire spectrum is a function of omega which can be plotted we shall typically look I will show you the figure in the next class, but anyway let us quickly look at the reference this was given in 1957.

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Semi empirical formula for seismic characteristic of the ground bulletin of earthquake engineering institute university of Tokoyo University of Tokoyo 308 to 325 sorry Tajimi a structural method of determining the maximum response of a building during an earthquake proceeding a second world conference of earthquake engineering quto volume 11. So, this is what we have a Kanai Tajimi power spectrum given in the year 1960 by both this people we will talk about the basic assumptions used by this gentleman in estimating the earthquake forces the maximum earthquake force on a given system we will take up a simple example and try to solve and show how this force can be computed using this spectrum easily then we will move on to in your classical structure like artificial towers etcetera any doubt in this lecture we have discussed about 2 issues here 1 is the drag linearization.

We have showed how linearlization can be done and how why is it a necessary in a given system second is, how to compute the earthquake forces caused by the ground sea bed movement of the sea bed displacement in terms of this power spectrum given by Kanai Tajimi which is known as Kanai Tajimi power spectrum and which is essentially approved for buildings. But nevertheless recent studies on offshore engineering have used this spectrum for calculating the forces in the given members because of sea bed moment.

So, there is a very interesting homework for you to undergo that let us try to find out what is the maximum disturbance caused to any one of the offshore platform the recent past because of the earthquake there is a very interesting history available in one of the platforms in offshore which had a very close epicenter in the gulf of Mexico, which was disturbed highly by the earthquake caused in the recent past. So, try to find out which is that system or which is the platform which is disturbed heavily by the recent in the recent past by the earthquake signal in the sea bed then you will really appreciate how this system was actually.