Dynamics of Ocean Structures
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Lecture – 28
Modal participation and missing mass corrections

We were talking about the Static Correction. In the last lecture we wanted to know that if you truncate the modes from n hat to n what could be the loss of the contribution of x arising from the higher modes to the (Refer Time: 00:32) total contribution. So, we started with this equation we said this in my classically equation of motion, where x is the response can be represented as a modal participation of the mode factor and that of q which is weightage of the mode. Once we agree that x can be represented as a displacement q, obviously x double dot and x dot can be represented as phi of q dot and phi f q dot double dot respectively substitute back.

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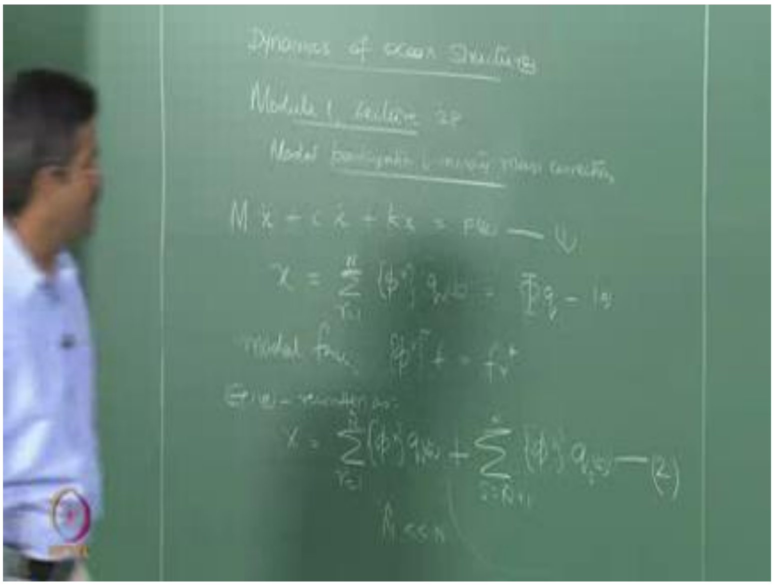

I get what is called modal force, after I pre multiply this equation with phi transpose and I already have a phi here. So phi transpose m phi will be a specific value, phi transpose k phi will be a specific value, phi transpose c phi will be known to me. And I am talking about mode and force which was given to me as let say if phi vector transpose it is called as f r star which called as modal force. When I pre multiply this equation phi and transpose I will get this value also phi r transpose for rth vector is on modal force on r.

So, one is interested to know what is a contribution of the higher missing modes in the total overall response that is what we are focusing at. Then now I split this response of equation 1 a. Now equation 1 a can be rewritten as in two parts; that is instead of having from r to n I have r to n hat and then n hat to separately as two we sets of equations because this is an algebraic sum linear super positions I can split this. So, x can be rewritten as summation of and summation of r equals 1 to n hat s equals n hat plus 1 to n phi r q r of t and phi s q s of t; I can split this. I think in the last derivation I think equation number 4 or 5. Let us call this as equation by in this case as 2 that it is a continuation; that is a equation number 2.

What I am interested is to know what the contribution of this specific component on x is because I am going to neglect this. I do not want to consider the modes beyond n hat where n hat is much lower than n. I would not to neglect the higher modes, but I want to see what the influence of this, in this is.

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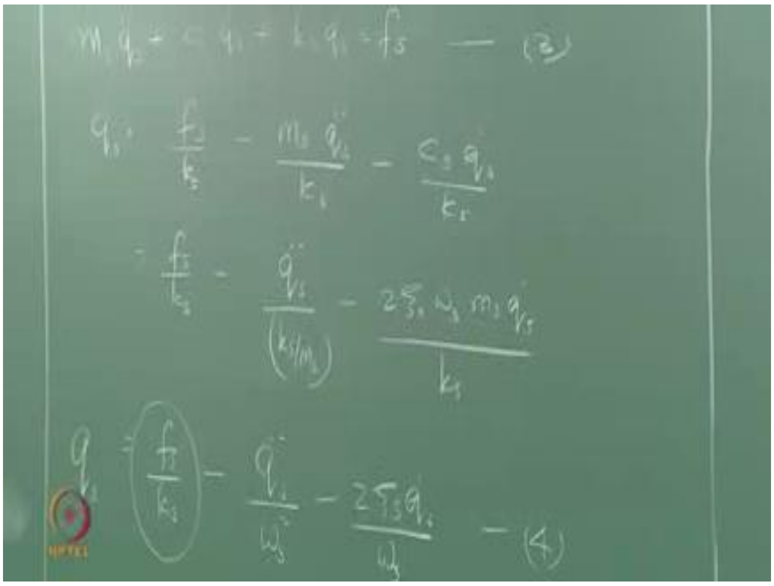

They have also said it is going to be a static correction because m s q as double dot, because now I am writing equation of motion in s frame that is the degree of freedom from n hat to n. I can find q s as f s by k s minus m s q s double dot by k s minus c s q s dot by k s which can be f s by k s minus q s double dot by k s by m s minus 2 zeta s omega s m s q s double dot by k s. Which can be f s by k s minus q s double dot by omega s square minus k by m s omega square omega by omega goes away the 2 zeta s q s dot by omega s, which we had in the last equation also we have the same.

So obviously, this becomes the dynamic comprehend because this is the velocity in acceleration this is a static component. And you see this component keeps on decreasing because as for higher frequencies this value will keep on increasing so negative term, therefore from the static response or from q s which is the response of s modes which is from n hat plus 1 to n on the overall response it will keep on decreasing. Therefore we can say the contribution from the static correction is dominant in this case; call equation this as 3 and this as 4.

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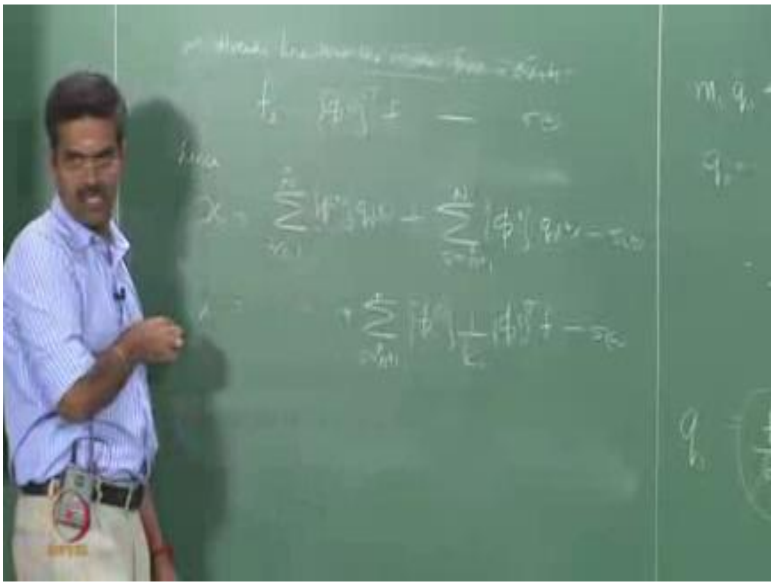

We already know a modal force is given by which is f s I am talking about s modes which is from n hat plus 1 to n. Similar to that of this same borrowed from equation 1 a, but in terms of s. Hence you can call this as equation 5 a. Hence, x which is equation 3 in terms of q s rewritten now as summation of r equals 1 to n hat and s equals n hat plus 1 to n phi r q r of t. Now, I am going to be replace this let say q s of t same way. Q s is of course having a static correction of this order did not neglect. I will expand this portion plus phi s. Q s is f s by k s I will say 1 by k s. F s of course is product of this phi transpose of f; I will call this equation number 5, let say 5 b and 5 c.

In the last lecture I think we stopped here and we want to show that how this correction can be modified so we will extend this from here, so this is my x. We are anyway not bothered about the first part of it, because first part we anyway executing and calculating x as a summation of contributions of q r and phi r's there is no argument in this. The argument will be now on what would be the influence of the missing values from n hat plus one t n on x that is the argument here.

Now, if you ask me a question that if I am going to find out what is the influence of those forces corresponding to n hat plus 1 to n on the total response, then obviously this will demand me to estimate phi s. It means I must have all the mode shapes with me. If I am having all the mode shapes with me why should I look forward for the one which is contribute in to the error of the total problem. So, my advice is not to compute or not to spend computational effort towards estimating omega s and phi s where s is varying from n r plus one t n. I want to truncate the calculations of a computation still n hat only where n hat is much lower than n.

So, a procedure should not demand from me that I am bound to estimate phi s. What I would do is I will rewrite this part of this equation as the total flexibility, because I am taking about 1 by k s, I am talking about k s inverse that is actually the flexibility matrix of the entire system. If I know the flexibility of the entire system I subtract the flexibility of the system from r is 1 10 hat I will actually get this, because I am talking about this missing mass where it demands flexibility of the remaining modes which is s part of it; s part is from n hat plus 1 to n, but what I will tell is I will take out the flexibility of the entire system which I call as k inverse I will subtract the flexibility part of this I will get this is, is it not. So, that is what I am indirectly doing.

Therefore, without the estimating omega s and f phi s I will use only omega r and phi r, but I will find out the value of this contribution on x; that is what I will write now here.

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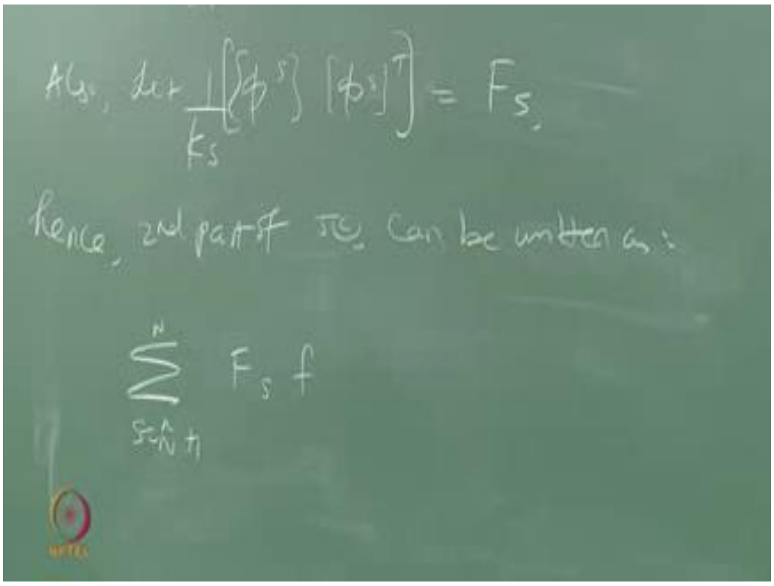

Since, b r truncating the modes till n hat the procedure shown in equation 5 c demands estimate of phi s which is redundant, because I will not have phi s with me I will have only phi r. Also let phi s phi s transpose of f or let say by k s is called as f s. Hence, second part of equation 5 c can be written as summation of s n hat plus to n f s of f.

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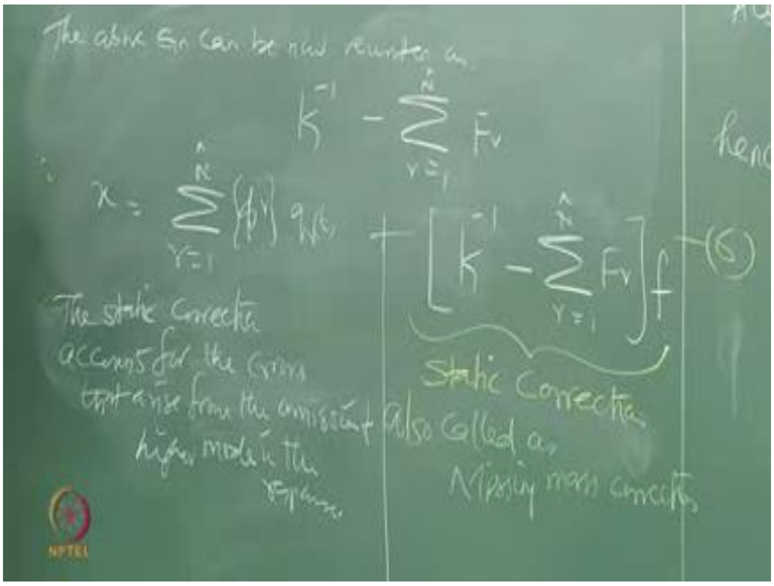

1 by k s of phi s transpose summation s n hat plus 1 to n can be written as 1 by k s of course is k inverse. Remember k s is only for those values of s modes which is from n hat plus one to n, whereas k is a stiffness matrix to entire system which is having the

modes from 1 to n total. So, from the total flexibility I subtract summation of r equals 1 to n hat 1 by k r phi r phi r transpose and I have these values with me k r I have, phi r I have and phi r transpose I can compute. By this way I can easily find out this component of equation 5 c which require for me which can be the value of f s.

Hence the above equation can be now rewritten as k inverse minus r equals 1 to 1 hat when I call this as f r, because already I said if you know subscript for the entire system I call this as f s. Now look compare f s with this the one and the same x of the subscript is r now, so I can call that is f r. Therefore, x is now summation of summation of r equal to n hat phi r of q r of, summation remove this we simply say this is going to be k inverse minus summation of r equals to 1 to n hat f r of f; this actually f s of f.

In this case f r of f, that is a modal force this is from n hat to n hat plus 1 t I call s this is from r to n hat I call that as r so one and the same the modal force I got the new x now. This particular term in this equation 6 it is cost static correction, is also called Missing Mass Correction. Now, the static correction accounts for the errors that arise from the omission of higher modes in the response. Let us apply this and see how I can compute x. We will take two examples.

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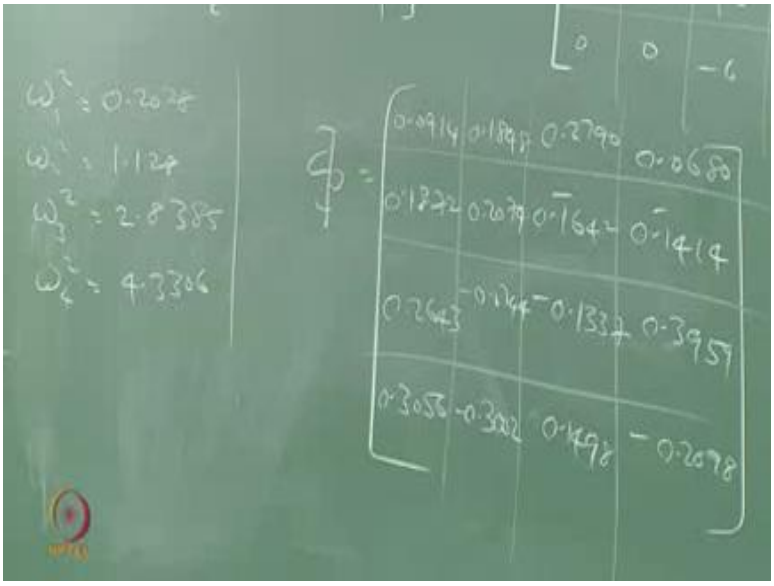

I have a system whose mass and k matrix and frequencies mode shapes are given to me let say mass is this value, so 4 by 4 system 8 8 4 and 4 all other elements are 0. Maybe we can take this as a multiplier of 10 power 7, 10 power 10 whatever maybe the value we are not bother about the multiplier; let us talk about k a m. And k matrix is and omega 1 0.2028, omega 2 square 1.128, omega 3 2.8385, omega 4 4.3306.

Phi matrix 0914, 1872; it is not symmetric actually slightly there is a variation, 3002 at the second mode. There is a negative term here also this is also minus, so 1 2 and 3. I want to work out the modal participation factor. In this problem I actually have all the 4 modes and 4 frequencies, but I want to know do I have to include all of them for calculating the x of value I want to see.

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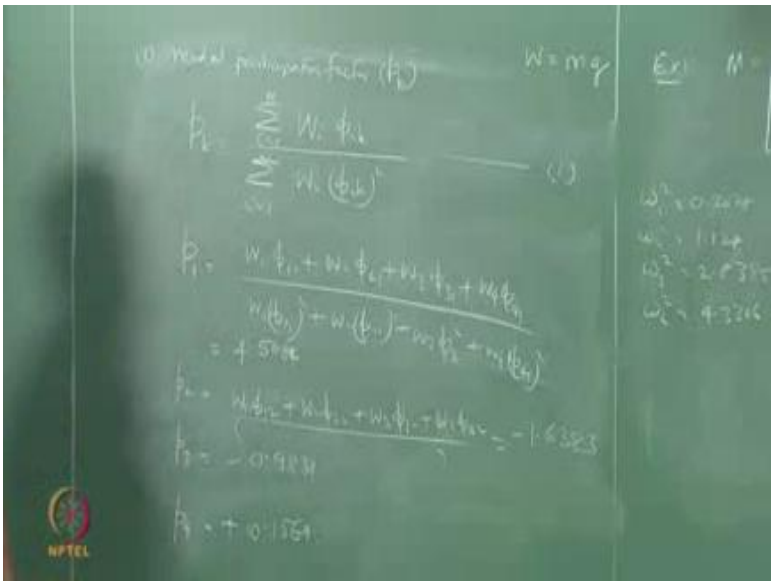

So, let use workout the modal participation factor which we derived in the last lecture it is given by p k participation factor of the kth mode it is given by; let us calls equation number 1. If we really wanted to find p 1 I can expand this, k is 1 only i is count so I should say w n phi 11 plus w 2 phi 21. You will see the second counter always remain as 1, only the first count will change plus w 3 phi 31 plus w 4 phi 41, w 1 phi 11 square w 2 phi 21 square w 3 phi 31 square w 4 phi 41 square.

Obviously, look at the second substitute unity it means you are referring to the first column of this vector which is actually the corresponding first mode shape of the system that is why it is called the modal participation of the first mode. So, now you have w use, of course you have mass w is m g you will multiply g here you multiply g here they were cancels so does not make in difference in the calculation anyway, so let us substitute them w 1 w 2 w 3 w 4 we have, we have the first column can be give me what is p 1. Just for a establishing the leadership w 1 is 8 8 into 0.0914 plus 8 into 0.1872 plus 4 into 0.2643 plus 4 into 3.056 divided by 8 into 0.0914 square and so on so forth. We will just substitute them and get me the value.

Similarly get me the values of p 2, p 3 and p 4. First let us find out the value of p 1, how much is this?

Student: 4.508.

Is it a positive? So, can you give me the value of p 2? Now if you substitute this the count is going to change for k, so if we rewrite this equation here it will be w 1 phi 12 plus w 2 phi 22 plus phi 32 plus phi 42 divided by some of the square of this. So, only the second count will change now you are looking for the second column of phi. Now we can expect the negative values also because the second column onwards your negative number also depending upon the contribution of that in the total the sum of p k 2 3 4 can also become negative and so on.

This comes to minus 1.6383, p 3 it comes to minus 0.9831 and p 4 becomes positive of 1569, so these are the modal participation factors. Ultimately, I wanted to actually know what is the contribution of the mass to be added to the system. Because, this factor multiplied by the mass contribution will give me the total response in terms of force acting on the system. Let us try to find out the modal mass. Now I find out the modal participation factors alone.

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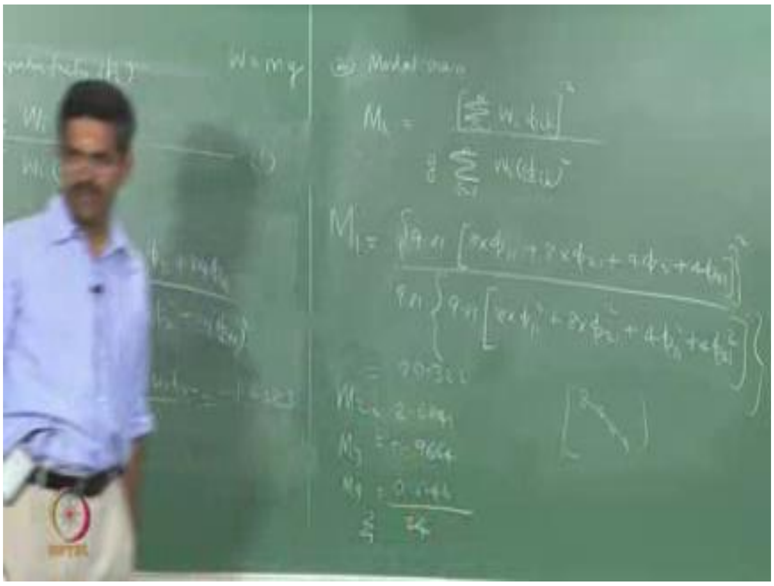

I can find the modal mass, because modal mass will now tell me an idea how much mass should I include. Modal mass m k that is the modal mass of the kth mode is given by i k square. These are w's therefore there is a g here. So, I quant to find m 1; let us try to find m 1, that is a first modal mass I am talking about mass w therefore, it is m g I have only mass matrix with me so I should say 9.81 into 8 or I can simply say 9.81 of 8 into the first mode I think write on the value which is I do not have the value with me so the first mode that is phi 11 plus 8 into phi 21 plus 4 into phi 31 plus 4 into phi 41 of square divided by further 9.81 of 8 into phi 11 square plus 8 into phi 21 square plus 4 into phi 31 square plus 4 into phi 41 square which gives me the modal mass in the first mode as.

Similarly, if you try to find out m 2 m 3 and m 4 I will write down the values for a convenience here, the summation of this it should be actually equal to 24 because the mass matrix are the values of 8 8 4 and 4 it should be equal to 24 actually. I think it should make up 24; 20 this is 23, 23.9 it should be 24.

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Let us try to find out the contribution of m 1 versus m total that is 20.322 by 24 in terms of percentage it comes to approximately 86 percent just check this. How much? 80.

Student: 84. (Refer Time: 28:27).

84.6 percent. Obviously, we can find out for all. So, what we want convey through this is, either you find the modal participation factor multiply this factor with the corresponding response in every mode get the total response or check the modal mass contribution in the total system and try to see up to what mass I must include. See remember this is about the mode truncation. This is inclusion of inertia force in respective modes, what we call as modal forces. Either you check this or you check this, but I want you show you both because from these numbers you actually could not make out what is the contribution of this. Except that you will know that some modes will contribute to the negative contribution to x.

So, one physically please understands that all the modes will not add up for the final response; some of them contribute positive, some of them contribute negative also is it clear. Therefore, they do not just add up linear super version does not mean always it is adding up there can be compromise also, because mode shape is actually plus and minus both right. But if look at this participation m 1 m 2 m 3 on the contrary of p 1 p 2 p 3 p 4 here you can easily make out up to which mode I must consider in this analysis. So, if you add the m 2 versus m total also you see that 11 percent. So, m 1 m 2 alone will contribute to close about 95 percent in this problem. Either you can use m 1 alone to compute x which will be equal to the total response or m 1 m 2 which amount more or less hundred percent of the system.

So, there is no need to work out all omegas and all phi's though we had in this problem we found out omegas and phi's, but it were not required to find out all omegas and phi's. Because you really appreciate to know m 1 I need to know only the first modes please understand that. To know m 2 I need to know only the second modes, I need not have to know the third and 4th modes. You can go step by step. First find out the first mode or using Stroud law or Alerts find out the first mode and frequency find out m 1, find out p 1.

If you are able to convenience 90 percent closer in the first mode itself stop there. If we are not go for the second mode by influence coefficient method iterate get orthogonal modes get the second mode and second frequency then get m 2 and p 2. So, you can proceed like this is. Is the working out all omegas and all phi's and then truncating them keep on truncating them parallel so that you can save lot of computation, is it clear that is a advantage here.

We will do one more problem is any doubt here is it clear, how we are employing the modal participation factor or mass or modal mass in the total response, how we are significantly understanding the physical meaning of this contribution in total x of t. Are we able to understand this? We will do one more problem and see how x can be calculated. Now we are only truncating the modes in terms of mass participation, now we will see how x can be calculated because ultimately I am interested in the response of the system.

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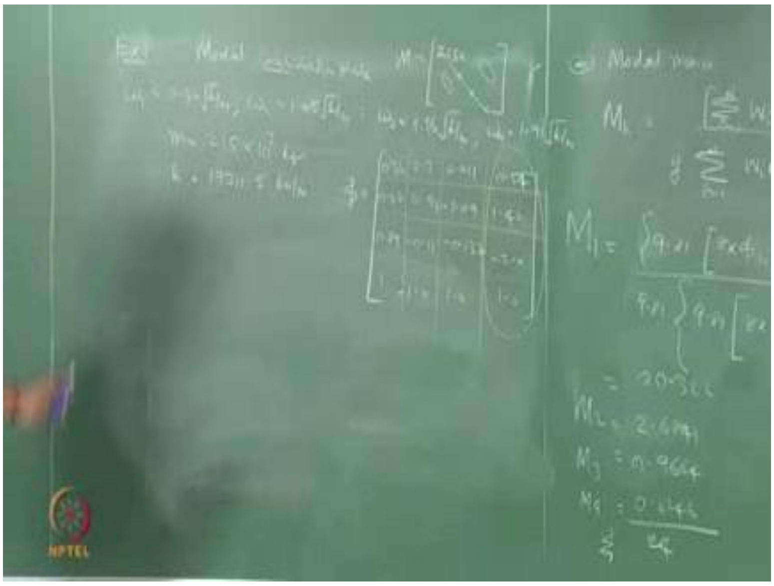

So, we will do example 2 where we will talk about over modal combination rule. Let say m is the mass matrix of 4 by 4 where the diagonal elements are all 215 tons all of them same and this is there and omega 1 square. So, m is nothing but 215 tons or 1000 kg and k kilometre per meter. So, I get so many frequencies, mode shapes will be corresponded to this matrix, I am not very sure about this particular mode shape f should be 3 0 crossing I am not sure remaining all are.

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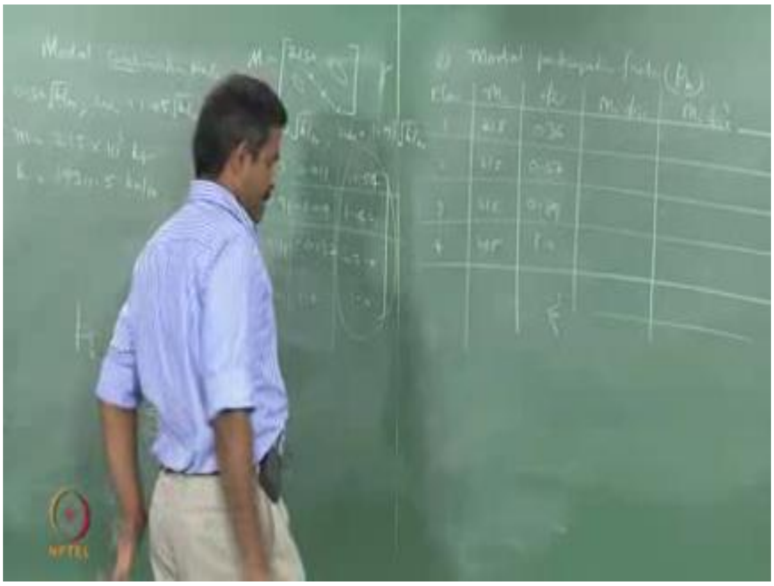

So, let us try to find out the modal participation factors, p k. Let us open a table this is easy to do it in tabular form. Mass point 1 2 3 and 4 let say m i phi i let say phi i 1. So, in this case 0.36, 0.67, 0.89 and 1.0 we need m i phi i 1 then I need this square of that to work out p k, because p k is given by the equation. Can you get me these values by multiplying simply and summation of this ratio of these two to give me my p 1; can you find out this what is p 1?

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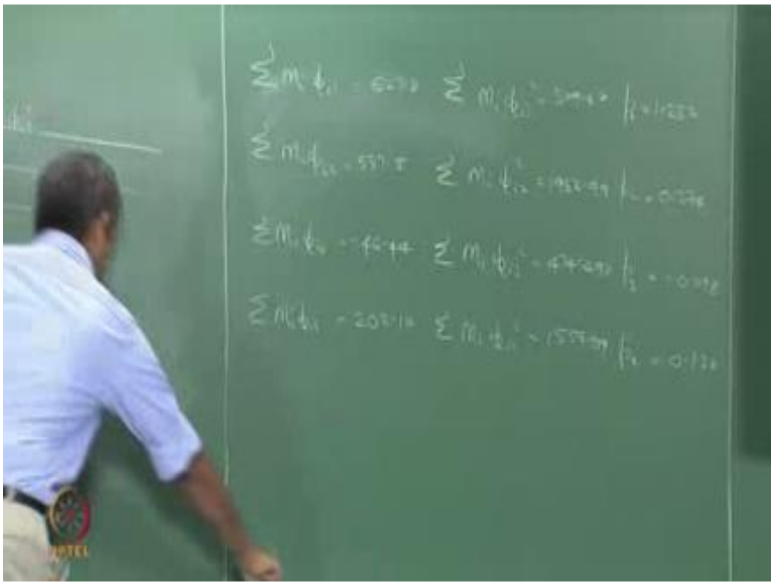

So, I should say sum of m i phi 1, sum of m i phi 1 square and therefore p 1. Sum of m i phi i 2, sum of m i phi i 2 square and therefore p 2. Sum of m i phi i 3, sum of m i phi i 3 square therefore p 3. Sum of m i phi i 4, sum of m i phi i 4 square therefore p 4. It is 627.8, am I right? 509.67. So, 1.232. Can you find out for the second mode? This is 537.5. All these numbers are getting indicative you must check them yourself; I may also make some numeric mistakes, 1962.99; 01274. Similarly, m i phi i 3 minus 46.44, on p 3 is negative 0.098. Are these numbers tallying or not? Once I know this the second step can compute the modal mass participation.

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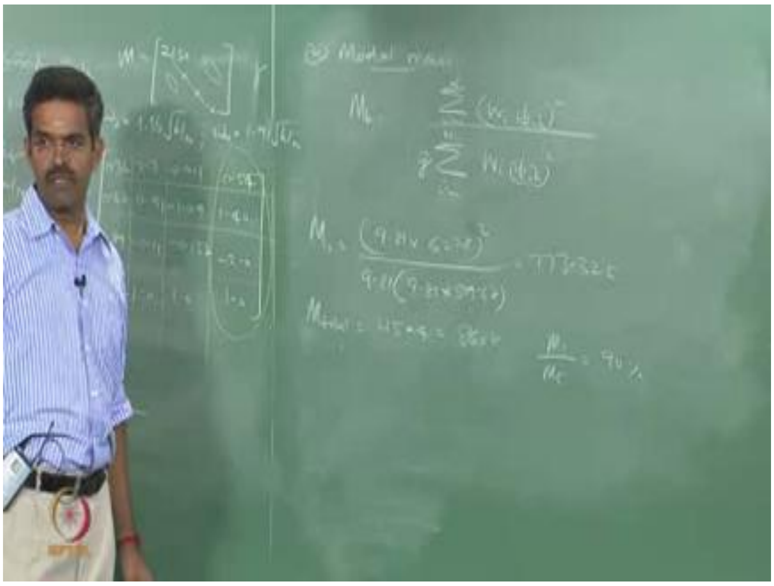

It is given by a general equation let us try to find out m 1 directly from this data. It is easy for us to know do it from here see how you are doing it, it is easy because I want that sum it is available here, which can be 9.81 of 627.8 divided by 9.81 into 9.81 of 509.67. It directly gives me the value of m 1. How much is this? 773.32 tons is that is it ok. Where the total mass is is 215 into 4 it is 860, is it not.

So, m 1 by m total is about 90 percent you see that, it is close to the around 90 percent. So, only first mode itself is sufficient for to compute x of t in this case. Now, let us see what are the different combination rules available in literature to compute x, because I know the modal participation factor. Now there are two things to understand from p f k.

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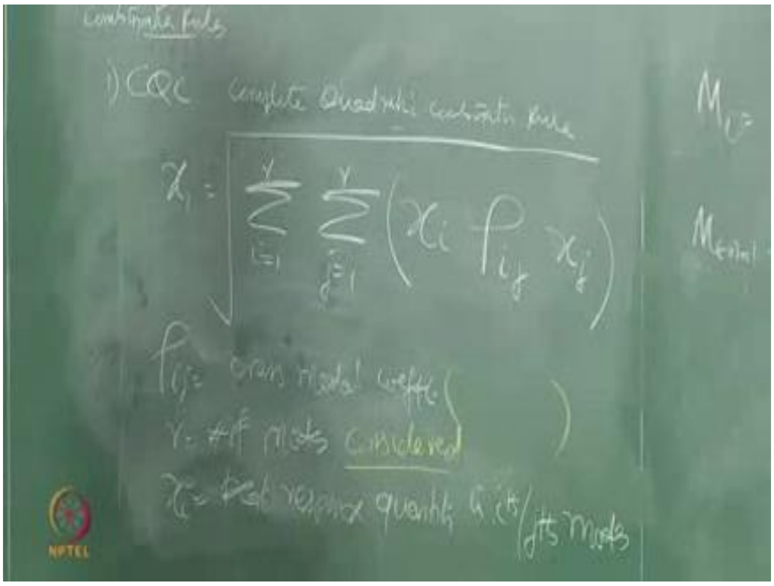

There are two things; one p f k significantly tells b in physical terms in that the modes not only contribute positive to the response with the auto counter produce response. If the mass moves to the right there may be possibility because of the vibration one of the mass can move to the left, therefore the total overall response may get compromised also. It gives me that meaning, so that is a physical interpretation.

The second interpretation is if I combine this with x on a specific mode etcetera I will get the contribution of x of that mode in the overall response that is why it is call modal participation factor. Whereas the modal mass indicates me how much mass should I include in my system. So, this number will tell me at what p k I must stop. This will give me idea about the truncation; this will give me an idea about participation of x in the final response. There are two different which both are important.

Once we know this there are different combination rules available to compute x, because now I have modes response in each mode; what is the response in first mode, what is the response in second mode, third mode, and etcetera. I also know what the contribution of that mode is in the overall response. So, I also know how many modes I should include in my system. Now I have three data’s independent. I have response in each modes separately, I have the contribution of each response in the total response separately and I have how many mode should I include in my response I have the data. Now, I want to combine these all the three information to find out the overall response I call this combination rule.

One combination rule is CQC rule; Complete Quadratic Combination rule. Interestingly more international course including IS 1893 talk about this kind of combination rules very well in advance. We have enough data available in the Indian standard course for finding out the response for any lateral system subjected to lateral loads. I am talking about earthquake engineering course which is IS 1893 which gives me the combination rule never the less for any lateral force this rule can be applied. It is not only related to any kind of force which being in earthquake process. Any lateral force the system vibrates, system develop frequency in mode shape I want to combine like I am use this force complete quadratic combination rule.

So, this says that if I want to find x 1 is square root of 2 summations I equals 1to r j equals 1 to r x i rho i j x j. Where, rho i j is called cross modal coefficient. This is actually an indirect verification of Maxwell Betti's reciprocal theorem which we discussed way back in some of the lecture here, phi 12 and phi 21 we talked about that. This is actually the cross modal coefficient you will see when you work out they will be equal. And of course, in this count r this is a number of modes considered; this will tell you how many modes we are considering for the analysis. And x i is the peak response quantity in ith and jth, because there are two x i and x j modes.

Now, there is very interesting question I want to ask you anyway I am not going to solve this I give you the value you can know the value phi a j i will give you the equation rho i j. So, you can solve this and try to find out the x value.

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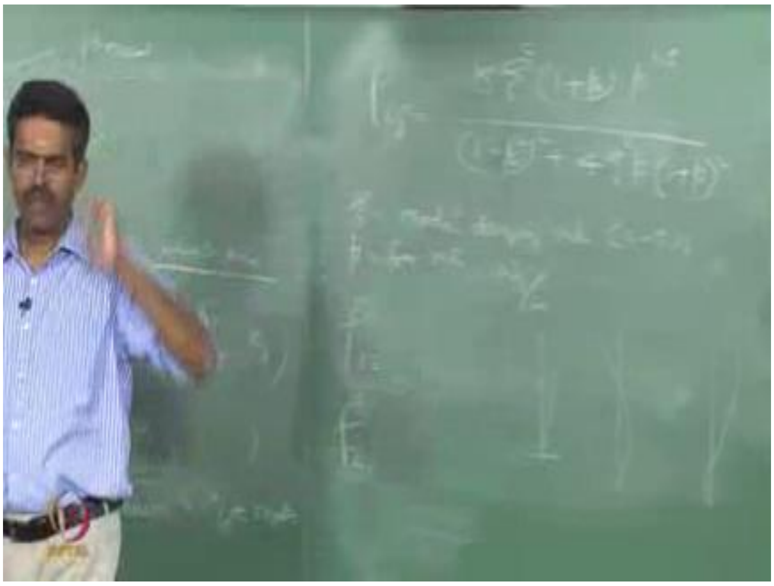

The cross correlation coefficient sorry, modal cross coefficient is given by 8 zeta square 1.5, I am sorry this is 1 minus. Let me check the equation first, where zeta is called modal damping ratio. Now, it is very important here for you to recollect in Rayleigh rates or in Rayleigh damping in the car you see the modal damping it should be equal for the uniform it is a classical damping. If it is non-classical they may not be equal in different modes. In this case we are seen my percentage may be 2 to 5 percent, we mean it is a classical damping.

Beta is the frequency ratio which is omega j by omega i. Where omega i is ith frequency and omega j is the j s frequency. So, if you really if you want to find rho 12 let us say i j, so beta is always j over i, I want to find rho 21 it is 1 over 21 and so on may be careful in substituting these values. So, once you get this number you know the response of peak value in each mode separately use this relationship to get the overall response of CQC you have got other rules also; sum of root of square SRSS rule and so on so forth there are many rules available. I would request you to urge to suggest I mean read a classical text book on the modal combination rules which will help you to find out the estimates or refer to my text book which I am dynamic analysis in offshore structures. We talked about that we also done an example on this. Please look at the example try to find out how you get the x 1 values.

So, it is very important to know for us that where do we imply modal truncation in terms of the rules applied here. This will give me how to estimate the number of modes required for the analysis once I fixed that number I will then find out the cross modal coefficient combination for the different modes response and try to find out the answer which is going to be the sum of the final response. On the other hand physically if I have let say two modes, where these are the two responses of the tip m 1 and m 2 tip m 1 and m 2 respectively this should give me hypothetically the final response of the system which we do not know actually depending upon the summation of the these two.

So, this is a physical interpretation of the responses in two different modes if they are widely spaced, if they are closely spaced there are other rules are available in the literature. Anyway we have no time to discuss that still I have got good references given to you please read them. And there are many research papers are available in checking which method is applicable to what kind of closely spaced modes that is an intrinsic research people have done in this. So, modal combination rule itself has undergone a very intrinsic research in earthquake engineering or in structural engineering. Please read those papers which I referred in the NPTEL website, so you will know how, what is the weightage of CQC? CQC is one of the well applicable combination rule applied to estimate the final response.

So, our job is to estimate the final response. Now what we are confused is will this response we contributed from all the modes or only few modes. Now, we answered in this in the past two lectures that how we can truncate the modes and how can we get the combination. Now we terminate the lectures on module one now. We have in module one you spoke completely about dynamics of offshore structures we started the introduction to floating I mean structural systems in offshore platforms, how form based design dominates, and how the platform geometric evolved from the fixed type from the shallow waters to that of d potter platform and now we are talk about ultra deep water platforms. When I talk about conceptualization of platform structural design we generally not have the data about the frequency and the response behaved in the system, therefore we need to know the mathematical models of finding out the frequency and the mode shape of this. We started by single degree we attempted to solve in the multi degree. We know there are different methods are available to write equations of motion. We wrote equations of motion derived stiffness matrices. We also understood when the mass matrix will become diagonal, to make it diagonal how to select the degrees of freedom. If they are not diagonal what is implementation of that final equation of motion. How to get stiffness matrix? How to get flexibility matrix without inverting them? And we also saw how many modes we must consider, how to estimate all the modes and frequencies by different methods which is computationally efficient and you will give all the methods will give you the same answer more or less.

So, we found out them and ultimately we said that yes the modal modulation rules can suffice me to find out the response in a given system for. So, the next module will be address on fluid structure and wave structure attraction in detail. We will pick up about 6 7 example platforms starting from the jacket structure, articulated towers, multi like hinged platforms, triceratops, TLP's, FSRU’s and FB source and spar. We will discuss them in detailed we will derive the mass and stiffness matrix and damping matrices here. We will plug them in a software run the analysis and show you the results and interpret the results from the research paper directly. So, when we read a paper you know what do they what modal of dynamic modal as they followed I will show you online how it can be done here so that the second module will focus on that completely which is dedicated to FSI; Fluid Structural Interaction.

Once we understand the basic dynamics and application of design in dynamics and FSI applications on dynamics, then we move on to stochastic dynamics and see how we can estimate fatigue damages in a given structure using stochastic dynamics models. Which in advance method of doing dynamics analysis offshore structures, when we talk about non-linearity in dynamic analysis is to part and we will stop there. That is the idea what we have. I think now we are running about 28 lectures here I will have another 22 or 24 lectures more.

So, we will close around 52, 54 lectures in total to complete the entire module, so the registration for examination is open. Generally if you look at the application of the dynamics in structures people generally discuss maximum part of the application here. I take diversion from here in earthquake engineering, wind engineering, etcetera we will also divert from here take our understanding of dynamics to ocean structures from the next class onwards.

So, it is important that we must brush up these understandings. There are many text books are available. Most of the text books focus on dynamic analysis and design in basics, but anyway my text book addresses the design concepts also parallel. And I have given a lot of applications examples in the text books. The exam registration is open it will open till 31st March for the people in India and then and people in Abroad open from 1st April onwards. The exam will be likely on 10th and 17th of May which will be contesting about to 2500 to 3000 people in international board.

So, we will have golden silver and bronze certificates qualified by and certified by IIT for qualifying this examination. So, I urge that you must register for the examination so that you get the benefit of this complete course in terms of it is credits stands worth.

Thank you.