Dynamics of Ocean Structures
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Lecture – 26
Duhamel's Integral

So, we will talk about the Duhamel's integral in this lecture, which is lecture number 26 on module one on NPTEL online course on dynamics of ocean structures.

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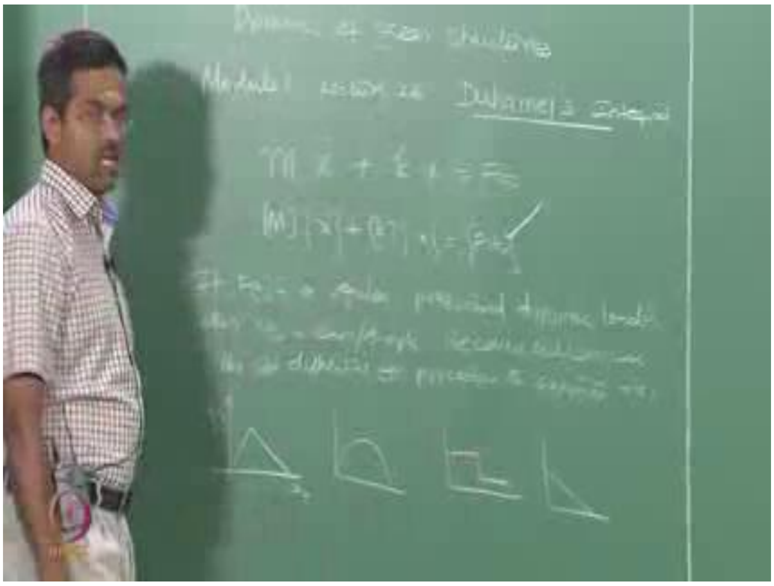

We already know for a single degree or multi degree freedom system; we can simply write the equation of motion for an undamped system with or without forced vibration like this. Of course, in multi degree, this will become a matrix and this will become a vector, which can be of any size of n-th order; where, n is the degree of freedom of the given system. We already know the essential dynamic characteristics of a given system; it is actually to find out the frequency and mode shape of the system. So, mode shape will give you an idea where the mass is getting displaced relatively for a given frequency of vibration under a given force of free vibration. Give an idea whether the displaced portion of mass is effective to compute for the participation of the ultimate displacement of a given system. We know that inertia force is dominated in the given system.

Therefore, we always worry about the mass displaced portion what is the mode shape on a given system.

Now, when we talk about the focus on f of t; if f of t is a regular – if f of t is a regular prescribed dynamic loading that, estimating the x of t is simple. Then x of t estimate is easy, rather it is simple because one can use the standard differential equation procedure to estimate x of t. But, all the time, f of t may not be a prescribed dynamic loading. Now, what is meant by a prescribed loading? For example, if the loading history is well defined and so on, so forth. If the loading history is fully defined; then, we call this as a prescribed loading. In that case, it is easy for us to estimate x of t using the standard differential equation procedure, because we can easily find out the complementary function from the given problem and the particular integral for a given f of t and so on, so forth. We can divide this into two parts and easily find the first part of the problem separately, second part of the answer separately, which we did an example in the last few lectures ahead and we explained this.

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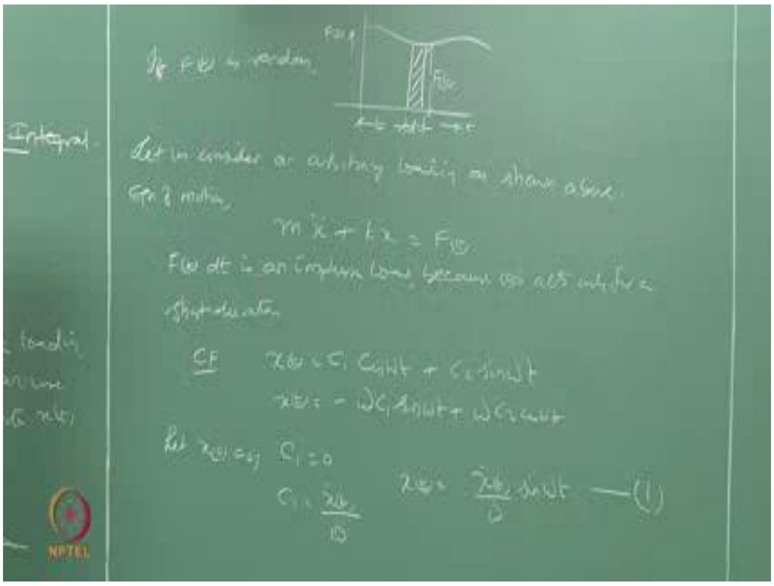

But, if it is a random loading; if f of t is random; for example, I can write f of t like this. That is not prescribed loading; then how do you find actually the response of the given system, which is difficult? In such situations, people try to use an integration procedure given by Duhamel. Let us see how this works out. So, let us consider an arbitrary loading as shown above.

Let us say at time t for small impulse duration of dt, and this value becomes f of t. So, it is an arbitrary loading, which is explained in the figure as above. Equation of motion preferably for a single degree is know to me as like this; f of t into dt is an impulse load because it acts only for a small duration. And, the complementary function of this particular problem – we know is given by let us say c 1 cos omega t plus c 2 sin omega t and x dot of t is minus omega c 1 sin omega t plus omega c 2 cos omega t. Let x 0 be 0; that is initial condition; which makes c 1 as 0 and c 2 will be x dot of t by omega. Hence, x of t will be x dot of t by omega sin omega t. That is the equation number 1.

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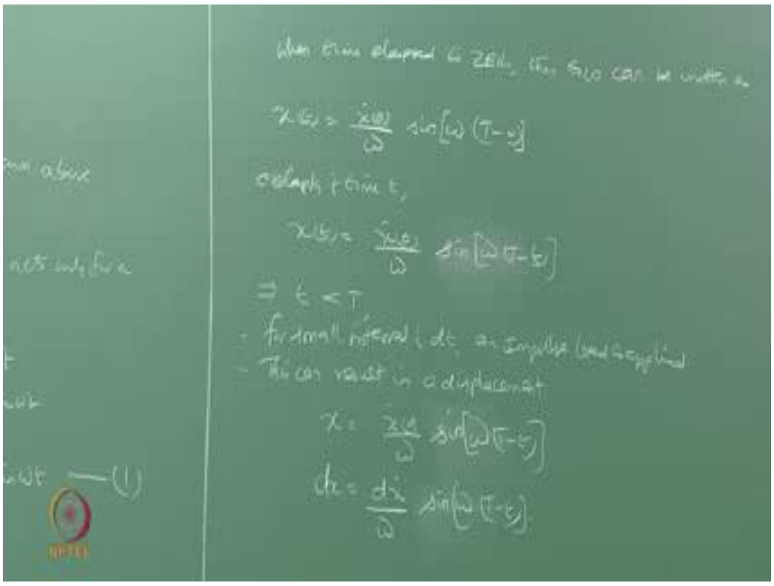

Now, this equation when time elapsed is 0; then, equation 1 can be written as – time elapsed is 0 means let us say apply this equation at time t is equal to 0. There is no time elapsed happened in this system let us say here. So, when time elapsed is 0; then, the equation 1 can be written as x of t is x dot of t by omega sin omega t minus 0; where, I can even correct this as x naught 0. Then, let us also correct this as x dot 0 – x dot 0. At elapsing of time t, x of t can be – this implies that, t is less than T; that is, this equation is valid till up to a specific capital time T beyond which this equation is not valid. So, what

I am trying to do is I am trying to convert these continuous to discrete parts, where this result can be applied on specific intervals of t.

So, this implies that t is – smaller t is less than capital T. This equation is valid. For a small interval of dt, an impulse load is applied. For a small interval of dt, an impulse load is applied; spike is applied to the given system. This can cause in a displacement. Let that displacement be x, which is x dot zero by omega sin T minus t, now, in this given displacement, I am looking for an incremental displacement because of incremental time dt. The time dt is incremental. So, I am looking for an incremental displacement. So, I say dx. Call this equation 2.

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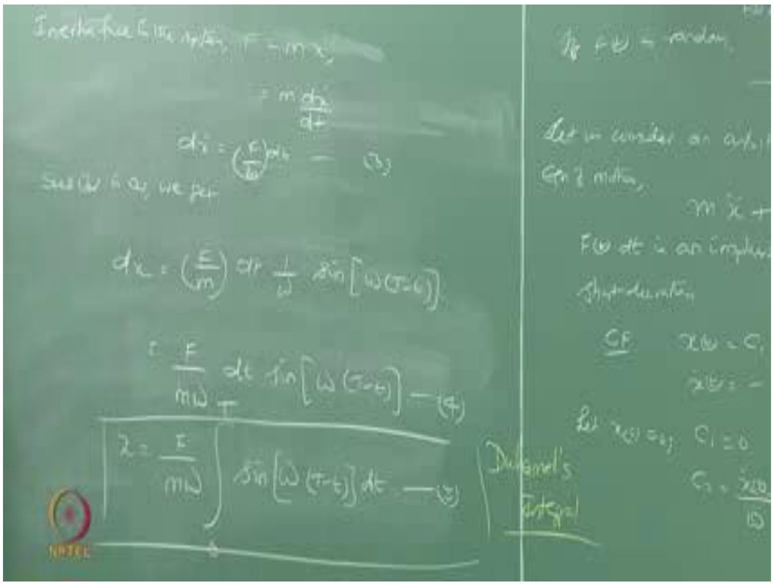

Now, we know the inertia force in the system is mass into acceleration, which can be mass into dx by dt because dx dot is the velocity; dx dot by dt will be the acceleration. So, dx dot can be f dt by m or let us say f by m of dt – equation 3. Substituting 3 in 2, we get dx is equal to f by m dt 1 by omega sin omega T minus t; So, f by m omega dt sin omega T minus t – equation 4. So, if we really want to find the total response, I must integrate this for the whole segment of 0 to T. This is what we call as Duhamel's integral.

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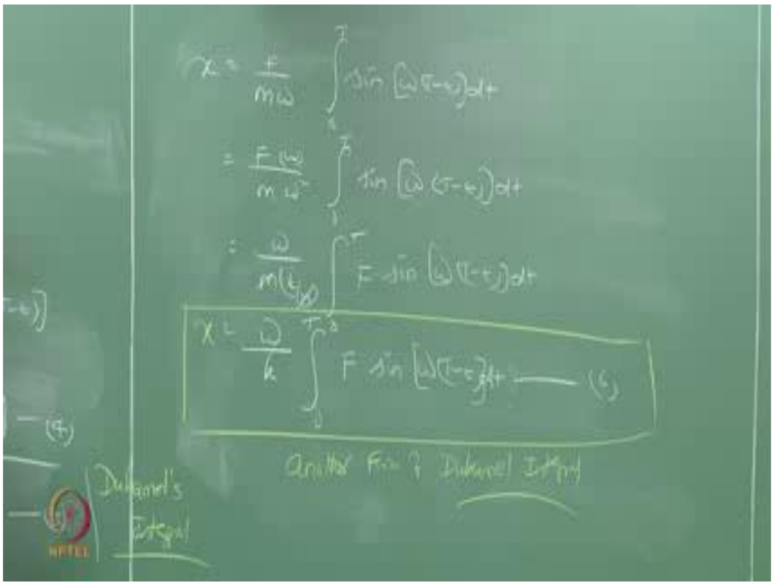

We can also write this integral in other way – f by m by omega 0 to T sin omega T minus t dt f omega by m omega square 0 to T, which can be omega by omega by m k by m. This is also another form of Duhamel's integral. Now, for a given prescribed loading, it can be easily obtained – the response can be easily calculated using this integral directly. I will show you an example now how this can be used directly to compute the response. We will take up prescribed loading examples, which demonstrate easily. Therefore, we can apply this function to find out the response for any random loading also. So, we will take an example now. This is called Duhamel's integral. Both are different forms of Duhamel's integral. This is just a modification of the existing one. So, what you actually want in this case is you must know what is the defined value of f of t. If you have f of t, you can easily find. And, what is the period of application of f of t? If you know that, you can easily integrate it for different parts and try to compute and find the response of the given system.

Now, we will take an example and show how Duhamel's integral it can be directly applied to a problem, which is of course a prescribed loading. But, we will try to apply a Duhamel's integral for this. We have already solved a similar problem in earlier lectures of this module. But, we will again resolve this slightly in a different manner.

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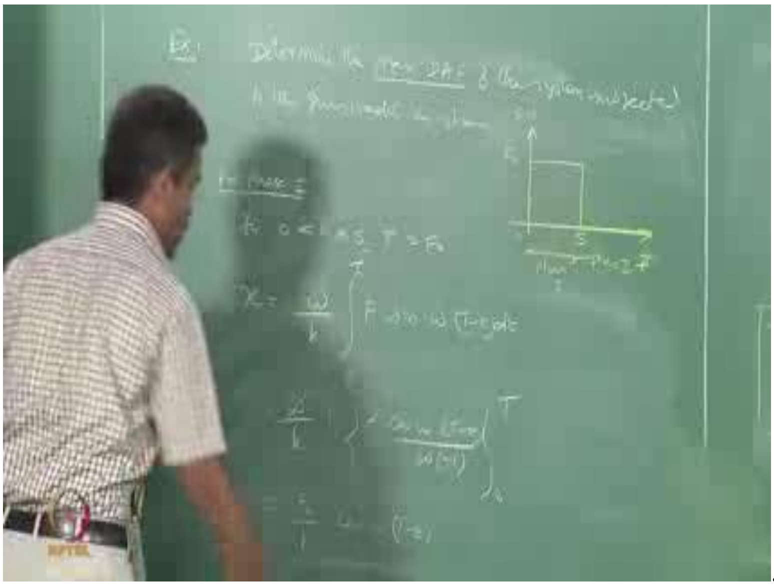

We will have an example now. The example problem shows – determine the maximum dynamic amplification factor of the system subjected to the given loading as shown. The loading is this, which is of course, a prescribed loading, which is easy to obtain, but I am taking this to demonstrate Duhamel's integral, which is plotted for F of t against t – capital T. The amplitude is F naught. We have already done this problem earlier. This varies from 0, s and T. So, this is phase 1; this is phase 2. We have already done this problem. This is phase 2. So, we want to find the maximum dynamic amplification factor for the given problem. So, we are interested in finding out the DAF value – maximizing DAF value. So, let us say for phase 1, for 0 less than s – less than t less than s, F is equal to F naught. And, to find x, we already know it is omega by k integral 0 to T F sin omega T minus t dt. So, integrate this omega by k F minus omega cos T minus t divided by omega of minus 1 0 to T. So, this omega omega goes away; and, this minus minus goes away; i get F 0 by k of F 0 by k cos omega T minus t applied to 0 to T.

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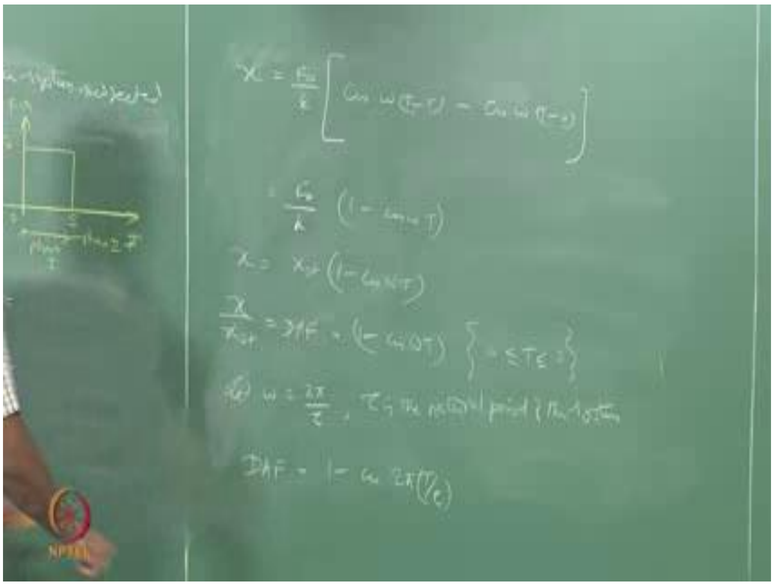

Which will now become cos omega T minus t minus cos omega T minus t, which will become F zero by k – 1 minus cos omega T, F 0 by k is x static. So, x by x static is DAF. This is true for 0 less than T less than s. That is validity for this T. Let omega be 2 pi by tau; where, tau is the natural period of the system. Therefore, DAF will be now equal to 1 minus cos omega – 2 pi T by tau. I want to maximize DAF, because I am interested in finding out the maximum dynamic amplification factor. So, let us take away this.

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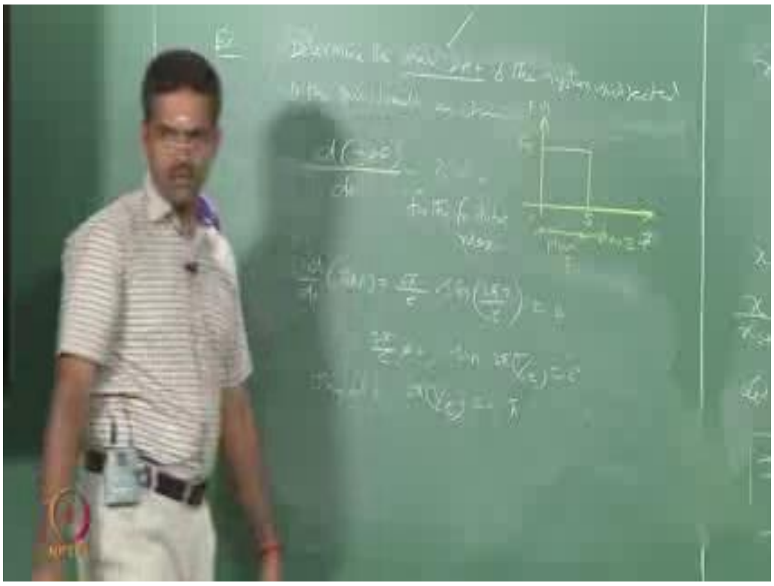

So, to maximize any function I must differentiate the function with respect to time. Let us find out this. So, this is my DAF. Now, differentiate this function with respect to time and equate that to 0 for the function to be the maximum. So, if we do that, d by dt of DAF will now become 2 pi by tau sin 2 pi by tau. Set that to 0 or we see 2 pi by tau cannot be 0 because omega is not 0. If omega is 0, there is no vibration. Since 2 pi by tau cannot be 0, sin of 2 pi by T of tau is set to 0. This is true only when – this is true only for 2 pi by – 2 pi T by tau is varying as 0, pi, etcetera. So, we cannot take 0, because it has to start at specific vibration value. Therefore, take it as pi.

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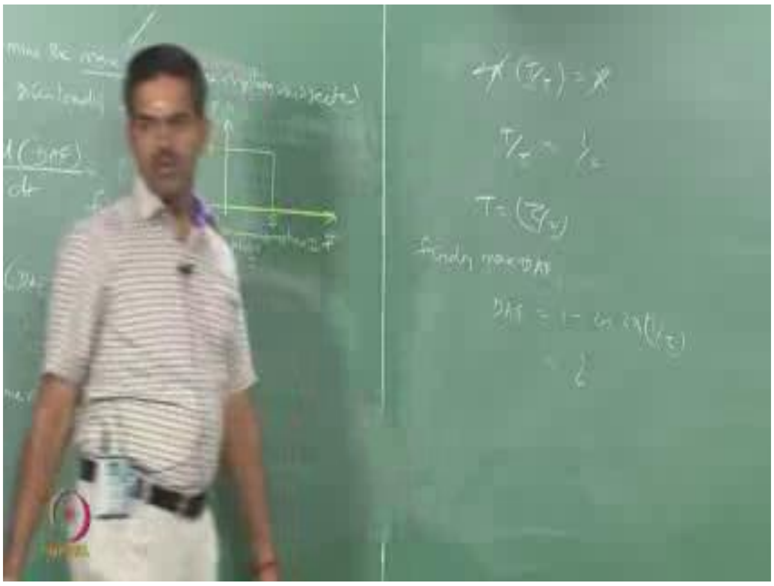

So, T is tau by 2. So, when you are looking at a time, which is equal to half of the natural period of the system, you get the maximum DAF for the given loading. That is what the physical interpretation of this is. We can check this. And, find or finding maximum DAF. You have the equation for DAF already though I rubbed it. You have the equation for DAF. Substitute T is equal to tau by 2 and find the maximum DAF. So, D is 1 minus cos 2 pi by tau. So, T is tau by 2, which will give you the value as 2. So, it means the excitation is getting doubled at a time, which is equal to half of the natural period of the given system for phase 1. So, for a prescribed loading, if we really wanted to find out the response, we can easily use Duhamel's integral straight away to find out the response like this.

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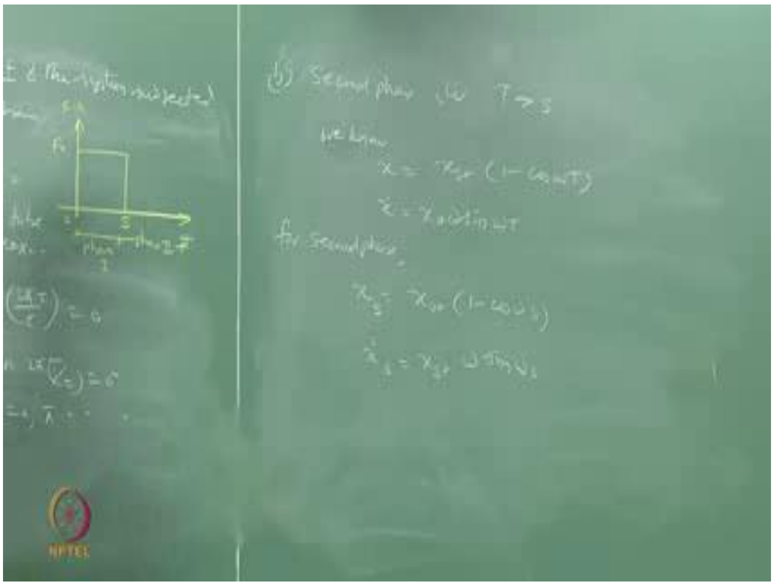

So, the second phase b – second phase, that is, t is between s and T. Or, for any time or you can even say for any time T greater than this – for any time T greater than this, is true because function is now getting repeated. So, any time greater than T if greater that s, this function should be true. So, we already know x is given as we know x is x static 1 minus cos omega t; we already found out. So, one can also find x naught. Now, for phase 2 or for second phase, x will be at x s, which will be x static of 1 minus cos omega s, because instead of t, I am substituting s because at the end of the first phase, s is equal to t. Similarly, x dot s is x static omega sin omega s.

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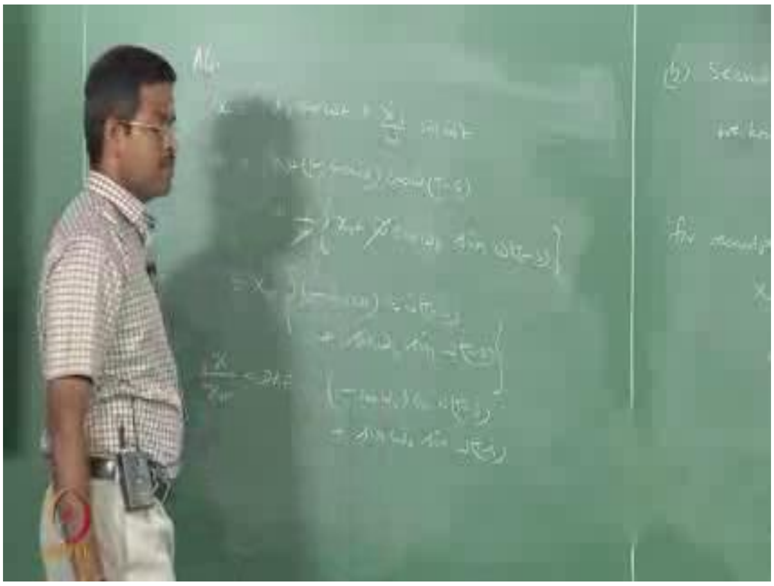

Also, the general response x was given by x s cos omega t plus x dot s by omega sin omega t. That was general equation for finding response. So, now, I have x s and x naught s here; substitute back here - x s t 1 minus cos omega s cos omega T minus s because this period of t is beyond s; up to s this is valid – plus 1 by omega x s t omega sin omega s of sin omega T minus s. This can be now x s t of 1 minus cos omega s cos omega T minus s plus sin omega s sin omega T minus s. So, x by x t, which is DAF is simply 1 minus cos omega s of cos omega T minus s plus sin omega s of sin omega T minus s.

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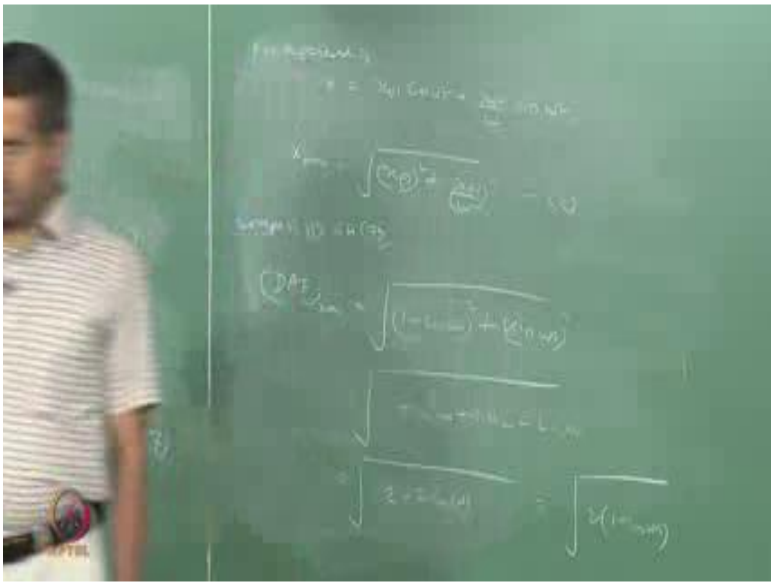

So, for the general equation of x is equal to x 0 cos omega t plus x dot 0 by omega sin omega t, x max is root of squares of the amplitudes – x 0 square plus x dot 0 by omega whole square. That is how you found out actually the maximum value of x. So, therefore, comparing I will call this equation number – what could be the number here? 7. So, I call this is 8. Now, comparing 8 with 7, I can say DAF max can be square root of 1 minus cos omega square plus sin omega s whole square expand. So, you can simplify this trigonometric relationship.

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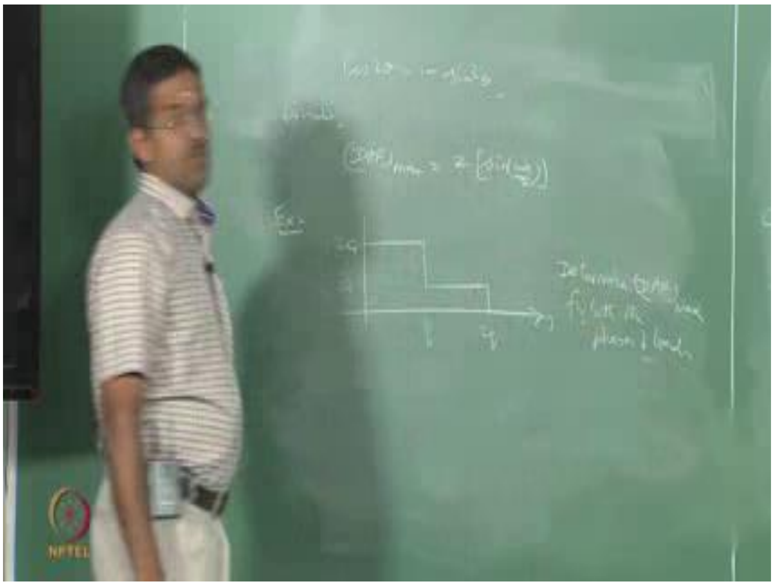

Cos 2 theta is 1 minus sin square theta. So, I have values here in terms of – so, I can say 1 minus cos square – sin square theta root. So, I can find this and tell me what is the DAF max you get. So, I get DAF max by substituting DAF max 2 sin omega s by 2. Is that okay? So, in this problem, we have applied Duhamel's integral for a prescribed (Refer Time: 28:22) of course for this given problem to estimate the value of maximum amplitude factor for a given prescribed loading. You can easily use Duhamel's integral. We have done a similar problem earlier, but I divided the problem into two parts; I use a conventional value to find out x. Then, we added. In this case, we found out separately, because I wanted to find out DAF max in both these cases separately. So, I can use Duhamel's integral. It is a direct application for this problem. So, you can try this example and see where we are able to do this.

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Now, friends this book available on online now by Springer. It is talking about dynamics analysis and design of offshore structures, which is authored by me. The interesting part of this I want to show you. If you look at the same typical content of one of the chapters, which talks about one of the forces; whatever we discussed in the class in terms of different kinds of loads, wind force, the expression for aerodynamic admittance function, the dynamic admittance function for computing aerodynamic loading, different spectra for calculating wind forces with their expressions.

Then, wave spectra, regular and irregular waves; different modification is given by (Refer Time: 30:42) Chakrabarti, etcetera; different wave theories, application of different wave theories suggested by Sarpakaya and Issacson; different spectra used in offshore structures – Johnswap, ISSC spectra, modified single peak spectra, modified equation of Morrison's theory, then, computing the maximum forces on a given member; then, applicable to different wave theories for different c d c m range.

The current forces, earthquake loads and applications and dynamic (Refer Time: 31:20) variation taken from research paper of myself and Gaurav. Then, Kanai-Tajimi ground escalation spectrum used for calculating the earthquake force on TLPS; ice and snow load – their (Refer Time: 31:33) spectra, ice force spectrum; then, marine growth, damping estimates, dead load, live loads, impact load factor, general design requirements for steel structures, allowable stress method, limit state method. Then, approximate guidelines for jacket piles in terms of bottom founded structures for different kinds of load combination, design criteria, leg size and jacket bracings, deck framing size, fabrication loads as suggested by DNV, lifting forces using erection and construction, load-out forces, transport forces in terms of numerical value; then, different kinds of motion of floating objects during installation, then lifting and launching and appending of jacket stresses, accident loads, are discussed in detail.

And of course, there is a very good exercise given at the end of each chapter as you see here. There were about answers given for about 50-60 questions, numerically solved. And, all explanations given in simple terms is available for all the questions including the discussions for wave theories, etcetera. If we look at the table of contents of this book; so, we talk about introduction to offshore platforms including the new generation platforms. So, BLS, triceratops and FSRUs; different environmental forces, which just now you saw; introduction to structural dynamics, where we are somewhere here now – Stodola’s and mode truncation; we will be touching this shortly in the next lecture – static correction. Damping estimates – we have already done; Rayleigh's, Caughey, etcetera. We will talk about FSI – fuel structure interaction in the next module in detail.

Then, I will talk about stochastic dynamics in the third module in detail. All will be given in one stretch with application to preliminary analysis and design starting from design of triceratops, free-floating experimental studies on tethered triceratops, springing and ringing responses, evaluation of platform geometry, analytical model of TLP, damping of triangular TLP, ringing and springing responses experimentally, analytically and numerically. All given in single book; where, the advantage is you can always purchase this book chapter-wise also. So, if you want to only purchase one specific chapter, you can purchase that. So, I request that you please follow this as an important textbook for this course. Shows restrict for the examination for this online course.

So, this book is available on stands. This 119 Euros is actually a price, which is world wide. Of course, VAT will be applicable separately for India. So, the price may be around I think 90 Euros you can talk to Springer or write to them. We have requested for a sample copy of this for the library of IIT madras; but, still I want that you should try to access this because this has got about 64 color illustrations, which explains the form evolved to design of offshore platforms in its geometrical terms. So, this is a very interesting book, which combines dynamical analysis and design; which talks introduction to structural dynamics as we discussed so far. With this application of offshore structures, where we derived the stiffness mass and damping matrix for about six type of structures in detail and proved them and validated them experimentally, analytically and numerically with the quotes given in the book. So, this is a very interesting and single combination of the entire subject in one shot, which is now available in the stand. So, if you are interested, please purchase or recommend your organisation to purchase this book.

So, any questions, please post it to Springer. As such there is no – I think concession available on this price. But, this book will be very useful, because about 200 examples are solved in this book, which are useful for the examination online course on this particular subject.

So, in the next class, we will talk about the modal correction and modal response, truncation factors for multi degree of freedom system. And, that will be the last lecture in the first module. Then, once we complete this, we will move on to fluid structure interaction, where we will talk about application of dynamics and ocean structures in detail. Then, once that is understood, we will move on to the advanced topics in the structural dynamics, where you talk about stochastic dynamics, fatigue and fracture failures and reliability theories applicable to structural dynamics in detail with some solved examples. So, any questions, please post it to NPTEL.

Thank you.