Dynamics of Ocean Structures
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Lecture – 24
Caughey Damping

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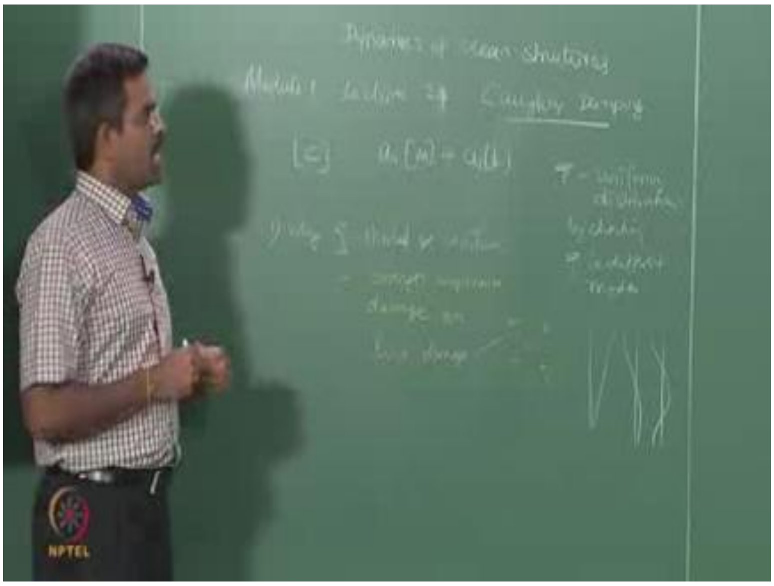

So, in the last lecture we discussed about the Rayleigh damping, where the model was very easy to evaluate a damping matrix, which is proportion of m, and some proportion of k matrix. So, for a classical damping to be done for a structural problem, or a model, is essential that the damping ratio should be uniformly distributed for the entire structure.

We ensure this, the uniform distribution of damping, by checking damping in different modes. So, that can be couple of interesting questions on this; one why damping should be uniform throughout the structure. Now the answer is very simple, if you do not allow damping. Let us understand damping is dissipation of energy, whenever energy dissipated there is always a loss, essentially the loss will be strength compromise, and it can also be a damage etcetera. Suppose if we do not allow damping uniformly for the entire structure, which is a prerequisite of a classical damping, you are unnecessary promoting the local damage.

If the damping in the first deg mode is very high, compared to that of the second and third mode, we all know that the mode shape will have an indication of relative position of mass points in a given system, under lateral forces. So, in the first mode all responses or all mass portions has displaced from the equivalent position, either to the positive or to the negative ,either to the right or to the left of equivalent position.

So, if the damage is maximum in this first mode, the mass which is displaced by the maximum amount, will invoke the maximum damage, the mass displaced by a lesser amount will invoke the minimum damage. So, generally the tip displacements in a cantilever stack like this, if invokes the maximum damage, could cause instability to the entire structural system. On the other hand if we are able to impose uniform damping, there may be a compromise of this mass position, when you super impose all the three modes, because this is to the positive, this is to the negative. This may be also to the negative you will be not known. There may be some compromise on one or compensation of two on the other. So, the net effect of damage indices or strength compromise, when you superimpose all the modes with equivalent amount of damage or damping may save the structure.

So, you must ensure that damping should more or less be uniform which is also a prerequisite of a classical damping. So, mass proportional damping alone, stiffness proportional damping alone was not established in this factor, because experimental studies show that both of them, either capacity do represent the actual damping phenomena. Therefore, Rayleigh combined them and made a suggested a model where a proportion of mass matrix and a proportion of stiffness proportional matrix a damping is taken, and c can be worked out and we also demonstrated in an example in the last lecture, that how zeta can be distributed, and can be seen that whatever modes you consider for analysis; not necessarily all the n modes, you may consider j modes, where j is less than n. So, within that given j modes we ensure that the damping ratio is proportionally equal, not exactly proportionally equal to equivalent all the modes.

So, having understood that in the last lecture, that Rayleigh's model is applicable very much for mass and stiffness proportional, just remember mass proportional is inertia dominant, stiffness proportional re centering model; one is a design aspect, one is the payload or the feed aspect, both are important for offshore structures. We are going for a new generation of forms for depotron and ultra depotron explorations, where we do not and we do not insist on having a massive system, because installation and commissioning decommissioning will be expensive, whereas, we also want a flexible system, because I want to take the band of the frequencies or the time periods of the structure, in two distinct bands.

So, that the expedition frequencies do not fall on this band, so I avoid the resonating response. And we are making the structure flexible even at resonance I can always exit the resonance band easily, because there may be a damage adapted to the structure or cause to the structural member. If we exit the damage mode by the exit the resonance mode and damage can be minimized. So, one is interested in any way having a flexible system, as well as mass system at the minimum payload required for drilling and exploration. So, both the models combined together as a recommended value by Rayleigh which is very good agreement with the experimental studies which can be applied to offshore structures.

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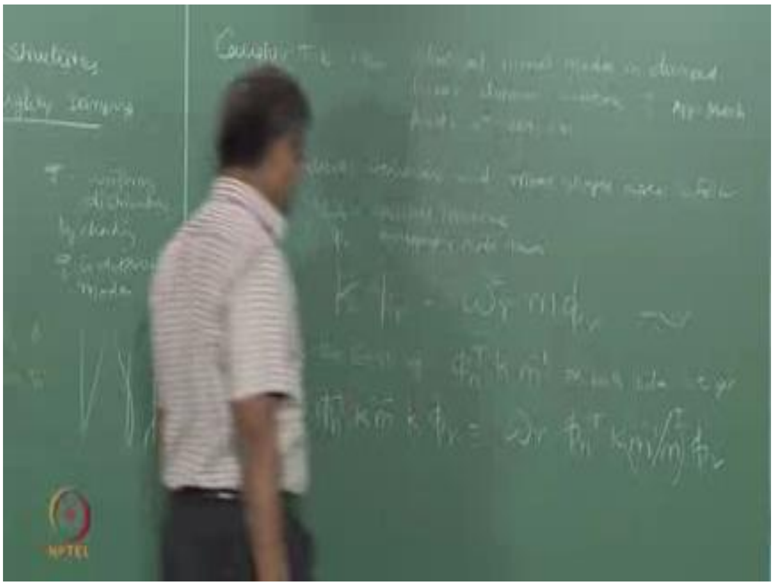

Now, the added advantage on Rayleigh model, Caughey developed a model in 1960. So, he developed a model in 1960, which is an extend study of Rayleigh. So, today in this lecture I will talk about how an improvement was made on Rayleigh damping, and how the damping ratio can be ensured using a Caughey's model. Most importantly estimating of the damping matrix in a given dynamic analysis for a structural system, where the new generation forms are involved, if I got a classical form of geometry; for example, a rectangular rectilinear structure, curvilinear still shape etcetera, where the geometric forms are more or less free defined. There is absolutely no difficulty in using the classical damping model, because there is always a mass uniform, distribution of mass for the entire structural system, because that is how the system is generally evolved. Whereas an offshore structures the forms are very novel, where as we looking for the form dominated design, where we want to have a flexible system, where can be commissioned decommissioned easily, payload cannot be compromised.

So, it is actually a hybrid combination where, the feed dominated form based designs are invoked, for depotron and ultra depotron system. Therefore, one cannot look for a classical method of 2 z to omega n, where damping can be more or less uniformly distributed. We must ensure that by forming a damping matrix, we may show that it is having a uniform distribution of damping in almost all the considered modes of analysis, not all the modes present in this system.

So, considering this, let us say, let the natural frequencies and mode shapes agree as follows, omega r or the natural frequencies of, and phi r are the corresponding modes. So, we know k phi r can be also written as, because we know k is omega square n, let us call this equation number one. Now pre multiply this equation phi n transpose k m inverse on both sides. So, we get let say phi n transpose k m inverse k phi r omega square r phi n transpose k m inverse m of phi r, and this is identity, which can be said to for n not equal to r orthogonality. So, this implies phi n transpose k m inverse phi k phi r is set to zero, equation two. Let us do it like this, for n not equals r. Pre multiply equation two again phi n transpose k m inverse square. So, initially it was to the power one.

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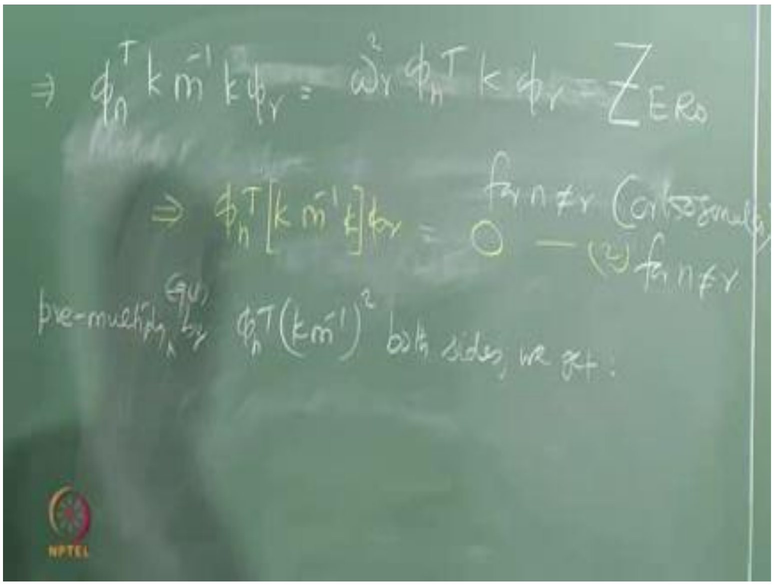

Now, I am doing it for the power two both sides, we get pre multiplying equation one, this equation.

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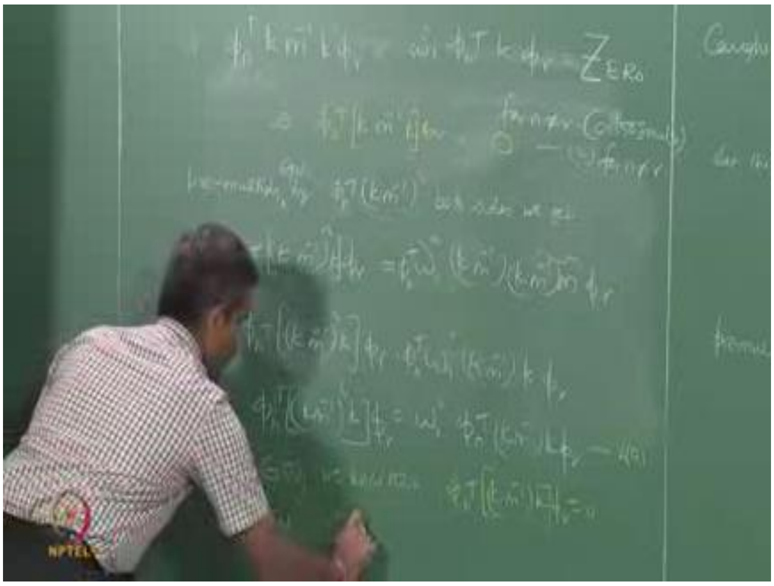

So, for m transpose k m inverse square k phi r equal to omega square r k m inverse k m inverse m phi r. Let us separate this for multiplication sake, this becomes identity;

therefore, phi n transpose k m inverse square k m inverse square k of phi r is omega r square k m inverse k phi r phi n transpose. You have to multiply phi n transpose of this. So, phi n transposes omega r square and so. This can be also written as omega r square phi n transpose k m inverse k phi r which is equal to. Let say phi n transpose k m inverse square k of phi r. Now we know that, I call equation number two a from equation two, we know that, equation two says that phi n transpose k m inverse k phi m transpose m inverse k phi r and phi n transpose is zero actually, phi n transpose k m inverse k of phi r is zero.

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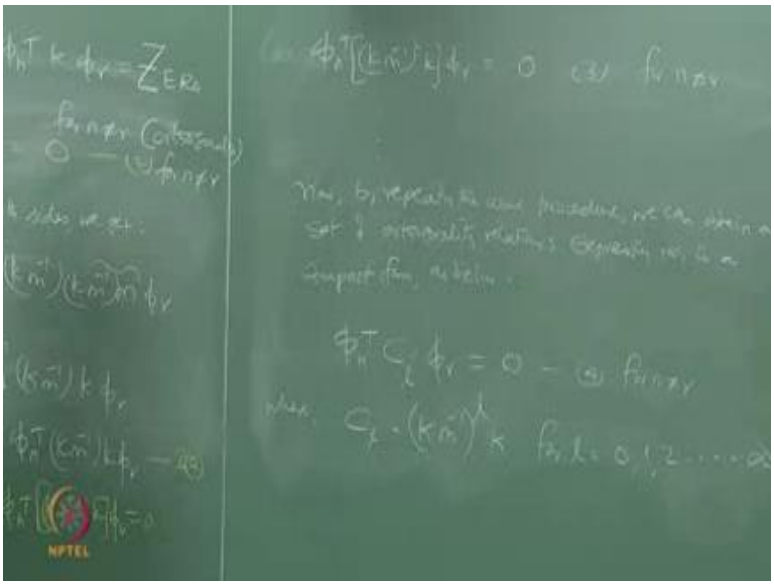

Substituting in equation two we get phi n transpose k m inverse square k phi r is actually zero, for of course, n not equal to r. So, from the given equation of k phi r equal to omega square m phi r, I keep on pre multiplying with the first power of phi n transpose k inverse to the second power of phi n transpose k phi inverse etcetera. We keep on getting a sequence of variations, which I can generalize. So, keep on doing this now, I can generalize this by repeating of a procedure, we can obtain a set of orthogonal, orthogonality relations, expressing this in a compact form as below.

So, what we are trying to do is, we are trying to repeat this set phi n transpose k m inverse k phi r zero, I will keep on getting for every multiplier of phi n transpose k m inverse 1phi n transpose k inverse 2 n etcetera. So, now I can say phi n transpose c l phi r is subscript, let me write it like this c l phi r is zero, where of course, this equation is valid, for n naught equals r. We are talking about orthogonality property if there equal to become one, where c l can be k m inverse to the power l of k for l equals 0 1 2 etcetera till infinity. Now, one can ask me a question how this valid for zero, if I put zero you will get the equation number one back again. The original where we are started you get the equation back again. Let us rewrite c l slightly in a different manner.

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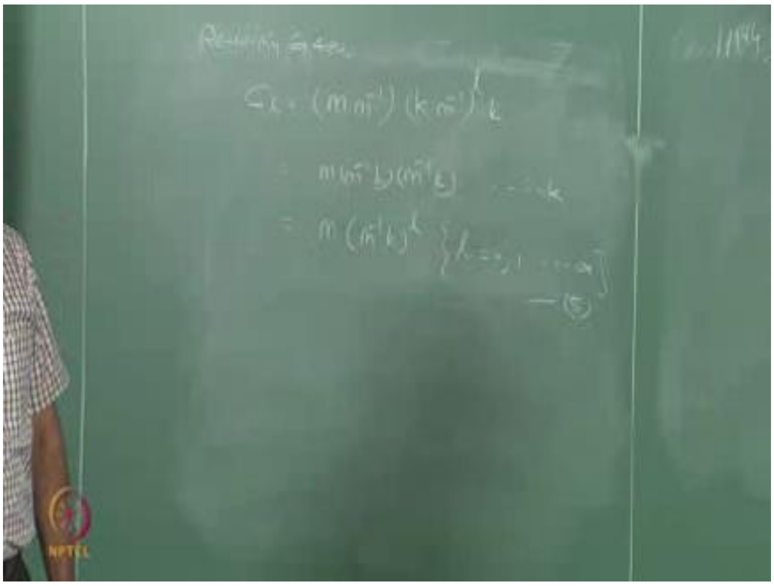

Rewriting, I will call this equation four a, rewriting equation four a, c l is equal to m m inverse, because it does not matter its identity k m inverse l k. So, I expand this l, for different values from zero to infinity, I expand this, which is equal to m m inverse k then again m inverse k, because when it keep on writing this n number of times I get k inverse again and again, keep on clubbing them and so on. Last term will again be m inverse k this k is there. So, you get the same term back again, which will end up in k, which will be m m inverse k to the power l where l can be infinity.

I call this equation number five. So, if you look at the difference of equation number four a and five, we are working again on the coefficient c l, which is derived from the set of orthogonality relations, where 4 a says c l is only a multiplier l number of multipliers of k m inverse of k, where as there is m multiplier is also available here, and the k is gone, because that is clubbed, because 1m from here and 1k from here, we got l number of terms and this m is here and that k goes away. So, it is rewritten now. Similarly, let us hold this at this level, take another equation. Similarly, now we have been working with m and k, I want to work with k and m in a pre multiplied order.

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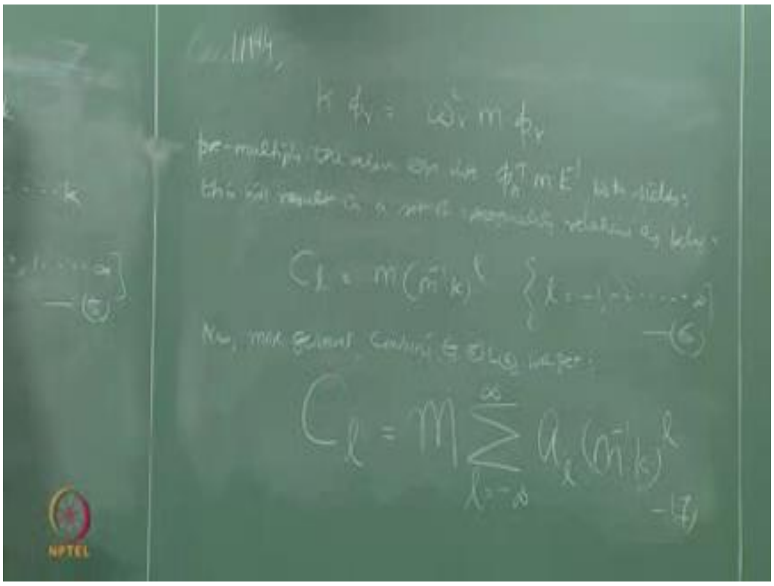

Let see how we will do that. We started with k phi r, is omega square r m phi r, we started with this equation which is equation number one. I pre multiply this with pre multiply the above equation with phi n transpose m k inverse both sides. This will result in a set of orthogonality relations, as below. Now you can say here when I multiply this, this k inverse k look identity, whatever we had on the r h s of this equation, will all now move to the left hand side of this equation. So, you get the same expression back again c l will be equal to, I will ask you must do this and see, c l will be m m inverse k of l, you get the same equation, but now l varies from minus 1 minus 2 infinity. So, I have got now the variation of this coefficient from minus infinity to plus infinity, I will call this equation number six, this is equation number one.

So, do not write this is six. Now more general combining equation five and six, we get let see what do, we get c l can be written as, there is m is not in the multiplier, so m out summation, l varies from minus infinity to plus infinity, a l m inverse k of k, equation number seven. When you compare the equation number seven, with that of five and six, the only confusion is start coming here is, what could be this coefficient for the corresponding value of l. Interestingly this will lead to the Rayleigh coefficient back again, see how.

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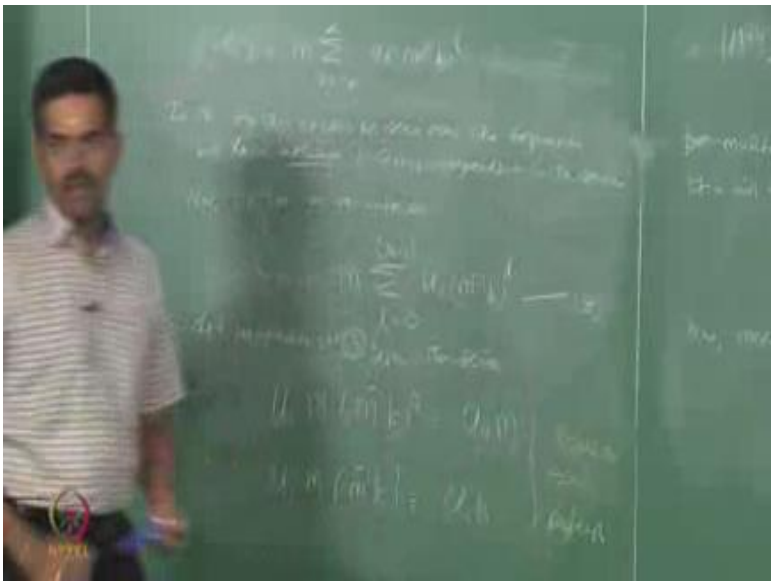

So, let say c l is m l minus infinity to plus infinity a l m inverse k of l. In the above equation, it can be seen, that the expansion will have at least n terms independent in the series. So, what is the advantage of this? The advantage is, now c l can be rewritten as, you will see that after certain numbers of cycles of l, more than n that is n plus 1or n the values will be repeated. So, at least there will be n independent terms, but I am interested only to catch the independent terms, what does it mean is. If I looking for a contribution of k and m as a damping matrix in your Caughey damping coefficient, after specific Eigen number, the contribution of k and m will get repeated.

So, I am not interested in knowing those contributions are repetitive in k and m. I want to know what is the independent contribution of k and m, of the coefficient multiplied and c, even Rayleigh also focus the same thing. So, I am now looking at the independent n terms in this expansion of series, and now I can say c l can be written as m expansion, leaving the negative terms. Why I am saying n minus one, nth term will be repetitive. Call this equation number eight, because this is same as that equation I just written for my convenience. So, we are now estimating the damping coefficient, using Caughey relationship, we started with the orthogonality principles. So, we picked up the basic equation where k phi r is omega square m phi r, where as k is equal to omega square m omega square is k by m we know that. We picked up that and this equation is identically equal we know that.

We pre multiply and post multiply or pre multiplied in both the sides by a specific value to find out, for a power of pre multiply of m inverse k what is the effect of the coefficient of this matrix, in terms of this multiplier l and l. So, that is what we wanted to know. We understood that the series is expanding from minus to plus infinity in a broader range, but it is understood from the series that all terms in this series will not be independent, only n terms will be independent.

So, n minus 1we want to capture, from zero, we leave the negative part. Now if you substitute l is equal to zero I get a zero and k by m. This is more or less looking familiar to the Rayleigh's coefficient (Refer Time: 27:0) l 1. So, it is k by m, because if a is zero this goes one. So, there is no power etcetera. Let us see how this expands. So, let us pick up first three terms in the series. Let us consider first three terms in the series, a zero m m inverse k of zero, which is nothing, but a zero m. Similarly a 1 m m inverse k of 1, which is a 1 k, these two terms are same as that of Rayleigh, because it is summation, you have to add these terms. So, c will be actually equal to a naught m plus a 1 k. So, on we already have these two terms in Rayleigh.

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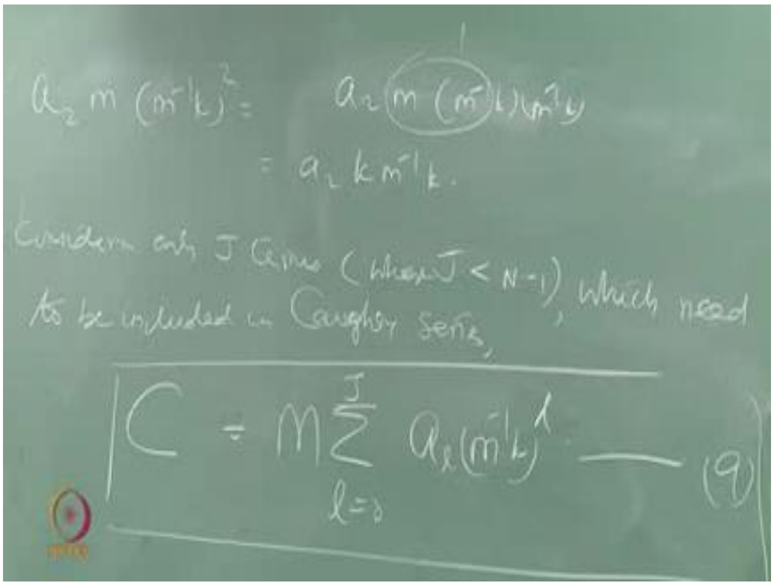

Caughey extended this for one more series; see how it happens; a 2 m m inverse k square will a 2 m m inverse k m inverse k, which becomes a 2. This is unity k m inverse k. So, now, we have considered this two and n independent terms, but there is no guarantee there is no necessity that, in estimating the damping coefficient you must consider all the modes which are independent. You can truncate the most j where j is less than n minus 1. So, now, considering only j terms, where j is less than n minus 1 which need to be included in Caughey series. So, now, see will be equal to the damping coefficient matrix will be equal to m l varies from zero to j a l m inverse k l, call this equation number 9; that is my damping matrix. Actually, need to check the zeta value in every mode that they should be uniformly distributed we are talking about the whole argument and the classical damping, applied to the structural systems.

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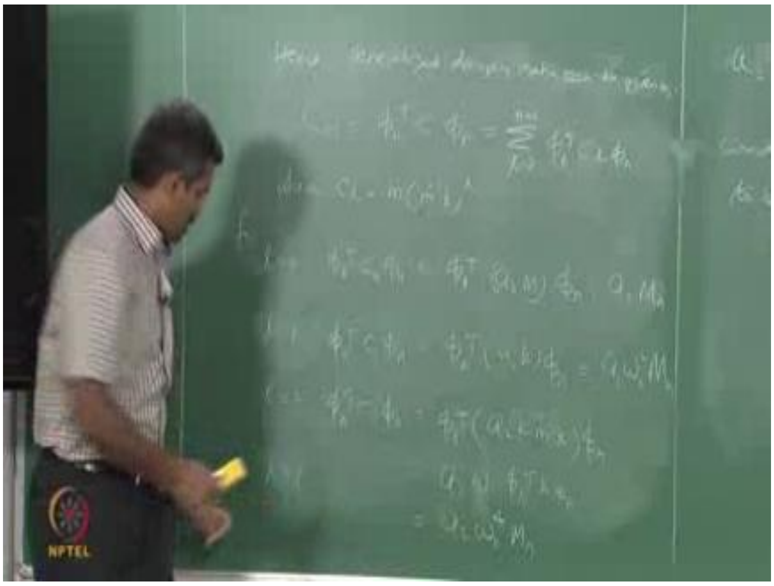

Hence the generalized damping matrix can be given by c n is phi n transpose c phi n that is the generalized damping matrix which is now written as l is equal to zero to n minus 1 phi n transpose instead of c, it is c l phi, where c l is given by equation nine, if it is for j terms or equation a if it is a n minus 1 terms where c l is m of m inverse k l is it not. So, for l equal 0 l equals 1 l equals 2 l equals 3 phi n transpose c l; that is c naught phi n which can be simply phi n transpose c 0 for l is equal to 0, we know it is a 0 m we just now calculated a 0 m of phi n we already know phi n transpose m phi n is a naught m normalized mass matrix.

Similarly, phi n transpose c 1 phi n is phi n transpose c 1 is a 1 k of phi 1 phi n transpose k phi n is of course, is normalized k, but I am looking at this as phi n transpose k phi n. If you remember it gives me the Eigen values back omega square. So, I can write this as a 1 omega n square of m n. Similarly for l is equal to 2 phi n transpose c 2 phi n will give me phi n transpose c 2 was a 2. I think this was this c 2 a 2 k m k inverse a 2 k m inverse k a 2 k m inverse k of phi n k inverse here. So, a 2 phi n transpose, or let us say omega square n phi n transpose k phi n which gives me a 2 omega n 4 m n. Now I can rewrite this equation, because I am keeping on substituting for this expression for different values of l to n minus 1, I am keep on substituting and keep on getting this l is equal to 2 3 4, you can keep on doing.

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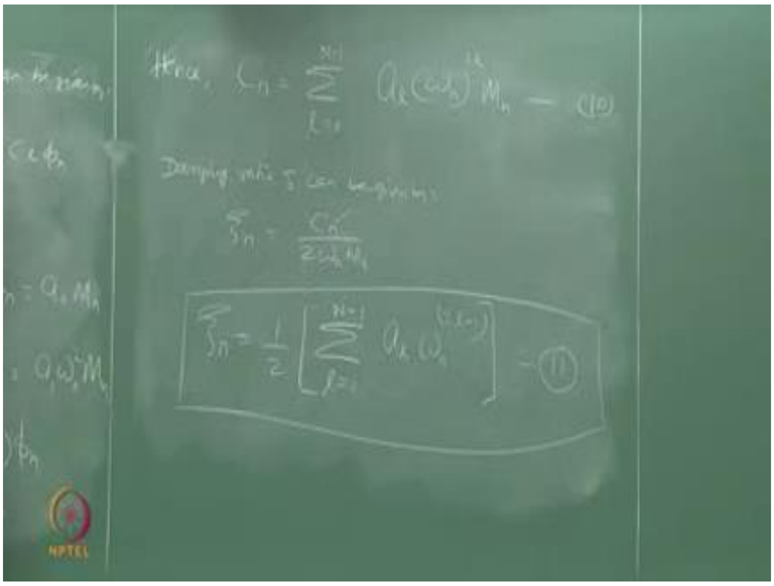

Can be written as summation l is equal to 0 to n minus 1 a l, because for l 0 l 1 l 2 keep on getting a 0 a 1 a 2 as a multiplier of omega n to the power l if l is 0, it is there is no power. So, it says omega n to the power of 2 l that is all for l is equal to 2, this becomes 4 of m n. I call this as equation number, I think nine or ten. Why is to compute zeta, I want to work out the damping coefficient zeta. Now, the damping ratio zeta can be given by zeta n is equal to c n by 2 omega n m n that is the classical expression.

So, now, zeta n can be given by half of this half l is equal to 0 to n minus 1 a l this m n and this m n goes away this omega n goes up. So, omega n to the power 2 l minus 1, which becomes equation number twelve, sorry eleven which is my general expression for zeta. So, if you know zeta for every value you can easily fill up, and find out the damping matrix here, because you already know omega n s. If you really want to extend your study till the entire frequency, or you can pick and choose j which is less than n as per your choice may be you have gotten ten modes, but you only want to save three modes. We can work out only for three modes or you can truncate the number of modes. There are some critical observations on this equation, let see what are the critical problems with this Caughey damping model.

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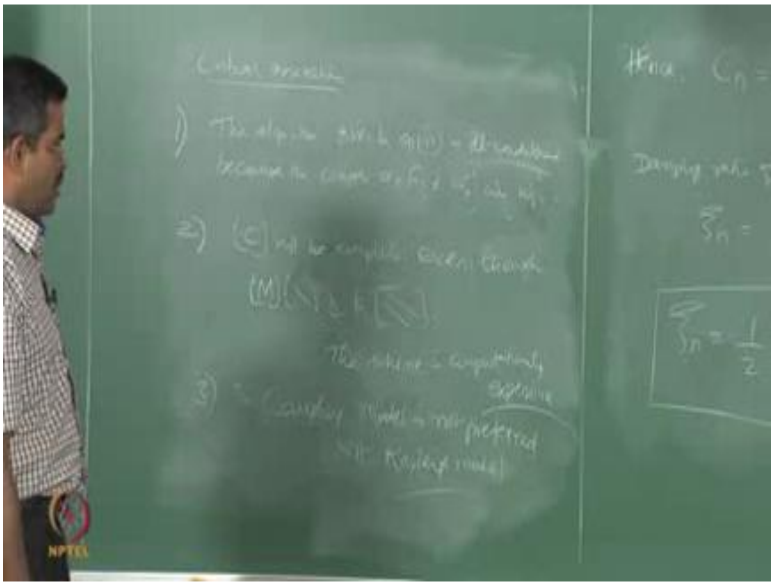

There are some critical observations, let see what are they the algorithm given by equation eleven given by equation eleven is ill conditioned. Now why it is ill conditioned, because their coefficients or functions of omega n to the power minus 1, if l is 0 omega n l is 2 means 3 etcetera. So, you are actually missing certain frequencies in between its not capturing all the frequencies. So, its ill conditioned 2 c matrix will be full will be complete. You may ask me what is the beauty about this even though m is diagonal and k is banded. It means this scheme is computationally expensive; therefore, caughey model is not preferred with respect to Rayleigh model. So, these are critical observations in Caughey systems. So, we will take up an example next class and work out and see how this can be applied and how you can find the coefficients a 0 a 1 and a 2.

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Because you must understand ultimately c n or the damping matrix c will be a 0 m plus a 1 k plus a 2 or something. So, if you know the value of a 1, I mean a 0 a 1 a 2 etcetera. You already have the values of k and m; you can easily multiply them and get c. So, our focus is to estimate the given problem, how to value this - a 0 a 1 a 2, we already have the equations for a 0 a l etcetera. We compute this and show how this matrix can be obtained, how zeta can be evaluated. In the next example we will see that, then this the third method of estimating damping, which is based upon the model truncation and frequency, I mean model participation will talk about that in the following lecture. So, that we complete the discussion on estimates of damping any doubt here, the second model by which you can analytically estimate damping ratio in a given system, which is a substitute for the classical damping technique especially applicable to offshore structures.

Thank you.