Dynamics of Ocean Structures

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Lecture-23
Rayleigh Damping

Now, in this lecture we will talk about Rayleigh damping.

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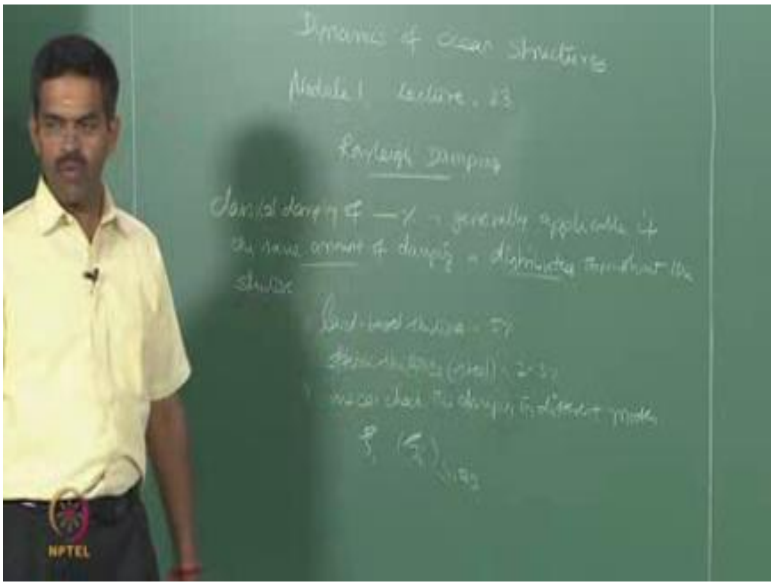

We have already seen that classical damping of let us a certain percentage of that of the critical is generally applicable if the same amount of damping is distributed throughout the structure. And there are two questions can be asked here, what is generally that percentage of damping which is generally assumed these structures. In land base structures generally it is assumed as 5 percent, in offshore structures if it is steel you can take it as 2 to 3 percent.

The second question which is asked is how we ensure that the same amount of damping is distributed throughout the structure, this is the first one. The second one to ensure that the same amount of damping is distributed throughout structure one can check the damping in different modes. So, instead of having a single zeta value we now have set of zeta values, zeta i - where i varies from 1 to n; where n is not the number of degrees of freedom, n is a number of degrees of freedom which want to include for competition. For example, you may have 15 degree freedom system problem, but you do not want include all the 15 degrees, you want to include only 3 4, I will come to the point and later how much is the truncation of the modes later, but let us say we decide to include only 3 modes.

So, I must ensure that the damping ratio is almost constant not exactly same it will not be exactly same, but it lies in the closer range in all the modes chosen for analysis. So, if that happens there is a zeta 1 is about 5 percent, zeta 2 is about lets a 4 percent, zeta 3 is about 6 percent, lets a you are in the close range. Then one can clearly say that the damping is distributed more or less uniformly throughout the structure. Now, one can ask a question indirectly from the second question saying that how do we ensure just by seeing zeta is equal in all the modes which are considered for evaluation, how can we ensure that the damping is distributed throughout the structure. Damping is an estimate of dissipation of energy, you give an excitation to the system, the system keep on does not keep on vibrating in the infinity it stops after sometime.

There may be many reason, for this the first foremost reason is the structure as got a self inbuilt stiffness which will invoke the structure or restore the structure to the recentring capability number one, number two that can be media like air, water which also exerts external energy which stops the vibration of the system. Now all these will happen only by stopping the innovation inertia motion of the system, because as we know in case of even D' Alembert's principle, the principle says that if really want to control the system write in equation motion apply equal and opposite inertia force that is D' Alembert's principle is actually based upon.

So, when I say the system is vibrating we are worried only about the dissipation or we focus only on the dissipation of the inertia force in the given system one can says a inertia force should be mass proportional, is it going to be significant offshore structure because mass is significant in offshore structure; x double dot may not be for a fixed structure. If you talk about a complaint structure x double dot will also be significant, but mass maybe compromised, mass may not be as high as that of the fixed structure or fixed flat form.

But still as we all understand the top side low of a given system cannot be compromised, whether your system is floating or semi complained or complained or fixed, the top side activity which is meant for production drilling all need to be meant you cannot compromise them. Therefore, you cannot really sacrifice a large amount of value in the in terms of mass, but structure can be made flexible. So, x double dot can be larger if it is flexible in certain degrees of freedom whereas, mass may not be very low but it significantly present therefore, inertia force will be always representative value in a given system. Therefore, if you are able to identify that the zeta distribution is there in all the modes this indicates that model participation is an indirect representation of the mass points because mode is nothing but the relative displacement of the mass points at any degree of freedom for a given frequency of vibration.

Therefore, modes are very closely associated to mass whereas, frequency are not associative mass alone but also with the stiffness. So, if you really wanted to look at the mass proportional discussions is of looking at the frequency I think one should look at the mode shapes, because mode shape will give you a relative disposition of the mass point for any frequency of vibration of the given system therefore, if you ensure that the zeta is distributed equally, not exactly equal in the range of value for all the chosen modes for analysis. You can always indirectly say that the dissipation of amount of damping available in the system is more or less uniform throughout the entire structure. That is the reason why we are focusing on motion.

Now the question comes how we estimate zeta at every mode. If you look at single degree of freedom system we knows a task given by (Refer Time: 06:47) which is 2 zeta omega n, a single value because omega is only k by m only 1 k and 1 m therefore, there is no problem we have only 1 zeta. Whereas, in a multi degree of freedom system you got different k, different m because they are matrices now we have a different zetas now, we must have a policy how to estimate this. Rayleigh has come out with the very interesting discussion estimate of damping, analytical estimates of damping let us see that now.

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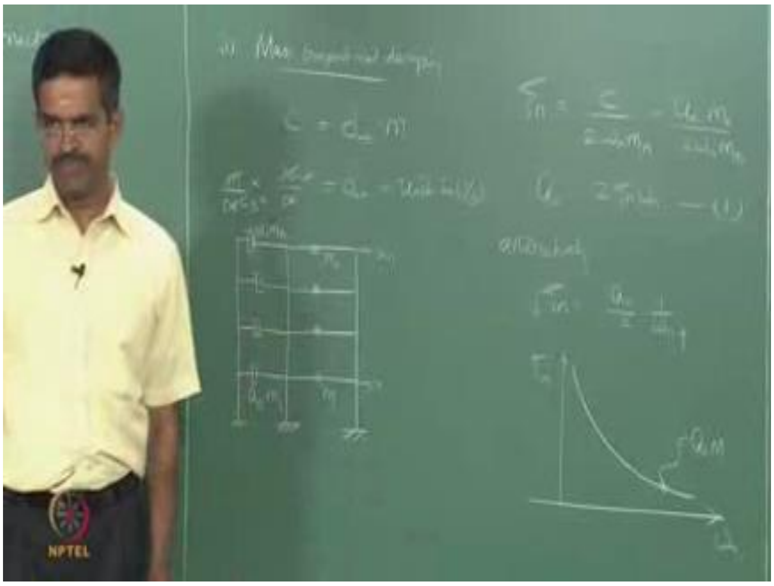

There are two arguments in damping before Rayleigh proposed damping the two arguments is one can be mass proportional damping. We have know damping constant c is now expressed as a 0 of m, mass proportional damping; where a 0 is the proportion to constant, but interestingly see the unit for damping co efficient we know it is Newton per meter per second where as we say what is the unit of a naught which will be second square. So, the unit of a naught actually is one over second. So, it is not a constant. There we call this proportionality constant, but is what a unit.

Now how this will be excise in a given system, let us have a multi storage frame I am taking an example of a buildings system it can be even an offshore jacket also. Let us say number of stories we all know and we all mutually agree that I lump the mass at every floor and I measure the displacement at every floor because there are advantages of doing so in dynamic analysis. So, if I call this as mass 1 to mass n I call this as x 1 to x n. Now I want to indicate stiffness will be given by this column members, mass points are already there I want to indicate the dash pot in the given model, the dash pot I am indicating symbolically like this.

Let show this is a 0 of m 1 this is going to be a 0 of m. So, there is a proportionality constant which will be multiplied with respect to the respective mass of that floor which imposes some damping value to this system to arrest or to dissipate the energy which is cause because of inertia compound of given system, that is now proportion to mass only. We are not bothered about the restoring capability of the column members which is otherwise imposed by the stiffness of the problem.

So, let say this mass is simply tied by anchors or wires and they do not have an axial stiffness etcetera or they are very highly negligible. So, I want to impose the damping or dissipation of energy of these particular inertial system for a given vibration of the system therefore, I want to calculate or model mass proportional damping in every floor or every degree of freedom as I indicated here. Now we already know that zeta n is given by c by 2 omega m n, so where in my case c is equal to a 0 m by two omega n m n therefore, a 0 will be actually equal to 2 zeta n omega this calls the equation number 1.

Alternatively, we know that alternatively zeta n will be equal to a 0 by 2, 1 by omega n. On the other hand for every higher frequency, every higher frequency the damping ratio will be inversely proportional. So, I can plot this quickly for every higher frequency the damping ratio is inversely proportional. So, I can call this model as a 0 m model which is mass proportional damping. The second argument what people say is of course, the dissipation of energy exercises stoppage of vibration or controlled to the response of the vibrating system, this control can be also achieved by reentering capability of the stiffness of the column members.

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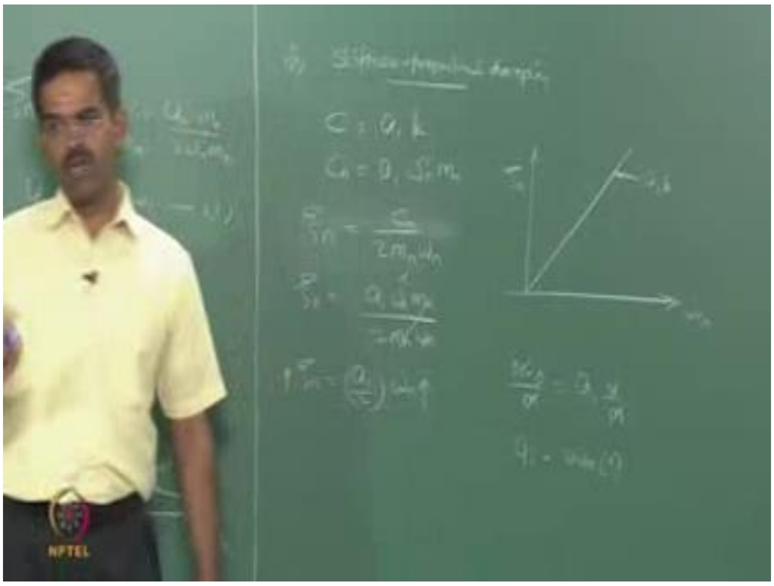

Therefore, the damping now, can also be proportional to stiffness alone forget about the mass. So, stiffness proportional damping. So, now c is going to be equal to a 1 of k. So, c is equal to a 1 of omega square m. So, let us say a 1 of k a 1 omega square mn. So, zeta n is equal to c by 2 mn omega n lets the cn. So, zeta n is equal to a 1 omega n square mn by 2 mn omega n. So, zeta n is a 1 by 2 of omega n.

So, for every increase of omega n zeta n is increasing. So, this model is contradictory to that of mass because in the mass proportional damping for every increase in omega you get decrease in damping where as in stiffness proportional damping, for every increase in omega as a direct increase in zeta. So, if I try to plot this it will look like this, the linear one which is stiffness proportional damping. I am deliberately taking the constants as a 0 and a 1 for our understanding; let us see the units of this. This is Newton second per meter I want to know the units of a 1 and we know this is Newton per meter. So, a 1 will have units as in second, though they are called as damping constants but they have units; a 0 has units in second to the inverse or s power minus 1 where as a 1 has units directly as seconds.

So, here there are two (Refer Time: 15:16) models one is decreasing with increase in omega n other is increasing with increase in omega n. Now people measure damping experimentally also, where a hybrid representation of dissipation of energy which is occurring from the mass contribution and also from this (Refer Time: 15:36) contribution because they have platform is existing or a structure is existing if you try to give a free vibration to a the structure will come to stand still after sometime you can keep on measuring the envelop by giving an unit displacement initially and we know how to get the ratio (Refer Time: 15:52) decrement we know zeta.

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So, people experimentally evaluated zeta and found that it is neither mass proportional nor stiffness proportional. On the other hand to be very specific it was not decreasing or increasing omega at the same time (Refer Time: 16:06) increasing with increasing omega also. There has been a mixture of this. So, Rayleigh found out that and proposed a new model for damping which is very well applicable to offshore structure, there are two reasons for this.

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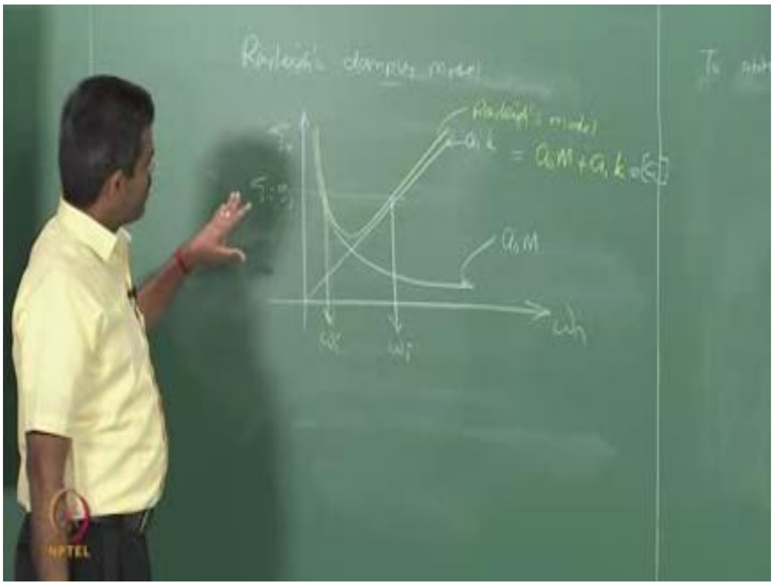

So, Rayleigh proposes a model like this, this is what we have proposed. Our argument actually was to ensure that the damping is effectively there in the analysis we must ensure the dissipation of energy distributed throughout the structure that is called classical damping. And we also said in the beginning of the lecture that if you want to ensure there is you must check that the damping ratios in different modes are almost equal. So, look at here this equation for a specific value of zeta for any specific value of zeta let us a zeta i you will get two omegas. If for two vibrating frequencies if you are able to get the same damping ratio, it is understood that this representing the classical damping of both mass and stiffness proportional. And this also ensures that the dissipation of energy is uniformly distributed for the entire structure.

Now here I have shown only omega 1, omega 2, if I want to take omega 3 you must prove that omega three which is also closer to omega 1 or omega 2 etcetera all the three or all the four or the n frequencies more or less have the same damping ratio. If you are able to show that then the model from which you will estimate c because this going to be c (Refer Time: 18:56) this going to c matrix if k and m matrixes. The model from we will estimate c can be used in the structure analysis for finding out the response of this system because now this is going to be representation of the mass proportional and stiffness proportional, hybrid damping because none of them who have to individually for offshore structures. So, we want to go for a combination.

So, Rayleigh suggested this and it is a very popular model which is used for buildings and of course, there are some literatures there are some researches where people applied this model for estimating c. You will see most of the dynamic papers in offshore structures will always assume c as classical damping which is 2 percent or 5 percent of that of the critical. But Rayleigh showed that the classical damping of either mass proportional or stiffness proportional that is what 2 zeta omega n means omega n m or combinations of m and k they do not work, see we cannot simply have a percentage of that order. You must ensure that, that damping ratio what we (Refer Time: 19:59) analysis should be uniform distributed for the entire structure, you have to show that.

So, estimate c, after finding a 0 and a 1 contributions for on m and on k respectively. If we use that c instead of 2 zeta omega m model then that will ensure that the dissipation is uniform throughout the entire structure. Now the question is how do we get a 1 and a 2 or a 0 and a 1, m and k I know; we all know that how to get m and k by this time. For a given problem you should know generally in all the exercises I give you the last lectures I given you m, but I am made you to workout k at least.

In the next module where (Refer Time: 20:40) is dealt we will tell you how to calculate m and how to calculate k or derive k also. So, you will have a good idea how to we get these matrixes k and m and how do we calculate a 1 and a naught if I tell you this using Rayleigh model then one can propose a new damping matrix which is classically adopted for offshore structures because it is an hybrid combination of or hybrid representation of mass and stiffness.

One can ask me a question why in offshore structures one should go for a hybrid combination of mass and stiffness. In land base structures is the system is very flexible for example, talk about thin electric port of diameter maybe 100 millimeters steal, but length of the pole is about lets 6 7 meters, the system is very highly flexible. So, when the system is highly flexible recentering of the tower under wind action to come to the equally position is practically impossible because it is very thin. So, if at all it has to come it has to come only based upon the tip mass of the system. So, it is mass proportional damping or imagine a building with columns of 600, 600 square the building does not vibrate at all because a column has got very high bending stiffness. So, even try to push the column using an earthquake or an wind load the structure re-centers automatically because though the mass is also there, but stiffness invoked or recentering capability invoked by the columns or stiffness based damping is very significantly representative in the building therefore, they come to recentering positions or dissipation of energy takes place.

So, in these kinds of structures either mass or stiffness proportional damping may workout, but in offshore structures since we have talking about the complaint platform and hybrid structures for the recent invention in deep waters and ultra deep waters we are talking about the system where the super structure is massive whereas sub structure is highly flexible. So, if it is flexible invoking a stiffness based damping is not a successful idea if it is not very massive invoking a mass proportional damping is not a very good idea. But system is an hybrid combination of both because the system is designed in such a manner only about three frequencies are very light or very low and three frequencies are very high for example, TLP in surge sway and yaw the periods are very high closed around 100 where as in roll pitch and heave the periods are closed to around 2 to 5 seconds. The periods are low omega is very high because it is inversely proportional to omega.

So, we have two distinct combinations either a very high omega or a very low omega. So, I must workout using both the combinations, I should not look only either of them because they will not work I have got more the combination present in the system. So, I am looking for a hybrid model proposed to be Rayleigh therefore, this damping matrix which is proportional mass in stiffness is a very useful and intelligent application for offshore structures where you are talking about complained systems. When you talk about let us say fixed type of offshore structures where it is mass proportional or stiffness proportional may workout like buildings because the response given by the system or response shown by the system and the lateral action of waves and winds etcetera maybe very less, it is insensitive. Whereas those kind of structures or not in the let us say practice of evolving because we all looking for structures which can suit exploration in deep and ultra deep waters.

In that case I must look for a system which can have a representative combination of both mass and stiffness. So, the problem rounds out to understand or make me to understand how we get a 0 and a 1, but it is very clear if you want to (Refer Time: 24:29) this model you must have the damping ratio constant at least for two frequencies. So, if any problem I supplied I must showed in the problem for omega 1, omega 2, and omega 3 zetas almost equal, if I show that then this model is accepted to me. So, let us see two things one how to estimate a 0 and a 1 derive then take an example and show how omega 1, omega 2, omega 3, or having zeta 1, zeta 2, zeta 3 almost equal. If you show that then the problem is solved, let us do that now.

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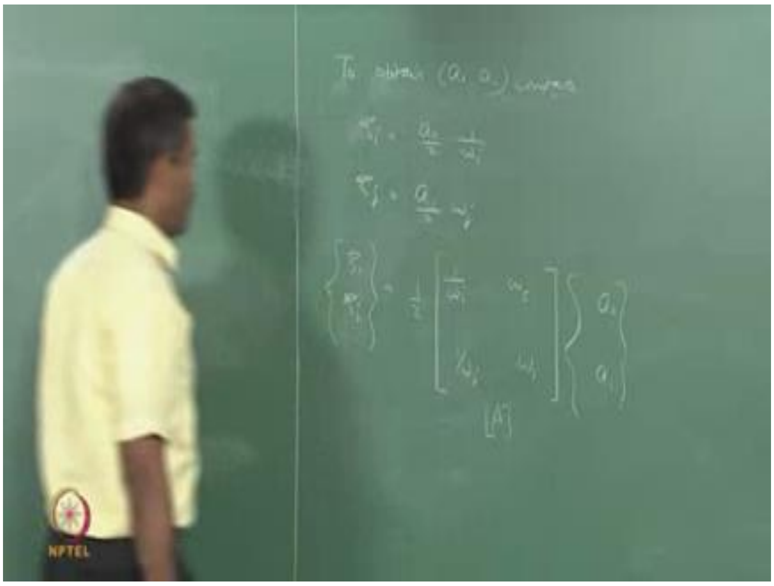

So, now, to obtain a 0 and a 1 constant, I put the word constant here, but you dimensionally have some units. Let us for our derivation we call this is omega i and we call this omega j instead of 1 and 2 it is omega i and omega j. So, can I write here zeta i comma zeta j because zeta is same though there are two frequencies. Remember these frequencies is projected from the yellow line not neither from white nor from line say projected from the yellow line that is the Rayleigh model. So, omega i and omega j two frequencies, but whether I call them as zeta i and zeta j is practically going to be same, is going to be a 0 by 2, 1 by omega i because we know this lets a zeta j is a 1 omega let us write down this as a matrix form zeta i zeta j 1 by 2 is common anyway a 0. So, 1 by omega i omega j, 1 by omega j (Refer Time: 26:43).

Zeta i is only omega i by 2 plus a 1, a 1 pi omega i because zeta i will have contributions from omega i and omega j both, similarly zeta j will also have contribution omega i omega j both. I call this matrix as a matrix (Refer Time: 27:07). So, if you really wanted to find the constant vector a 0 and a 1, I will invert this matrix multiply this to this value I will get a 0 and a 1. These values are known to be or I assume them two person three person by trial and error I will assume because you do not know from this behavior at what percentage of zeta you get 2 omega say, you do not know because if it is mistakenly select this you got only 1 omega. So, have to select the zeta an according value. So, that you at least get two omegas. So, you have to assume this, it means this is known to me.

For a value zeta i and zeta j known to me or zeta known to me I can easily find this because I know for a given system omega i and omega j, I can use there are five methods available to me I can easily find out them and use this and get. Let us see the equations of a 0 and a 1 can we invert this matrix and try to get my a 0 and a 1 value quickly is two by two matrix. We have to invert this matrix and try to get a 0 and a 1, I write down the values here a 0 and a 1.

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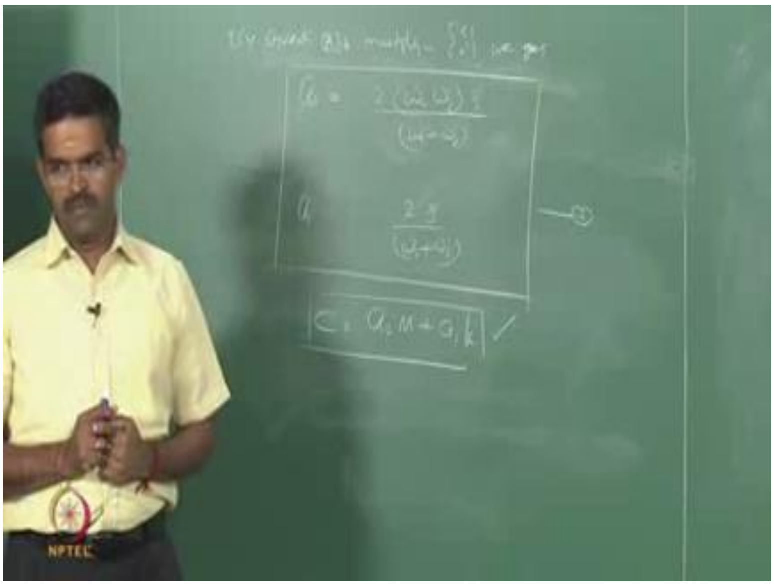

So, these are the two values it cause equation number 3, this is equation number 2. So, I will write here by inverting a and multiply with zeta 1 zeta 2 vector we get, please check whether we really getting this, you have to get this it is very easy. I am not showing it, for complete that here I can wait for a minute complete that you must get this. This is please correct this; please correct this equation. So, once you know this c can be easily said as a 0 m plus a 1 k which is because a 0 and a 1 are known. One may ask a question how do we know a 0 and a 1 omega i and omega j are known to me, I assume zeta therefore, I know these two constants I know the mass matrix and k matrix I will substitute them get the c matrix which I will use from a analysis.

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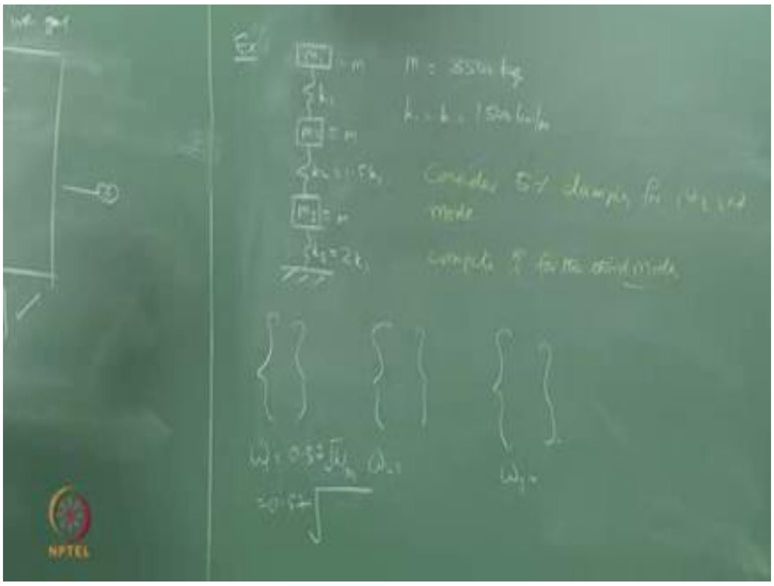

So, will take up an example problem and show how this can be proved. So, let us a three degree freedom system all this three values are m where m is equal to 3500 kg, this is k 1, this is k 2 and k 3 is equal to 1.5; k 1 where k 1 or k 2 is equal to 1500 kilometer. So, what is asked is very interesting, consider 5 percent damping for first and second mode; compute zeta for the third mode that is what we want to know. So, one can use many methods to omega and zeta i mean omega and (Refer Time: 31:52). So, let us say I have the values this is 0.57 root k by m which is 0.57 of 1500 into 1000 you got the SI units for stiffness is Newton per meter.

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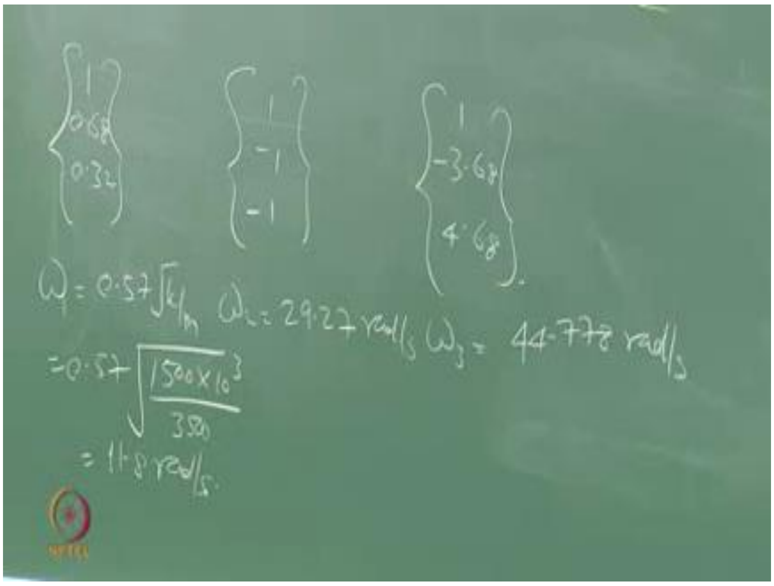

If mass is in kg, so I get 11.8 radiance per second, please check, this is 29.27 radiance per second 1 minus 1 and 1; no, both this will be one zero crossing, there is only one zero crossing. Third one is 44.778 and the mode shape is 4.68, these are my frequency, this is my mode shapes.

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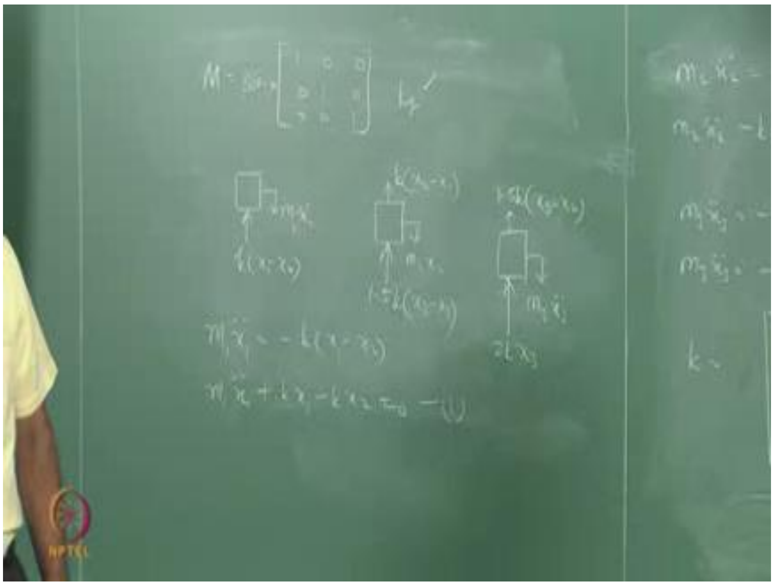

Now, let us say for using this equation of a 0 and a 1 I want mass matrix and stiffness matrix. Mass matrix I have I can directly write which is m of in so many kgs we know this, m is given to me as 35 unit kg this the mass matrix. Stiffness matrix I do not know. So, I have to write the equation (Refer Time: 34:25), I will get stiffness matrix let us quickly do that. So, m 1 x 1 double dot is equal to minus of x 1 minus x 2, m 1 x 1 double dot plus k x 1 minus k x 2 is 0 - is the first equation.

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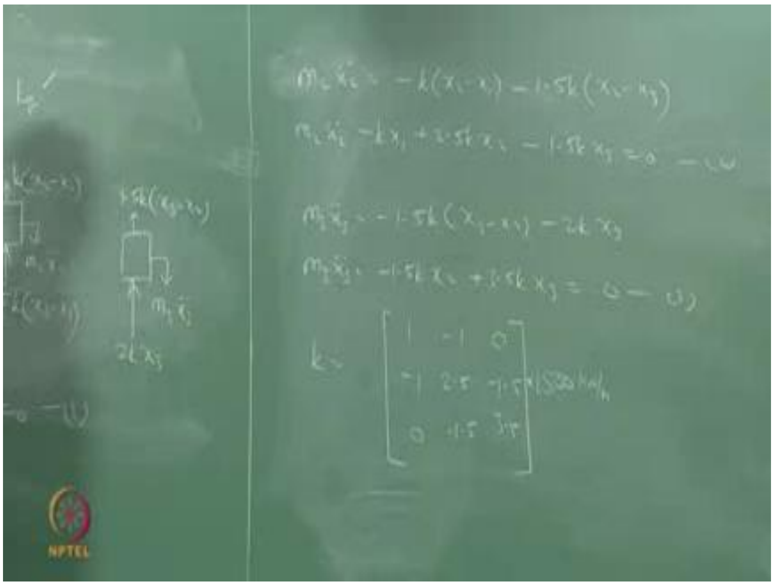

So, let us say, is that of course, I can remove this and substitute into 1500 kilometer per meter I can remove this and substitute 3500 it is I get a mass and k. So, I have the equation by a naught and a 1, I have m and k substitute and try to get the value of a 0 and a 1 quick, get me the a 0 and a 1 value. Now a 0 is given by 2 omega 1 omega 2 that is ij and omega 1 plus omega 2 of 0.5 because I am a signify percent.

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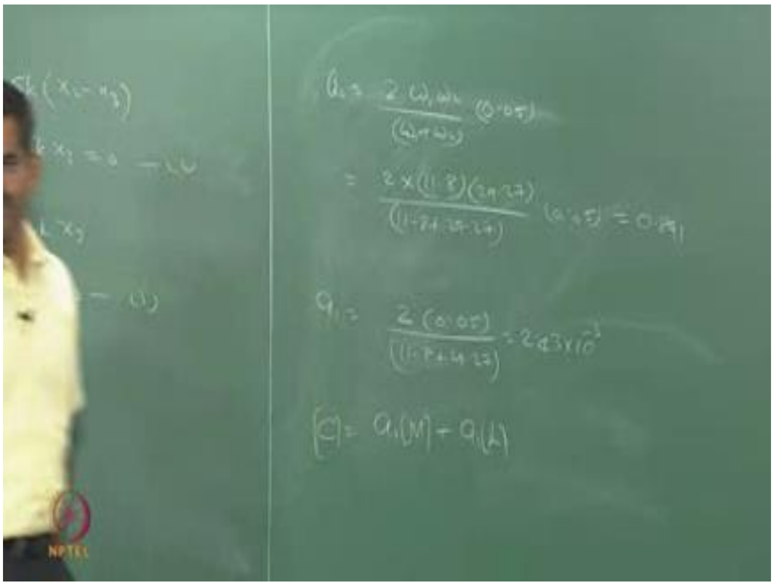

So, what are the values of this? 2 of what is omega one? I forgot it rubbed it.

Student: (Refer Time: 38:35).

11.8 into (Refer Time: 38:40).

Student: 29.27.

Can you give me the value of a 0 let us write a.

Student: (Refer Time: 39:16).

0.841, OK.

Student: (Refer Time: 39:22).

Now c is a 0 m plus a 1 k where m and k are matrixes and c will also be matrix. So, can we get me the c matrix?

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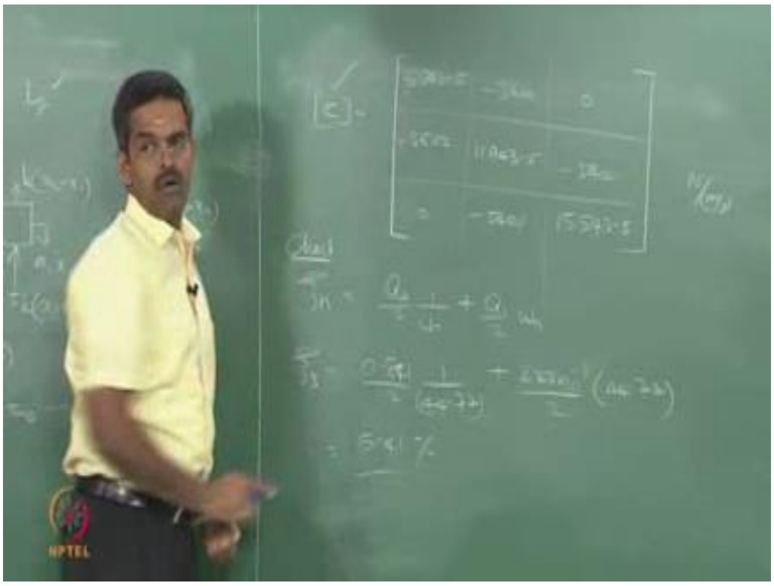

So, c matrix you can check up the value I am writing it here c value. So, let us check zeta n, a 0 by 2, q by omega n plus a 1 by 2 omega n that is a general equation for zeta n. Now I want to check zeta 3, a 0 we know the value it is 0.841 by 2. I know omega 3, what is omega 3?

Student: (Refer Time: 41:11).

a 1 is 2.43 10 power minus 3 by 2 of 44.77 can you give me what is the value of zeta 3 percentage.

Student: 6.41.

Six point?

Student: (Refer Time: 41:32).

This is more or less equal, we start over 5 percent for zeta 1 zeta 2 (Refer Time: 41:40) 6 percent in zeta 3. So, now, this model of c would ensure uniform distribution of dissipation of energy with the entire structure whether all the three modes as more or less the same damping ratio. So, Rayleigh model will propose you this kind of hybrid mixture of mass proportional and stiffness proportional damping which is a very powerful tool to applied to offshore structures because a new generation of the platforms are both capable of recentering as well as mass proportional damping, you can use this one.

And it has been experimentally also determined for offshore structures that the damping ratio does not decrease with increase in frequency, the (Refer Time: 42:25) does not increase with increase in stiffness both of them do not agree. So, there is a mixture. So, this model will try. So, your worry was how to estimate c for a given system k and m are known to me, if now c for a damped vibration frequency or damped vibration system you can always find the response we have the standard equations available with us we can do that.

So, you will see most of the focus and research (Refer Time: 42:48) dynamic analysis we will focus only on estimating of mass k and c and they will go on to a numerical method maybe (Refer Time: 42:56) or some other algorithm t estimate x of t. So, we will talk about that later as an applied problem in second module, but let us know how to estimate these three characteristics of equation of motion – that is how to estimate the mass matrix, how to estimate a derive a stiffness matrix or how to derive a influence coefficient matrix from where we can derive a stiffness matrix and how to get acceptable model of a c matrix; it is never proportional how to two zeta omega n, it is having contributions from mass and stiffness both. So, it is a very interesting and very valid example.

You will find this kind of application very rare in the present scenario in research in offshore structures, peoples still use classical damping. This is a new idea where people generally propose this and it is a very interesting proposed by Anil Chopra, it is a good model. So, I think this is useful and I have applied this in some of the examples in TLP it works very well and that damping estimates are reasonably good compared to experiments. So, this model is good and this is how a new area of research is being introduced to you for estimating c matrix.

Thank you.