Dynamics of Ocean Structures
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Lecture - 21
Numerical Method for MDOF - Rayleigh's Method

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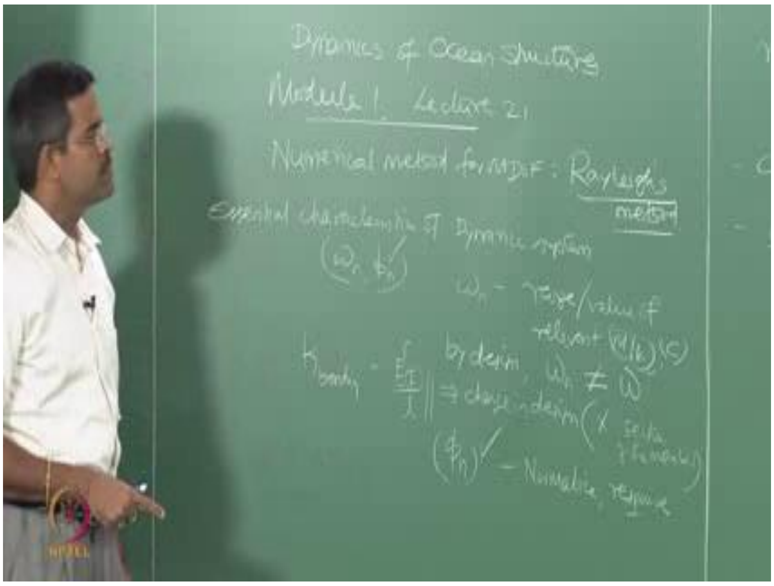

So, in this lecture, this is the 21st lecture in module 1 on dynamics of ocean structures. We are now continuing to discuss about the numerical methods which can be used for multi degree of freedom system. As we all know the essential characteristic of a dynamic system, is to obtain omega and phi n. Now what is the principle used of getting this fundamental frequency in the corresponding mode shape. The fundamental frequency will tell you the range, or an appropriate value of the relevant mass, and stiffness of the system. Therefore, by design, you can keep omega n far away from the excitation frequency. So, the structure will not resonate in the design.

On the other hand, alternatively if your omega n or the fundamental frequency, comes closer to the frequency of band of the excitation force. Then you have the liberty to change the stiffness of the system. So, let us see what are the properties of a system stiffness of the system stiffness; we are looking for the bending stiffness now. Actually it is a function of e i by l. So, either you can change the modulus of elasticity of the material, or essentially decrease and increase unsupported length of the member, or cross section dimensions. So, essentially it is amounting to change in design. Because design is nothing, but getting the cross sectional dimensions.

Alternatively, you can also compromise on the mass the top side of the system, so that you can restrict the top side mass for operational reasons. So, at the excitation frequency does not be in the same band as that of the natural frequency of the system, but we all understand very clearly the system will not remain in the single frequency band for a longer time. Once the system gets into a non-linear response mode there will be a damage initiated to the system. Therefore, the stiffness will change therefore, omega will be keep on continuously, and changing the system will not remain in resonating band for a longer time that is by virtue of the design.

So, this is about the omega n which is one of the important characteristic of the dynamic system, which we are trying to work out numerically by different methods. Now what is the significance of phi n? If I am able to get the mode shapes of the given system, either 1 or n number of mode shapes depending upon the degree of freedom operated. I can easily normalize them, with simple computation I can find response of the system. Now one may ask me question why to find the response of the system I required the mode shape. Because mode shape is the relative disposition or displacement of the mass points in a given system, but anywhere response is not the dispersion of the mass points. Response is the displacement of the system with respect to the force acting on the system, the force can be the excitation force, for which the system is subjected, to it can be wind load it can be wave load and current etcetera interestingly for the mass being very high in a given offshore platform, inertia forces will be significantly present in the system.

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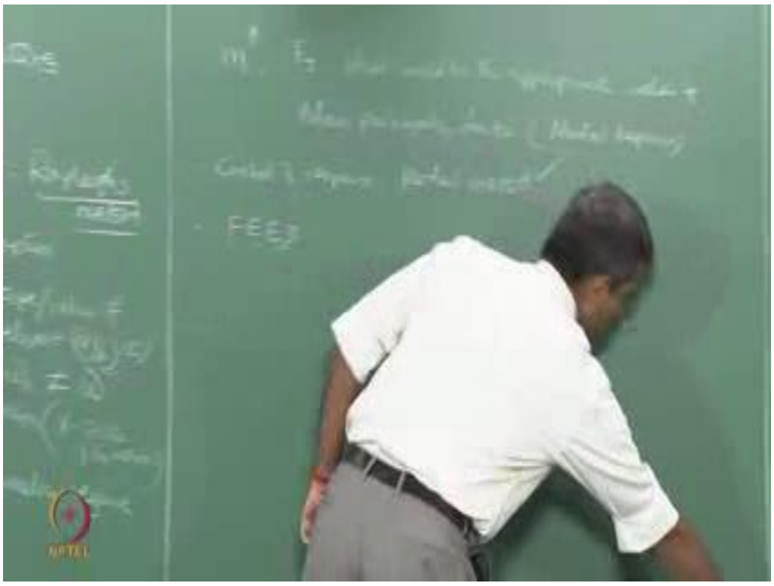

So, if you know the mode shapes in advance for the given design, or the chosen design by you. You will know what would be the appropriate order, of the mass participating factor. What otherwise we call as modal response. Now the modal response method is got very significant applications in structural engineering. Especially if we talk about control response control of response of the structures, modal methods even today place a very significant application.

Now, one can ask me a question why one should talk about the control of response. We already said that we are talking about the system highly flexible in nature because rigid systems in offshore platforms will cost you more. Therefore, we are talking floating structures semi submersible complain structures, which are highly flexible in nature it means they have got large periods. So, the system become flexible therefore, you cannot afford to have large displacements. Because you have you cannot then improvise the functionality of the design for example, drilling etcetera it is not possible. Because risers will be keep on moving the risers may fail, but not the platform therefore, you cap you cannot allow large displacements on the deck therefore, you should control the response on certain degrees of freedom especially in heap degree of freedom in compliance systems, which is one of the stiffest possible, degree heap responses may result in tether pull out which can effect stability of the system.

Therefore response control becomes a very important target, in offshore structures in the reason passed in research. When we are talk about response control you cannot minimize the response only by altering m and k. Because if you alter m and k, you will get into a different band, you may land up in a resonating band. So, you cannot only play with a characteristic of m and k alone. You can partially play with c, but we all know c is also a part or a product or a factor of m and k. Because 2 zeta omega m is what c is. And omega is again a part of k and m therefore; you cannot also play with c in a major parameter.

So, you have to introduce an external agency which could control the response. In such situation modal method plays a very important role. So, if you really wanted to apply a modal response technique, I must know all the modes in advance in design. Therefore, obtaining mode shapes also become significantly important, in dynamic system. These are all happening only even before the platform is thought of commission. It is also happening in feed level. Feed stands for front end engineering design. Before even we look at the conceptual design of the platform even at the feed stage, when the conceptual design is initialized we start working on omegas and phi's. Therefore, omega and phi are determining the essential characteristic of the dynamic system, becomes very important in structural dynamics by enlarge. Therefore, there are very many methods which have been employed and used by different researchers in the literature, to find out essentially this essential characteristic, which are important in dynamic system.

Today we will discuss about a new method, again for us which is Rayleigh’s method Rayleigh Ritz concept is very often used in continuous systems, where system can undergo more or less a linear deformation, but method suggested by Rayleigh Ritz is essentially analytical. This method is also extended further in early 70s by Rayleigh’s method, which can be used for numeral’s. So, we will talk about the analytical method later in the next lecture, but in this lecture we will talk about the numerical method suggested by Rayleigh, and I want to demonstrate a problem again as usual pick up a problem, solve a Rayleigh solve a Stode law solve a insufficient coefficient solve Dunkerleys how do I compare?

So, that we also try to find out the validation of a new method, discussed in the class with that of the old methods, which we already k. Now, therefore, I am establishing a confidence level in your mind. We can follow any method all will give you the same answer that is the point. So, there is no question of choosing any method. All methods will lead to the same answer the question is the comfortability how you enjoy, in selecting a method for your problem.

So, we will take up an example now, and try to demonstrate how to get omega and phi for a given problem, using Rayleigh's numerical method. Rayleigh reads us an analytical method also, but that way we will see later. So, as usual I will take up a problem and then try to demonstrate it with the help of the problem, as seen here I will take up the 2 degree problem.

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So, let us say an example, illustrated example one on Rayleigh’s procedure, x 1 and x 2 m and 2 m stiffness of the springs, are 3 k and k respectively. So, I want to find omega and phi for the system, using Rayleigh’s procedure Rayleigh’s method. So, let us open up a table.

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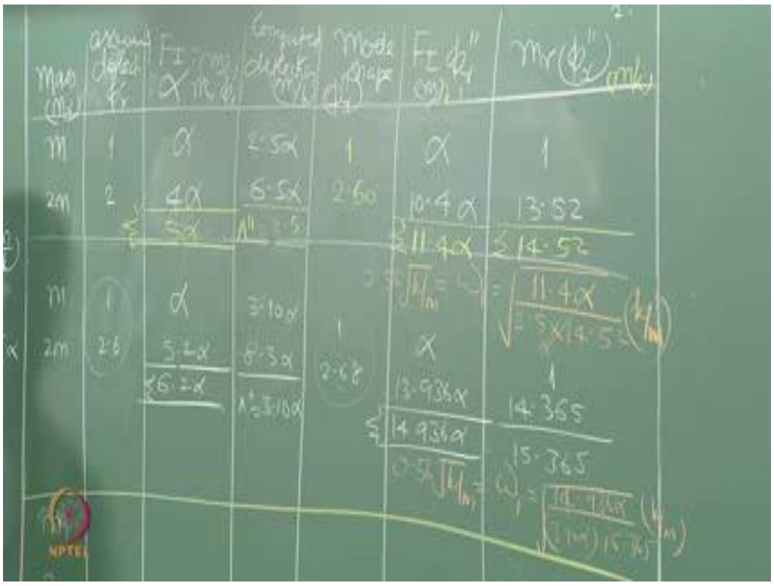

And say the mass value; I call m r, r, and stands for the degree of the freedom. Next is assumed deflection. I call them as phi r; again i stand for the degree of the freedom. Oh phi dash r that is the first level of deflection what I am assuming. Then I can calculate inertia of force which is equal to alpha m r phi r 1, first level. Then I calculate the computed deflection then I found the mode shape. Now I call this as phi r two. This is the assumed deflection, let see be very clear this is assumed deflection. This is computed deflection from the computed deflection, I take a proportional value out, I will get the mode shape. Then I will compute the new force which is f r, f r or the f i into phi r double prime, double prime stands for the new value. Then I find m r phi r, double prime square. These are the variables I have in restored in Rayleigh's method.

So, we know that I have got 2 mass points m and 2 m. These are the 2 mass points I have m and 2 m. I can assume the deflection as 1 and two. Let us say I am looking at the fundamental mode shape therefore, I am talking all the positive values 1 and 2. So, I must multiply with this some coefficient alpha, multiplied by m multiplied by 1. I got m alpha similarly alpha multiplied by 2 m multiplied by two, I get 4 m r alpha, if you consider m as a common multiplier here. Let us say then I can write this also, as alpha and 4 alpha.

So, let us sum this. Let us sum this, 5 alpha. So, this value, I want to enter this value. Here star value. The star value will be equal to the sum value divided by the stiffness of the first spring. Please take it as 2 k. Please make a change here this is 2 k. Please make a change in the problem this is 2 k, and this is k. Yeah this is m and 2 m. So, 5 alpha divided by stiffness on the first spring, because I am looking at the response of the first mass. So, I only the first spring which is 2 k which will be 2.5 alpha of course, m by k will be multiplied here. Because m is already in the numerator you can see here, and k will be now in the denominator.

So, I can simply say this as m by k as a multiplier out. I can rub this and enter the value as 2.5 alpha. Now I want to enter the second value. That is the say double star value. The double star value will be equal to in the cumulated deflection because this spring has already deflected, or this mass as deflected depending upon the capacity of the spring, when you apply an inertia force here. So, I add the deflection first which is 2 point alpha m by k you leave, because this is a multiplier out we are having 2 point alpha plus.

Now, the inertia force is subjected to the first mass, which is alpha, will not influence the deflection of the second mass. So, I will now talk only about 4 alpha, divided by the stiffness of the second spring now k. So, I will get 6.5 alpha. So, I put 6.5 here I will rub this write 6.5 alpha, here now I take a multiplier, because I want to I am going get the mode shape. I want the multiplier, then the multiplier be called as a double prime some notation given by Rayleigh, as 2.5 yet let a double 5 be a 2.5 in that case this, will become 1 and the other one will become 2.60.

So, now this is nothing, but a product of this column, and this column. Because it is phi r second prime. Only the mode shape this is going to be simply alpha and this going to be 4 of 2.6, 10.4 alpha. So, again the sum, which will be 11.4 alpha. And you can also say, there is a m multiplier here, m multiplier here. Because this is f i of phi double prime r. Phi double prime r do not have any multiplier. Because m by k is out anywhere there and so on.

Now, let us get this value which is m of phi dash r square. Which will be in this case of course, 1 for the first row the second one, will be 2.6 square multiplied by 2 which is 13.52, now, the sum again which is 14.5 two. So, I have m by k here automatically m by k here also automatically, now I want to find out omega from here. Now omega 1 value will be equal to the inertia force product, divided by the mode shape multiplier of the mass participation factor, which is 14.52, the whole root. So, of course, k by m will also be present out, is only a multiply. Then here is also an alpha is 2.5 alpha. That is how you get one here there is an alpha. Here which goes away actually? So, the alpha will not be there. There is a 2 point by alpha here that is how we get 1 as 1. Otherwise you get alpha there also 0 point.

Student: 56.

Let me write here 0.5 6 root k by m. So, we started with the an assume deflection 1 and 2 where as we got a deflection of 1 and 2.6, we are not converging. So, let us do the second iteration. So, m, 2 m, now the assume deflection will be 1 and 2.6. So, can you do this? So, this becomes alpha, this is 5.2 alpha, this is 6.2 alpha. So, if you want really enter this values single star. I will just once again explain the single star will be equal to in this case 6.2 alpha, by the stiffness of the spring which is 2 k.

So, you get a multiplier of m by k out, an alpha which will become 3 point. Similarly if you want to find this value, this vale will be actually equal to this deflection, plus 5.2 alpha by the stiffness of the lowest spring which is unity. Which is 1 times of k which is 8.3 alpha . So, now, I can get a prime of 3.1 alpha. So, this becomes 1 and this goes to 2.68. You know of course, this column is inertia force multiplied by the new one which is always alpha. And if it is 5.2, into 2.68, let us sum this. So, this is going to be m of the new value which is again 1 and 2 m of a new value, will be 14.365.

So, 15.365, the new omega will be 14.936 alpha, by 3.1 alpha of 15.365, the root of k by m, which gives me 0.56. Is it okay? So, we start with 1 and 2.6 with a 1 and 2.68. So, of course, we have got 1 decimal convergence. Let us do one more iteration. So, now, m 2 m, 1 and 2.68 can you get me the values.

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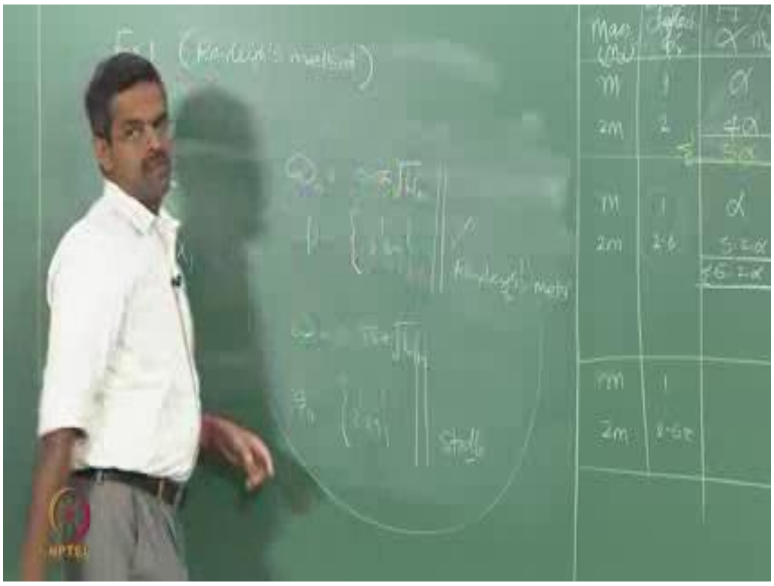

So, I get omega natural frequency as 0.56 k by m, because that is not changing and the corresponding mode shape what you will get after convergence, will be 1 2.69. That is what you will get please check whether it is or not this is by Rayleigh that is due by Stodola so, 2 k then m k 2 m.

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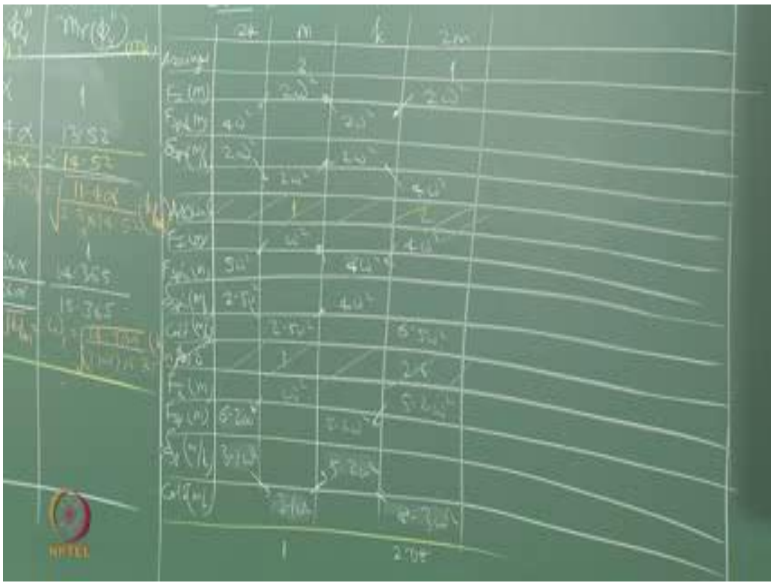

Let us assume deflection as this as 1 and this as 2, inertia forces. So, let us multiply m out here. This is going to be 2 omega square. This is also going to be 2 omega square. And then the corresponding spring force, let us say force on the spring. Spring deflection the force by the stiffness, I get m by k here now. So, 2 omega square. 2 omega square. So, I can find the band value as 1 and 2 we start at 2 1 1 and we get 1 and 2. So, we have to again iterate. So, the assumed deflection will be the same value and inertia of force now will be the mass multiplier which is going to be 2 of 2 4 omega square, and 1 of 1 omega square, that is the spring force. Then the deflection of the spring which is m by k 2.5 omega square 4 omega square.

So, the calculated deflection of m by k, 2.5, 6.5 which is going to be 1, 2.6, this is the banded value now. Which is again not converging we start of a 1 and 2. We got 1 and 2.6. Let this be the assumed deflection then the inertia force with m multiplier 5.2 omega square, 1 of 1 omega square. Then the spring force with m 5.2 omega square, or 5.2 plus 1 6.2 omega square, 3.1 omega square. So, 3.1 omega square, it is 5.2, this is OK.

So, 38.3 omega square. So, the band value is this is 2.68. So, if you do one more iteration, you will see that the mode shape will become 1 and 2.69. I want you to find the corresponding omega get me the corresponding omega. This will become 1 and 2.69, if you are simply copying the calculation from this without even straining yourself as the calculator, be sure that you will never understand anything in dynamics. We guaranty you will never understand 1.567. So, we get 0.567, root k by m. Because this is a multiplier of m by k, here we equate and inverse we get k by m automatically, here which is we can see here they are exactly matching.

Now, let us do the same problem with influence coefficient method. I will remove this. So, the influence coefficient method, we should first write the control matrix and the alpha matrix, which are influence coefficients. So, this is the problem. Let us write down this this.

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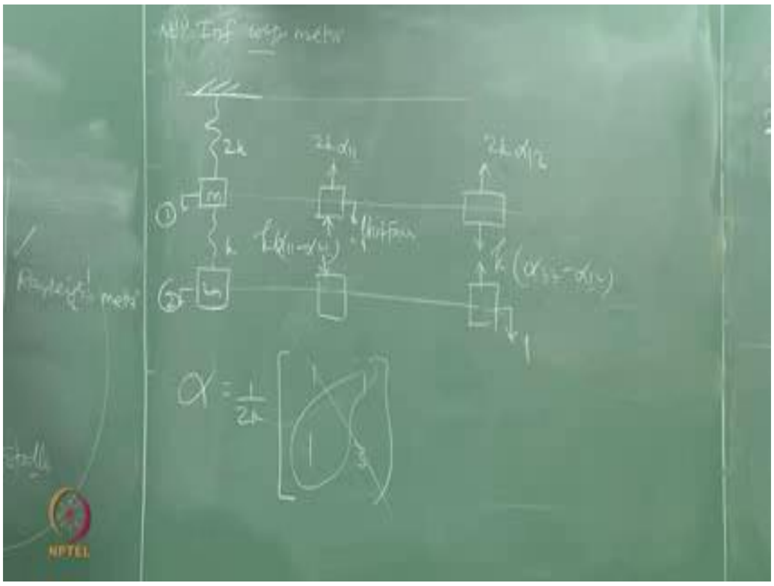

Is the third method, I am solving influence coefficient method. So, I have the problem 2 k and k m, and 2 m first degree, second degree. Unit force unit force we are talking about influence coefficient which is flexibility coefficient actually. So, whenever we move this mass down this spring will get compressed we push it up.

So, mark the pair, k of alpha 1, 1, 2, 1. When this mass moves down this spring will pull it up 2 k of alpha 1, 1. And the second subscript is always 1, indicating that we are applying unit force to the first degree and the first subscript is located depending upon the location where handling the problem. Similarly, here I move the mass down this spring will try to pull it up, I always mark the pair, when this is pulling up this will also have the same manner. So, the stiffness of the spring multiplied by alpha 2, 2 minus 1 2, because this spring connects to 1, 1. The first subscript is always where you are applying the force second 1, where it is connecting. That is how we are done here also. And this mass is moved down the spring will pull it up back again.

So, 2 k of alpha 1 2 the second subscript is always 2 because I applying force in the second degree. Now I write force equation I rub this.

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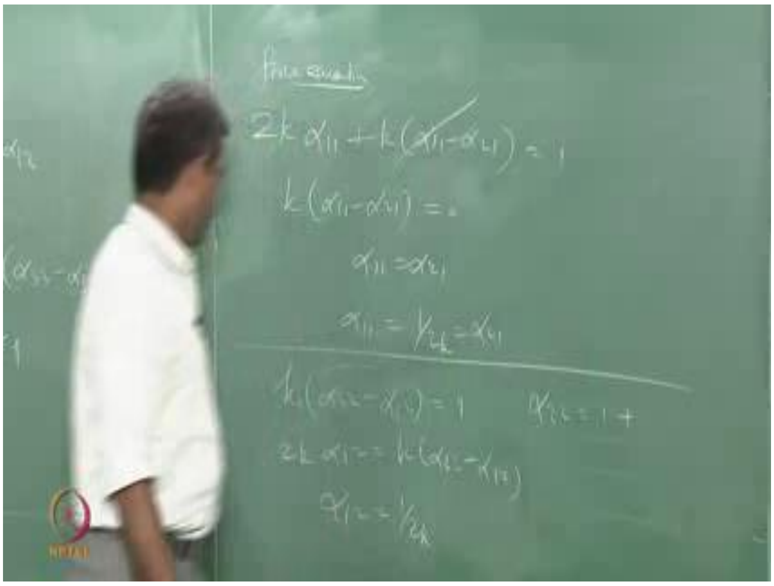

I will take up here this one; let us say 2 k. 2 k alpha 1, 1. Plus k of alpha 1, 1; Minus 2, 1 is actually equal to 1, because they are in the upward direction this one in downward. Now come here k of alpha 1, 1, minus 2, 1, is set to 0. K cannot become 0. This implies alpha 1, 1, is alpha 2 one substitute back, here this goes away, and alpha 1, 1, will automatically become 1 by 2 k, this is also same as alpha 2 1.

Now, let us do the second one, there is a force here. So, k alpha 2, 2, minus 1, 2, should be 1. Can move up here, 2 k of alpha 1 2 is k of alpha 1 2 minus 2, 2, minus 1, 2, k of alpha 2, 2, minus 1, 2, actually is 1. So, I can simply say this value is 1 over k, substitute here k, and gets canceled. I get 1 over here therefore, alpha 1 2 will become 1 by 2 k. So, I can find alpha 2, 2, which will be alpha 2, 2, is going to be 1 minus or 1, plus or 1 by k plus because this k goes there plus alpha 1, 2, which is 1 by 2, k 3, by 2 k.

So, I get my influence coefficient matrix. Now which will be the alpha matrix the denominator in all the case are 2 k. So, I say 1, by 2 k 1, 1, and 1, 3. You will see this always symmetric is always diagonally dominant, and it can be inverted. That is how we get a stiffness matrix for this. Now I want the control matrix for iterating this for influence coefficient scheme. I need not to go back to the original fundamental, we can direct write the control matrix, which I am going to do now. I will remove this.

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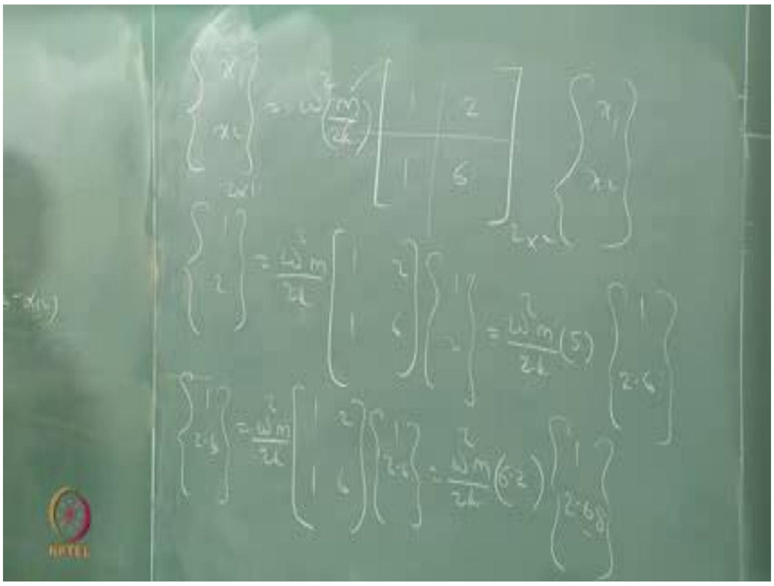

And now say x 1, and x 2, that is the first iteration scheme, omega square m by k, in my k the denominator 2 k, therefore, 2 k of a matrix, which is going to be 2 by 2, why because this is 2 by 1, I have a compatibility of 2 by 1 here.

Therefore this is 2 by 2 square matrix. I want to enter these values. The original matrix, I have is 1, and 1 and 1, and 3. So, I multiply this value or this column, with m 1, which is simply m. So, I get this 1 in the first column multiplied by m alone, I get 1 and 1 here, because m is already here, the second column 1 and 3, where the second mass is two. So, I must get 2, 1 6 here, this is my control matrix now. So, start with the same iteration of 1 and 2, and see what happens. Quick give me the vector, multiplier five. So, second, 6.21 and 2.68. So, the first decimal is matching second. So, keep continuing.

Let us see, omega you are getting continuous the next iteration, tell me what omega you are getting.

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How much is the omega? So, you get the vector as 1.2 0.69. Get the exactly same vector and omega will be 0.56. You can check now can you give the value of Dunkerleys because you have the influence coefficient matrix there. Dunkerley's equation Dunkerleys value how much.

Student: 5 3 5.

5 3 5.

Student: 5 3 5.

Interestingly you will see all methods now. I will discuss one more method later, all will be actually exactly converging. All have the same values 5 6 5 6 7, 2 6 9, 2 6 9 and 0.56 in all methods will lead to exactly the same solution. So, some more examples, and some more methods applied to multi degree of freedom are available in text books which you have referred. You please see the link in the NPTEL website the text book is opened for sale now. It is available in the stands published by springer. I will show you a small brief work out walk through the in the next lecture.

Now, available for sale, you can purchase chapter wise also. I have got lot of exams about I think 70 solved examples available in the text book on first chapter. So, those who are interested, you can please see the text book and see work out more examples, and how and of course, there are some proofs why Stadola will converge to fundamental degree, or fundamental frequency why influence coefficient method will land up in correct iteration. If you take the mode shape correctly are available in the text book for your reference, because shortage of time. We are not discussing the references and derivations here again; you can get back to the text book. And understand how they are proved mathematically and easily find out from the text book.

So, in this lecture we learnt couple of important things.

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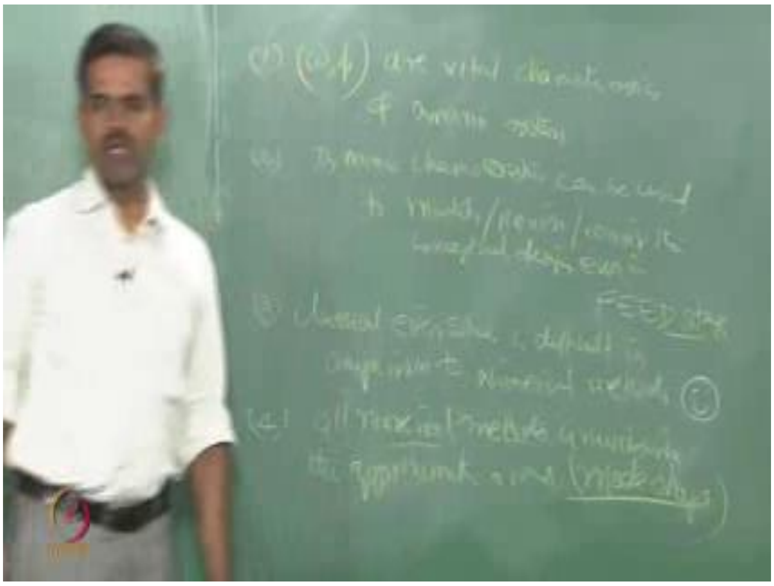

Let us summarize that. Omega and phi are vital characteristics of a dynamic system. These dynamic characteristics can be used to modify, revised, revisit, the conceptual design, even in feed stage. So, analysis dynamic analysis and design are coupled. Classical Eigen solver which is considered to be one of the powerful analytical method to finding omega and phi, fails to give the values as simple as numerical methods is difficult. I put it like this in comparison to the numerical methods ok.

Now, I can make this statement only with one effect. This statement is true only with this icon and icon is the method start by me. So, if you know these methods it is easy otherwise difficult numerical methods are difficult. Actually the fourth all numerical methods circumscribe the approximation, around what can you fill up this blank, all numerical methods circumscribe the approximation, around the specific technique, what is that mode shape. Everywhere you assume only the related displacement of the mass point in all the methods.

So, please redefine dynamic basics. Dynamics does not focus on frequency it focuses only on mode shapes actually. That is very important it is a new definition, what we are coming out in this. Class dynamic characteristics has got 2 components one is of course, the frequency, other is the mode shape, but people generic focus essential frequency to talk about resonating response, but we focus on mode shape because next lecture, will talk about opening on model participation factors, there you will know how mode shapes are very important for understanding, the physical meaning of mass participation is a overall response. So, mode shapes did not, should not be, it is because of this reason that mode shapes are very important, which people realized later. Dunkerleys method became unpopular. You must realize why it is. So, Dunkerleys is also giving frequency which is closely matching. Why this method is unpopular because it does not give me the. So, called mode shape and you may always ignore mode shapes saying that sir, it is only a factor of relative displacement of mass points this will tell you, what will be the contribution of respective mode in the final response.

For example, if this is the first mode shape, this is the second mode shape, third mode shape, and fourth mode shape etcetera. If we want to know what is the final response, if I am able to find the first mode shape 80 percentage of first mode shape 20 percentage of this second mode shape etcetera, as the final response and higher modes do not contribute, I may not land up in working out the higher modes at all. So, that is why fundamental modes are very important. There we will talk about the participation factor later. Which becomes a very vital characteristic in seismic analysis; it becomes very vital why seismic analysis. Because seismic analysis focused around inertia forces and in our platform inertia force become dominant, we will be focusing on this kind of problems where the participation factor becomes very important.

So, numerical methods there are couple of more methods mechanical impedance technique, we have discussed them in the text book. So, you please refer into the literature and try to find out, how these methods can also be useful. So, never circumscribe yourself only on a specific method, you should be thorough in solving all the methods, and most importantly in couple of last few classes, including today, I have demonstrated a single problem may be 3, or with 2, solving the all the methods in 40 minutes.

So, if you say in an examination, you will be not able to solve one problem, at least by one method in 10 minutes. It becomes highly unacceptable to be. Because I have explained and demonstrated same problem, I have solved all the problem by all the methods in 40 minutes all the 4. So, you should be able to do much fast in the examination, because you are preparing for that. So, do not tell me numerical methods of finding omega and phi are difficult. They are very easy providing now the scheme. So, I have given you lot of short cuts how to remember Dunkerleys equation.

How to remember control matrix for influence coefficient; how to verify whether the control matrix, or the influence coefficient matrix is same as that of the stiffness matrix, from the stiffness matrix. How can write equation of motion all as been coupled actually in the whole lectures of last few, and today we are discussed about Rayleigh's numerical method.

In the next class, we will talk about Rayleigh Ritz analytical procedure, if time is there. So, we will explain how this can be useful for continuous systems. It is very interesting and the essential focus now gets emphasized, on an important concept which is called mode shape. And most of the cases in most of the literature in dynamics people do not bother about mode shape, which is I have actually a wrong character. People only worry about frequency. Mode shapes are very important do you have any difficulty or doubt. So, please look at my solved examples in the textbooks. So, for any research papers what we have with the reference try to get hands on experience on solving all these things. And try to have a time control. That is very important you should be able to solve the problem, in a given time of about 10, 15 minutes, by hand using a simple calculator at least for a 3 by 3 problem.

Thank you.