Dynamics of Ocean Structures
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Lecture – 20
MDOF - Stodola Method – Examples

We will look into the 20th Lecture now in Module 1. We will solve one more example in Stodola, we will compare the results what we got from stodola influence coefficient method and Dunkerley. We will also give you a coding in this class that how we can invert a matrix just because you want to know the stiffness matrix from the flexibility matrix let us try to do that.

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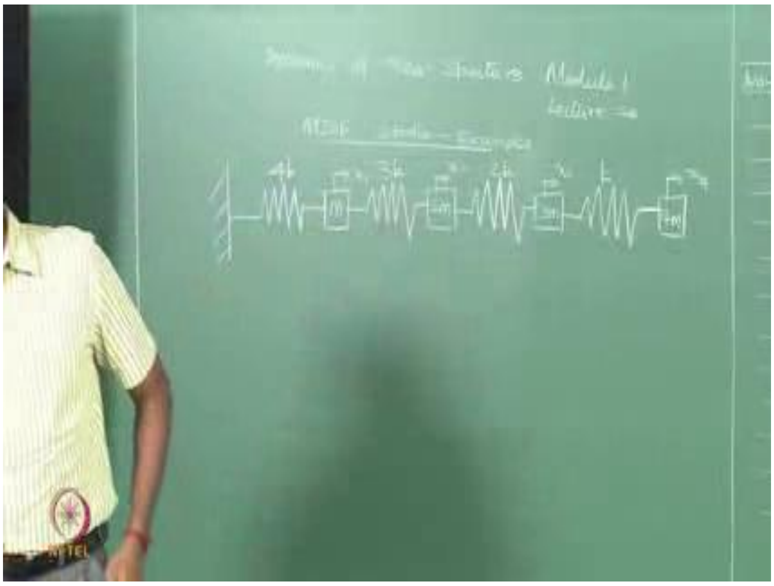

We will take an example - now in stodola let us try to solve this problem. We will take a 4 degree freedom system problem. Let us say this is 4 k, 3 k, 2 k and k. This is m, 2 m, 3 m, 4 m. This is x 1, x 2, x 3 and x 4. So, we will first solve the problem using stodola and try to find out the fundamental frequency and mode shape for this problem. (Refer Time: 01:28) deflection.

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Only enter the deflection. So, let us enter the values of k and m. I am entering the values as 4 k that is k 1, then m, then 3 k, 2 m, 2 k, 3 m, k 4, m. These are the values of respective m and k values given in the problem.

Let us start; I am interested in working out omega n that is the first fundamental frequency. So, all mode shapes will be positive. Let us start with the displacement expression on the mass as 1, 2, 3, and 4 all are positive value there is no 0 crossing, so we are expecting to get the first frequency. Let us find out the inertia force, let us take the mass out. We know that inertia force is proportional value of the displacement as omega square x i. So it is going to be 4 omega square, m I have taken out here before the multiplier of 4 is coming in. So, 6 omega square is 2 and 3, 3 and 2 again 6 omega square, 4 omega square. So, let us say 4 omega square this is the spring force and mass out, spring deflection divide the force by the stiffness of the spring.

Let us say m by k is out I am taking k out here so this becomes 5 omega square. So, let us say calculated deflection m by k constant out. Is started with 4, 3, 2, 1, whereas we are getting 1, 2, 3 close to 4. So, let this be the assumed deflection now. Let us continue with the second iteration (Refer Time: 04:58) banded value now. You are not matching therefore one more iteration this is the first scheme of iteration. Assumed value is 4, 3, 2, 1 obtained value are 1, 2, 3, 4 approximately I have converted them to assume now I am doing the second iteration.

Let us continue this now. So spring force. So spring deflection, calculated deflection. This is the second set. This becomes to be assumed deflection. That is the ratio, so there is no need to do write m by k here. So, we start with 12.066, 3.066 and 3.866. Anyway we got the convergence of this except for this case, so let us try to now revise it further.

Give me the values of the third and fourth iteration that is we getting the convergence in the third iteration or not. What are the values of third iteration? One, 2.0.

Student: 4.35.

4.35. So, this is the third iteration. Let us do one more. Can I have the values for the fourth iteration? Values for the fourth iteration, can you give any values of the calculated deflection also it is required because then only we can work out the ratio. Can you give me the values of the calculated deflection in terms of m by k?

Student: 10.373.

10.0.

Student: 373, 41.0 (Refer Time: 10:52).

41.0.

Student: (Refer Time: 11:02).

So, the ratio now comes to 1.

Student: (Refer Time: 11:24).

Let us say anyway this is converged, this is converged, and this is also more or less converged. In fact, this is also 3, 5 and 4 let us say you can try one more. Let us assume that this is converged value because, so now we can compute omega.

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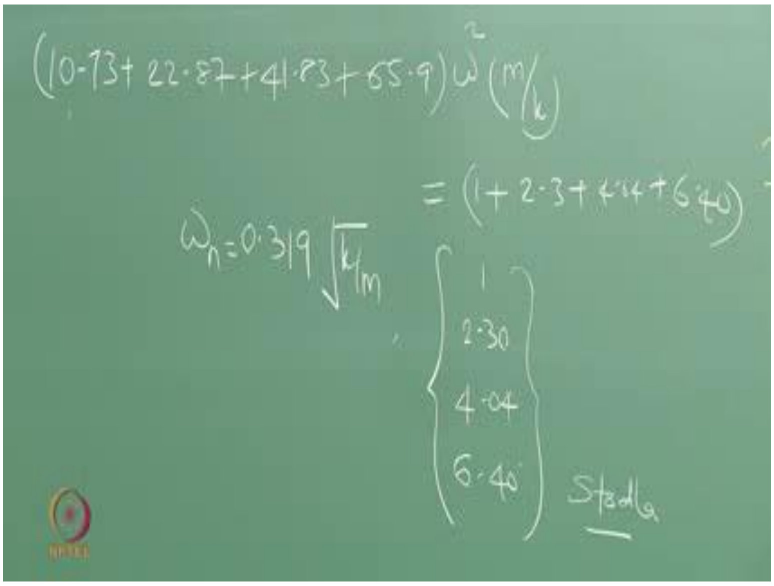

So, 10.73 plus 22.87 plus 41.83 plus 65.9 of omega square m by k should be equal to 1 plus 2.3 plus 4.04 plus 6.4. Can you get me the value of omega n, k by n? And the corresponding mode shape is 1 2.30, 4.04 and 6.40 that is what I got from stodola. Is anybody who has got difficulty in following this table or this method? We have demonstrated one problem on 3 degree one on 4; this is a similar way you can do that, any difficulty.

Let us derive the alpha matrix for this specific case so that we can have a Dunkerley's as well as the influence coefficient results also.

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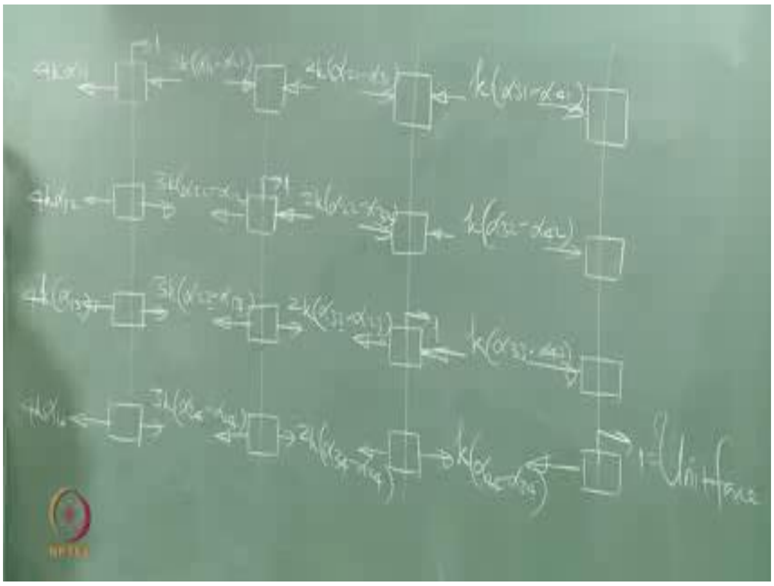

Let us give I have unit force here, in this case unit force here, in this case unit force here, in this case unit force here, so the first degree, second, third and fourth degrees respectively. When I tried to pull this mass towards the right by giving unit force this spring will oppose the mass. Let us start from here. This spring will oppose the mass, so mark both the directions parallel for the spring. This is going to be 3 k of alpha 11 minus 21.

Similarly, when this spring pushes this mass, this again will oppose because the mass will try to move to the right. So, the stiffness of the spring is 2 k of alpha 21 minus 31, because it is connecting to 13. Similarly, when this mass moving towards the right this spring will try to bring it back, so 4 k of alpha; similarly this spring will try to push the mass to the right and this spring will oppose the mass movement the stiffness of the spring is k this is alpha 31 minus 41. All will happen with the second subscript as 1, because we are giving the unit force in the first degree.

Similarly, you give the unit force in the second degree. The mass will move towards the right, so this spring will try to push it back and I am marking both the arrows parallely and the corresponding stiffness is related to 2 k, so 2 k of alpha 22 minus 32 because it is connecting 2 and 2. So, when this mass is moving towards the right this spring will try to pull it back, this spring the 3 k 1, so 3 k of alpha 22 minus 12 because it is connecting the coefficients of 2 and 1. And the second subscript stands for the unit force given at the second degree. When this mass is moving towards the right this spring will try to bring it back 4 k of alpha 12. So, this mass will try to move to the right because this spring is pushing k of alpha 32 minus 42.

Similarly, apply unit force to the third degree the mass is going to move to the right; this spring will try to push it back. The stiffness of the spring is k alpha 33 minus 43. So, the mass is moving to the right therefore this spring will try to bring it back 2 k of alpha 33 minus 23, 3 and 2 are the degrees of freedom. The second subscript three stands for the unit force given in the third degree.

Similarly this mass is moving to the right now so this spring will restore it back, so the stiffness of the spring is 3 k alpha 23 minus 13, because these are the coefficients which are connecting this spring 211. The second subscript stands for the unit force applied in the third degree. This mass is moving to the right now, this spring will try to restore it back the stiffness of the spring is 4 k that is what is here and alpha 13. The second subscript 3 stands that the unit force is applied in the third degree.

Similarly, give unit force in the last degree of freedom. This spring will try to restore it back. Stiffness of the spring is k alpha 44 minus 34. So, this mass will move to the right therefore the spring will bring it back stiffness is 2 k alpha 34 minus 24 because this spring is connecting 3 and 2. Similarly, the mass will move to the right this spring will try to bring it back, the stiffness of the spring is 3 k and the coefficients are alpha 24 and 14. The mass will move to the right this spring will try to bring it back, so 4 k of alpha 14.

So, the second subscript 4 means that this unit force is given at the fourth degree of freedom, whereas remaining all coefficients are respectively marked with the degree of freedom. Once this is done we can write the force equation I will remove let us take the first degree and write the force equation. Let us pick up here.

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So, 4 k alpha 11 plus 3 k of alpha 11 minus 21 should be equal to 1. 3 k of alpha 11 minus 21 should be equal to 2 k of alpha 21 minus 31 at the second here. 2 k of alpha 21 minus 31 should be equal to k of alpha 31 minus 41 that is the third equation here. The last one k of alpha 31 minus 41 is 0 there is no other force. This implies that k cannot be 0, so this implies that alpha 31 will be equal to alpha 41. Substituting this back here this goes 0 which implies that 2 k alpha 21 minus 31 will be 0, 2 k cannot be 0, so this says alpha 21 will be equal to alpha 31.

When I say alpha 21 is equal to alpha 31 I put it here this now sets to 0 this means 3 k cannot be 0, so alpha 11 will be equal to alpha 21. I substitute this relationship here this sets to 0 because they are equal, so I get alpha 11 as 1 by 4 k. So, which all will be 1 by 4 k, because alpha 1 minus 21 is it 31 and 41; I got the first column of the influence coefficient matrix which will give me 1 by 4 k.

Let me write down the matrix here 1 by 4 k, 1 by 4 k. Just for our understanding further I will write one more equation for the second degree, For our understanding I will rub this.

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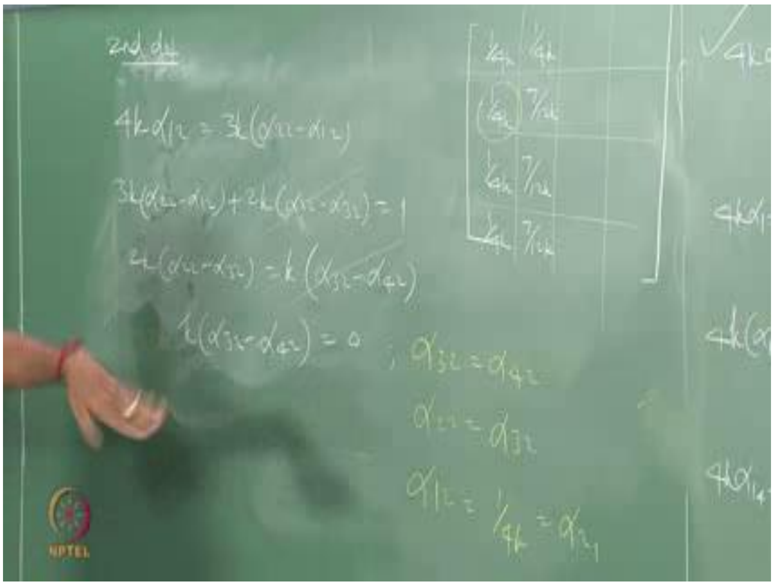

So, let us do it for the second degree here, 4 k alpha 12 which is for second degree of freedom. 4 k alpha 12 will be equal to 3 k of alpha 22 minus 12. 3 k of alpha 22 minus 12 plus 2 k of alpha 22 minus 32 will be 1. 2 k of alpha 22 minus 32 will be equal to k of alpha 32 minus 42.For the last one k of alpha 32 minus 42 is set to 0, k cannot be 0. This implies that now alpha 32 will be equal to alpha 42. Substitute back here this goes away 2 k cannot be 0 this implies that alpha 22 will be also equal to alpha 32. This makes this term as 0.

So, alpha 22 minus alpha 12 is 1 by 3 k I substitute that here, so 3 k gets cancelled this becomes 1. So, alpha 12 will be 1 by 4 k which is as same as alpha 21 which is here. So, the matrix is completely symmetric. I substitute back and get alpha 22 and then 32 and 42; I get the second column of this matrix now which is 1 by 4 k and 7 by 12 k. Once I get alpha out of 12 here you can substitute back in this expression and you can find alpha 22. Once you know alpha 22 you can find 32 and 42 which are going to be same I get the second column.

Now similarly I can get the third and fourth column I want you to write down them, but anyway I will fill up the matrix here. If any doubt for anybody in deriving the influence coefficient matrix directly like this form the force equations. Any doubt for anybody here? So, I will write down the remaining two columns also.

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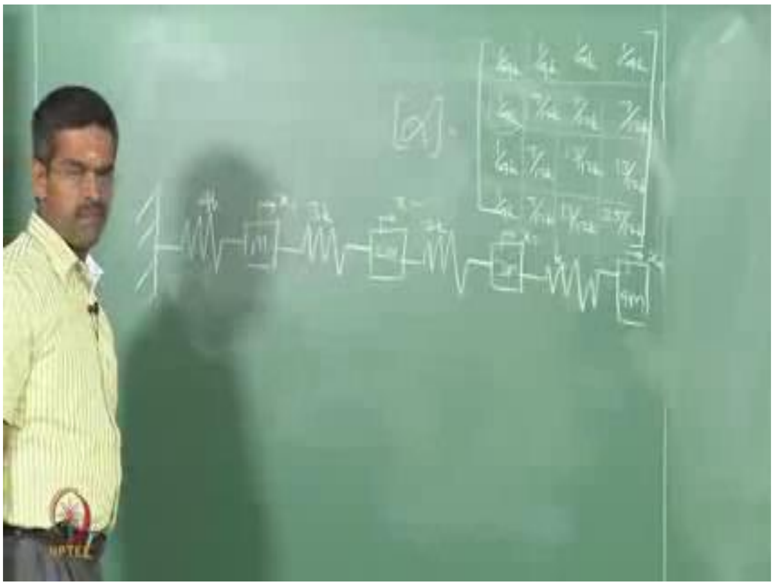

1 by 4 k, 7 by 12 k, 13 by 12 k, 13 by 12 k. 1 by 4 k, 7 by 12 k, 13 by 12 k and 25 by 12 k that is my so called influence coefficient matrix. It is nothing but the flexibility matrix; I can invert this matrix and get the stiffness. Let us try to get the stiffness matrix for this problem before we proceed further. Let us try to draw the problem again m 2, m 3, m 4; m 4, k 3, k 2 k and k; first degree, second degree, third degree, and fourth degree. Let us apply Newton's law and try to derive this stiffness matrix, because I am going to write the equation of motion now. So let us draw this separately.

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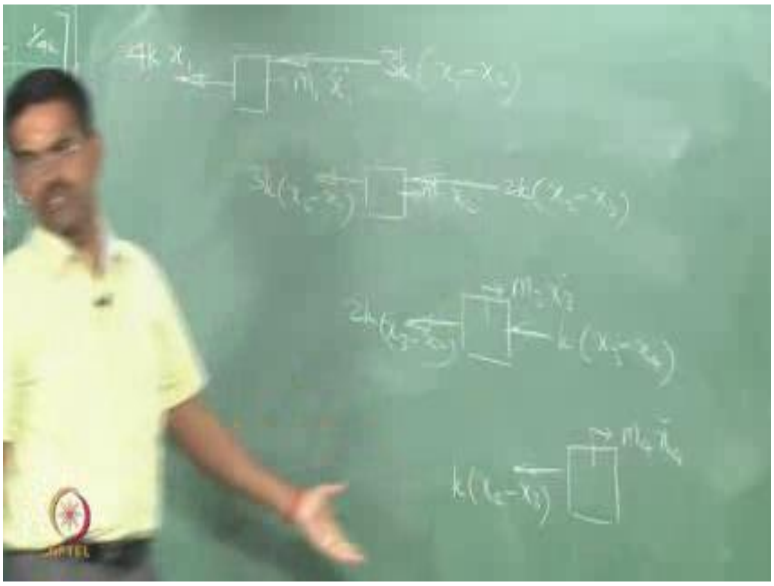

So, this is going to be m 1 x 1 double dot m 1 may be m, the restoring force going to be 4 k of x 1 and this will be going to push the mass back so I will get a force here which is 3 k of x 1 minus x 2. Force stiffness into displacement gives me the force.

Similarly, I can do it for the second mass also let us say this is m 2 x 2 double dot this way here. When I move this, this spring is going to pull this back; so this is going to be automatically 3 k of x 2 minus x 1. We already this in algorithm you start from x 2 apply this coefficient first and then the next one next. Similarly when you move this, this thing will push it back; so 2 k of x 2 minus x 3 because it is connecting 2 and 3.

Let us take the third mass. Let us say this is m 3 x 3 double dot moving this way I am going to restore it which is going to be 2 k of x 3 minus x 2 and this spring will push it back, so k of x 3 minus x 4. The last mass which is going to be m 4 x 4 double dot, this spring will restore it back it is going to be k of x 4 minus x 3. So, I have the four secular equations here now with Newton's law. Let us write the equations of motion and from that we can pick up this stiffness matrix easily. So, let us do the first one.

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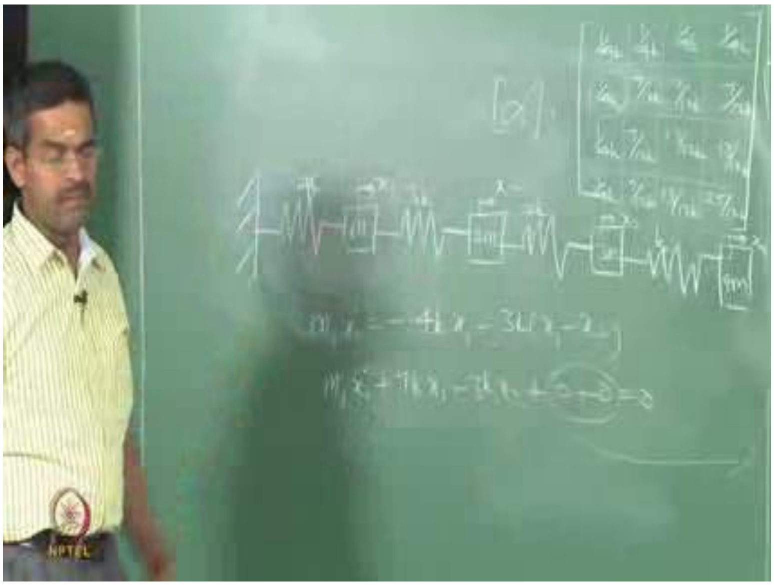

M 1 x 1 double dot should be equal to minus of because they are all opposite 4 k x 1 minus 3 k of x 1 minus x 2 if you give me m 1 x 1 double dot plus 7 k of x 1 minus 3 k of x 2 plus 0 plus 0 is 0. These are corresponding to x 3 and x 4, there is no value here.

Similarly, I can do it from the second; third; fourth I will get the stiffness matrix. What I want you to do is invert this and see whether you are getting the same stiffness matrix as you get from here. Now since it is 4 by 4 and n by n it is difficult to invert it by hand because you require some mathematical support.

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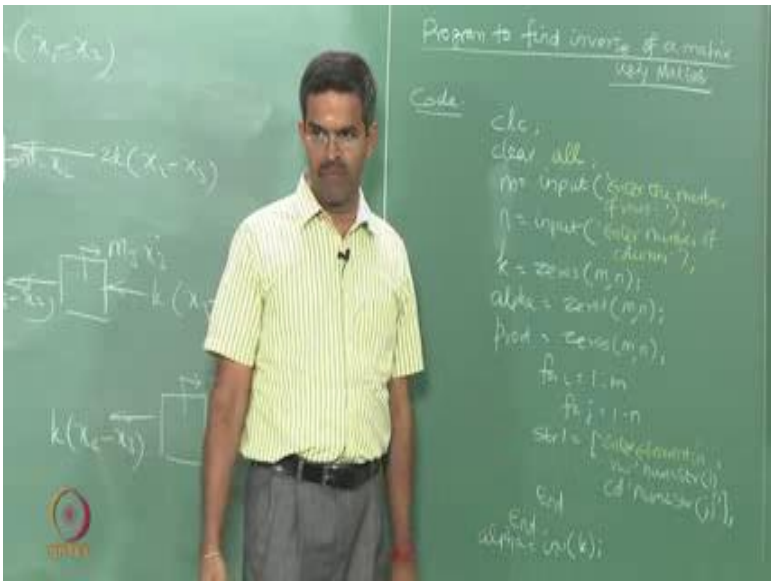

So, there is an equation available here, which is a simple program which can be used for inverting a matrix using a MATLAB program. You can use this MATLAB program; I will just read the program easily. So, the code is this clc is clear screen, clear all. All these yellow written here are all commentary statements. So, clear screen, clear all, m is the row and n is the column because enter the number of rows and enter the number of columns. In this problem you will say 4 and 4. Then k 0's initialization of m and m because sometimes there may be some 0 error available in the system it may take some values for m and n initialize them. Then you call a new matrix alpha which is the inverse of a given matrix. And let us say initially all values in this matrix are 0; all values are 0's so it.

Then you start I also want the product of this because I am going to check whether the inverted matrix and the original matrix product becomes an identity matrix, so I have a one more variable prod again initialize them. So, I run a loop for i and j, i is the row and m is the column because it is varying from n and this is n, so m is the number of rows and n is the column enter them and alpha matrix will be inverse of k. So, you will be able to get the matrix very easily. You can check this and run this program.

And you can check whether the k matrix obtained from the equation of motion here. I am sure all of you know how to write the equation of motion for the remaining three and get a k matrix. So, get a k matrix and check whether the inverse of this is as same as this. Do not be afraid that there are two values 0 here will it be problem you just see I checked it, it is perfectly it will be an inverse perfectly no problem on that.

Now our job is if I know the leading coefficients of this alpha matrix I can quickly find Dunkerley's frequency. Let us try to get the Dunkerley's frequency.

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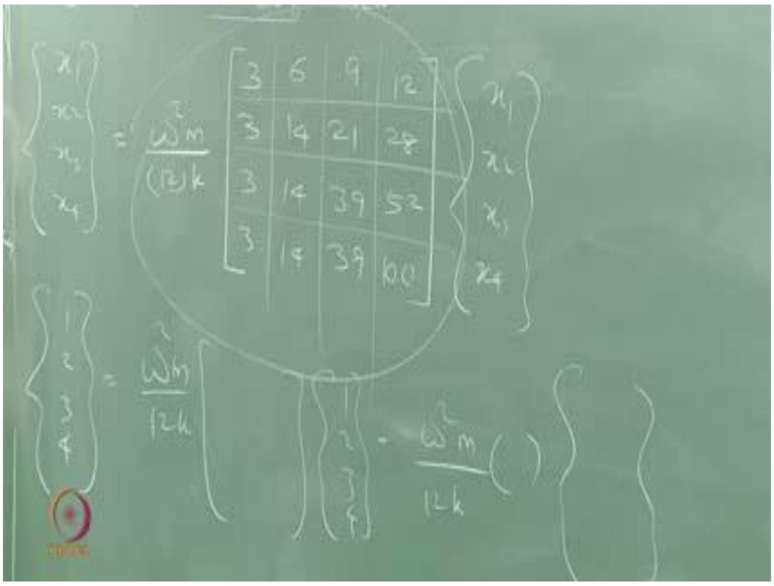

So, 1 by omega square is m i delta i in this case is i to 4. Can you give me the Dunkerley's value? M 1, m 2, m 3, m 4 are available to you and you already know the delta values leading diagonals, can you give me the value of Dunkerley's frequency which will lead to omega n as some value of; how much?

Student: 0.316.

0.316, everybody is getting the same answer?

Student: 0.2694.

It will be 0.27.

Student: 0.27.

So, that has we have got 0.31 there 0.27 here, it is again matching. Now the major problem starts with influence coefficient method where I want to set the matrix for iteration. So, what we have been doing is we are writing x 1, x 2, x 3, x 4 algorithm from that we multiply m i‘ s and try to get the control matrix which is set for the iteration. Now I will tell you here a shortcut how the control matrix can be directly written for influence coefficient method.

So, I want the control matrix now directly for the influence coefficient method. Let us write the control matrix here, so this is the vector which is getting iterated. We all know that omega square m by k will be a multiplier here in this control matrix. So, there is a 12 k denominator here, let me take this 12 out of x 1, x 2, x 3, x 4. This is a control matrix now for the first frequency. The moment it takes 12 k out there is a multiplier of numerator 3 here. There is a multiplier of numerator 3 here; this 3 multiplied by the m 1 gives me only 3. In all the cases 3 is out therefore 3.

So, in this case there is a 3 numerator here, because I have taken 12 as the denominator out here there is a 3. I multiply this with m 2 which is again 2, so I get 6 here, whereas, in the remaining 3 the 12 k is already in the denominator so no problem, so 7 with 2 14 remaining all 14. Similarly, go back to the third column there is a 3 numerator here multiplied by the 3 m, so I get 9. Remaining all I have 12 in the denominator there is no multiplying the numerator available 7 3’s 21, 13 3’s 39, 13 3’s 39.

Let us go to the fourth column I have 4 k I have 12 k here, so the numerator 3 multiplied by m 4 so 12. Remaining denominators are 12 so no issue, no multiplying the numerator 7 4’s 28, 13 4’s 52, 25 4’s 100. This matrix can be obtained in minutes. So, be careful in multiplying the terms.

So, let us do the same iteration of 1, 2, 3, 4 as we started with stodola with the same of 1, 2, 3, 4 and see what do we get.

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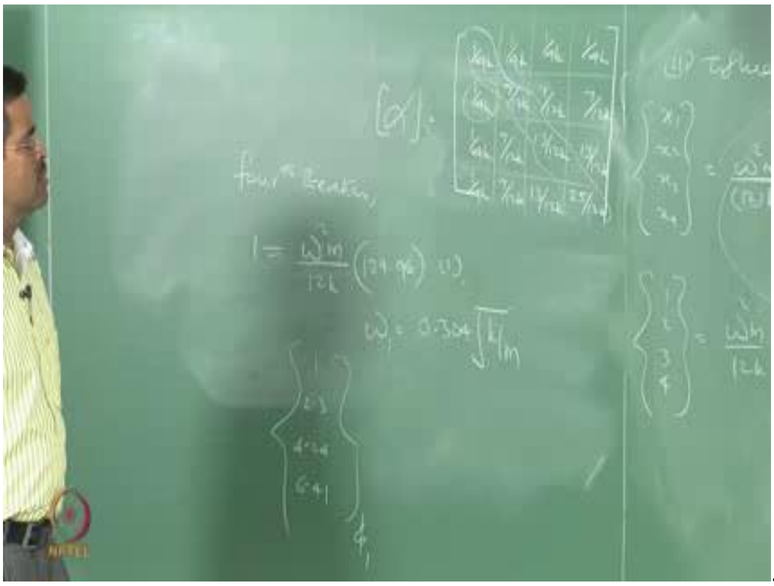
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After four iterations I get a multiplier which is 1 equals omega square m by 12 k of 129.96 of 1 which gives me omega as 5. What we get is 5. If you compare the results of Dunkerley as 0.277, then stodola is 0.31 and iterate influence coefficient method omega 1 and phi 1 as 0.304 you see all of them are almost in the same range within an error of about 5 to 7 percent. All the three methods can be easily used. So, that is a very interesting assignment for you.

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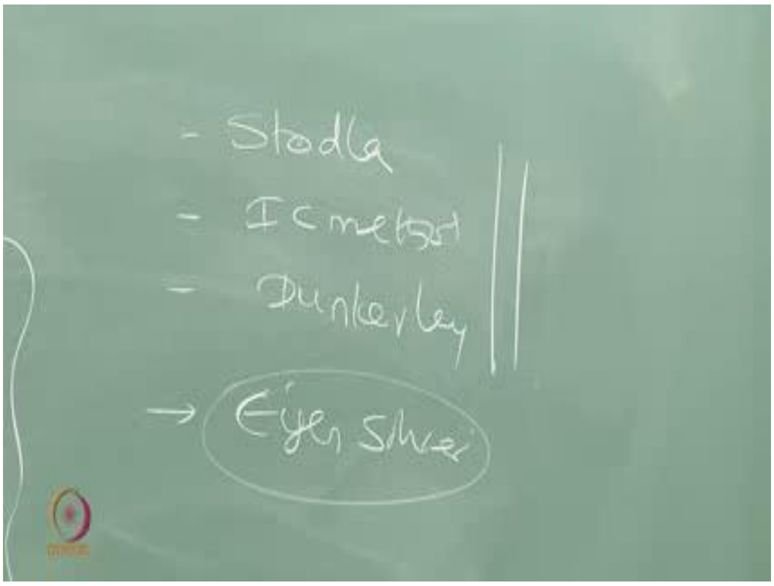

You have got to write a code in MATLAB which will help you to solve the problem when stodola, problem on influence coefficient method, problem using Dunkerley, and also a code for a classical eigen software. Of course, the classic eigen solver code is available in the MATLAB as an inbuilt protein, but these three are not available. So, you have to submit this code to me within three working days from today by email to me directly. We will have weightage of this in the final exam later. Within three working days you have to submit the code directly to me or email to me. So that I must have a problem of solved example of the problem taken in the class which we will solve by all the four will give me all the values and compare the percentage error between the methods.

Only first three submissions will be taken remaining I will not consider; only first three will be considered depending upon the time just complete it pdf it and email it to me. First three entries right or wrong only will be considered, fourth entry I will not consider. No evaluation for the fourth entry onwards only, first three I will take we will see. So, any doubt here?

We will move on to the next method in the next class. So, we have concerted about four methods which are very interesting which can be used for multi degree very comfortably. All are computer programmable, all are comparable, all results can be worked out using a calculator and I have solved the problem in the blackboard in 20 minutes using a fourth degree freedom system problem. One should be able to solve 6 degrees in about half an hour if you have a computer code. In few minutes you will be able to get omega and phi which is one of the major problem in many of the new generated structural form problems in offshore structures, because you would not get omega and phi so easily.

For getting omega and phi you must have mathematically model them, go to software available, do a finite element modeling, then try to do a free vibration and get omega and phi. It is a long process, but here it is very simple for a given form if you are able to idealize them as a spring mass system model this you can easily get omega and phi as a first stand value we can also compare them with four methods we can see which is the correct value. You can see whether the structure is exciting as a designer within the frequency band of a wave input so you change the form. So, we are talking about dynamic analysis and design together now, so it is very easy for us to know.

Thank you.