Dynamics of Ocean Structures

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Lecture – 19
Stodola Method (For MDOF Systems)

We will talk about the 19th lecture today on module one, on dynamics of ocean structures. Today we will discuss a new numerical method, which is given by Stodola which is applicable to elastic systems, as far as multi degree of freedom system is concerned. The one main advantage this system has, or this method has is, it will give you the fundamental natural frequency of the system automatically and the mode shape. It will give you a path.

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So, this gives you the fundamental frequency in the corresponding mode shape. And in this example in this class I will show you. I will pick up an example and demonstrate that I will solve the problem by all the three methods, namely Stodola which I will explain here, influence coefficient method which I will again explain, and Dunkerley method. You will see all of them are highly comparable then you get the same answer in all the three cases. So, this method is very interesting easily computer programmable. It is not problem specific, it is highly generic. This method has concept based on the proportional displacement of the mass points.

In fact, to be very specific we can say of the lumped mass points, under inertial forces. So, you have to assume a proportion of displacement between the mass points, and try to converge it. So, again you will see this method has got a very good correlation and understanding, between with that of the influence coefficient method, even in influence coefficient method also, you actually assumed displacement vector, and try to iterate the vector for it is convergence. This method will also do exactly the same thing, but it does slightly in a different manner, we will see how it can be done. This can be applicable to elastic systems. On the other hand the displacement offered by the system is not very large. So, let us take up a problem and demonstrate this method step by step. So, the problem is, the basic problem what we will have here, which I will demonstrate in all the three methods.

So, let us say I have a spring mass system. So, I have three mass points, let us say m 1 m 2 and m 3, let us say k 1 k 2 and k 3 or the stiffness of the springs, and the degrees of freedom are marked here as x 1 x 2 and x 3 as shown in the figure. Just for our understanding we will keep k 1 k 2 and k 3 same. I will do another problem where they are varied, just for understanding. And let m 1 m 2 and m 3 be same as m for this problem, but I will take another example where this is four k this is 2 k and this is k, we will just see how it can be easily handled, there is no confusion. Now we will demonstrate the Stodola method, by assuming a proportional displacement between the mass points at respective degrees of freedom.

Let us try to find out what will be the restoring force offered by these respective springs, to the respective mass points in the respective degrees of freedom, and try to see whether these vectors assumed in the beginning of the iteration are getting converged. There is a mathematical proof available in the literature the Stodola will converge to the natural frequency in mode shape. So, there are two issues; one is the Dunkerley, and the other is Stodola, we will deny this after we understand the method later. There is a mathematical proof available, where Stodola will converge to the fundamental of the lowest possible frequency in the corresponding mode shape. We will talk about that proof later; first let us understand the method.

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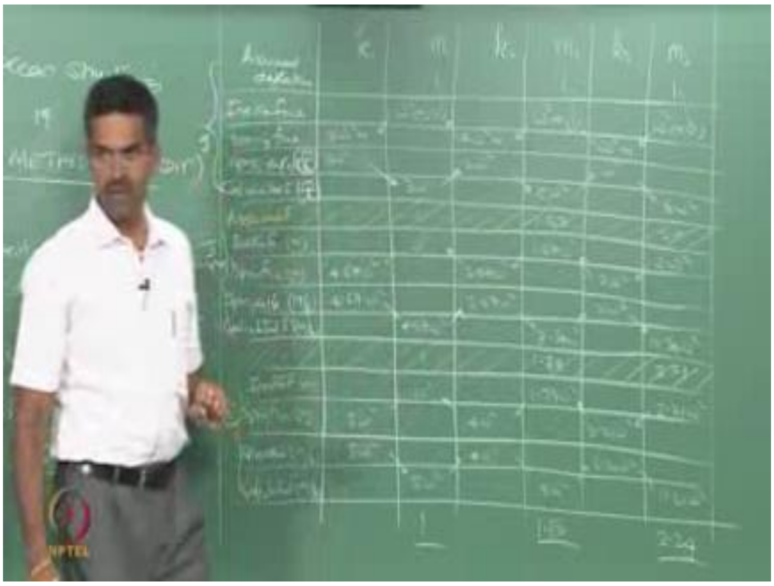

So, the method goes as a tabular form. Let us try to draw the tabular form here. Let us say assumed deflection. Let us say the stiffness points are k 1 m 1 k 2 m 2 k 3 and m 3. In this case they are all going to be k s and m s let us write down the values they are as such. Now I want to assume the deflection at the mass points, because I am always arguing about the proportional displacement of the mass points. So, in all these examples there is commonness. The commonness is the degrees of freedom are generally measured at the point where the mass is large; that is a very common practice in dynamics. So, since we do not know the proportion of the mass; that is what is the proportion of m 2 and m 3 versus m one. I mean which is heavier, we have no clue. Let us assume that all of them are going to be displaced by the same proportion, which may not be correct, but only one catch here is all of them are positive. It means I am looking for the fundamental frequency, I am not allowing the change of sign or a zero crossing in the mode shape.

So, I do not know I will start with 1 1 1. So, let us say the inertia force m x double dot is the inertia force, x double dot is omega square x I for a given harmonic excitation which already seen except for the change in sign; that is out of phase, acceleration and displacement will be out of phase, because they are displaced by nine I mean minus nines they are out of phase. So, we look at the absolute value, we are looking only for the proportional displacements between the mass. We can look for the absolute value because all of them are going to be negative. We need not have to look at the negative sign. So, I should say now as omega square m of 1 omega square displacement is the acceleration minus sign I am not considering; m is this m. Strictly speaking this should have been m 3, but since m 3 and m are same I am writing simply m. Similarly omega square m of 1 omega square m of one, this one is this displacement, had a I have 1 to four I would say 421 here at, so one and this.

Let us try to find the spring force; mass m 3 is the last point which is extended from the system. So, let us start from the spring k 3 here. So, simply say this is my spring force which is omega square m. Then when I go to k 2 the spring force of k 3 will also get added to it, two omega square m 3 omega square m. Once I have the spring constant k, I have the spring force f. I can easily find the spring deflection, which will be 3 omega square m by k one, but it is k. So, I can write m by k outside here, I can simply say, it is 3 omega square. Now you may ask me a question sir here the masses are all 1 1 1 and k is all 1 1 1 or k. So, you are able to take out a constant here. What happens if this is m this is 2 m or this is 4 m, and this is k this is 4 k and this is 6 k, still m by k can be taken out that multiply them (Refer Time: 10:07). On the other hand it is very important for us to establish the proportional mass of m 2 and m 3 in terms of m 1 all the time.

Similarly, the proportional ratio of k 2 and k 3 in terms of k 1, it means I have a common multiplier this is only to facilitate the unit's of iteration. If you have a larger unit iteration becomes difficult, just to facilitate, because mass and k values will be very high, in the real problem right 10 power 6 or. We need not have to have a multiplier of this order here, we take it out, keep that value as a multiplier, and we can use it later. So, get only the proportional inside. So, k has gone here. Similarly this will be 2 omega square, this will be omega square m by k is gone out. Now I want to find the calculated deflection, let us say m by k is available here. When I start doing the calculated deflection I will start from the first degree here, because the deflection of the second and third degree will be cumulative from the first. So, start from here.

So, this is going to be 3 omega square m by k is available anyway here, then add this and take it here as 5 omega square, add this and take it here as 6 omega square. Now I have assumed the deflection of 1 1 1 of the mass point, I got a deflection of something as 3 omega square 5 minus 6 omega square. I want to find the proportion of this, because this is also a proportion only. If I say this as one this will be 1.67, this will be 2, let us go for a two decimal convergence. You can also go for n number of convergent digits. So, this becomes my assumed deflection for the second iteration. Why I am banding this, because we start up with 1 1 1 it is not converging at 1 1 1, it is coming to be 11.67 and 2, they are not converging. So, I have to go for a next iteration. So, can we repeat the same set of procedure again here; let us try to fill up the table. So, I have inertia force here, I can take multiplier m out. So, this should be 2 omega square, this should be 1.67 omega square, this should be omega square, I have taken m out, then I can work out the spring force.

Let us say again m out. So, this is going to be 2 omega square 3.67 omega square 4.67 omega square. I want to find the spring deflection; I take m by k out here. So, 4.67 omega square 3.67omega square 2 omega square. Now I want to compute the calculated deflection m by k constant out here 4.67 omega square 8.34 10.34, I want to find the ratio of this 1.785. Now let us say 792.21. So, we started with one 1.67 and 2.0, but we landed upon 1.79 and 2.21. So, this again banded. So, let us do one more iteration. Kindly fill up the third scheme, this is three, this was two, and of course, this was one. So, inertia force, spring force, spring deflection, calculated deflection. So, 2.21 omega square 1.79 omega square omega square. So, 2.21 omega square 4 omega square 5 omega square, let us say five omega square four omega square 2.21 omega square 5 omega square nine omega square 11.21 omega square ratio becomes 1.80 2.24.

One can verify this data, once again back by putting this and see in the fourth case, you will get back the same value as 1 1.0 at 2.24. You can otherwise also see here this is converging, this is more or less equal to this, the second digit is only the variable, and the first digit is converging. So, it will converge. Now, let us quickly look at the discussion of this particular table. Now let us look at the crooked points where we can go wrong, where we can go wrong when making the table. You should always enter the table in the same format as x 1 x 2 x 3 on the other hand x 1 with k 1 and m 1 x 2 with k 2 and m 2 x 3 with k 3 and m 3; therefore, x 1 x 2 x 3; that is the first order, which we will follow. The second order will be always m and k should be expressed as a ratio; you must take m and k common out, and enter only the proportional values of m and k here. The third which is very tricky, now at 1 1.67 and 2 at the second iteration, if you would have rounded up this to 1.7 let us say and did, you will never get convergence, you have to maintain the value as it is.

No truncation should be done at this level for the next iteration, never, try to continue with the same value as it is. And the fourth very tricky difficulty is, spring deflection calculated deflection, there are two deflection terms here; one is entered at the mass values, one is entered at the stiffness values; that is why I am deliberately writing, this is spring deflection, this is deflection of the mass point please understand, both of them are deflection only. One I am writing only at the location of stiffness of the springs, one I am writing at the location where the mass is you are always finding the proportion of the mass, but not the proportion of the stiffness of the springs please understand this, both are deflections only.

This is another problem where generally people make a mistake. Now, from this how do we get omega that is our important issue. Now the original deflection, is actually not 1 1.8 and 2.24, it is actually 5 9 and 11.21 omega square m by k, which are converted to ratio of 1 1.8 and 2.24, it means if the frequency of the mass 1 is 1, mass 2 is displaced by 1.8 of that of one, and 2.24 of that of mass one at a specific omega; that is the meaning of this. Therefore, these two values should be connected by the multiplier of m by k and omega square. Let us do that here.

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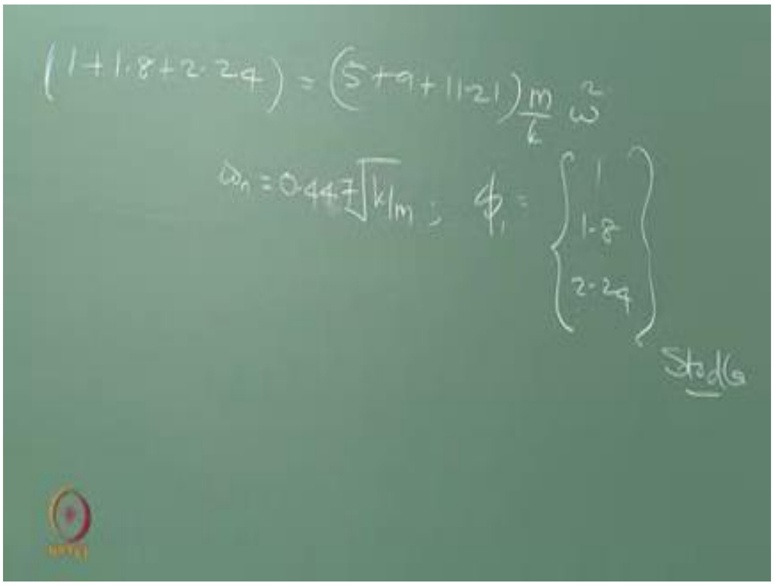

So, 1 plus 1.8 plus 2.24 should be equal to 5 plus 9 plus 11.21 of m by k omega square. So, let us get omega some value of k by m, how much is this value. Let us say very specifically, is it 0.48 or 0.447 something.

Student: 0.447.

K by m, this is my fundamental frequency, and the first mode shape, is 1 1.8 2.24, this is what I got by Stodola. As you know in engineering practice it is very simple, if you are proposing a new scheme of iteration or any numerical algorithm you must always show, the validations algorithm with respect to the established existing algorithm; that is always a practice in engineering. When Stodola was introduced already influence coefficient method, and Dunkerley method were in existence. So, one has to actually compare these values with both of them which we will do now.

We will see how they are comparing, people you may have a question in the exam that a table of this order is given to you with convergence, omega and phi are given to you, some of the values in between are erased, you have to fill up that. If you know this table thoroughly you can do that, otherwise you will not be able to do it. I will just remove some of the values in between the table for a ten degree freedom system problem let us say, which we cannot imagine by easy calculation, unless you know how is the procedure.

So, that is why I am telling you please follow the table carefully how the hierarchy was developed, and what is the reason why we started from the third x 3 and then came to x one, because that is a cumulative deflection of the spring; that is how it is done. So, now, I want to calculate the fundamental frequency, but not the mode shape from Dunkerley. So, I must have the delta matrix with me, I can directly derive the alpha matrix which is also useful for influence coefficient method, from that I can pick up the diagonal elements I can also find Dunkerleys frequency from there. So, let us try to derive the influence coefficient directly; that is alpha i j. Alpha i j is otherwise called flexibility matrix, where for unit force we are trying to find the displacements at different degrees of freedom, keeping all other degrees of freedom constraint.

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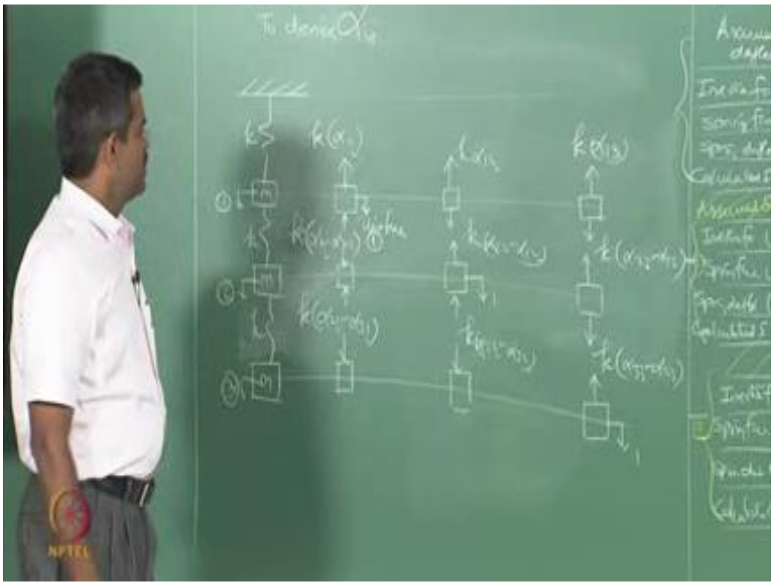

So, to derive alpha i j which are called influenced coefficients. We have already given in algorithm in the last class; let us try to derive the alpha matrix directly. There are two ways of doing this; one you can convert them into equivalent springs and keep on deriving which is a lengthy procedure, we can directly derive alpha i j by forming an algorithm as we discussed in the last class, let us try to do that, because I did that problem slightly faster in the last class, let us repeat that for this particular example, so that let us understand how this was derived. So, this is my first degree, this is my second degree, this is my third degree.

Let us say we will keep all of them as ks, and all of them as m in this example. So, let us apply unit force here, unit force I am talking about alpha i j for stiffness during displacement k, here it is force. The moment I try to apply unit force here, please watch carefully here, because if you are able to understand this I think, the first module essence is completely understood by you, because it is very difficult for you to understand this otherwise, please understand this carefully here. So, when this mass is moving down by an unit force this spring will try to push the mass up. So, I mark the arrow in this fashion. So, it is always marked as a pair. When this mass moves down further, again this spring will try to push it up, so I am marking it as a pair. When this mass goes down this spring will try to pull the mass up, I am marking it here.

Of course here there is no displacement it is a fixed and I do not need any connection here. So, this value will be alpha, the stiffness of the spring multiplied by the displacement which is alpha 1 1 minus 2 1. This is going to be stiffness of this spring alpha2 1 3 1. This is going to be stiffness of the spring alpha one. So, the second subscript here will all be unity indicating that we have given unit force at the first degree of freedom. The first subscript here will all refer to the respective values of degrees of freedom, accordingly to where we are measuring it. Similarly let us do the same procedure here, unit force here. So, when this mass is moved down, this spring will try to push it up. So, it is always marked as a pair. When this mass moves down this spring will try to push it up, always marked as a pair. When this mass moves down, this spring will try to push it up.

So, this is going to be k of alpha 2 2 minus 3 2, alpha 2 2 minus 1 2, k of alpha 1 2. The second subscript all will indicate to showing that, we have given unit force at the second degree, and the first subscript all will refer to the respective degrees of freedom as we have measuring the forces. Similarly, the third one give unit force at three, when you try to pull the mass down, this spring will try to push it up. So, always mark it as a spare. Similarly, when this moves down this pushes up, mark it as a spare, when this moves down this will push up. So, stiffness of this spring multiplied by, stiffness of this spring multiplied by, stiffness of this spring multiplied by, this is one. So, I want to write the force equally of equations for this, let us do it here.

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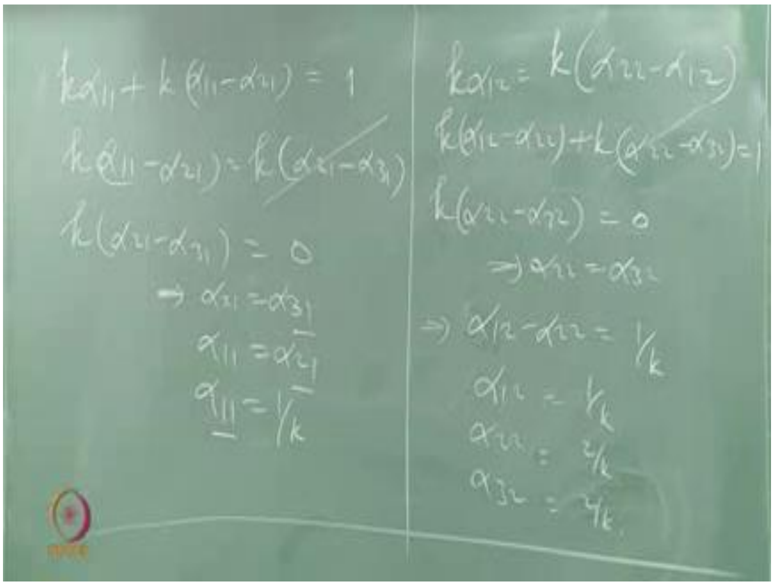

Let us pick up this k of alpha 1 1 plus k of alpha 1 1 minus 2 1; that is these two should be equal to one, why because these two forces for this mass point are upward, but this is downward. Similarly here, k of alpha 1 1 minus 2 1 should be equal to k of alpha 2 1 minus 3 1, these two, they are opposite, and here k of alpha 2 1 minus 3 1 should be set to 0, because there is no other force. So, k cannot be zero therefore, this is zero, this implies that alpha 2 1 will be equal to alpha 3 1. Substitute that back here this term goes zero, which means k cannot become zero; therefore, alpha 11 will be equal to alpha 2 1, substitute this term here this term goes zero alpha 1 1 will be 1 by k . So, I am now getting 1 1 2 1 and 3 1.

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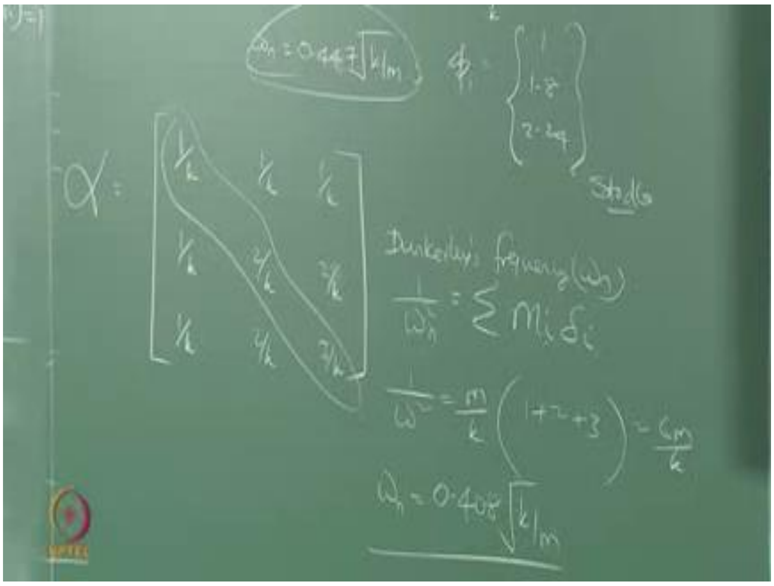

The first column I have got all will be one by k. So, let me write down the matrix here, alpha matrix. Similarly let us do it for the second one, let us pick up here k alpha 1 2 should be equal to k alpha 2 2 minus 1 2. I am talking about these two then here k of alpha 1 2 minus 2 2 plus k of alpha 2 2 minus 3 2 is 1, then last k of alpha 2 2 minus 3 2 is 0, which implies that k cannot be zero. So, alpha 2 2 should be 3 2 substituting back in this equation, this term goes away, because they are equal. So, again alpha 1 2 minus alpha 2 2 is 1 by k, I am substituting it here, alpha 1 2 minus alpha 2 2 is 1 over k, I have substituted that value here. So, can you give me the value of alpha 1 2 and alpha 2 2, there are two equations now here, I will substitute this back here. I know alpha 2 2 is alpha 3 2, can you give me alpha 1 2 2 2 and 3 2 as if you know these two this can be equal.

So, this is 1 over k this is 2 over k and 2 over k am I right. So, let us write down this as 1 over k 2 over k and 2 over k. Similarly can you get me the value of the third column writing the force equation for this third case, third column. So, this is going to be 1 by k 2 by k 3 by k am I right. So, Dunkerley says. So, where the mass points all m 1 m 2 m 3 equal to m and delta is nothing, but 1 by k. So, 1 by omega square will be equal 2. So, only the leading diagonal elements, because I (Refer Time: 31:44) elements only. So, 1 plus 2 plus 3 which is 6 m by k, can you find what is omega n.

Student: 4 1.

41, what is the third decimal. So, there are more are there, verify. Let us quickly do this for influence coefficient, for first degree alone.

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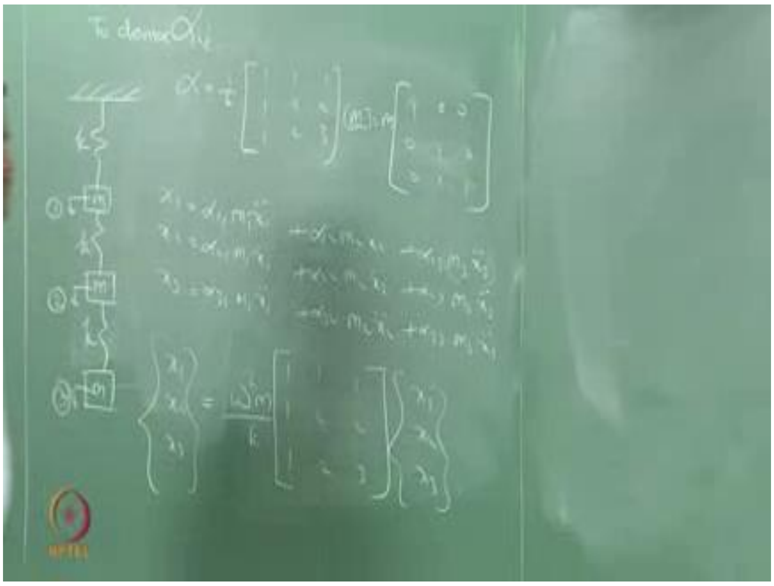

So, alpha matrix is available here one by k of 1 1 1 1 2 2 1 2 3 and mass matrix is m 0 0 0 m 0 0 0 m. It can be also m of let us say 1 1 1. So, x one x 2 x 3 alpha 1 1 2 1 3 one alpha 1 2 2 3 2 alpha 1 3 2 3 3 3 m 1 x 1 double double (Refer Time: 32:22) that is the algorithm. Then I can replace the inertia force as an equivalent displacement force as omega square x I i get a vector, I write directly the control matrix. The control matrix now is going to be x one x 2 x 3 which will be equal to, omega square m by k. Let us start with the first iteration as 1 1 1, and get me the multiplier of omega square m by k of the vector; the multiplier is 3 and 11.672 and so on, let us continue. You will see that there is a very close agreement between these vectors with influence coefficient method with that of Stodola, exactly you can say.

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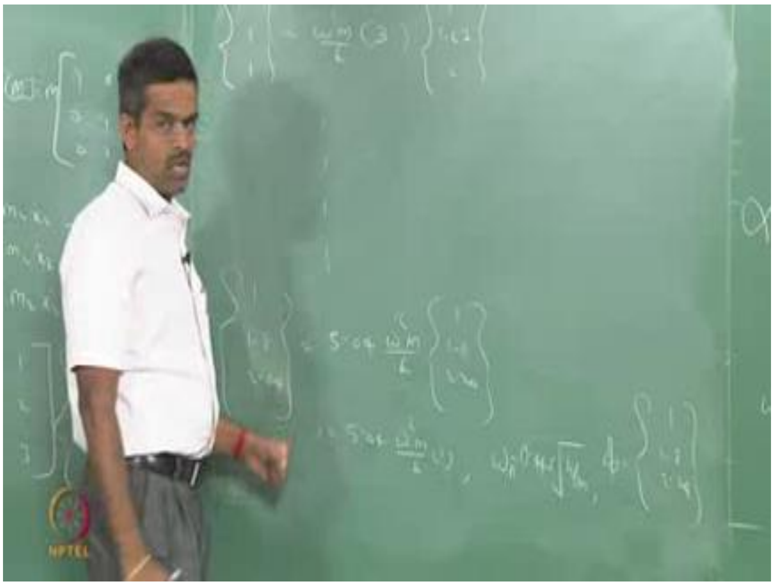

You just converge you will see as you proceed, you will get the final vector as 1 1.8 2.4, 2.24 sorry, with a multiplier of 5.04 omega square m by k of 1 1.8 2.24. So, I can now read one is equal to 5.04 omega square by k of 1 which gives me omega s and the phi value is, and this value is going to be how much.

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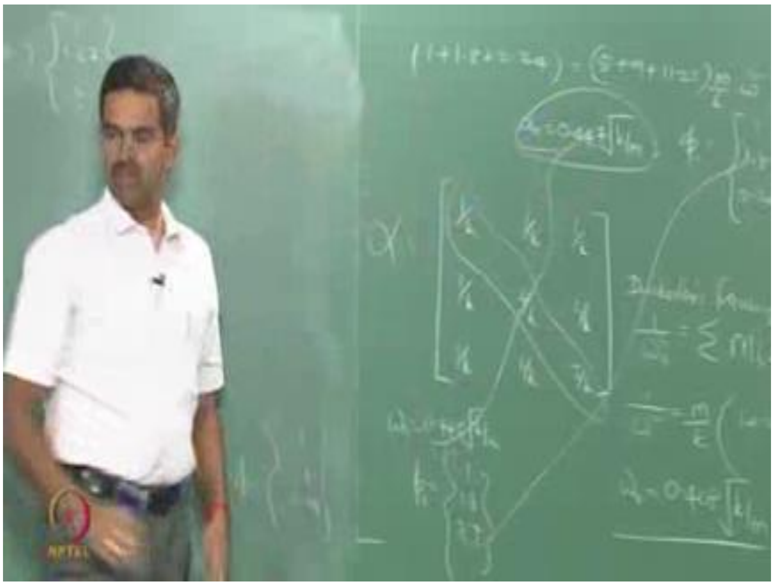

So, this value is going to be 0.445. So, this Dunkerley and omega here, is 0.445 k by m and phi one is 1 1.8 2.24, is exactly same as that of Stodola, and this is exactly same as that of Stodola. So, there is a complete convergence and matching between the three methods. The next example next class we will take up one more example of four degree of freedom system model, and again demonstrate the problem with Stodola influence coefficient method Dunkerley you will see that, this method will be very promisingly converging for higher degrees of freedom. Now Stodola is considered to be one of the most effective tools, of finding the fundamental frequency in the mode shape which has supersede the use of Dunkerley, because Dunkerley had only one difficulty, it does not give me the mode shape.

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Now there is a very interesting question asked to you why Dunkerley cannot give me the mode shape, what is the problem? So, that is a very interesting question you have got to answer that, why Dunkerley is not focused on mode shape, and influence coefficient method and Stodola are comparatively matching, but Stodola is more popular, because Stodola focus only on the fundamental frequency in mode shape, and this method is highly generic and the convergence is guaranteed and it is very fast, and higher modes if you are not bothered about one can easily find, the lowest possible mode with Stodola.

Now, I already said you that the influence coefficient methods convergence or the time taken for iteration is totally depend on, what is the vector you have assumed for iteration scheme here, this is true in Stodola also, but the convergence of Stodola will be much faster? Why, two questions, why, Dunkerley is not focused on mode shape and why

Stodola, though is also dependant on the displacement proportionality of the mass points, but still Stodola will converge faster than influence coefficient method why, there is a problem in the algorithm, please look into that and see tell me what is the answer for this.

Any doubt here in three methods now. We have got three methods now, I will allow another two more methods subsequently discussed in next classes. So, you will have five methods, there are around seven methods available in the literature, out of which we have spoken about five. I have already given you the Eigen solver also, if you include that as also a method, we have got four methods now available on hand, to solve any multi degree freedom system problem in general. Three an Eigen solver classical; four methods we have, I will have two more methods Rayleigh Rit's and Holzhauer.

So, three more around six seven methods are available on hand, you can use any one of the methods to find out omega and phi. Any doubt for anybody in these particular algorithms all the three. So, you must practice the problems by taking more number of let us say spring mass systems, and note down the time how much time you take. So, on an average you should not exceed or may be about 10-15 minutes to solve a single problem. If I have been asked to solve all the three mode shapes and frequencies in influence coefficient method or maximum 5 to 7 minutes by hand, we want to find out only the fundamental frequency in the mode shape; that is a time limit you must train yourself to solve this problems by hand using calculator within about 5 to 7 minutes for three degree freedom system problem.

So, try to have a practice on these problems and there are many numerical available in the textbooks, try to put your idea on thoughts on that, try to solve them and try to answer those. Two questions what I asked, why Dunkerley is not giving the mode shape, and why Stodola is converging faster than influence coefficient method.

Thank you.