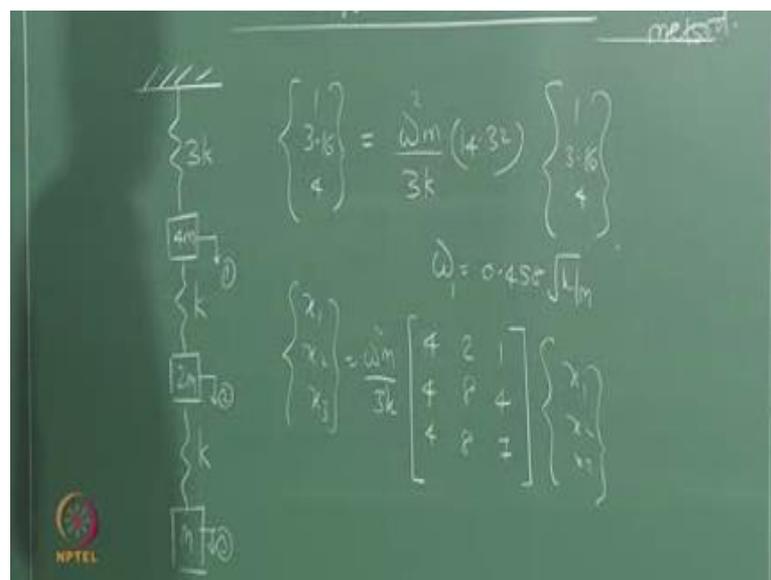


Dynamics of Ocean Structures
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Lecture – 18
Numeric Example: MDOF influence Coefficient Method

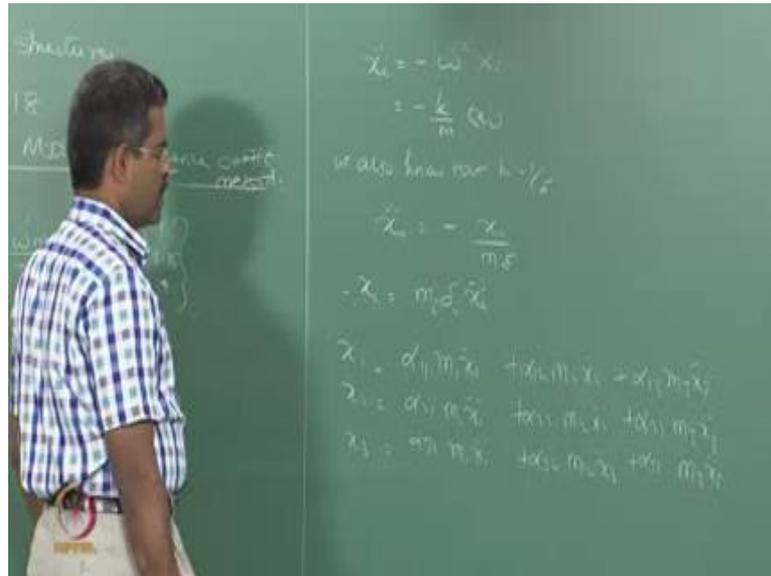
In the last lecture, we discussed about the Dunkerley's Method and Comparison of the First Frequency with the Influence Coefficient Method. We derive the first expression for omega 1 from influence coefficient method. We also explained you why this method is called Influence Coefficient Method, because the entire system is under the influence of the inertia force caused at different modes of vibration.

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So, we try to derive what, we call as the delta or the alpha matrix which is called Influence Coefficient Matrix.

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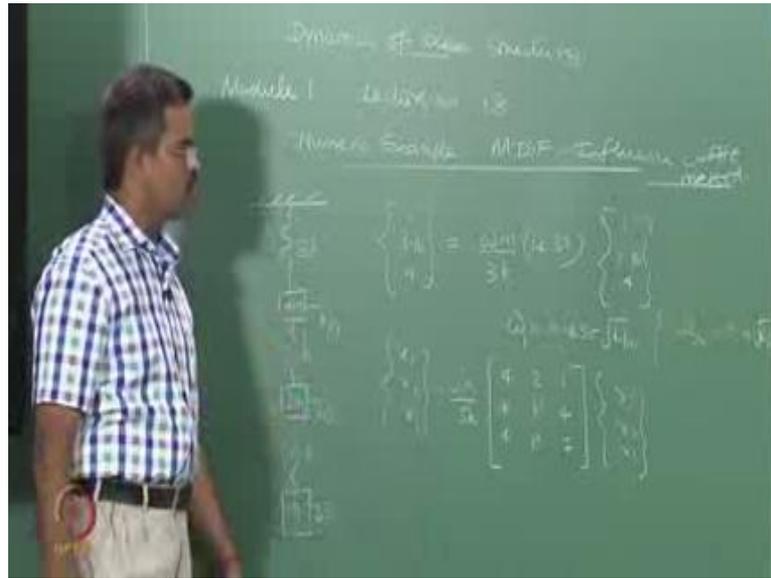


So, we had use this algorithm \ddot{x}_i is minus omega square x_i , which is minus k by M x_i because, we know omega square is k by M and we also know that k is 1 by let us say δ . So, \ddot{x}_i becomes minus x_i by $M \delta^2$. So, minus x_i is $m \delta^2 \ddot{x}_i$ and we have got this algorithm x_1, x_2, x_3 $\alpha_{11}, \alpha_{12}, \alpha_{13}; \alpha_{21}, \alpha_{22}, \alpha_{23}; \alpha_{31}, \alpha_{32}, \alpha_{33}$ $M_{1 \times 1} \ddot{x}_1, M_{1 \times 1} \ddot{x}_2, M_{2 \times 2} \ddot{x}_3$.

You will; obviously, see that the iteration will have to have with the alpha and M matrices. We have the M matrix for this problem separately done alpha matrix for this problem separately calculated then, we multiplied then we found the control matrix which is actually this matrix we multiplied. Because when say essentially the alpha matrix was $1 \ 1 \ 1 \ 4$ M is the M value.

Therefore, the first column has become $4 \ 4 \ 4$ and so on. We got this control matrix, now we have the vector on both sides which is an iteration vector, I assume the vector and keep on iterating it unless the, one which is assume the one what you derive, converges we say that it is converge at this vector and now, equated the 1 value as 14.32 multiplier of this one value and we got omega 1 . As this value in terms of k by M and its compare, this a very well with the Dunkerley Value, which is 0.4 root k by M 39 something a 0.4 which is a non iterative just only taking only the leading diagonal values of the alpha matrix we got this value.

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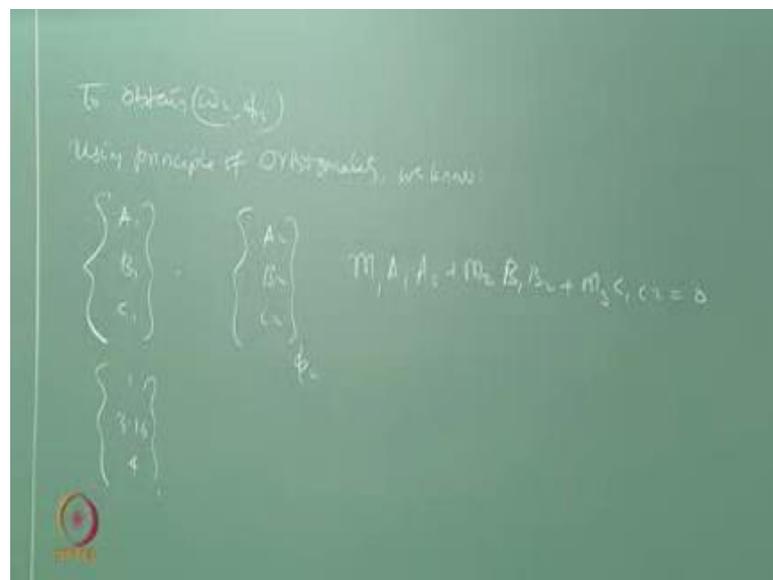
Now, the question is how to take it forward to find ω_2 and ω_3 . So, for this particular problem this becomes ϕ_1 and this becomes ω_1 . So, we got the first frequency in the corresponding mode shape and I can say the first frequency confidently because the mode shape is on positive value. We already know for a first mode shape there is 0 crossing.

Therefore, this called as a first frequency and we now understand Dunkerley's also gives me the frequency which is a fundamental frequency, which is also tallying more or less with the value I want to now find ω_2 ; ω_3 because this problem is a 3 degree freedom system problem. I must know the remaining ω (Refer Time: 04:08) we also said in the earlier lecture that once we have the set of ϕ 's with you if they are orthogonal to each other it becomes advantages with respect to mass matrix because, the calculation become simple. When you talk about the modal participation factors later, what we did is we found the vector from Eigen value problem and then we normalised it with respect to mass matrix. We got normalise vector by solving the Eigen solver, we do not get or we did not get the normalised or diagonalized vectors.

We normalise then, orthogonalize them with respect to mass matrix and we got the value. So, using that approach now in this particular problem I want to directly derive the orthogonal vector ϕ_2 and orthogonal vector ϕ_3 , which is orthogonal going to be with ϕ_1 and ϕ_2 . Now I am going to have a matrix of ϕ_2 , vector ϕ_2 that is the

Eigen vector which will remain orthogonal with phi 1 and so on and so forth. So, I am going to use that principle we already know that x_i and x_j will be equal to 0 if i and j are different if x_i and x_i product is 1. If i and j are 1 same, we know that value principle of orthogonality, we apply that logic here and try to get the second vector and the corresponding frequency please understand. In this method; in Influence Coefficient Method, the focus or the crust is not on omega it is not it is only on phi, where as in Dunkerley the focus was on omega and not on phi. So, you must understand the derivation in terms of the algorithm by different methods.

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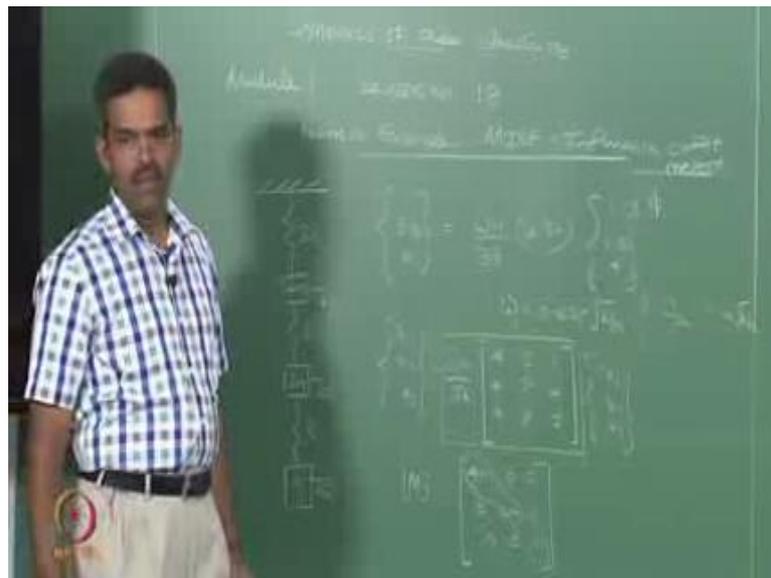


So, here the focus is not on omega it is on phi that is why we are iterating phi similarly since the focus for the entire set is on phi. I am trying to orthogonalize phi first and get the omega 2. Now to get omega 2 to obtain omega 2 phi 2, We know it is a pair it is an unique combination. We will principle of orthogonality using principle of orthogonality we know, let us see what do we actually know; I have a vector which is A_1, B_1, C_1 . I have another vector which is A_2, B_2, C_2 . Let us say this vector is nothing, but 13.16 and 4 and the vector which I do not know that is phi 2 is this vector, which is A_2, B_2, C_2 . Now I want orthogonalize these 2 vectors either respect to mass or with respect to stiffness. The advantage of doing mass matrix in this problem is mass matrix is diagonal because the degrees of freedom are mark in the location. Where mass in lumped. So, half diagonal multipliers will become 0 automatically, and second demerit in this problem is I do not have stiffness matrix for this problem. Already I told you to verify the influence

coefficient matrix and inverse of that we could have verify. So, stiffness matrixes do not have with me.

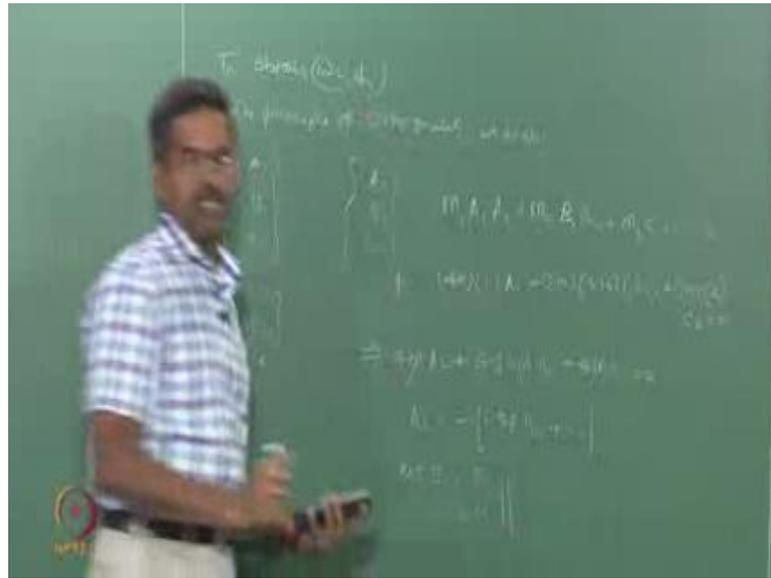
Therefore, I cannot diagonalize or orthogonalize these 2 vectors; respect to k because I do not have k because I need k to multiply them, I do not have k here in this problem I have derived k, but I am not having k here. Therefore, I am using m. So, let us write this condition M_1, A_1, A_2 plus M_2, B_1, B_2 plus M_3, C_1, C_2 is set to 0. Why first? Why M is getting multiplied because M diagonalize in this specified to mass matrix I do not have k I could do with k also. But I do not have k here. So, can why this is 0 because there are cross products, we already know x_{ij} i, j all are equal the product is should become 0. Actually if they are orthogonal to each other, so I am deriving a vector which is remaining orthogonal to a known vector conditionally. When they are multiplied with mass matrix that is what the condition here is.

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Let us do that. So, mass matrix we already have here in this problem, I will write down the mass matrix. Here these are the linear diagonal elements which are diagonally dominant and positive remaining all are 0 in this problem.

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So, $M_1 = 4M$, $A_1 = A_2$, I do not know similarly $M_2 = 2M$, $B_1 = 3.16$ and B_2 I do not know plus $M_4 = C_2$. So, let us explain this equation $4M$ of A_2 plus $6.32M$ of B_2 plus $4M$ of $C_2 = 0$. Therefore, I can say A_2 is minus of 1.58 because M goes away in any way, B_2 plus C_2 is it. Let us say; let B_2 be B_2 , C_2 be C_2 and this statement looks very funny. This is required I will tell you why there are 3 unknowns; here what are the unknowns A_2 , B_2 and C_2 , we also know that A_2 and B_2 and C_2 are not independent. They are relative because this is a mode shape mode, shape is nothing, but it is a relative displacement of mass position for a given frequency of vibration they are not unique. If I know 1, if we know the proportion; I can remaining find remaining B_2 and C_2 . So, I have only 1 equation I have 3 unknowns that is A_2 , B_2 and C_2 .

So, it is very interesting I cannot solve this problem therefore, I have founded 1 equation the second and third equation I am generating myself saying let B_2 be B_2 and C_2 be C_2 . Now you may ask me what is the advantage of writing like this, I can convert this now into a matrix form see how I am doing it.

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$$\begin{Bmatrix} A_2 \\ B_1 \\ C_2 \end{Bmatrix} = \begin{bmatrix} 0 & -1.58 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} A_2 \\ B_1 \\ C_2 \end{Bmatrix}$$

Let us say A_2 B_2 C_2 vector can be 0 0 0 minus 1.581 0 0 1 of A_2 B_2 C_2 is that am I reading the same value in a matrix, from there is a minus here because this negative is multiplied to both the values I have got. Now a matrix which is new, now we have got apply this matrix multiply with existing control matrix. The existing control matrix is this first why, we have to multiply the answer is very simple the vector A_2 B_2 C_2 should remain orthogonal with that of in existing vector, which is derived from this condition only this vector is true this vector is true only for this condition.

So, we this the control condition, which makes this vector true if this vector is taken as a multiplier here then, I must use this multiplier with the corresponding multiplication of this vector or this matrix. So, let us now do that I will remove.

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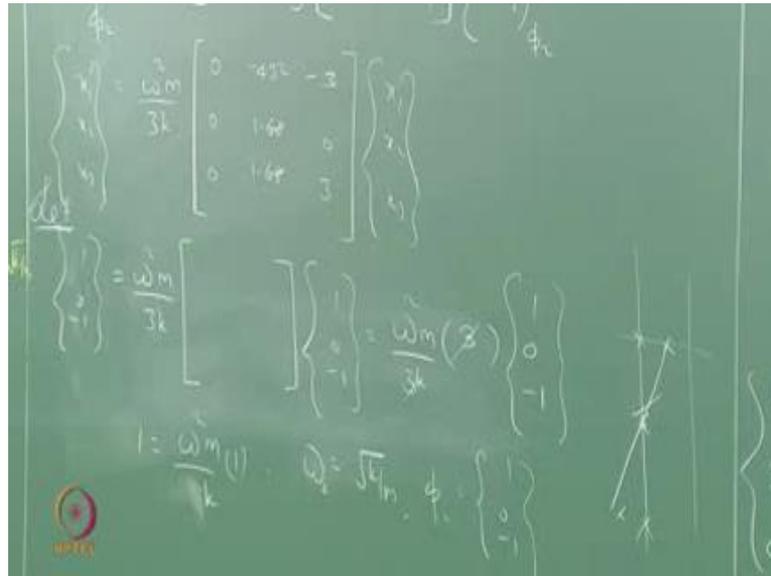
$$\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \frac{\omega^2}{3k} \begin{bmatrix} 4 & 2 & 1 \\ 4 & 8 & 4 \\ 4 & 8 & 7 \end{bmatrix} \begin{bmatrix} 0 & -1.58 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix}$$

$$\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \frac{\omega^2}{3k} \begin{bmatrix} 0 & -432 & -3 \\ 0 & 1.68 & 0 \\ 0 & 1.68 & 3 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix}$$

So, I am now looking for the new vector instead of calling A2 B2 C2, I will call this as conventionally x1, x2, x3 conventionally I call this as, let us say for our understanding phi 2 second vector Eigen vector.

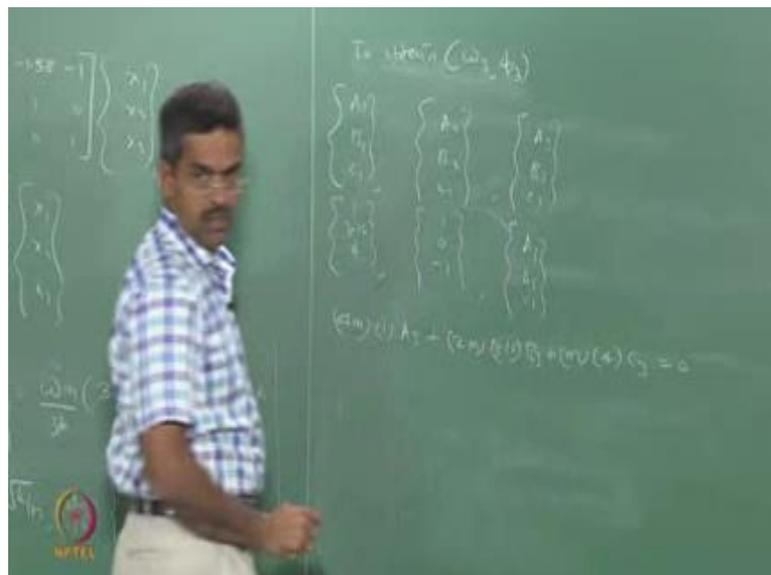
We should be now equal to omega square M by 3k of 4 2 1; 4 8 4; 4 8 7; 0 minus 1.58 minus 1 0 1; 0 0 0; 1 of x1, x2, x3 for our understanding, this is right. Now I got an iterative scheme, now I have got left hand side vector right hand side also A, vector I do not know either of them. So, I will assume 1 I will get the other 1 by convergence I will write to quantify them first let us multiply this 2 matrices and get a single matrix. Which is omega square M by 3 k of x1, x2, x3. Now we now we are talking about the second vector. There is 1 0 crossing, let us assume a vector as 1 0 minus 1 these assumption. So, I am not writing this matrix back again. So, omega square M by 3 k of this matrix multiplied by 1 0 minus 1. Let us see what do I get I get some multiplier of a vector tell me what is a vector we are getting.

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What is a multiplier? You are getting back, the same vector fortunately for this vector assumes. So, its converging because assume 1 and the derived 1 are same therefore, I can equate and read 1 is equal to omega square M by k because this is a multiplier of 1 on the other hand omega is root of k by M that is a second frequency and the corresponding mode shape is 1 0 minus 1 if you try to plot this is a 3 marks question let us say this is M this is positive this is 0 negative you get 1 0 crossing. So, it is a first 1 0 crossing means second mode n minus one. So, second mode let us try to get the third one.

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So, now to obtain omega 3 5 again principle of orthogonality, let $A_1 B_1 C_1$; $A_2 B_2 C_2$; and $A_3 B_3 C_3$ be orthogonal vectors. So, this vector already we know 13.164 this vector also we know this vector, we do not know. So, I should say 4M of 1 of A_3 4M is a mass matrix; M_{11} is this value A_3 is this plus 2M of 3.16 of B_3 plus M of 4 of C_3 should be set to 0 because they are cross products.

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The image shows a green chalkboard with handwritten mathematical equations. At the top, there is a vector $\begin{pmatrix} A_3 \\ B_3 \\ C_3 \end{pmatrix}$. Below it, two equations are written:

$$(4M)(1)A_3 + (2M)(3.16)B_3 + (M)(4)C_3 = 0$$

$$(4M)(1)A_3 + (2M)(0)B_3 + (M)(-1)C_3 = 0$$

These equations are then simplified to:

$$4A_3 + 6.32B_3 + 4C_3 = 0$$

$$4A_3 = C_3 \quad A_3 = \frac{1}{4}C_3$$

$$6.32B_3 + C_3 = 0 \quad B_3 = -0.79C_3$$

Similarly, the second equation I am now talking about these 2 communications; 4M of 1 of A_3 unfortunately these 2 terms will look alike this term is, from this multiplier to A_3 where are this term is M of this multiplier to A_3 plus 2M of 0 of A_3 plus M of minus 1 of C_3 , set to 0 I have got 3 unknowns $A_3 B_3 C_3$; they are relatively placed I have got 2 equation. Let first get this equations, 4M A_3 plus 6.32 B_3 plus 4M C_3 is M here I can take away M everywhere I can take away M everywhere.

Similarly, 4 A_3 is equal to C_3 . So, that tells me A_3 is minus 1 by 4 of C_3 let C_3 be C_3 I can get B_3 which will give me. So, all of them are now expressed as a multiplier of C_3 . So, I can now generate this in a matrix form I will retain this multiplier. This multiplier I will retain I will remove this.

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The image shows two equations written on a chalkboard. The first equation is:

$$\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \frac{\omega^2 M}{3k} \begin{bmatrix} 0 & -4.32 & -3 \\ 0 & 1.67 & 0 \\ 0 & 1.67 & 3 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0.25 \\ 0 & 0 & -0.79 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix}$$

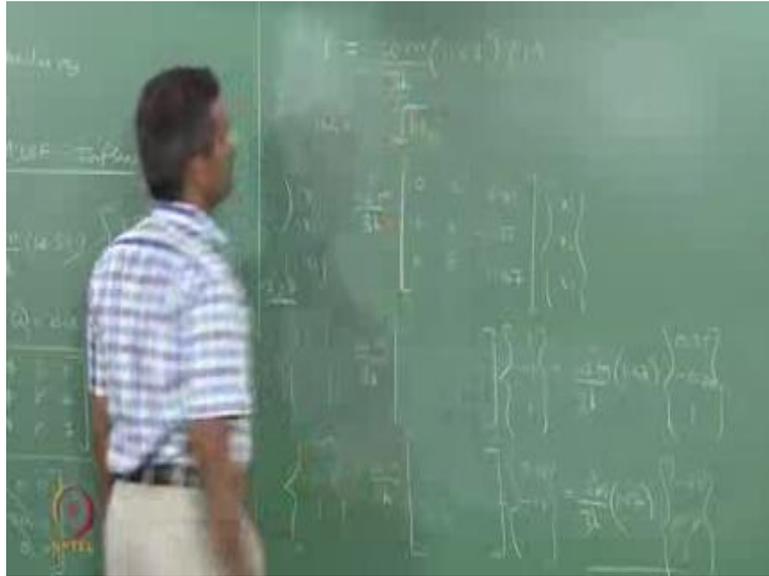
The second equation is:

$$\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \frac{\omega^2 M}{3k} \begin{bmatrix} 0 & 0 & 0.41 \\ 0 & 0 & -1.33 \\ 0 & 0 & 1.67 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix}$$

Let us call a new vector again as x_1, x_2, x_3 , but it is ϕ_3 which is equal to $\omega^2 M$ by $3k$ of $0 \ 0 \ 0$ minus $4.32, 1.68, 1.88$ minus $3 \ 0 \ 3$ multiplied by a new matrix which will again give me x_2, x_3, x_3 this matrix says a 3 is only a part of $C_3, 0 \ 0 \ 0 \ 0 \ 0 \ 0.25$.

So, let us get a new matrix; now I will rub this, I am just doing some manipulation please do it carefully, I will remove all this just to save some time just replacing them. So, get me the new matrix can you get me this column, the last column and we all know this if any 1 column is 0 or only 1 rho 0 that can be automatically put here. So, I will give only 1 column. Now can I say it is 0.41 , positive first value this $0.41, 2A$ to let us say 0.41 next is minus 1.33 , you please do not copy this you please check if it is wrong then we will correct this 1.67 you please check.

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Now, I looking for the third vector, I should have 2 0 crossings. Now I can consider this value as 1 minus 1 one that is from positive to negative, one crossing from negative to positive another crossing which is omega square, M by 3 k of this matrix of 1 minus 1, One which gives me omega square M by 3k with the multiplier a new vector, let us see whether new vector is same as the old one. Can I give me this multiplier on the value?

Student: (Refer Time: 22:23).

No multiplier.

Student: (Refer Time: 22:31).

Then you say it is 1. I will say this as 1, I will take this as 1.67 and this is 0.25 and this is minus 0.8. You can also set the other way no problem, but why I am writing this here because originally the last vector is a multiplier of C3. So, I am doing like this, now you see they are not converging because; the value is start with 1 minus 1 1, but I am getting something else. So, let us now iterate. So, 0.25 minus 0.81 of omega square M by 3k of this of 0.25 minus 0.8 of 4. So, omega square M by 3k multiplier a, new vector what is new vector and what is a multiplier? 1.67 is a multiplier and the vector is same. So, we are converged at 0.5 minus 0.8 and 1. Now I can expand this say 1 is equal to write it here I am using the last statement. One of this of this is equal to this. So, 1 is equal to

omega square M by 3k of 1.67 of 1 can get omega 3 as so much of k by M no, no its 1.34 right.

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The image shows handwritten mathematical work on a chalkboard. It includes the following content:

- Eigenvalues: $\omega_1 = 0.45 \sqrt{k/m}$, $\omega_{2m} = 0.4 \sqrt{k/m}$, $\omega_2 = \sqrt{k/m}$, and $\omega_3 = 1.34 \sqrt{k/m}$.
- A matrix $\frac{\omega m}{3k}$ with entries: $\begin{bmatrix} 4 & 2 & 1 \\ 4 & 8 & 4 \\ 4 & 8 & 7 \end{bmatrix}$.
- Eigenvectors $\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix}$ corresponding to the eigenvalues.
- A vector $\begin{Bmatrix} 1 \\ 0 \\ -1 \end{Bmatrix}$ and another vector $\begin{Bmatrix} 0.25 \\ -0.8 \\ 1 \end{Bmatrix}$.
- A small matrix $\begin{bmatrix} 4m & 0 & 0 \\ 0 & 2m & 0 \end{bmatrix}$ is also visible.

So, let us quickly write this values here' I think I can write down the values here omega 1 is this omega 2 can you give me the simply it is k by M, I suppose correct and the vector is 1 0 minus 1 1 0 minus 1 and omega 3 is 1.34 of k by M and the vector is 0.25 minus 0.8 and 1.

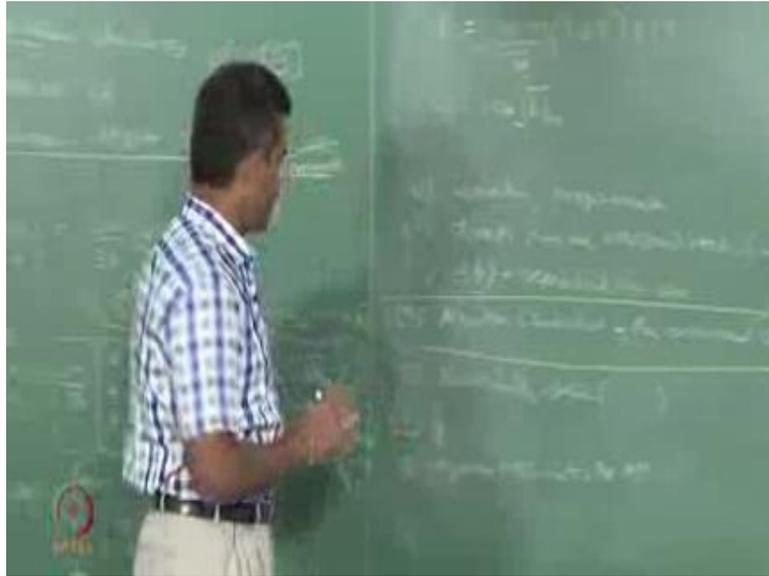
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- The image shows a list of four points written on a chalkboard:
- (1) iterative, programmable
 - (2) directly gives me orthogonal vectors ($\omega_k(m)$)
 - (3) $(\omega, b) \rightarrow$ sequential manner
 - (4) Algorithm is controlled by the assumed vector

So, if you look at these answers of all the 3 sets there were some observations. We can make with this method some of the observation or experience, what we have in solving this problem using this method is, one the method is any way iterative the moment I say any numerical method iterative. I can always program this 2 the method directly gives me orthogonal vectors with respect to mass matrix 3 the values of omega and phi are given in a sequential manner. What does it mean it always starts with the positive vector? So, omega 1 then omega 2 and omega 3, now you can very easily understand the whole algorithm is controlled by the assumption of vector had you assign a different vector, here you would get omega 1 as something which may not be actually the natural frequency system fundamental you get some other value instead of say 1 1 1 you would assumed 1 minus 1 0, some vector then you will get a different omega which be not be the fundamental frequency.

So, the whole algorithm is control by the assume vector this is a drawback of the system actually this will make the system or the algorithm user specific, we cannot generalise this vector a this is one of the you can say. In this problem you should know how to assume the vector. So, therefore, is very interesting if we really wanted to find out omega 1 assume a vector with all positive want to get omega 2 assumed the vector with 1 positive, negative and positive etcetera. So, you must have an algorithm such a manner that it should lead you towards the successive frequency, which is generated. If you are able to carefully choose your vector will give you the values in sequential omega 1, omega 2, omega 3 nicely no problem this method is highly numerically stable. Can you tell me why the main reason is this method does not invoke an inverse of a matrix are inversion is only multiplying a matrix.

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So, any method which does not ask for an inversion is always stable as far as matrix algebra is concerned.

Now, this method indirectly gives me the stiffness matrix, does not give me the stiffness matrix it gives me the indirect stiffness matrix indirectly maybe that is a demerit because I want always k and M as essential characteristics of a given dynamic system. M is any ways available to me in the system, but k is indirectly available to me this method also compares well with other numerical methods. One can say sir what are the other methods for the timing we know only 1 method that is Dunkerley. Therefore, it compares well, but I will also show you further method also will discuss will still compare very well the statement will be then way also valid. But now we know only 1 method Dunkerley. Therefore, it is agreeable with other numerical methods. So, these are some of the observation and experience as what we have gained by solving a problem in this now. This problem essentially as let us get back to the algorithm turn your pages and see this matrix not directly derived you derive only this matrix are, let us say you derive delta matrix using a linear or a series of spring system then you multiply that with a mass matrix and you got this matrix.

Now, I wanted to derive the alpha matrix directly for the given problem see remember the alpha matrix. In this case was not derived directly the alpha matrix derived with delta and the multiplier got this matrix. I would not derive the alpha matrix directly why? If

you can derive the alpha matrix directly I can always get k just by 1 step of inverse. So, can we derive the alpha matrix directly, for this given problem alpha is nothing, but inverse of k we know that for deriving k_{ij} we used to give unit deflection of forces for deriving α_{ij} , we give unit force and derive the displacements.

Let us try to do that is there any doubt in this method this method is well demonstrated as per as I am concerned that for a multi degree freedom system problem. I have taken 3 1 can take 4 also and you can try to experience the time involved in this method is very less though, it has taken much time. Because we are demonstrating the method with the writing example it takes time when you get to this method. You will be able to solve you can easily get than this does not require any essentially it does not require any computer program with the calculator you can get all omega and phi s, as of now there is now other method parallel to this which can give you all omega and phi with the simple calculator there is no other method. So, this method is very powerful in that way.

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$$[\alpha] = \frac{1}{3k} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 4 & 4 \\ 1 & 4 & 7 \end{bmatrix}$$

Now, let us try to derive the alpha matrix directly. First let us get what is this alpha matrix available to me we already have an alpha matrix of this, can you give me this value 1 by 3 k I think is 1 1 1, 1 4 7.

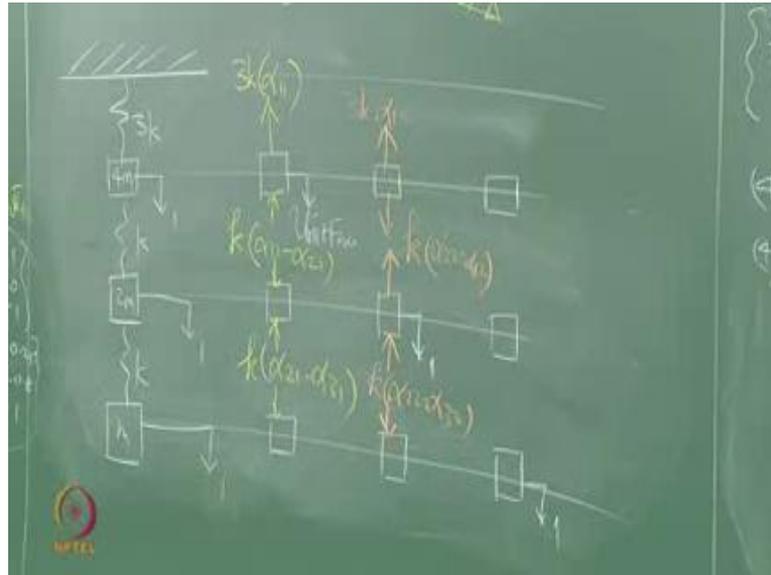
Student: (Refer Time: 31:39).

Yes.

Student: (Refer Time: 31:41).

1 4 4, 1 4 7 that is what we have a ready with is it not. So, I must get this matrix directly without the assumption of series of spring's etcetera. I may directly get from the first principle of alpha i j let us do that.

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Individually I am a marking 1 they does not mean that degrees of freedom they all the first second and third locations of respective unit forces has to be applied. So, let us do that in this method the common mistake what generally people make when, we carry over this control matrix from 1 to the next Eigen value. You should ignore the multiplier which will give you the value, if you carry over this multiplier also into the algorithm then you land up into it. That is where the confusion will start; this multiplier is nothing but your proportion value of the vector.

One does not mean anything it was actually 14.32, 3.16 of 14.34 4 times of 14.32. Since they are common multiplier I have plug it out. So, this is nothing, but a number for me which an amplitude, I am not bother about this I am always bother about the relative displacement of mass position. When the system is vibrating at this frequency I can either say 14.32, 14 into 3.16 14 into 4 or I can let us say ignore 14.321, 3.16 4 is give me the same physical meaning actually.

So, you should never carry over this multiply in the control algorithm which is a common mistake, but people generally do. If you do that you land up in a wrong omega only phi will be ok you will get wrong omegas you will not be able to know that because phi will be all right you get wrong omegas only. So, that why then, you will get; you may get the value lower than this which is not correct. So, be careful about that now let us carefully examine this now the second interesting problem, what you are confusion is how to mark these arrow directions for the given alpha $i j$ matrix is the very big confusion. Generally people have this confusion very common, let say this is unit force I am writing unit force just for our understanding here on wards, I will write only 1. I will write only 1 its means the force is unity not the displacement I am talking about alpha $i j$ matrix not $k i j$, $k i j$ is force in i th degree of freedom by giving unit displacement the j th degree of freedom keeping all degrees of freedom constant where as alpha $i j$ is give unit force and give the displacement.

So, these are all unit forces in the respective degrees of freedom. So, let us try to mark them, same logic as we used for k derivation when I am trying to pull this spring down by some force in the case the magnitude is 1. Just spring will upwards the force. So, let us mark this as $3k$ of alpha 1 1. Now very carefully listen this I think possibly this way of explaining alpha derivation or the base matrix derivation is not there I think in any of the textbooks is not there the only textbook, where you will find will be the book which I have written with springer which will be available by this public domain. There is no other book which will explain you this.

So, it does not mean that it is a unique or a very great technique to be followed in mass and turn is very simple actually, there is a problem here right. So, try to understand the arrow direction how you are marking be exactly with me. It is easy for you the moment I complete everything you will not be able to reproduce this until you understand the basics, it is for guarantee you will not be able to reproduce this you will always 100 percent make a mistake either in numbering number or in the arrow direction. This is for granted you can take it granted you will not be able to do it you have to be exactly mesmerized on the black board then only you will know how it is done.

So, I am giving an unit force here the spring will try to pull it back because, now I am multiplying the stiffness with displacement this give me the force actually correct because, alpha 1 is nothing, but the displacement coefficient I am talking about

flexibility matrix now, displacement coefficient multiply the stiffness with displacement because stiffness is nothing, but force per displacement when you multiply you get the force, this also actually a force only. So, I will write a force equation equilibrium later right. So, I am getting an equation of that all so $3k \alpha_1$.

Now, when I try to pull this mass now this spring will push it up as usual. So, I mark the arrow here and here parallelly I write the value of both of them common at the centre which will be the stiffness of the spring which is k multiplied by α_1 $1 - \alpha_2$ $1 - \alpha_1$. Similarly because this will spring pull this mass will go now this spring will push up this is going to be k times of α_2 $1 - \alpha_3$ 1 . Let us do it here for this unit force start from here. So, this way and this way is always a pair when, you put here and here. Because there is a common spring which connects this 2. Therefore, you have to mark the arrow direction of both of them, I think this we already explained in the trust problem either it is pushing or pulling it is not one end pushing or one end pulling it is never. So, therefore, I am marking the force in this spring actually that is what I am trying to do, therefore I need not have to mark the separately then think after a day and come to for this arrow direction I should do it parallelly because, this is a spring force which will be; obviously, the stiffness of the spring multiplied by α_2 $2 - \alpha_1$ 2 .

Now, when this spring is pulling this mass down this spring will pull the mass up actually. So, $3k$ of α_1 2 similarly when, this mass is pushed down or pulled now let us say this spring will push it back. So, the marking is like this which will be k times of α_2 $2 - \alpha_3$ 2 . Let us apply the same logic for the third column here. So, I am trying to pull this mass down by unit force this spring will pull it back. So, I am marking it here pulling this back and this simultaneously arrows are marked which will be k times of α_3 $3 - \alpha_2$ 3 . When this mass is being pulled down by the spring.

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$$k(x_2 - x_3) = 0$$

$$x_2 = x_3 = \frac{1}{3k}$$

$$\frac{1}{3k} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

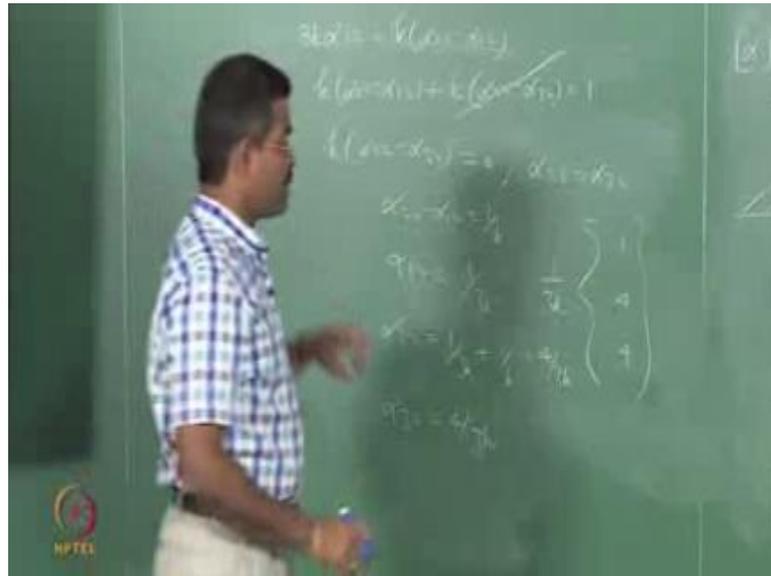
$$x_1 = x_2 = \frac{1}{3k}$$

$$x_1 = \frac{1}{3k}$$

So, this spring will try to pull it back which will be k times of $x_2 - x_3$. When this mass is pulled down by the spring this will try to pull it back $3k$ of x_1 . So, like let us say the force equations equilibrium equation for 1 by 1 separately, let us pick up this $3k$ of x_1 plus k of $x_1 - x_2$, sorry x_2 this value this value and this is equal to 1 why because these 2 are upward, this one is downward the second equation you written here which is k times of $x_1 - x_2$ should be equal to k times of $x_2 - x_3$.

The third equation is here for this mass which is k times of $x_2 - x_3$ is 0 which implies k cannot be 0 because stiffness cannot be 0 this implies $x_2 - x_3 = 0$. When I substitute back here this becomes 0 which implies $x_1 - x_2 = 0$. When I substitute back here it will say $3k$ plus $1 - 4k$. Therefore, x_1 will be 1 by $4k - 1$ by $3k$ because this goes 0 , this goes 0 because $x_1 - x_2 = 0$. So, this goes 0 by $3k$. So, this implies that $x_2 - x_3$ is also 1 by $3k$ and $x_3 - x_1$ is also 1 by $3k$. So, if you take 1 by $3k$ common the first column will be 1 by 1 which is as same as exactly here is 1 by $3k$.

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Let us spend couple of more minutes at least get this also. So, let us write this equation here $k(x_1 - x_2) + k(x_2 - x_3) = 1$ should be equal to $k(x_2 - x_3) = 1 - k(x_2 - x_3)$ plus $k(x_2 - x_3)$ should be 1.

$k(x_2 - x_3)$ should be 0, which implies $x_2 - x_3$ is same as $x_1 - x_2$. I am deriving this I am writing this equation from the third mass writing this equation from the second mass writing this equation from the first mass. So, when I do this goes away. So, we can say $x_2 - x_3$ minus $x_1 - x_2$ is 1 by k from this equation I substitute back here because this going to be 1 by k this becomes 1 therefore, $x_1 - x_2$ will be 1 by $3k$ substitute here $x_2 - x_3$ is 1 by $3k$ plus 1 by k . So, $x_1 - x_3$ is also 4 by $3k$ because they are same.

So, the second column is going to be now 1 by $3k$ common $x_1 - x_2$ is 1 $x_2 - x_3$ is 4 $x_1 - x_3$ is 4 which is as same in second column of what you have here similarly third, can also be written. Now let us quickly see where is the confusion there are many multiply and confusion in this problem the first confusion is, if you do not mark the arrow in combination or pair please take it granted you will never be able to solve this problem you never get alpha matrix at all, if you do not mark these arrows in pair you will never get.

If you what people generally do is like this they mark the arrows with respect to mass they pick up the mass they mark the arrow they say. For example, the mass is moving

down. So, this spring will push it up this spring will pull it up they have written, when they come here they are really worried. So, you never do that pick up the mass pull it down put the pair pull it down, put the pair put here. Similarly I start from here pull it down put the pair as it goes down put the pair goes down, put the value similarly this mass goes down, put the pair goes down, put the pair go here then only we will have to solve this you have to follow this.

Systematically you will not make a big mistake and in all these a second subscript will be one indicating the force applied to the first degree in all this a second subscript by mistake is also, two indicating force applied is second degree and third degree. So, this is a second catch. So, you do not mix match the coefficients then of course, you can carefully write down the values and solve them, you will be able to get the alpha matrix which was otherwise obtain by assuming the springs and series etcetera in directly, but this a direct method of finding out.

Imagine matrix methods was in position in practice in engineering only in seventies, even 70 years, before this method was in position or place Dunkerley did this actually. So, imagine how powerful was this mathematics tool to apply for dynamics we get back the same matrix here.

So, you can invert this which you must have already tested it will give me the stiffness matrix that is the problem. So, one need not have to actually go round about deriving this alpha matrix directly for the given problem. We can derive this alpha matrix then, write down the control equation get the iterative scheme set in and find omega and phi s successively depending upon how to you assume a vector the whole algorithm is control by the assumption of the vector. If you are able to assume the vector correctly you will get omega and phi correct. So, we will solve some more problems, when we couple of more problem by some other method then will compare the answers of that method with this method back again.

So, this method will give you all omega s and phi s in sequence this method is controlled by the vector this method is called influence coefficient method. Because it is trying to find out the influence of inertia force in every degree of freedom, all the vibration system of the model that is why it is called influence coefficient method, alpha is the designation used in the literature for this method was first proposed in a research paper unfortunate

on remember it. Now it is available in the NPTEL reference please see in early seventies people propose this method first, time the research paper this is not a classical method available in text books this is been borrowed from the research paper.

So, that is why it is very interesting and very powerful tool even today, it is valid because it gives me probably the easiest method which gives me all the omega and phi s instantaneously the easiest method and it is and computable with the calculate, you can do that is very fast we are done 2 problem say about 40 minutes time. So, you should be able to do about 8 problems in 3 hours in a given exam like this, any questions?

So, exam is now open for dynamics we have got about 1900 candidates who are registering for this course all over the world about 200, candidates from abroad and remaining all are from India about 30 to 30 percent are practising professionals. So, try to register for the exam, now available in Google.

Therefore, your denominator will be have a competition about 2000 candidates in the country are in the world and your percentile will indicate, A2 representation for understanding of subject in international scenario instead of getting a grade of s or a only with a class room of 30, who all the 30 does not know what dynamic is, but now you compute with 3000 or 2000 people whom all know what dynamics is then, you really know where do you stand actually that is a right way of getting percentile for any course. So, please register for the examination and get a certificate done through NPTEL.

Thank you.