Dynamics of Ocean Structures
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Lecture - 16
Orthogonality of modes Multi-DOF system models

We are working on the Classical Eigen Solver problem, where you have got a two degree freedom system like this. We will continue with the Lecture 16 on Dynamics of Ocean Structure, Module 1 on the NPTEL IIT Madras places.

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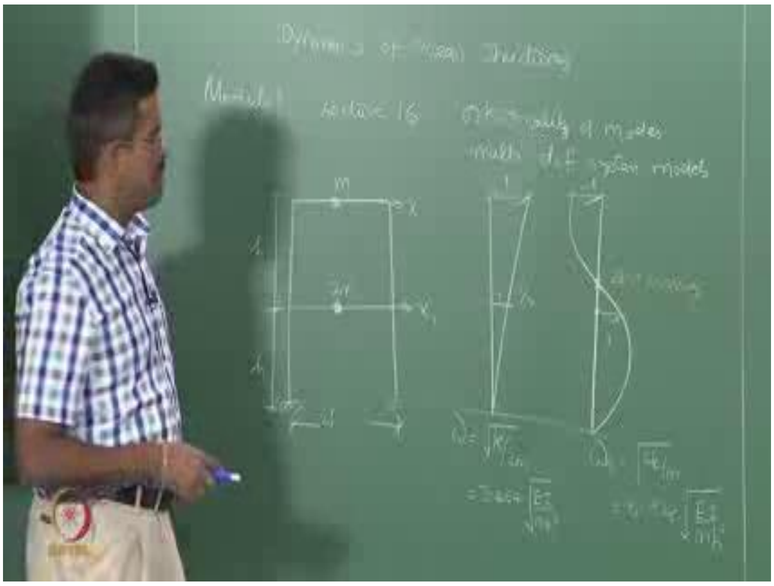

In the first module we are attempting to discuss the fundamentals of structural dynamics, where we will also give you some design guidelines and how the form based design can be used as a successful application with dynamics as the background. In various examples we have a already illustrated that fundamental structural mechanics can be used as a tool to compute some essential characteristics of dynamics given system.

Spring mass system is a very elementary example available for demonstration or modeling any multi degree freedom system. We picked up a two degree freedom system model in last class; we solved and wrote the equation of motion, we solve the classical eigen solver using a classical theory and found out the natural frequencies omega 1 and omega 2 as the values as indicated here. And we have also derive the stiffness matrix for the columns ei by hq, 24 ei by h q we know that how they have been derived from the first principles.

So, we also identified from the given set of frequencies and mode shapes, how to identify the fundamental frequency in the corresponding mode shape; it is a pair you cannot separate this. It means the mode shape of this has no association with this frequency and vice versa. For example, this mode shape has no association with this frequency. So, every frequency has got a very corresponding unique mode shape therefore they are paired, you cannot decouple them.

And we also said in the last lecture that mode shape is nothing but the relative displacement of the mass points where you are measured degrees of freedom. In the last examples we also illustrated to you that if you do not mark the degrees of freedom at the points where the mass matrix is or the mass is lump, what will happen to the mass matrix if we demonstrated. Therefore, in this example classically every design is used to always lump the mass at the point where the degrees of freedom are marked.

Essentially the reason is not to make the mathematics simple, but you are always interested in knowing the structural acceleration or the displacement of the mass point. In all structural system in any mathematical model you are interested to know either the acceleration or the displacement at the point where the mass is lumped. You are not interested knowing at this space point where the mass is not lumped, because we are always bothered as a designer what would be the maximum acceleration given to the flow or the mass that is why in all mathematical models the degrees of freedom are generally marked at the point where the mass is set to be lumped. That is the reason why we do so. But if you violate what happens to the mass matrix and stiffness matrix we already demonstrate in an example.

As we then said that the mode shape is a relative displacement of the mass position in the given system. Now, I will take you forward to use this mode shape as an application of Maxwell Betti Reciprocal Theorem. Maxwell Betti Reciprocal theorem has got a very great advantage in structural mechanics saying that, the force on a given system appeared one point giving displacement in other node will be equivalent to the force appeared the other node displacement should be in different node. I will explain this; I will use that concept for mode shape to understand and to explain how mode shapes can be used.

Now to carry forward this mode shape which is available as an eigen vector by solving the equation of motion as an eigen value and in vector I want to normalize this. Now, the question is; why do we have to normalize this? If we normalize the mode shape there are advantages when you do the mathematical modeling. And also most importantly the physical advantage is you can use indirectly the Maxwell Betti Reciprocal concept. See it is very clear that Maxwell Betti Reciprocal concept in mechanics can be used only for small displacements, whereas in this can the displacements are only quantified in relative value. We really do not know whether the one means what, how much is the millimeter or centimeter you do not know.

So, one cannot really say whether the displacement is minimum or less within the elastic displacement system, we do not know that. But, Maxwell Betti Reciprocal concept can be applied here. To make that applicable we have to essentially normalize these modes. Than what is normalization or what is orthogonality?

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We need have the modes orthogonal each other. Let us see what orthogonality is. If phi is a mode shape in my case it is half and 1, one can have it as 1 and 2 also; it is question of only multiplier out. Whatever the multiplier is taken out it is no consequence only the related displacement as a consequence, either you can say 1 as multiplier out in the case this is 1 and 2 and so on so fore does not make a difference it is only relative displacement between the two points. And phi 2 in this example is 1 and minus 1. You can also say minus 1 and 1 does not matter because it is relative. And we already know that for second mode there will be 1 0 crossing, for nth mode there will be n minus 1 0 crossing which will enable you to really find what is the corresponding number of the frequency in a given multi degree freedom system model.

Now put together in mathematics we call this Capital Phi Matrix, which can be written as half 1 minus 1 sorry 1 and minus 1, nothing but putting the vectors column wise and making a matrix that is called Eigen Vector. Now this is a mode shape matrix, each column represents the corresponding mode shapes. The moment I say phi i transpose phi j is equal to 1 for i equal j equal 0 for i not equal j then, the modes are orthogonal. Now let us first understand whether the available modes are orthogonal or not. If they not orthogonal then you make them orthogonal. Once we understand that we will talk about what is advantage of making the orthogonal. First let us see are they really orthogonal are not, let us check.

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Now, I want to check the orthogonality of the modes. So phi 1, if I expand this let us say phi 1 transpose phi 1 should 1, phi 2 transpose phi 2 should be 1, phi 1 transpose phi 2 should be 0, phi 2 transpose phi 1 should be 0; because if an i and j are not equal it should come 0, if i and j are equal it should become unity. So, let us apply this here. I want find phi 1 matrix its vector is given to me which is half and 1, phi 1 transpose is half and 1 because this is 2 by 1; two row on one column is going to be one row on two column that is why I have transposed it.

Let us find out phi 1 transpose phi 1, let us see what happens to this. So, it is going to be half 1 multiplied by half 1. You can see this is 1 by 2, this is 2 by 1 there is a compatibility therefore you can multiply and you will get a unique value which is 1; only one number. That value should be equal to identity or 1 if it is orthogonal. We can check this which will be 1 plus 1 by 4 which will be 5 by 4 which is not equal to 1, is it not.

Similarly, one can also try with phi 2 transpose phi 2 which is 1 minus 1 of 1 minus 1, which is also not equal to 1. So, they are not orthogonal; these modes are therefore not orthogonal. I want to make them orthogonal.

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Let us borrow the mass matrix of this problem which is 2 m 0 0 m the off diagonal terms are 0 because, the degrees of freedom are measured with the point where the mass is lumped. The linear diagonal elements are always diagonally dominant therefore you can always invert these matters if we wish so. Now, one can normalize the eigen vectors or otherwise called as mode shape with respect to either mass matrix or stiffness matrix.

Let us try to normalize with respect to mass matrix. Let us say m 1 is a number which I now will get as phi 1 transpose m phi 1. So, let us expand this half 1 2 m 0 0 m of half 1, you can see the compatibility 1 by 2 2 by 2 2 by 1 is compatible, compatible ,you will get an unique value which is let us say m 1. Can you find what is m 1? First we will multiply these two then post multiply this vector you will get m 1; 3 by 2.

Student: M.

M. So, 1.5 m. To normalize phi 1 divide phi 1 by root of 3 by 2 m, that is phi 1 star which is normalized will be root of 2 by 3 m of half and 1.

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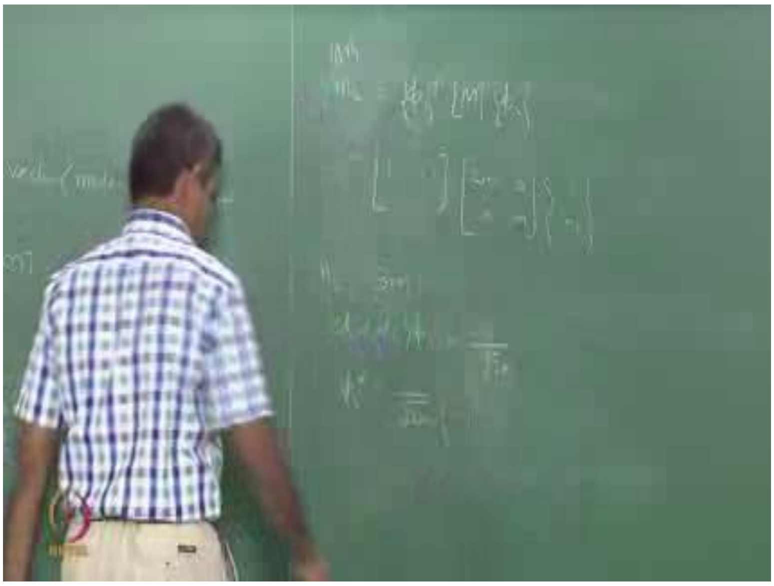

Similarly, let us try to find m 2. Similarly, m 2 is going to be phi 2 transpose m phi 2; this is a vector, this is a matrix, this is the vector, which will be 1 minus 1 2 m 0 0 m 1 minus 1 which will give me unique value which is m so divide phi 2 vector by root. Therefore, phi 2 star which is a normalized vector will be 1 by root 3 m of 1 and minus.

So, let us say I have normalized vectors which is this value and this value. Let us check is a really satisfying the orthogonality condition or not.

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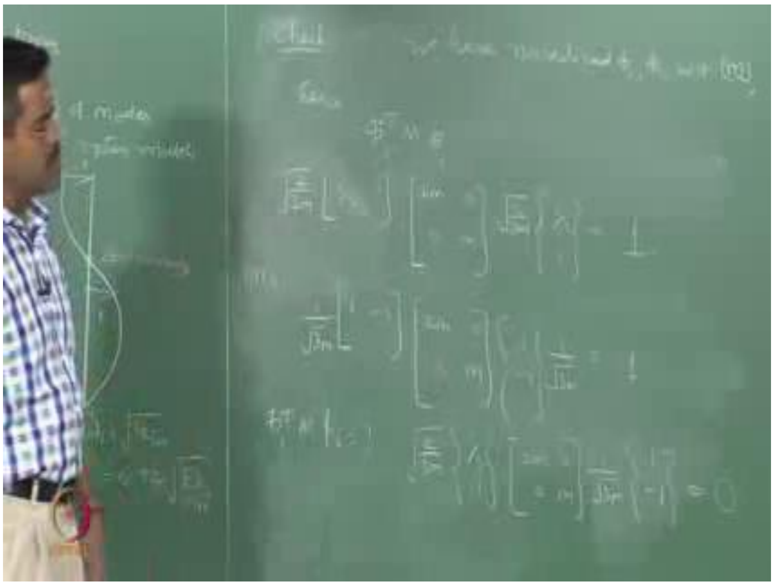

Since, we have normalized phi 1 and phi 2 with respect to m hence, phi 1 transpose m phi 1 should become unity. Try, I can put a star here because we have used star there so you know the value. Can you give me what is this value? Similarly 1 by root 3 m of 1 minus 1 2 m 0 0 m of 1 minus 1 1 by root 3 m will be 1. Now let us check the cross product also. Let us check phi 1 transpose phi m phi 2; let us see what happens to this. Because it should become 0 let us see. So, root of 2 by 3 m of half and 1 2 m 0 0 m 1 by root 3 m, it is becoming 0 because the m will get cancel automatically you will get 0. So, that is the check that phi 1 star and phi 2 star are the normalized vectors. As such when we solve a eigen solver problem the vector what you get may not be normalized. We have to normalize it respect to m or respect to k.

I will take a one example later that if you normalize respect to m what to be the advantage and normalize respect to k what to be the advantage. Generally people prefer to normalize mass matrix because, the diagonal elements in mass matrix are generally 0 in a given problem. Therefore, the computation becomes less expensive. So, people prefer to normalize the matrix or the vector respect to mass matrix because it becomes easy computationally when you apply in the equation because more easy. We will talk about that slightly later.

Now the fundamental question is arising in mind is a vector have obtained from the Eigen solver I have normalized it fine so if we just multiply this with 2 over root 3 m or

1 over root 3 m I will get a new vector still the relative displacement remains same. How I will physically now understand the advantage of normalizing it compare to that of non orthogonal modes. Now we will take the help of Maxwell Betti's reciprocal law.

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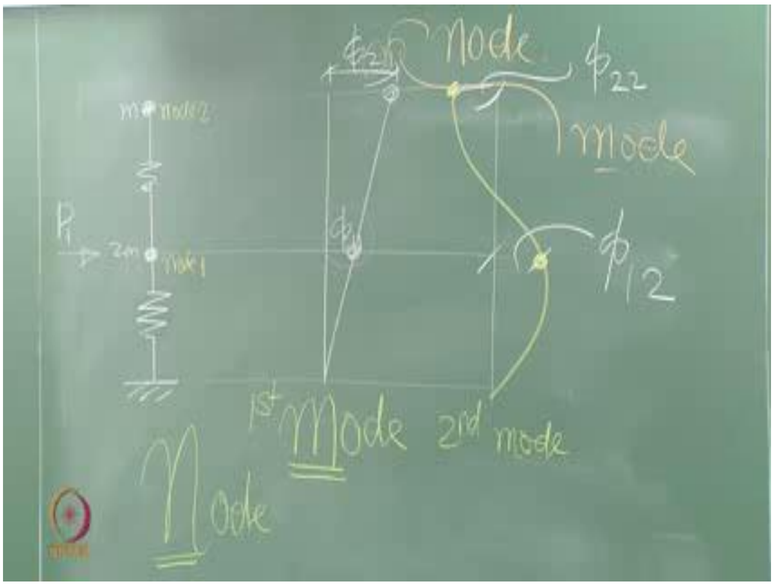

Say I have a system is a strict model of this like this, let say this is 2 m and this is m replaced by some spring, replaced by a spring. When we apply a force to the system in the lateral direction I call this as p 1. Then again a fundamental question comes in mind is if I apply a force in lateral direction the springs are provide in the vertical direction how are the accounter for. These are not linear springs, they are bending stiffness. We are talking about EAVL for these springs.

So, when the stick bends the restore, therefore the bending stiffness because stiffness is a character related to restoration in a given system. If I have any model which restores the equilibrium position then it is always a stiffness, it is not axial it is bending I have used EAVL here. If you look at the derivation later what we did in the last class we have used EAVL for other equations. When I pushed the model by a lateral force from the left to the right as p 1 applied to the node 1, let say this is node 1 this is node 2. I am making it very clear this is node, this is mode. The mass is displaced this is mode. This is the first mode and the second node.

Let us say this is orthogonalized modes. Now let us explain the advantage of orthogonality applied to dynamics here.

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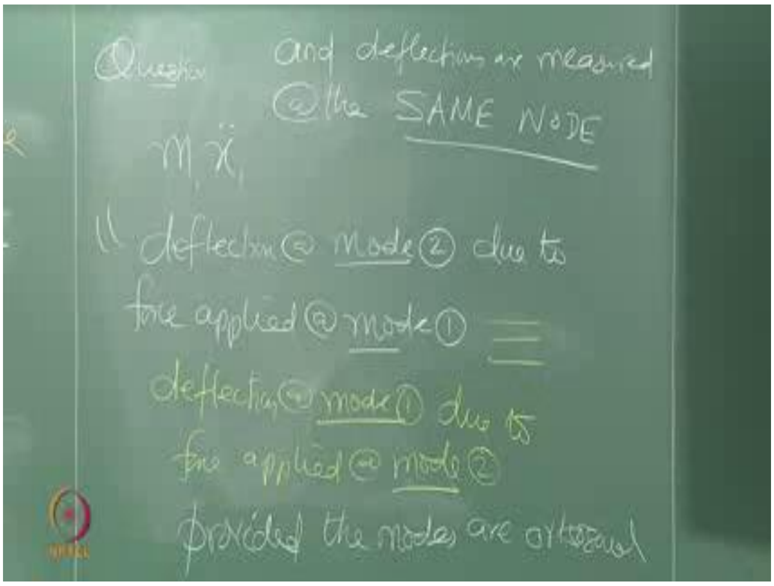

Let say I have a question; in this problem I have got only two modes because there are two degrees of freedom, if I have got n degrees of freedom practically speaking n story building because every story will have a mass and mass point will represent x 1, therefore n story let say hundred story building I have hundred modes. One hypothetically if I have looking at is solved all the hundred nodes and normalized them and I have all under modes in pack ready.

But I have a question here; let say if the load p 1 applied at node 1 displace the system by phi 11, this notation is simple the first indicates the node the second indicates the mode. So, by that logic I can call this as phi 2 1. The first letter indicates the node, the second letter indicates the mode. Similarly, I can call this as phi 12 this as phi 22 these values are known to me. The question what I want to ask is, if I apply the force p 1 at node 1 it displace the system in the first mode of vibration as phi 11 which is known to me. I wanted to know what will be the effect of this force on the second mode third mode forth mode nth mode, I want to know.

Maxwell Betti Reciprocal theorem says if I know the displacement offered by a deflected profile because of load applied with 1 node it will be equivalent to the displacement caused by the force applied it same node but different mode. On the other hand phi 21 and phi 12 should be equal. If I able to establish that I do not have to find the second mode at all I get the first mode I can keep on finding the displacements or response of the system for all the forces in all the modes; that is an advantage here. Maxwell Betti Reciprocal theorem is applicable only when these modes are satisfying the normalization with respect to mass.

Now the question comes why, because Maxwell Betti Reciprocal theorem in static is applied only for a force. But here we are not talking about the force; we are talking about the acceleration caused to the system which results in this vibration. So, my force here is m 1 x 1 double dot that is my force actually. So, p 1 is not the problem here, the p 1 is not applicable here it is m 1 x 1 double dot, so I am applying an inertia force to the given system. Therefore, I must normalize the vector with respect to mass.

When I do this then deflection at mode 2 due to force apply at mode 1, all are mode only no node. Deflection at mode 2 due to force applied at mode 1 same as that of deflection at mode 1 due to force applied at mode 2. Now, the argument comes here because Maxwell Betti Reciprocal theorem is focused on a point of application which is the node in this problem.

There are two parameters a please do not get confused about this. This is an interpretation of Maxwell Betti's law for dynamics. Say may be a new perspective of understanding. Many textbooks may not agree with a statement, but this is a well known establish fact when you talk about modal response parameter which will explain in a problem later in the same module end. So, there you will realize the advantage of this.

Now, Maxwell Betti Reciprocal theorem in statics has got two parameters varying; one is the point where you are applying the force what we call as the node, other is a defector profile of the beam or the section which is the mode. Now here the explanations are only on the mode, provided the modes of orthogonal and write here deflections are measured same node; that is very very important. So, this gives me a great advantage of understanding the response of the system for forces in different modes. Now here the forces under context are not the lateral force it is inertia force, because we are talking about vibration. The deflection here is not the rotational or transmission deflection of the beam, but this displacement which is relative of the mass points in a given system. So, that is how we can interpret.

Let us physically understand this statement, because very important to understand this statement. Now, what actually I am interested is the following, I have got a two degree freedom system model where the mass is lumped at 2 m and m as shown in the figure. I have mathematically idealized the given frame with two springs and two mass has indicated here. And the whole structural system is now under vibration at two frequencies the omega 1 and omega 2, if at all they vibrated omega 1 and omega 2 under the given system the shape of mass displacements relatively will look like this.

Now, I am able to understand this once I have all the 2 modes plotted. If somebody ask me a question back again what would be the deflected position of the mass m 1 when the force is m 2 x 2 double dot. I do not have to look at the second mode I can simply say it is going to be the same as the value at m 1 x 1 double dot. This will enable me to truncate the higher modes to the lower modes which we call as model truncation possibilities in higher modes of degrees of freedom. So, whenever more modes to be represented in given problem I need not have counter for all the modes I can filter few modes and still get the same essence of representation from the few modes provided these modes are all normalized. So that is a catch here.

So, we are extending the Maxwell Betti's reciprocal rule and advocating it indirectly for a vibration problem. And there is a basic violation here which engineering committee will definitely challenge. Maxwell Betti reciprocal law is applicable only when the deflection is small, but here we do not know whether they are small or large they can quantify. That is only a law which you are trying to, I am not saying that Maxwell Betti's reciprocal theorem is applied to dynamics I am not saying that. I am taking that advocacy and applying it for my understanding because this will help me to truncate the higher modes in higher md of problems later. Ideally speaking I can have all the modes and all the frequencies which are very difficult to find in certain cases.

So, I can easily use only the lower may be couple of modes or till 3 till 2 what we call modeled truncation. When will you truncate the mode we will talk about that later, but one can truncate the modes you need not have they are all the mode shapes. Then you may ask me a worry sir there is a mode shape which is truncated, but unfortunately I want to know what is the displaced position of the mass at that mode because of the force applied at m n x n double dot, because this mode is not available to me. You can get back and show that this will be as same as m 1 x 1 double dot at the nth mode it is easy, relatively it will be same. So, that is the advantage what you have. This is the very interesting phenomena for design, because people are talking about the maximum tip displacement in design, this phenomena can be easily applied for that. That is the advantage what we are talking about orthogonality of force.

Now we will move on to multi degree freedom systems models, where I really wanted to find omega n's and phi n's by some numerical method, because as I go higher and higher in this example you saw we have been solving a quadratic equation. If you go for a third degree you will solving a cubic equation and cubic equation solution will have at least one value to be assumed by inspection. It is very easy in a mathematics saying that x cube minus 3 x square plus 4 x minus 5 you can easily by inspection find x is minus 1 is the root, but here we are talking about the number of 3.464, 4.948 it is very difficult to find them by inspection.

So, there should be some numerical tools available in the literature which will help me easily to find out omegas and phi's for n degrees of freedom model other than eigen values in eigen vectors, number 1. Number 2, eigen value and eigen vectors can be applied or eigen solver can be applied to a problems when the mass and stiffness matrix, because it is k minus omega square m determinant when the mass and stiffness matrix are diagonally dominant and the convergence is sure. But in certain cases your mass and stiffness matrix may not be directly dominate. In that case this will lead to a very catastrophic solution mathematically. You are not able to the solution of all the modes, whereas numerical methods can give you this step by step.

So therefore, we are now attempting to find an approximate solution not accurate because accurate is, of course only this approximate solution but I will show you in some examples where this and all numerical methods will attempt here will give you exactly the same answer; it will give you exactly third digit answer will be same. Therefore, you can appreciate how these numerical methods are so intrinsically define therefore they will have no error at all, and they are very easy to follow. And very interestingly many of the textbooks and authors do not illustrate them with many examples of our choice. They will take up a same spring mass problems and establish in all, which I will to variety of problems for you to understand one by one all the methods thoroughly. Then I will take a couple of problems and solve the same problems by all the methods and I will show you that the answers are converging by any method you want and all of them are easily programmable also.

On the other hand, if you really have a structural form of your choice for research you can pick up any one method write the coding in MATLAB or in C any language and you can find omega and phi which is the vital characteristics of any dynamics system for which standard solution is not available from software. Because, to find a standard solution for this problems from the software you are going to mathematic in model this problem first in software where you are not comfortable with it, but you can be comfortable writing the coding for the numerical method easily because it is step by step algorithm, very easy to understand, there are codes available also in fact I can give you all the C codes and MATLAB codes here in the codes itself, just simply plug in substitute the values of m k etcetera in minutes you will get your omega and phi automatically.

As number of omegas and phi's, all omegas and all phi's you will get where as in certain cases it will be difficult, but there is a problem in these methods the convergence of the method because they iterative in nature it is only dependent on the observation or the initial assumption what you make, that initial assumption needs experience. So this is where people cannot apply this generic to all problems. Therefore, software do not follow this method because that initial assumption need to be given by the interpreter who is the user of the software who as absolutely no idea what dynamics is all about.

Therefore, you cannot unfortunately people who know about dynamics also cannot give those values because they are not solved by the authors in the textbooks; therefore they are really fed up. If you have a new structural form where you do not have the skill of numerically modelling them you cannot find omega and phi so easily using a computer or using software. You cannot also solve using classical eigen solver because they are n number there may be a problem here, then where will you land. Many of the research problems stop here because the structural form taken by the candidate or the researcher is not solvable for by finding the fundamental characteristics of dynamics problem. Therefore they stop there, they do not take it forward or they try to find out approximately wrong solutions and they go with design and the whole design can become absolutely wrong.

So, numerical methods are very important. Generally people see numerical methods as a side line for the original mathematics which is not correct, because in this case I am re insisting the numerical method will give you exactly the same answer as that of the classical analytical techniques. There are three method; experimental, analytical, and numerical. Experimental is what you conduct experiments and get the value. Analytical is using a code a b c's and gets the value. Numerical is modulating in FEM and get the value. Analytical anyway of demonstrator for 2, I will not do it for two more, but I will do all the problems in numerical make you to understand experimentally omega and phi cannot be obtained; only damping can be obtain.

So, now we will resolve and we will agree that numerical methods are interesting for multi degree of freedom system models.

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So, let us talk about numerical methods only to find omega n and phi, I put a bracket like this because they are couples they are married. The foremost method which comes in mind is Dunkerley. Dunkerley is given a method for estimating in the fundamental frequency omega 1. He did it in 1895 that is a beauty of this method; it is more than hundred years old. Imagine on those days matrix methods was not available in engineering, computers were not there, calculators were not there, only slide rules were there probably I do not know 1895 wave and slide rules were there are not. He has given an algorithm. We will solve the problems using this algorithm. We will pick another numerical method using computer technique and solve the problems, they answer exactly in the same.

In fact, before we do this method I think we should stand salute to this fellow because he has given a wonderful technique which is quick and simple from the basics you will never make a mistake. In any problem you have if you are able to work out Dunkerleys fundamental frequency most of a design is done for the form. So, we must know what my fundamental frequency is. We will see how this method is developed by Dunkerley.

This method is applicable to discrete systems only. If the mass is continuous you cannot apply this system you have to have a lumped mass. He said simply that 1 by omega square is 1 by omega 1 square 1 by omega 2 square 1 by omega n square. N here does not mean natural, n is the nth degree. Let k 1 is the spring constant. We know that k 1 is 1 by delta 11, where delta 11 etcetera or called flexibility coefficients because stiffness and flexibility are inverse of each other.

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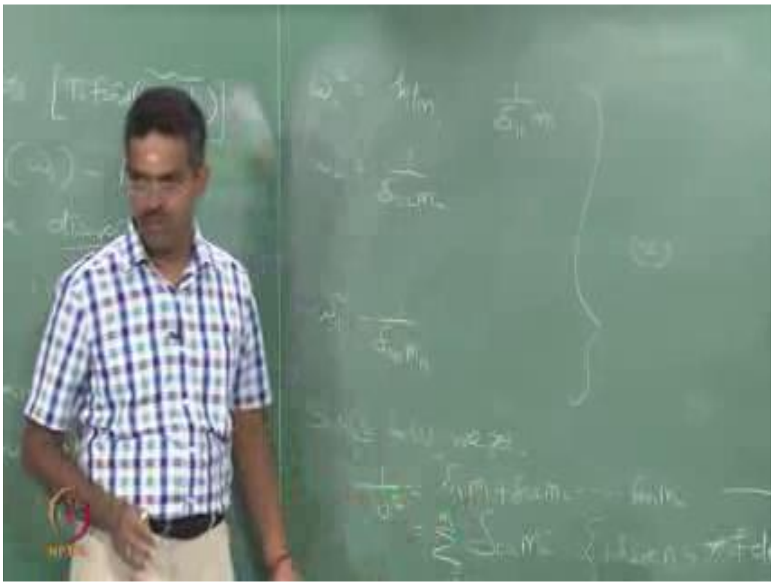

So, omega square or omega 1 square is k by m or k 1 by m 1 and k 1 is 1 by delta 11, therefore this is 1 by delta 11 m. Similarly omega 2 square will be 1 by delta 22 m, omega n square will be 1 by delta nn mn. I call all these equations as equation 2. Substituting 2 in 1 we get 1 by omega square will be delta 11 m 1 delta 22 m 2 delta n n m that is sum of delta ii mi i varies 1 to n. Where n is the number of degrees of freedom; I will call this as equation 3.

Let us physically understand what Dunkerley has proposed. Dunkerley has proposed to find out or helped you to find out only one frequency which will have a contribution from all frequencies. Now you know all the mass, because mass is one of the essential characteristics of the given dynamics system. The mass is not there inertia force is not there therefore you do not have to dynamic analysis we already said that. So, in equation 3 by some method hook or crook if I know how to work out delta ii I will get omega. So, I am interesting in knowing how to work out delta ii. If I know how to find delta ii for a given system I know m 1 m 2 m n I can easily find omega square take a root I get omega.

So, in this factor he has not focused on deriving stiffness matrix at all, he is helping us to find out the influence coefficient. Let us try to find or understand an algorithm how to get delta ii or delta 11 22 etcetera for a given problems. To understand this I must take problems and demonstrate how to get delta 11 22 etcetera then only I will be able to solve this.

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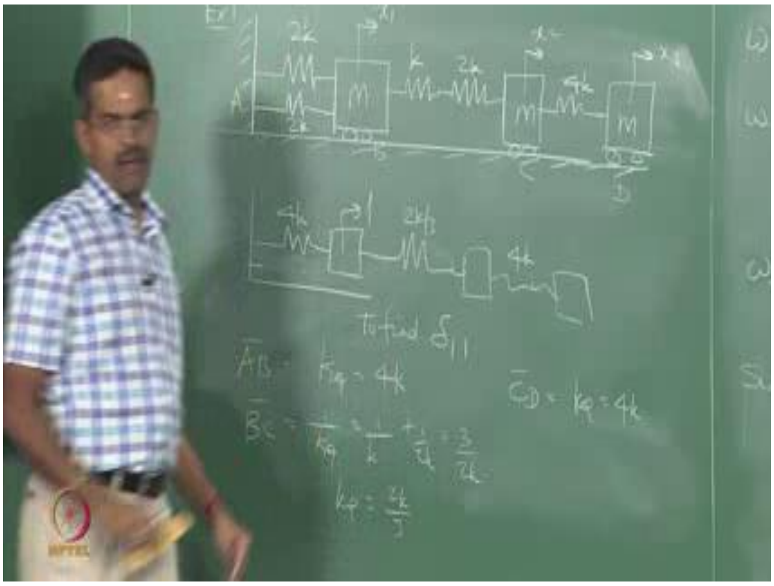

Let us take one simple example. So, just for a self check you should be able to write equation of motion for this in a matrix form using Newton’s principles as well as using energy method you can always try. What Dunkerleys propose is very simple; he is interested in finding out the deflection for unit force that is how stiffness will become deflection if the force is unity because stiffness is force per unit displacement, but if the force becomes unity the displacement becomes stiffness directly which is inversely connected. So what he did is, you give unit force to the system. And of course, the system continues unit force at this point at x 1; to find delta 11.

Let us mark this segment as A B C and D. Segment A B has got two springs in parallel I can always find k equivalent of this segment and simply the sum of these two which is 4 k. So, I will replace this by single spring which is 4 k.

Similarly, segment B C as got two springs in series I can easily find the k equivalent 2 k by 3. For the segment C D I will replace with the single spring which is 2 k by 3 and this will have a single spring which is simply 4 k, so k equivalent is straight away equal to 4 k.

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So, we know k is force by displacement. Force is unity in this case. Displacement is the coefficient. K is k equivalent of the respective sector. If I am giving unit displacement to the first sector I will get delta 11 which is nothing but 1 by k equivalent of the first sector which is 1 by 4 k. Now I want to find delta 22, because in this equation if you see omega 1 by omega square is equal to sum of delta ii m i, so I want 11, 22, 33, I do not want off diagonal elements in this matrix I want only the elements along the diagonal 11, 22, 33, and so on.

Now, to get 22 I must get unit force here as I did for 1, the moment I give unit force here I want k equivalent of the remaining part of the problem. So, now I have got two springs 4 k and 2 k by 3 in series, so k equivalent for the segment or sector B C to find delta 22. Now, the system is 1, but this is going to be k equivalent nu, this k equivalent will be 1 by 4 k plus 1 by 2 k by 3 so I put it like this here which is 7 by 4 k which is 4 k by 7. And delta 22 as you see from this equation is inverse of 1 by k equivalent I get 7 by 4 k.

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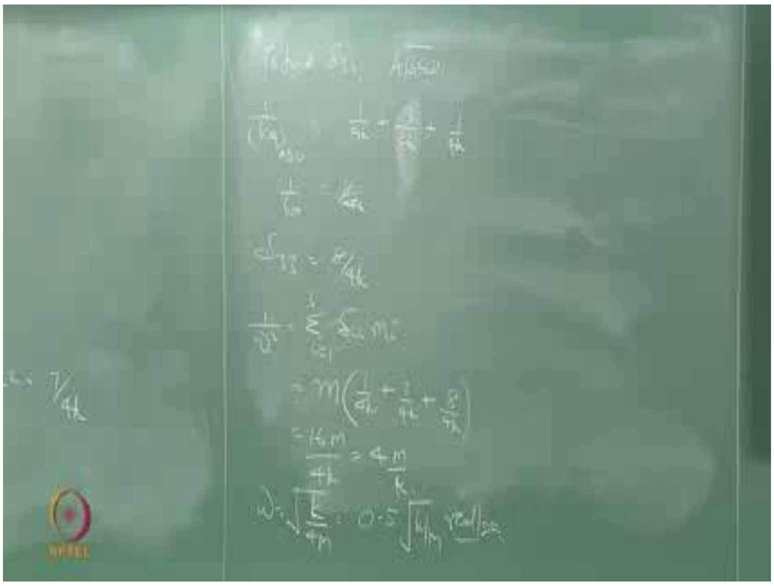

Now I want to find delta 33. Now to find delta 33 apply this concept for the entire segment A B C D. I have got three springs; 1 2 and 3 in series. Therefore 1 by k equivalent of the segment A B C D is 1 by 4 k plus 1 by 3 k by 2 plus 1 by 4 k; 3 by 2 k is it not. So, 1 by k equivalent is 8 by 4; therefore delta 33 8 by 4 k.

So, 1 by omega square will be sum of delta ii mi i 1 2 3 in this problem. So, all are m therefore m is taken out delta 11 is 1 by 4 k plus delta 22 is 7 by 4 k plus 8 by 4 k which is 16 m by 4 k which is 4 m by k. So, omega can be found out as root of k by 4 m which can be the value. So, Dunkerley has given the fundamental frequency of vibration of the system which is three degree in just simple steps like this.

Now, Dunkerleys has got a very major disadvantage. It I realises the spring as an equivalent system. It does not give me the mode shape corresponding to the frequency. It will give you only the frequency not the mode shape. Therefore this method is considered to be approximate technique of finding the frequency. Now, one will be interested to know what happens in off diagonal elements in my delta matrix, because it is talking only about the diagonal elements 11, 22 and 33; what happens to me remaining elements we will talk about that slightly later in the next example.

So, the home assignment is very simple; you must be able to derive the equation of motion for the given problem not from equivalent system, but from a regular system itself using energy method and Newton's method, and you must compare and you must get the same equations of motion for this. You can also solve this using a classical eigen solver problems and get omega 1 and see is it coming to be (Refer Time: 50:52) k by m or not. These two are take home assignments for yourself interest, but I will proceed forward in the next method that is make use of this and proceed forward for other numerical methods. Any doubt? Any question?

Thank you.