Dynamics of Ocean Structures
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Lecture – 15
Numerical Examples – Eigenvalue Problems

Welcome to the 15th Lecture on Module 1 on Dynamics of Ocean Structures. In this particular lecture we will talk about some examples on Eigenvalue Problems.

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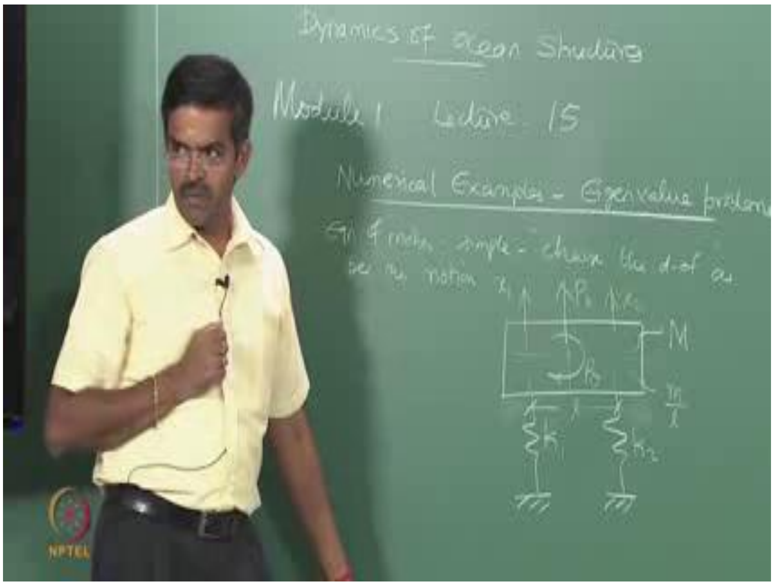

We have made couple of examples in the last few lectures to make you to understand how write equations of motion for a single degree and two degree, same concept for multi degree. How you can use principles of mechanics for finding out equivalent lateral stiffness matrix and how to get the frequency of vibration for an idealized single degree freedom system model. Now if it is a double degree or multi degree how will you actually get the omega that is the problem what we are going to address today. But there is a very classical difficulty in writing in equation of motion which we have experienced in the last few examples.

Now, equation of motion can become simple if you choose the degrees of freedom as per the motion. It means the choice of degree of freedom will help you to make the equation of motion simple. We will take an example where the degrees of freedom are not marked at the point where the mass is lumped. In all other real examples we had seen the systems where the degrees of freedom are marked at the points where mass is lumped, but we very well know that the number of points lumped does not qualify for degree of freedom degree of freedom is related to only to displacements. But fortunately or knowing we keep on lumping the mass at the points where degrees of freedom are marked, therefore mass matrix became always diagonal it was easy for us to solve.

Now we will take an example where if the degrees of freedom were not marked at the point where the mass is lumped or if the degree of freedom is marked at the point where the force is applied what happens to the equation of motion, what is the complexity; that is what we are going to talk about. We will take an example like this let us say, I have a system may be a pad or a deck resting on let us say two springs with two different stiffness. Just for understanding purposes otherwise you will not know if you put both as k we will actually not know which k I am handling in the equation. So, make it k 1 and k 2.

System has got some c g, this is the point. At this point which is the mass center I say my degrees of freedom in terms of force and rotation are marked. So, the force in the vertical axis applied at the c g point and the rotational motion is also applied, because spring has different stiffness so the pad will be keep on oscillating it will create a motion or a rotation about this point c g obviously; therefore we take that as the force P theta is the moment applied and this is the vertical force.

Whereas, the degrees of freedom are marked at the point where stiffness is applied, let us say this is my x 1 and this is my x 2. And of course, the mass is capital M with the total mass of course the continuous mass can be m by l ,and if the length is l then obviously the total m by l will give you the capital mass. Let us even consider this l between the points where this degrees of freedom is marked where let us say this is l let us not take it to be l. Hypothetically the points are at the end I have given a deliberate space for you to see they are actually at the ends.

So, there are some important observations in this problem. The problem shows that the forces P t and P theta are not applied at the point where the degrees of freedom are marked; that is the first observation. The second observation is the degrees of freedom are marked at selected in locations where stiffness is concentrated. So, from an first hand understanding we will know that if you select the degrees of freedom at the point where the mass is lumped you experienced the mass matrix to be diagonally dominant and half the elements we becoming 0. You can expect that in the stiffness matrix now. So, it is a very simple thumb rule, if your degrees of freedom aligned either with lumped mass points or with a stiffness points either of them will become half diagonal 0.

In the earlier examples you saw that force is applied at a point where the degrees of freedom are measured. For example, a frame problem we had x 1 and x 2 we had a lateral force. In the previous cases we had x 1 and x 2 and we had a spring force. So, we always mark the forces where the degrees of freedom were marked or selected, but in this particular example I deliberately violated this norm saying that my forces will be applied to the c g where the degrees of freedom are not measured at c g, deliberately violation let us see what happens to this problem.

If you have problem of this nature how will you solve. Before we solve this problem write equation of motion let us try to understand what the practical applicability of this problem is, where this problem applies. This has got two significant applications; one it can be a foundation resting on a soil where the soil stiffness is modeled as a (Refer Time: 06:34) equivalent springs. That is one classical example of geo technical application on dynamics.

The second can be you want to isolate this deck from the sub structure by some system. It means the vibration of the deck should not be transferred to the supporting system. It can be sea bed, it can be pipe or it can be any other secondary or primary system on which the secondary system is resting. The idealized example of this could be a typewriter in olden days where people used to keep a pad below when the typewriter is being used the table does not vibrate or you would have noticed the typewriter is kept directly on the table you would see the legs of the table will get loosened very quickly because typewrite posses lot of vibration in vertical and horizontal axis.

So, these are nothing but isolators which is a very common practice in earthquake engineering. It has got very good applications practically in many examples we will take an idealized model. Now we all know at this moment clearly that it is a two degree freedom system model a spring mass system, mass is continuous of m by l nature spring is available here. One can also convert it to an equivalent lateral stiffness of springs are in parallel you can always find k equivalent and you can always say the mass is lambda at this point and let the degrees of freedom also be lumped here, you can convert this to equivalent to single degree and solve. But I want to solve this as such as a two degree problem and I want to write equation of motion for this.

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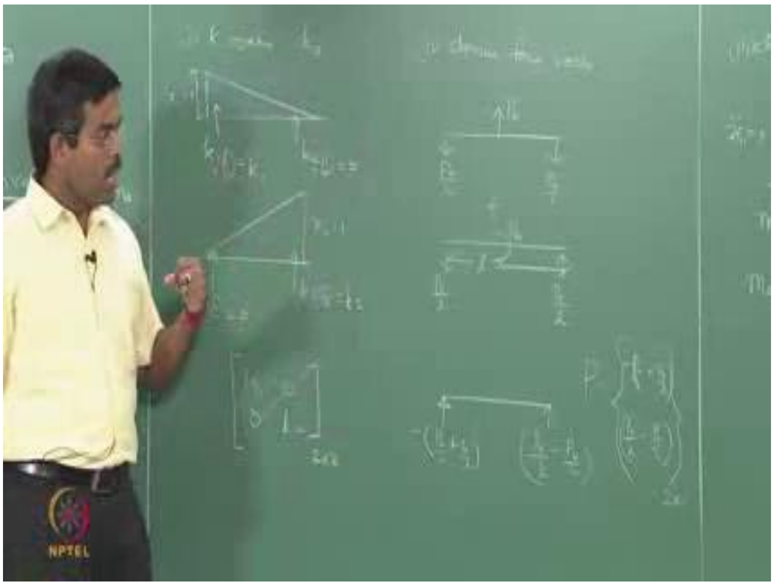

Let us first start with the stiffness matrix. We know k ij is the stiffness coefficient I want to find force at ith degree I give unit displacement at jth degree and I get the force keeping all other degrees of freedom (Refer Time: 08:26). Now, this is the spring point which is actually hinged so there is no moment. This displacement which is x 1 should be unity as per the derivation so I will get the force. The force will be applied vertically in the same direction because I want to apply this displacement I must pull this spring up, I must pull this spring up to give this displacement otherwise I cannot give this displacement x 1 up. So, I should say this as my k 1 1 row first column next and I must get this as my k 2 1 row first column next.

Remember stiffness matrix will be always derived column wise, you are giving displacement to the first degree getting the force at all other degrees 1, 2, 3, 4, etcetera, but the second subscript will always remain 1 because you are giving unit displacement at the first degree, so the first degree is here. Obviously, k 1 1 will be practically equal to my k 1 because k 1 is nothing but force per displacement. The displacement is known to be as unity I will get the force of the spring stiffness itself, is it not. And of course, the spring does not move at all, therefore this is going to be 0.

Similarly, I give unit displacement here and I want to pull the spring up, therefore this is going to be k 2 2 row first column next and this is also going to be k 1 2 row first column next, stiffness matrix is always derived column wise so the second column is now available to you. And for unit deflection or unit displacement in x 2 direction k 2 is the stiffness therefore this will having simply k 2 and this will be 0, because this spring does not move at all, no extension cost to this spring. So, my stiffness matrix now is a 2 by 2 matrix, why because there are 2 degrees of freedom if there are n degrees of freedom the stiffness matrix will be always n by n.

Remember the stiffness mass damping all will be always a square matrix. So, I can enter the values here k 1 0 0 k 2. You will experience immediately since the degrees of freedom I have marked at the points where the stiffness is focused the half diagonal elements are becoming 0 which was the practice earlier in mass matrix. Because the point where it is degrees of freedom are marked where the points fortunately where the mass was lumped, therefore mass matrix became diagonal now stiffness matrix became diagonal.

So, one can immediately expect that the mass if we derived now for this problem will not be the off diagonal elements will not be 0 should not be rather 0 that is the cross check. Let us say I want to derive the force vector. Now why am I deriving a force vector, because force is applied at a point different that of degrees of freedom. Now I want to know the contribution of P t at x 1 and x 2 the contribution of P theta at x 1 and x 2 locations they are different. If had x 1 and x 2 be in located exactly at here and here obviously x 1 and x theta then P t and P theta will be an simple vector. But, in that case it is not so because the contribution of P t at x 1 location x 2 location has got to be derived they are not applied. Let us derive this.

I apply P t here so obviously by symmetry we know this is P t by 2 and this is P t by 2. I have taken a very simple example for you to understand. Similarly apply P theta here, so it is a clockwise moment I must have an anticlockwise couple we already know this system of understanding which will be P theta by l and P theta by l, where l is the length between the forces or the reaction. Remember these reactions are actually offered by the springs, so the spacing between the springs is l, hypothetically the springs are at the ends, but I have shown you to make it understand that they are located in the system like this.

So, let us sum this two, because they are independent let us sum this two and try to find out. So the complete reaction will be minus of let us say my P t is acting downward so I am putting it upward because I want x 1 in upward direction I can write this as minus of P t by 2 plus P theta by l. Similarly, x 2 also I want in the upward direction the force should be in the direction of that of x 2, x 2 is upward. Therefore, I should say it is P theta by l minus P t by 2. So, my force vector is like this. The p vector which is the force vector is minus P t by 2 plus P theta by l, P theta by l minus P t by 2; my other force vector.

So there is no damping matrix here, because there is no damping applied to the system here externally. So, damping matrix need not be derived. This is 2 by 1, it is a vector two rows and one column. Let us derive the mass matrix.

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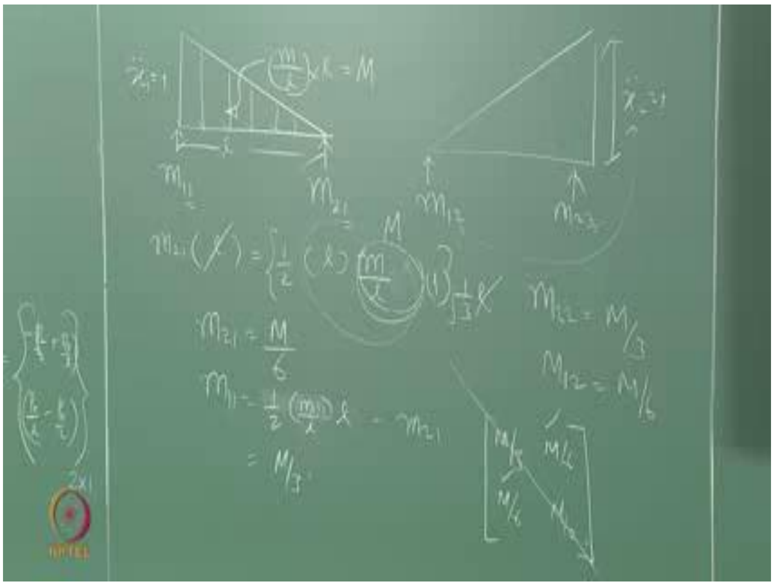

We know that mass matrix is not going to have 0 off diagonal elements, they are called off diagonal; o f f, they are not half off diagonal elements. They will not be 0 because mass is continuous here, whereas degrees of freedom are not measured to the point where the mass is supposed to be lumped which is the mass center or center of mass for center of gravity of the system. We will derive the mass matrix as the same fashion as that of stiffness. What we do is instead of giving unit displacement I give unit acceleration, same pattern I am just following the same algorithm but I do not give unit displacement for stiffness but I give unit acceleration for mass matrix.

So, all acting down I have reactions and this value which will act at the c g of this triangular loading will be m by l of l which is considered as capital M. I will call this as small m that is the mass matrix coefficient as 2 1 1. The second subscript 1 shows that I have given unit acceleration at the first degree. The first subscript 1 and 2 shows these are the respective locations where mass needs to be worked out because these are the degrees of freedom map in the problem.

So, 11 and 21 I want to find these two from principles of statics very easily. I will take moment of this point, so m 21 multiplied by the distance which is l should be equal to half that is the triangular loading base is l height is unity, but it is m by l of l of unity into one third of l. So, this becomes m this l goes away, so my m 21 essentially becomes m by 6 half l m by l this is unity there is no l here there is unity. So, this gives me the m value, m by 6. Half m by l of l into l minus m 21, so m by 3 into 1 sorry m by l into l or into 1 of (Refer Time: 18:05) let this l as a length; this 1 is the acceleration.

Similarly, for the second load you apply unit acceleration here which is x 2 double dot is unity and you get these values as m 12 and m 22, the second subscript indicates unit acceleration is at second degree and I will get m 22 as m by 3 and m 12 as m by 6. So, my mass matrix will be m by 3 m by 6, m by 6 m by 3. You will see that the off diagonal elements are not 0 and however the matrix is diagonally dominant the values will be higher along the diagonal, along the leading diagonal. It is call leading diagonal of the matrix. The other is called the off diagonal of the matrix.

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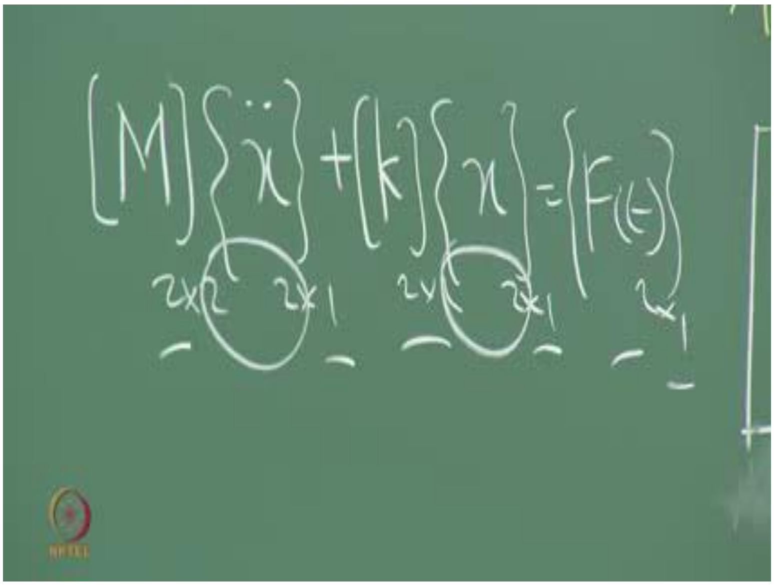

I have now k matrix, I have now m I have f of t I can even write the equation of motion simply as M x double dot k x f of t which is this 2 by 2 2 by 1 2 by 2 2 by 1 2 by 1. So, that is the total compatibly, the equation is written. Now we know how to how to solve this. We have to now examine how to solve this equation of motion and how to find the essential characteristics of this function like omega and phi. Because in the earlier examples is being single degree or idealized single degree we could find omega n very easily either by finding out equivalence stiffness matrix or by a single stiffness and single mass we can find omega n. But in this case they are all full matrices. All the time we are not able to idealize it to a single degree you must know how to solve for the natural frequency in the given system, what will be now called as classical Eigen solver problem. We will now enter into that.

Now any difficulty here, so we have learnt two things here, if the degrees of freedom are not selected carefully you will end up in a system where the stiffness matrix can become 0 the mass matrix will have off diagonal elements. But, one uniformity in all the problems is that the stiffness matrix and mass matrix in vector all will be symmetric and square more or less. But, in offshore structures it is interesting to know that off diagonal elements are the matrix stiffness and mass may become asymmetric also. One can easily argue a question that if the matrix remains all the times symmetric then I can easily singularize this matrix and use either only upper triangular or lower triangular I can save some times in solving this matrix, whereas in offshore structures the matrix that have to be solved is a full size because you will have asymmetric elements in the matrix.

So, where are those examples will come, because in this case you see matrix whether k not in this case in other cases either k or m they become more or less symmetric. But there should be some cases of structural forms of offshore structures which will invoke asymmetric non-diagonal elements. So, what those and why they became asymmetric that is very interesting to know. Remember friends only in offshore structure classical form of geometry asymmetric elements in (Refer Time: 22:11) are available. In all other structural forms they will be always symmetric only, so offshore structure therefore very special because we must know why they are asymmetric. I will come to that later in the second module.

Now let us understand simple problem first. So, now the question asked is, now I have my mass k and c and f of t which are available to me how do I get the natural frequency of vibration of this model; that is my problem what we call classical Eigen solver. This example which you discuss can also be discussed as a play field problem, where a person sits and plays here and the springs what you see here are the legs of the person who pushes and recoils. You must have used this game in the younger days it is always advisable that you play with your brother of sister, if your father and you play you will always either stay down or he will be always up. So, the recoil is very high.

It is is a very simple model which is actually having a application of dynamics, because the force is applied at this point the pivot, whereas the reaction is given in different point. That is a very interesting example what you can realize now at this age. So, all the games which have been practiced in India have got actually best engineering about 100 years back. That is why our social culture and our intelligence are appreciated all over the world, because we have applied engineering in even playing of games also, but the unfortunate part is they were not documented. We read them separately; therefore we do not know how to integrate them. But when one they integrate it becomes the patent of somebody else in abroad.

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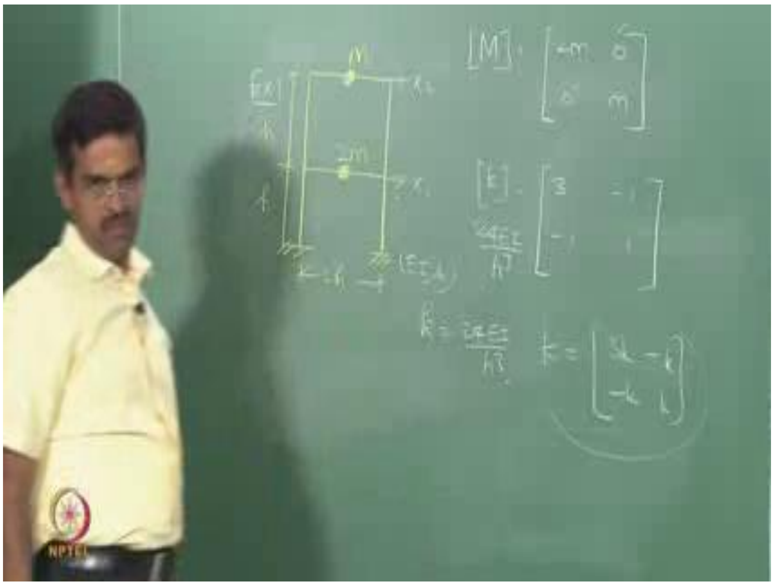

We will take example one now the Classical Eigen Solver problem. We have already drawn this and derived this equation earlier yesterday in the last lecture. I will borrow the same equations I will precede further. This was a single way two story problem with lumped mass at this points which were 2 m and m and the height of the story was kept same even this was kept as 2 h and the degrees of freedom were marked here as x 1 and x 2 we also had the mass matrix as 2 m 0 0 m. Now you will appreciate why I am writing

0 here because, mass is lumped at the same point where the degrees of freedom were marked.

Stiffness matrix remains symmetric EI and h being the property of the material in the members and we say 24 EI by h cube as a multiplier out we say this is 3 minus 1 1, is it so. So, I simply say small k as 24 EI by h cube that is my small k, therefore my capital K becomes 3 k minus k minus k and k. You will again notice here that since stiffness is not the point where the degrees of freedom are marked the stiffness matrix is in total and stiffness matrix is leading diagonal larger and it is symmetric. These are classical observations which you must keep on repeatedly observing when you keep on writing these matrices so that you know that you are not making any mistake, and you will also know that carefully how you will select these degrees of freedom so that they automatically get formed in a normal generation.

Now, I have equation of motion which is m x double dot k x as 0, because there is no force applied to the system. Then one may ask the question if there is no force then why displacement we all can talk about free vibration. We also know that even the force not applied can give a displacement and system will vibrate. So, I want to know the frequency of vibration or frequencies of vibration of this system.

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So, eigenvalue problem, now the first step is to find omega n that is natural frequencies on the system. Now, one can ask a question; how many natural frequencies will be given in the system if the degrees of freedom is n, the degrees of freedom in a given system are n you will have n number of natural frequencies. Of course this n does not stand for the number they stand for natural, the suffix n stand for natural. If you have n degrees of freedom here you will have n omegas the interesting part is all these n omegas all will be unique. Let us see how.

So, to find omega the classical eigenvalue problem says take a determinant of an equation which is k minus omega square m and set it to 0. That is the mathematical implication of finding eigenvalue solvers. Let us do that here determinant of 3 k minus omega square 2 m, I am substituting the k and m value respectively in the respective slots; m is 0 here therefore it is minus k minus k and k minus omega square m and set it to 0. So, 3 k minus 2 omega square m of k minus omega square m minus k square is 0.

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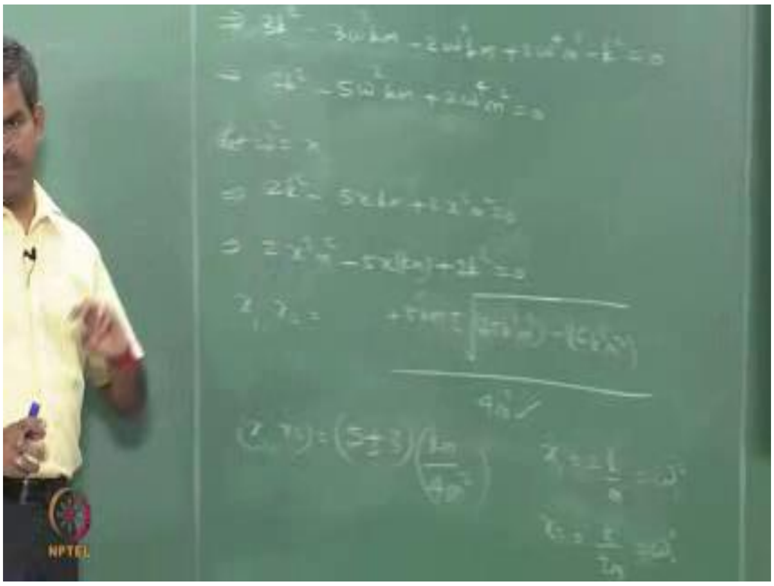

So, 3 k square minus 3 omega square k m minus 2 omega square k m plus 2 omega 4 m square minus k square is equal 0. Implies 2 k square minus 5 omega square of k m plus 2 omega 4 m square is equal 0. Let omega square be x. So, 2 k square minus 5 x k m plus 2 x square m square is 0 2 x square m square minus 5 x k m plus 2 k square is 0.

So, can you find the roots x 1 and x 2? The simple quadratic in x; to simplify you will get x 1 comma x 2 as 5 plus or minus 3 k m by 4 m square. This multiply and this multiply (Refer Time: 30:53) which implies that x 1 can be 8 by 4 2 k by m and x 2 can be 5 minus 3 2 by 4, so k by 2 m. We already know that x is omega square. So, can I say this is omega 1 square and this is omega 2 square? Yes, already x is omega square, so x 1 is omega 1 square x 2 is omega 2 square. These are the frequencies, because you know k and m, k is nothing but 24 EI by h cube m is m kg which is given in the problem. You can easily find omega 1 and omega 2 which we wanted.

But unfortunately the moment you start working on multi degrees along with the frequency one more pair will be generated that is called Mode Shapes. So, mode shapes and frequencies are like husband and wife.

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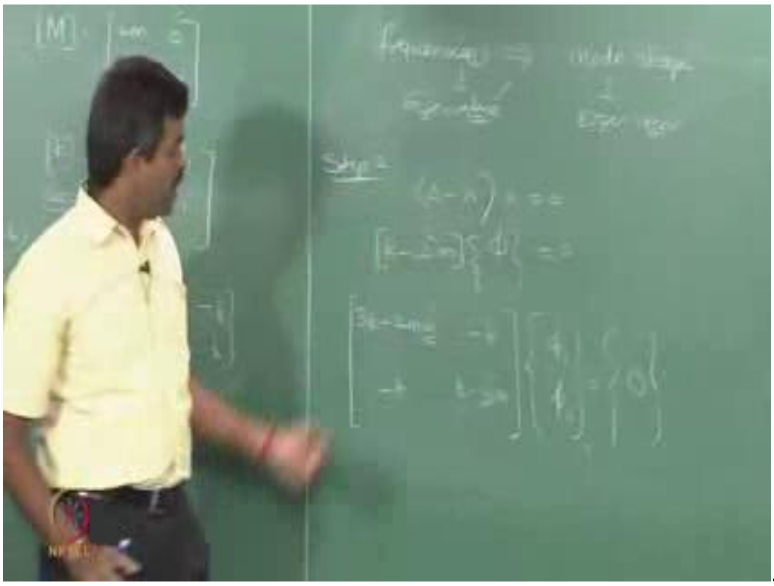

So, one you have frequencies; frequencies not frequency, if you have frequencies it will automatically generate a pair which is mode shape. Mathematically this is called Eigenvalue and this is called Eigen Vector. So, the term itself very clearly tells me that this is the number; this is the corresponding vector to that number. That used to call eigenvalue and Eigen vector. This is a vector corresponding to one number. It means every eigenvalue will have one vector. If there are two eigenvalues there will be two vectors.

So, the problem does not stop here. The moment you identify the husband we have to identify the wife also or you have identified the wife you must identify the husband otherwise it is dangerous. We have to identify both of them parallelly. Without one the other has no value, especially in India we believe strongly this. Therefore we have to identify this. Now we identify either the husband or the wife we really do not know, but we have to identify the corresponding counterpart, so we want to find the Eigen vector also.

Now, mathematically to find the Eigen vector the rule is A minus lambda x is set to 0. A is the matrix which is k minus omega square m. So, k minus omega square m matrix is what we call as the A matrix. Multiply this matrix with the vector which is phi vector, which is the Eigen vector is set that to 0. Let us see how we can do this. We already have this matrix with us, I have erased but still you can help me writing this matrix which is going to be 3 k minus 2 m omega square minus k minus k k minus omega square m. Multiplied by 2 vectors phi 1 phi 2 set it to 0.

That is a generic equation. Now remember in this equation we have the value of omega, so if you really wanted to find the first vector substitute omega 1 here you want to find the second vector substitute omega 2 here. I want to first substitute omega 1 and get phi 1; phi 1 is not this phi 1, capital phi 1 first vector I want to get. So, each eigenvalue will have a unique corresponding vector. Let us do this. So, let me write down the values of omega 1 square and omega 2 square.

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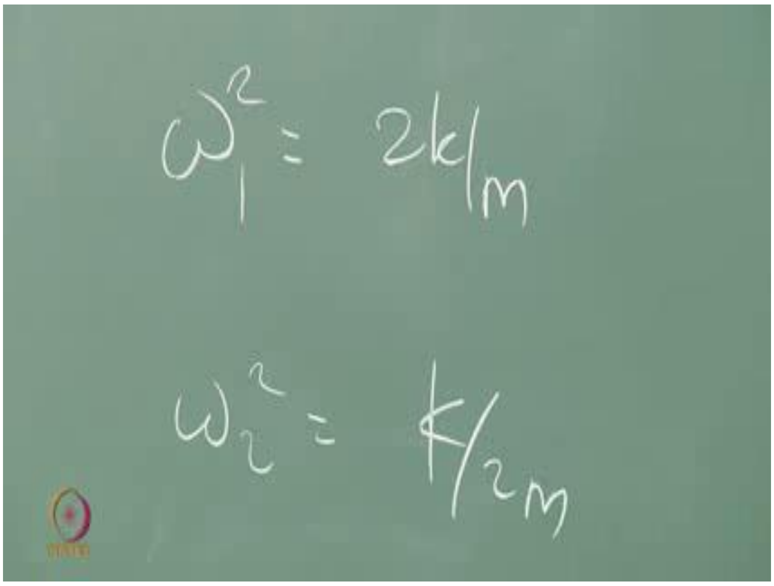

Omega 1 square is two k by m and omega 2 square is k by 2 m.

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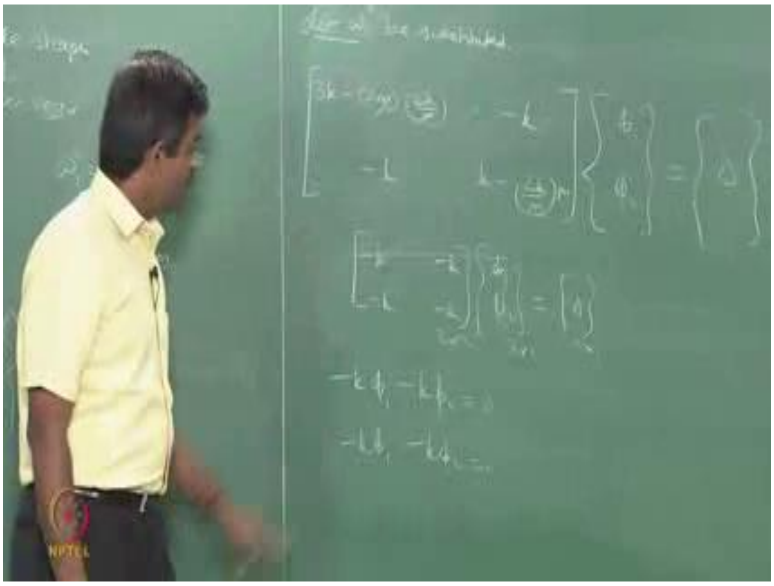

So, let us say omega 1 square, still remember we carefully notice this I am saying this is only an assumption that omega 1 value is equal to 2 k by m. One can ask me question amongst these two frequencies which is named as fundamental frequency or which is the first frequency in the first mode shift, unless otherwise you know the values of both you cannot find because these are nothing but the roots of the quadratic equation. Which root is cutting the x axis first we really do not know, because there are no algebraic value numeric values for this number. Let us first find the numeric value then we will say which is omega 1 and omega 2.

This omega 1 does not mean this is the first frequency I am pointing omega 1 because it is going to correspond to my so called omega 1 in my problem. Ultimately after you get both omega 1 and omega 2 then I must consult and say this is the fundamental frequency (Refer Time: 36:58), because now I have no numeric value for this depending upon the value of k and m you know this values can be different. Therefore, we cannot comment that. So do not get confused that when I write omega 1 this is the first frequency or fundamental frequency not like that, we really do not know that you have to estimate that. In this problem deliberately I have taken it as omega 1, but it will not be omega 1 I have deliberately taken like this.

Let us try to find out this. So, 3 k let omega 1 square be substituted, so 3 k minus 2 m of omega 1 square is evaluated 2 k by m minus k minus k k minus 2 k by m of m of phi 1 phi 2 is set to 0. If I simplify this m goes away 4 k 3 k minus 4 k is minus k minus k minus k this goes away minus k of phi 1 phi 2 is set to 0. Let me expand this. Row and column are multiplied; we know how to multiply them. We can see the compatibility, they are perfectly compatible. So, minus k of phi 1 minus k of phi 2 is 0 minus k of phi 1 minus k of phi 2 is 0. Both equations are same. Now there is a mathematical instability in solving this problem I have two unknowns phi 1 and phi 2, I must get two equations, but both of them are the same how I solve this. In this case please understand phi 1 and phi 2 are relative values they are not absolute, so I will assume one get the other in proportion of the 1. That is why they are called relative numbers. So, I will take away this I will just retain only this part.

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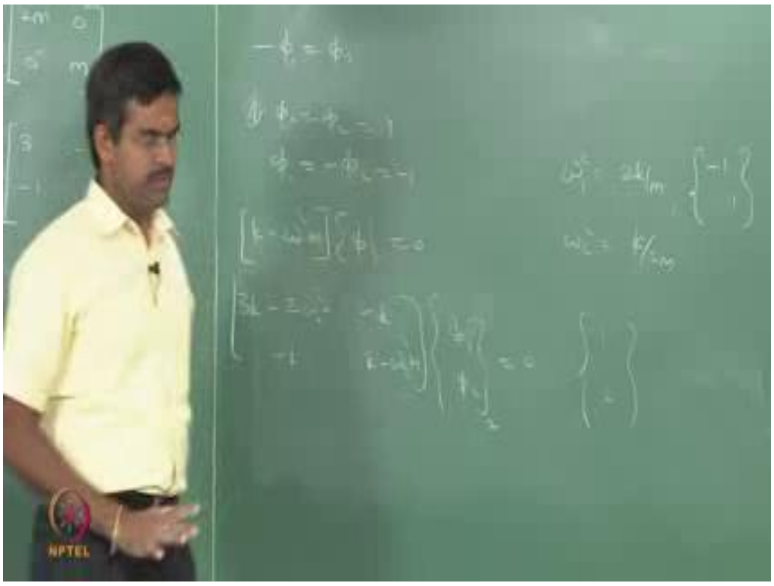

So, this implies that minus phi 1 is phi 2. Any equation you pickup this implies that minus phi 1 is phi 2. If phi 2 is phi 2 phi 1 is minus of phi 2 can I say this; if phi 2 is phi 2 phi 1 is minus of phi 2, I got the value. Let us say let phi 2 be 1 phi 1 will be minus 1 proportionate. So, my vector is very simple now for this the corresponding vector is 1 and minus 1, no this is 1 and this is minus 1. Phi 1 is negative of phi 2.

One can ask a very serious question here, sir you have taken phi 2 as phi 2. Say let phi 1 be phi 1 phi 2 is minus phi 1 my vector is going to be 1 minus 1; this is not same as this, is this correct both are alright, they are proportionate value. So, if you are happy on Mondays this way, you do it if you are happy on Tuesdays do it this way so does not matter. We have got the first vector corresponding to the first value. Now I want to find the same (Refer Time: 40:36) of second set. So, k minus omega square m multiply with phi should set to 0 and let us do this matrix again. So, 3 k minus 2 omega square m minus k minus k minus k k minus omega square m, I substitute 2 here multiply this vector with phi 1 and phi 2 set it to 0.

Please note this phi 1 is nothing to do with this phi 1, this is the second vector this is where the confusion will start. People will start substituting this values here they are messing up this is not, this is something different vector. Because remember that is why I said this is a husband of this wife, he cannot be a husband of this wife. This is very simple to understand physically like this. Provided this is applicable only to India I think but still it does not matter. Let us say phi 1 and phi 2. So, can you quickly find out phi 1 and phi 2? Then you have to give me like this, which is 1 and which is 2 because I am confused which is 1.

Now let us try to find out which is my fundamental frequency. There are two ways of finding out this. Pease understand this very carefully, this is the point where very intricate discussion and understanding is required. One way of finding out this is you substitute the value of k and m if you know get the numeric value. Now k I have 24 EI by h cube I will know the value of v (Refer Time: 42:49), I know the value of I the section is known to me, I know the dimension I can find, I know m I can find and get the numerical answer you can compare. But the easiest and correct way to find out is this when the vectors have got both positive representations there is no 0 crossing. Let us plot this.

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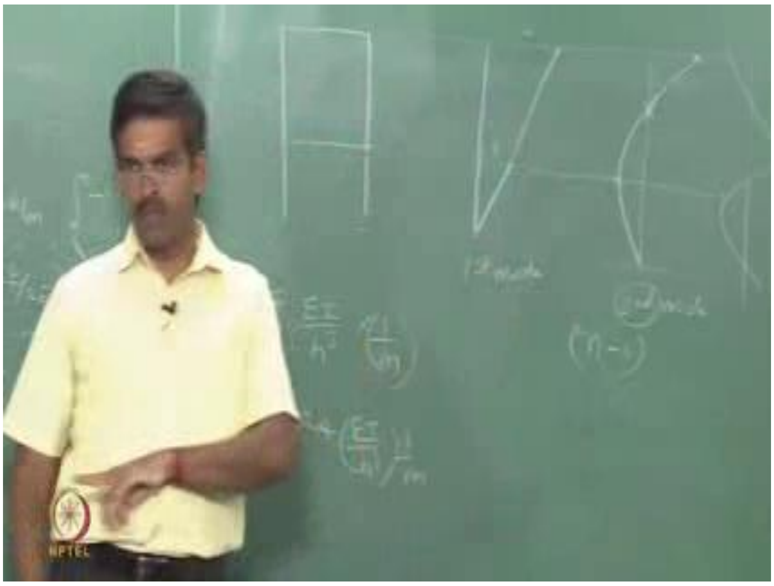

This is my structural system, Eigen vectors are displacements relative to each other. Why displacements, the x 1 and x 2 are actually displacements. Now I say here from this vector if the first mass is displaced by one times, the second mass is displaced by two times as per this vector. So, the deflective shape of the mass does not cross equilibrium at any stage. This is called the first load. On the other hand to be as the thumb rule first mode will have all positive or all negative displacements. There should be no crossing.

Now plot the second one. If the first mode is negative, the second mode is positive so I have a structure like this, it has crossed; this is second. Now, one can ask me a questions, sir let us plot the third mode in this case it is not applicable it has two crossing can we find out something related to this. If your mode is nth number you will have n minus one crossing. If the mode is the first number then no crossing, because 1 minus 1 becomes 0. Since, this is the first mode the corresponding wife is the first frequency. The corresponding eigenvalue is the first frequency which is this, so I can call this as fundamental even without knowing the values of k and m that is very very important to understand.

Characteristics in the fundamental frequency should be based on the deflected shape or the mode shape of the system not on the numeric value alone. Now let us see if I substitute value what happens. Let us pick up this 24 EI by h cube by m and this value is 2 into 24 EI by h cube by m. If I say EI by h cube m is a value of 1000, 15000 any number this value will be always (Refer Time: 45:42), which implies the same meaning it will the first mode. So, do not try to compare numeric answers and conclude the first mode or the fundamental frequency. Fundamental frequency is the first frequency at which the system will have no crossing from the equilibrium position. It is easy for us to identify. So, if I have n frequencies in a given system I will have n degrees of freedom I have n frequencies and all matrices mass, stiffness, everything will be a square of size n by n.

Let us quickly spent over 5 more minutes to give you a self (Refer Time: 46:23) assignment which I want you to solve. So, there are some self assignments questions please pay attention to this and try to solve them.

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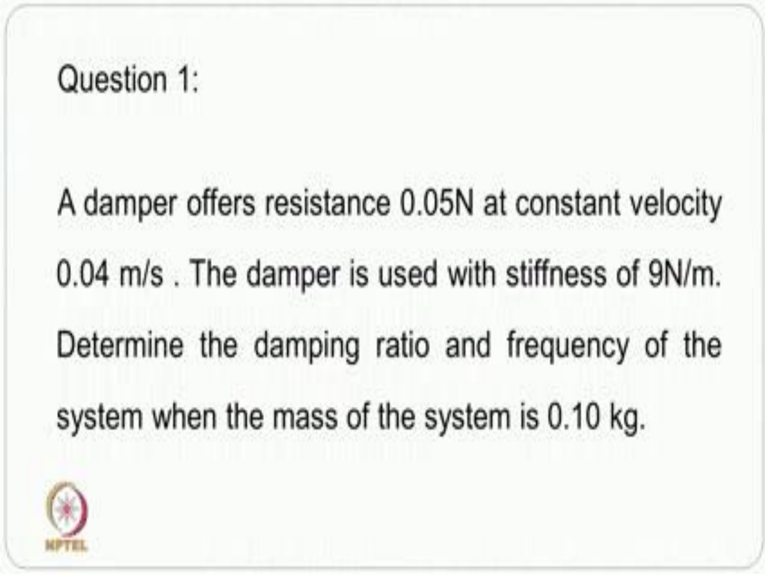

A damper offers resistance to the value of 0.05 Newton at a constant velocity of 0.04 meter per second. You can note down this values and try to solve them. The damper is used with stiffness of nine Newton per meter. Determine the damping ratio and frequency of the system when the mass of the system is 0.1 kg, of course a model study. Anyways the questions will be available posted in NPTEL we can see them later, but try to understand. Write down the known's and unknowns clearly and are you capable of solving this problem.

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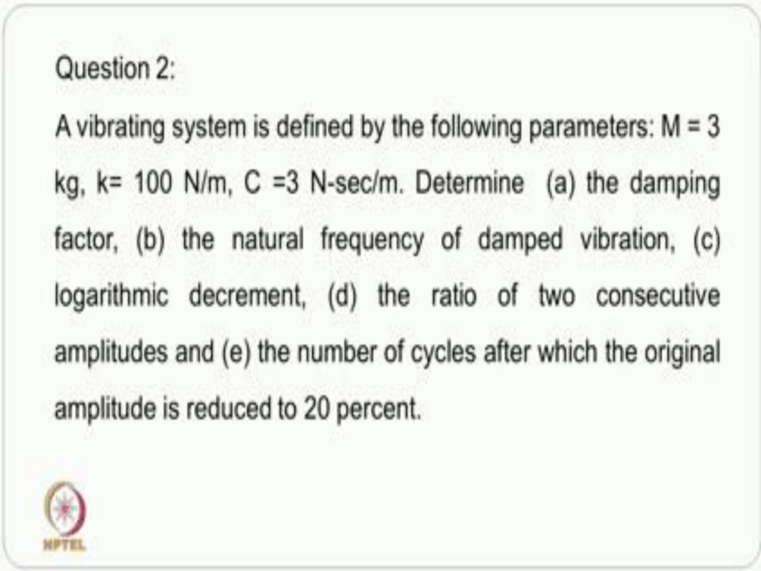

The 2nd question is on a vibrating system which is given by the following parameter mass is given, stiffness is given, damping coefficient or constant is known. I want to find the damping factor zeta the natural frequency of damped vibration omega d which you already solved the similar problem in the class one can easily solve this problem.

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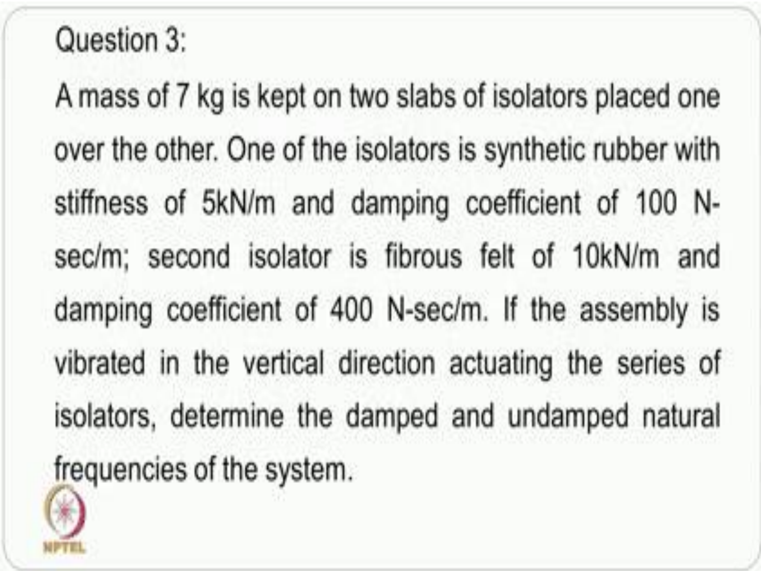

The 3rd question is again a mass of 7 kg is kept on two slabs of isolators. There are two slabs of isolators the mass of 7 kg is kept. One is a synthetic rubber stiffness is so much, damping coefficient is so such. The second is the fibrous felt stiffness so much, damping so much. We are trying to work out a related discussion with these two. If the assembly is vibrated in the vertical direction actuating the series isolators determines the damped and undamped frequencies.

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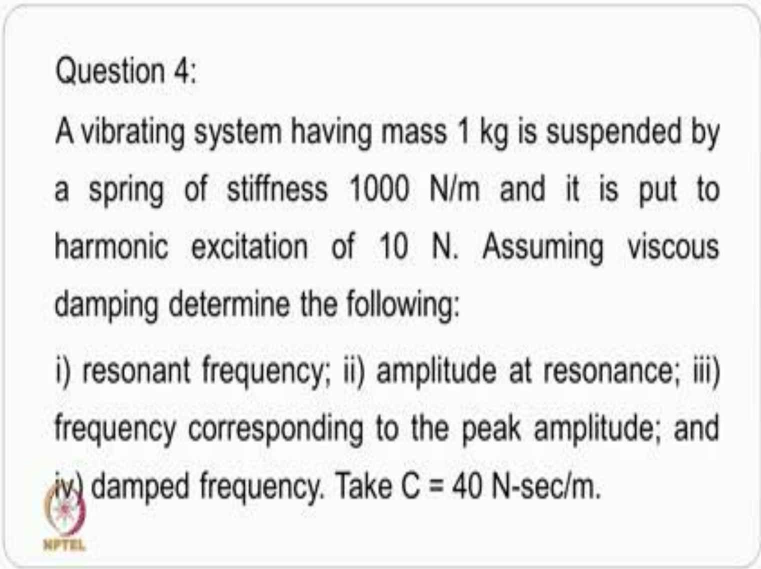

A vibrating system having a mass of 1 kg suspended by a spring of stiffness 10 or 1000 Newton per meter. He has put a harmonic excitation of amplitude of 10 Newton's. Assuming viscous damping find out resonance frequency it is forced vibration problem, resonance frequency, amplitude of resonance, frequency correspond to peak amplitude and the damped frequency omega d if c is known to you.

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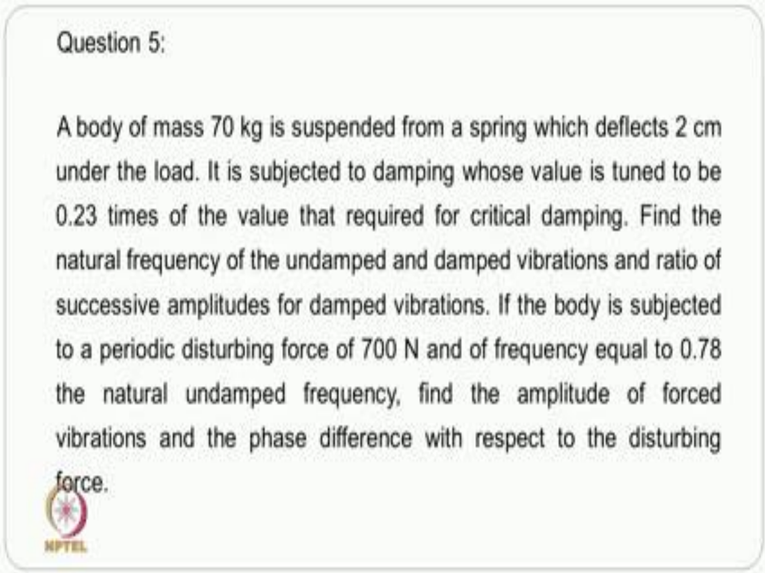

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The 5th problem; body of mass 70 kg suspended from a spring which deflects 2 centimeters under the given load, so the static deflection is known to me. So, initial displacement x naught is now given to me. It is subjected to damping whose value is tuned to be 0.23 times of that of the value the critical damping. So, critical damping you must find out. Find the natural frequency of damped and undamped vibration and the ratio of successive amplitudes from the logarithmic decrement equation. If the body is now subjected to periodic disturbing force of 700 Newton’s and of frequency equal to 0.78 times of omega d, then now find the amplitude of forced vibration and phase difference between the respective disturbing forces.

So, try to solve this problem and understand the answers. We will not solve this problem here; this is a self assessment you have to solve. So, kindly do not post any request of giving the solution for this in NPTEL. We will not give this; actually it will remain as an unsolved until the exam is done.