Dynamics of Ocean Structures
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Lecture - 14
MDOF - Numerical Examples

Now let us look at the 14 Lecture on Module 1.

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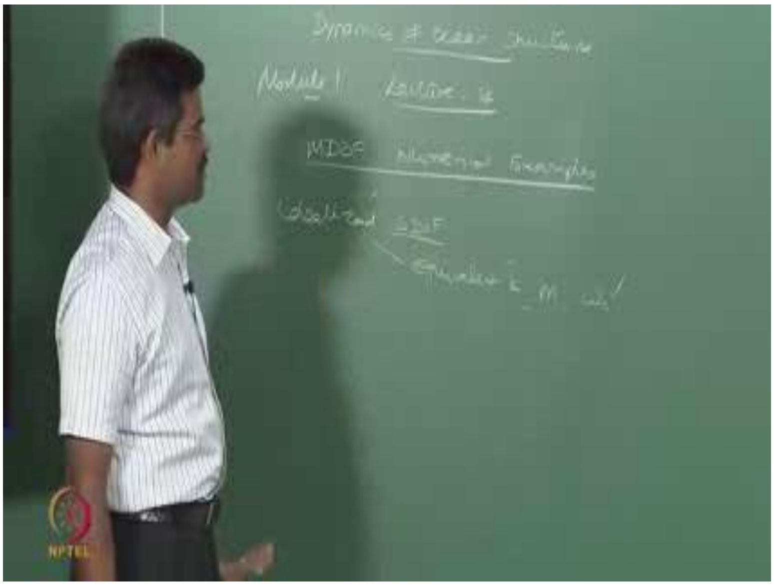

We already solved couple of examples make you to understand how for a given idealized single degree freedom system. The moment I say idealized originally the model is not a single degree, but you can always idealize. Idealize session comes with equivalent stiffness which we call as k bar which is a latest stiffness matrix. So, I know single mass and therefore I get omega n. Now this only a method by which the ideal helps you define out the approximate frequency of vibration of the system if it is idealized as a single degree freedom system.

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We will take another example now where the problem is solved from the first principles back again. Let us say how this is done. Let us say I have a spring mass system like this, it is subjected to a load which is F naught and F naught is an impulse function. (Refer Time: 01:41). The amplitude is F naught till q from 0 and above q for the infinite period there is no load. So, the load is acting only for a small duration of 0 to q, beyond q for t greater than q load is 0.

So, there are two cases here. I will split the problem into two parts; one is obviously 0 less than t less than q, two t greater than q. And this is also true for t less than or equal to q. For any value more than q then the function are very easy to remember. So, let us try to answer this problem into two parts and see how this can be handled. We know that the equation of motion for this problem is very easy F of t.

We know the complimentary function, solution on this problem which is x of t which we know from the first principles is A cos omega n t omega n t. Where A and b can be evaluated depending upon the initial conditions given to the problem. The initial conditions given are like this at t equals 0 x 0 is 0 at t equals 0 x dot 0 is also 0. Let us say these are the conditions given in the problem.

So, the system does not have an initial displacement initial velocity at t is equal to 0. So, the particular integral for this for the path of 0 less than t less than or equal to q is going to be F naught by let us say m D square plus omega n square that is what is it is. So, I can always say F naught as F naught e sin omega t and so on or F naught simply e 0 t, a standard function. Therefore I can say D square is 0.

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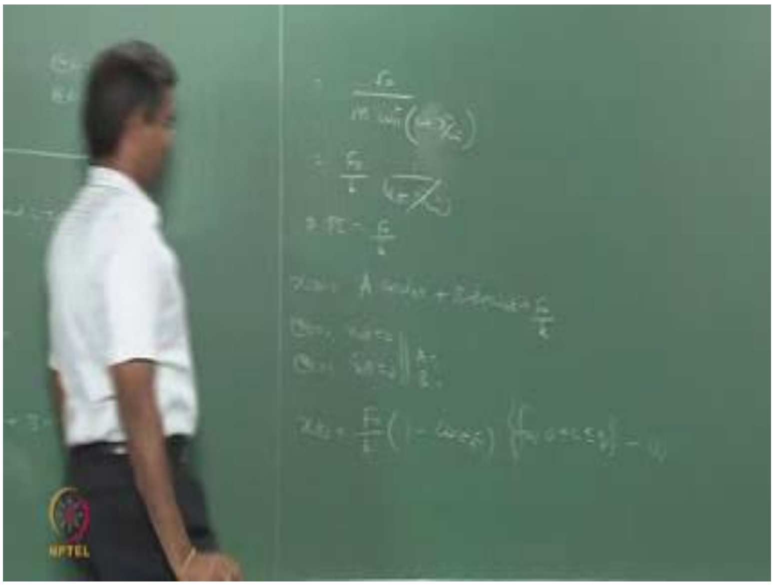

So, I can do like this which will be F naught by m omega n square D square by omega n square, which will give me f 0 by k I must put 0 here therefore my P I will be F naught by k, therefore my x of t will be A cos omega n t plus B sin omega n t plus f 0 by q. Now at t is equal to 0, x 0 is 0 at t is equal to 0 x dot 0 is 0 get the values of A and B.

Therefore, x of t can be known as find out the values of A and B it will be f 0 by k 1 minus cos omega n t for 0 less than t less than q. You can find A and B from this function you will get this. Let us call this equation number 1, first part of the answer. This only one segment of the answer I have one more.

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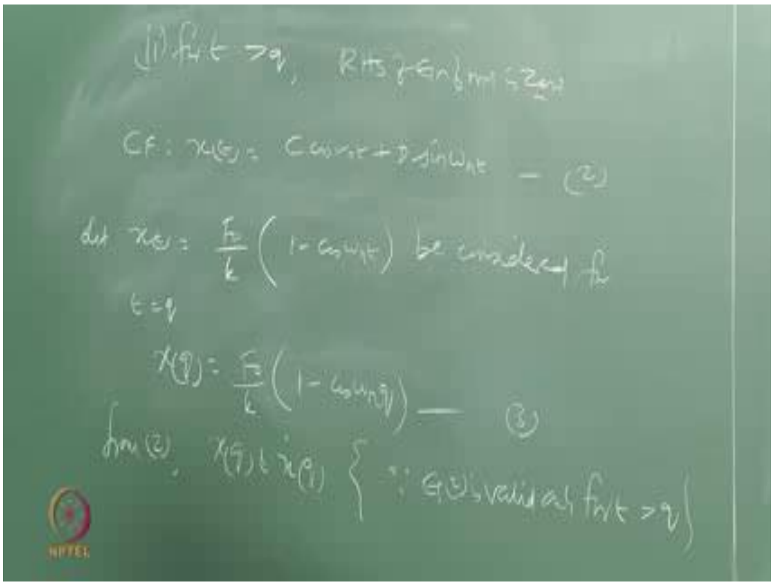

Now for t greater than q we know the right hand side of equation of motion is 0, there is no load. So therefore, only complimentary function will be there which is x of t which will be given as C cos omega n t plus D sin omega n t, because only complimentary function will be there, there is no P I. Now to evaluate C and D which are constants of this particular equation this requires initial condition from the first part, because at t equals q velocity displacements are not 0, there are some values so substitute those values. So, let x of t which is equal to F naught by k 1 minus cos omega n t be considered for t equals q, so x at q will be f 0 by k 1 minus cos omega n q.

So, I will call this equation number 2. This is of course equation number 3. Now from two I require x q and x dot q, because equation 2 is valid only for t greater than q. X q already have here, x dot of q also I can find because x of expression is there I can differentiate this and substitute and get x dot q. Once I know that substitute this condition this is the original equation and find x of t for the second part. Tell me what is the second part x of t.

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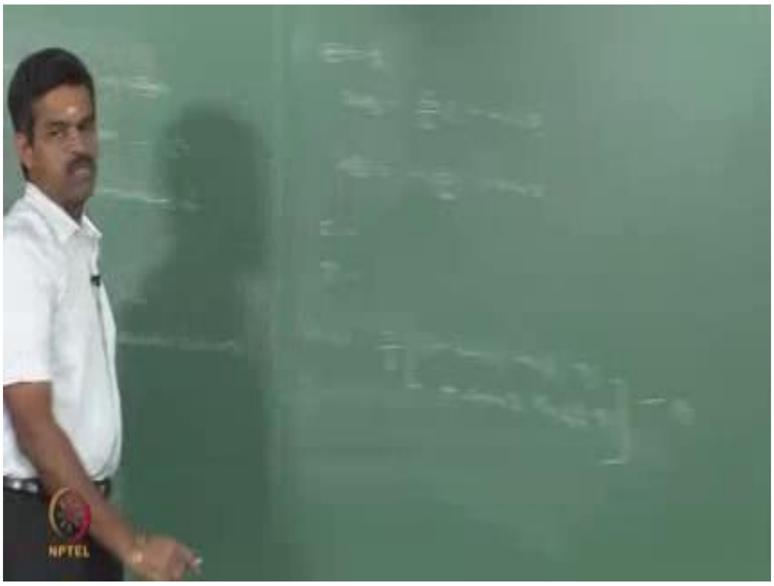

So, at t equals q x q is f 0 by k 1 minus cos omega n q x dot q if you differentiate and substitute you will get omega n f 0 by k sin omega, sorry. So, substitute back and find the C and D constants. And then you will know that x of t will be come after evaluating C and D f 0 by k 1 minus cos omega n q which is borrowed from here t minus q plus; that is the answer for second part.

So, you can also solve the problem from the first principles like this to find out the solution of x of t, where an impulse function is applied for a short duration of 0 to q and divide the problems into two parts.

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And this function can be of any nature, it can be a function which is rectangular, it can be triangular impulse; it can be an impulse of very short duration which is called a direct delta function; which we will talk about that slightly later and so on and so forth can be a step function.

So, you should be able to solve the response of a single degree for a given r h s of equation of motion by dividing them into equal number of deviations as they are possible to find. That is the idea of this particular case. We will take one more example, where I will take a two degree freedom system and derive k and m now quickly. Any doubt here? Is it clear?

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Another example; so single bay two story frame which is fixed at the bottom; and the single bay two story frame which is fixed at the bottom. I am taking very simple example to illustrate. In the second module you pick up original structure of offshore and you will know that how we can derive k and m for different kind of structures like the TLP's spar etcetera. I will talk about that in the second module later, but let us first understand how to derive for simple problems. Let us say the height of the frame is over h meters, the width of the frame is about 2 h meters in my (Refer Time: 12:08) it may look proportionate but this all the dimensions are. And I have a lumped mass here which is equal to m, I have a lumped mass here which is equal to 2 m.

Now we all know how to calculate this lumped mass in a given frame. Nothing but the dimensions of the beam and column, columns is taken up to half of this floor, so you know the length of the column or height of the column, you know the cross section dimension, you know the volume, you know the density get the mass, and you know slap therefore you get the mass focused at this particular point which is lumped at this area.

Similarly, there are some additional mass given to this floor therefore this is 2 m just to differentiate this. And we say the frame is subjected to later loading therefore I take the degrees of freedom in the lateral direction. Now there is a very interesting question here, I will come to that question later first let us draw the figure completely. Let us say stiffness of the column is k and this is 2 k this is 2 k and k.

So, to realize this how this happens; k is a bending stiffness in this case because when you push the frame the frame will bend. So you are looking for not the axial stiffness, but the bending stiffness. So, I should look for EI by l and not a e by l, in the last case we have explained that. So, l is fixed in the problem, e is also fixed in the problem for the entire material is almost common constant. The one which is causing more k should be the I, or unfortunately you can have a structure like this.

You can have a structure like this also let say l is phenomenally high which also changes k, which is not realistic because you not see a structure like this more or less the height will be almost equal. But in certain cases the bottom for height can be higher because of some specific requirements of functionality, but you not find this. Therefore, the major variable is k or the stiffness of the member will be essentially should come from I sometimes rarely comes from l or h, but never from e.

Therefore, in the practical cases these columns are having more stiffness. It means the cross section dimension of this column are larger compare to this, that is how you get more k at the bottom floors and get less k at the top floors. Ideally speaking the same column should go through and through, so k, but sometimes the k can vary because you have a higher loading in the bottom floor therefore the columns can be of a larger dimension, Whereas, it goes up and higher and higher the column dimensions can be lower than this and therefore, this is a realistic problems.

Now, there is a very interesting case here see generally in spring mass system you will notice that we always mark the degrees of freedom in the direction as that of the restoring capacity of the spring. For example, I have never marked x as a theta in the problem, because spring does not oscillate spring can only extend laterally not vertically in this particular case. It means the degrees of freedom are generally marked in the problem to match to the restoring capacity of the restoration component of the system. Whereas, in this case if I think that these springs are all of the same nature my x 1 and x 2 should have been vertical, but I am marking x 1 and x 2 in a lateral. It means I say the response of the system is proportional to the force coming on the system.

So, the force is pushing the system to the right, therefore the response should always in the right. But is there any miss match of the concept that the if the degree of freedom is in the same fashion as that of the restoring component it restores, whatever is the restoration here because it is only pushing it is pushing or it is only pushing to the side. So, it is the answer is very clear here. I am talking about this member which is having a bending stiffness I talked about this member which has an axial stiffness.

Your degrees of freedom are depending on the manner in which the restoration will happen. In this case it is going to bend back and restore. So, I am talking about bending stiffness. In these problems bending stiffness should be invoked and not the lateral stiffness. So, do not get confused that we are not marking degrees of freedom in the same fashion as that of the previous spring mass idealize examples.

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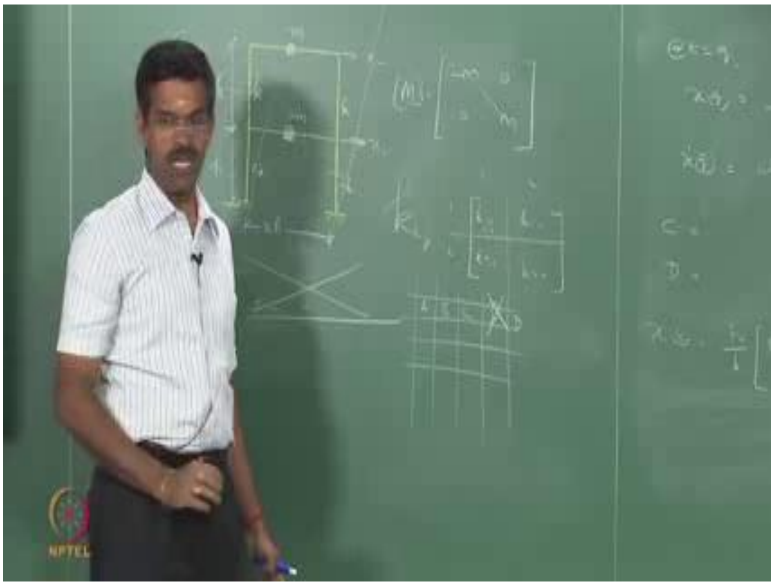

Now, the basic question here is I must derive the mass matrix and stiffness matrix. Mass are lumped, the moment I say mass are lumped and the degrees of freedom are applied to the same point where the mass is lumped then I can simply say the mass matrix will be a diagonally dominant leading element all (Refer Time: 17:31) element will be 0. But it is not the case all the time I will show you next example where this will happen. But in this case easy to write first degree 2 m, second degree m.

On the other hand if this should have been m and this is 2 m I would change this as m and 2 m, no problem. So, mass matrix can be written instantaneously for a given problem provided we can mark the degrees of freedom correctly at the point where mass is lumped. If the degrees of freedom are marked at different points where the mass is not lumped, then the mass component at this point should be worked out which I will do in the next problem.

So, carefully choose the points where you want to measure the displacements usually it is a practice to measure displacement is a floor level, but not a guaranty that you need not have to measure displacements middle of the column you can always. Therefore, it is always idealistic for an engineer to understand mass lumped at the floor. And interestingly I have not marked x 3 here, because this is all evident for people that x 3 will have no moment because it is fixed. And I would have marked theta 1, but I am not marking. So, you must choose the degree of freedom in such a manner which will explain the response of the system in a most simplest form.

So, do not create complication in choosing degree of freedom, because this will add or this will initiate complication through and through the problem. In dynamics make the choice of degrees of freedom at the simplest it should give me the physical representation of the response of the system for the given force. Now, one can ask me question, sir the force are acting also downward it is a (Refer Time: 19:07) structure. Why we do not have degrees of freedom in a wrong (Refer Time: 19:11)? Why here? So, (Refer Time: 19:17) of the structure will not have response because the structure is so rigid in terms of this response. Whereas, the lateral motion of the structure will be highly flexible.

So, you have to also choose which degrees of freedom will be important for the given system, which I have got to trace in the dynamic analysis. So, choice of the degree of freedom is yours. The moment you choose that ideally and intelligently lump the mass at those points to make the at least the mass matrix happy, the stiffness matrix will not be happy. We will see how it can be derived from the first principles again. So let us derive the mass matrix.

Now I want to derive the stiffness matrix. We already know the coefficient of stiffness matrix is like this, it is nothing but the force at ith freedom by giving unit displacement of the jth freedom or degree of freedom keeping all other degrees of freedom restrain that is a standard. So, stiffness is nothing but the force which is responsible to cause unit displacement in the other degree; keeping all other degrees extent. So, stiffness matrix is in this case going to be 1 by 2, because I have got 2 degrees of freedom.

So, I will be generating or deriving k 11, k 21, k 12, k 22 row first column next. You have to follow this like a nursery child. You must have seen in nursery classes people used to write A B C D in square pages A, B, C, if you write d teacher will scratch and say write D like this. Now you may wonder that why this practice was given. Now you see in all international practice (Refer Time: 21:59) asked to write you are always given a coded like this only. Have you ever seen a form where you write your name like this, and then people will write certain like this also. Wherever this coding is available you will write only in a form of a matrix is given to you. And they will also write a please legibly write within the squares, they will tell you to do this. It is easy to recognize.

Similarly, you have to have a mechanism by which you read the stiffness matrix, row first column next, row first column next. You must have this practice all the time. So, I am going to derive this column wise. I want to give in a displacement at 1 and find the forces at 1 and 2. Let us do that. You may wonder that why we are emphasizing on derivation of stiffness matrix all the time. We already know this. Can anybody answer this question? Why do we insist on deriving stiffness matrix all the time in these examples? It has got a very very valid answer. Very simple straight forward answer it is there, I have already told you n number of times in the class. Sorry, whatever may be right or wrong does not matter.

Student: So in the structure recently (Refer Time: 22:10).

My question is very simple, you are going ahead I am in the bottom. I am asking you in my examples why insist on deriving a stiffness matrix? I can write the stiffness matrix directly my job is to find omega n only why I am insisting on derivation of stiffness matrix and mass matrix why?

Student: (Refer Time: 22:33).

Sorry.

Student: (Refer Time: 22:35).

What is that?

Student: Equal stiffness conversion.

Equivalent stiffness conversion mode, no.

Student: It is a basic (Refer Time: 22:43).

No, the essential characteristic dynamic system is only mass and stiffness. If you cannot derive these matrices for a given system probably you cannot do dynamic analysis at all. You may know them so you will wait for somebody gives you this, the moment somebody gives you this you proceed further, but who will give you this? Nobody, you have got to derive.

There are special problems in ocean structures where you will generate your own form for which you are the boss nobody is going to tell you where are the degrees of freedom, what are the stiffness derivations, nothing is available in the literature you have to start from the scratch. So, we are starting from all the problem from the scratch and I will adopt the same principle to derive stiffness matrix for TLP, spar, triceratops, FSRU total in the second module; so you will know how it is been derived from the first principles.

Once you know those examples very clearly done, you can derive stiffness matrix and mass matrix by any form of your choice in research. That is the reason why we are deriving this stiffness matrix.

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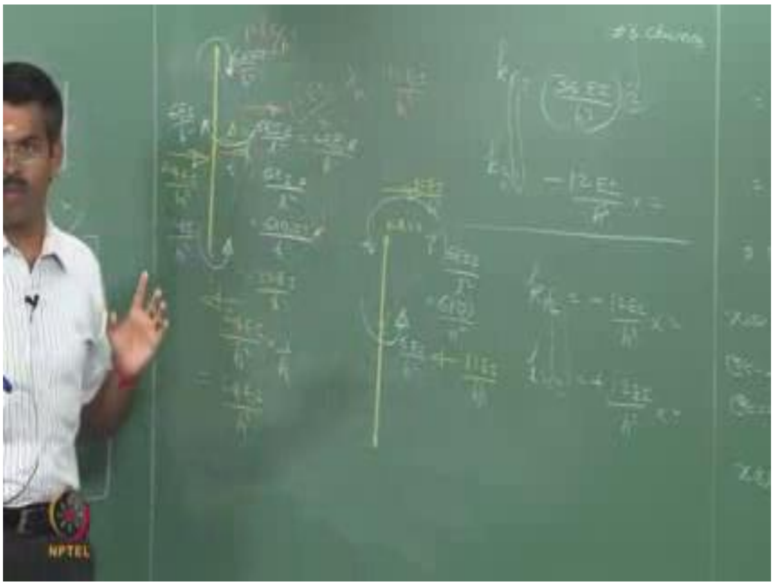

It may be look easy for you but still I will draw this column, this is the point I am just marking the points I should give unit displacement in the direction as that of the force. These are deflector configuration because both ends are fixed; I said all the joints are always fixed. And we already know this when you move it there is always a tendency for the structure to bring it back, we already marked this; similarly this way and this way. This was 6 EI delta by l square for that is a standard expression. As I apply this to this problem it is 2 E 2I delta is 1 by h square. So, 12 EI by h square can I write like this.

So, this is also 12 EI by h square, this will be again 6 EI delta by l square which is 6 EI delta I is only 1 because there is only 1 k that is why I said k is dependent on I not on l divided by h square. It is going to be 6 EI by h square 6 EI by h square. Now these are the moments I do not have any degree of freedom in the moments, I have degrees of freedom only in the lateral dimensions. So, I must convert them into equivalent forces because stiffness is nothing but the force for unit displacement; force along 1 and along 2. So, I must find the reactions, so this is an anti-clockwise couple. I must have a couple of this order which is clock wise because this anti-clockwise. So, 12 plus 12 24, so this is going to be 24 EI by h square by h that is why I do the couple. I can simply say this 24 EI by h cube then this also going to be 24 EI by h cube.

Let us go to the top one, this one. So, couple of 6 plus 6 12, therefore strictly speaking I should mark the arrow here at this joint because there is no space I am marking it here. So, it is going to be 12 EI by h square of 1 by h which can be 12 EI by h cube, this also 12 EI by h cube; the arrow because this is clockwise, this is got to be anti-clockwise.

I have to do similarly for the other column also. There are two columns. Now, I can write k 11 the force in the first degree for unit displacement given in the first degree itself will be the yellow one and the orange one both of them in the same direction, so 24 plus 12 36 into 2, because there are two columns. If there are n columns into n, so this 2 represents number of columns. Whereas, k 2 row first column next stiffness matrix always derive column wise, I am looking for this element now I have already have this element with me, I must look here. The direction is towards right by force is towards left, so minus into of course 2; this 2 is because of two columns.

Similarly, I can draw for the next column for the next two degree. I must give unit displacement as delta equals one. The deflector profile is going to be like this. We know that it will create the moment of this order since this segment of the column is not or this column is not deflecting there is no moment required for this column to restore it. So, this is going to be again 6 EI delta by l square, this is the standard expression let us substitute this for here which is going to be 6 EI because there is only 1 k into delta is unity by h square. So, this is also going to be 6 EI by h square. But I do not have moments as degrees of freedom; I have only the forces towards the right degrees of freedom. Let me convert this into forces this is anti-clockwise, so I can make it a couple of clockwise which will be 12 EI because 6 and 6 12 EI by h cube and 12 EI by h cube.

So, I should write now k 12 and k 22, I am deriving the second column and I am deriving now these two elements. So, k 12 I should look at this object here towards right my force is towards left so minus 12 EI by h cube of two columns. Similarly, k 22 I must look at this point I should look here towards right, so positive 12 EI by h cube of two columns. I can now write the stiffness matrix here.

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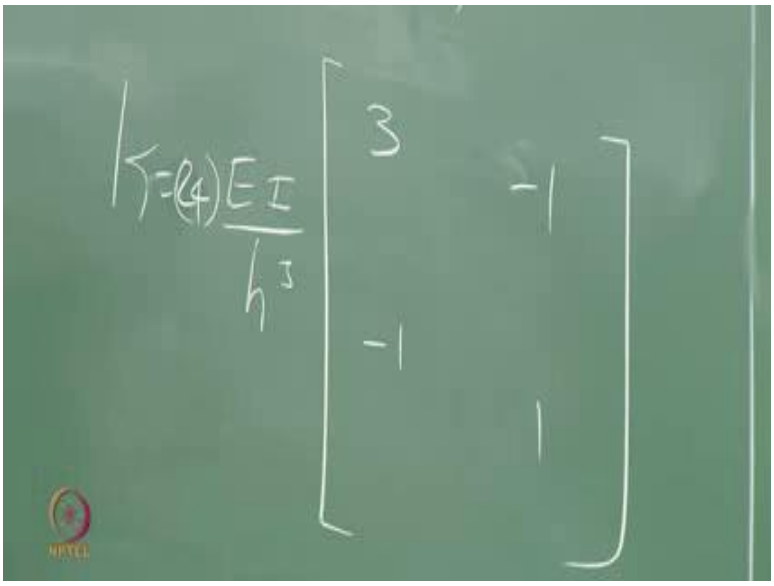

Let us take EI by h cube 24 as a multiplier, because you see this is 72 this is 24, 24 and 24; I take 24 out, stiffness matrix now.

So I have my k, I have my m; I can easily find omega. Now the question is if omega and k and m are 2 by 2 you can get 2 omega and 2 phi's. we will solve this problem probably after couple of examples more. We will retain the solution with us; we will have the same k and same m back again. We will solve omega and phi separately which are Eigen characteristic of this problem.

Ideally one can ask me a question, sir I can convert this to k bar or k equivalent and find omega n. If that is the case you must also have m bar. Now there are two m's here in the previous example if you turn back there was only one m. So, m relates to degree of freedom actually indirectly, so that ideal session was possible. But, in this case it is not possible because k bar will not help you because m is a 2 by 2 matrix. So, I have got to solve this omega and phi separately which I will do later, because I need more examples to understand this then I make you to the next transformation how to find omega n for a given problem we will do that.

So, now the problem stops here apparently that I have got k and m here how to proceed further how to get x of t we will talk about that later, but now we understand how to get k. Please understand the derivation of stiffness matrix is not for any specific problem, but it is independent of the problem, whatever you may be your form, procedure is same you look back all the examples. Whatever may be the form procedure to get k is same that is the beauty of the stiffness matrix in analysis, but I am using that advantage for dynamics.

So, one must strictly follow how to derive stiffness matrix for a given system from first principles give unit displacement get the force. If you understand this strongly any form you have you can easily derive this stiffness matrix easily. You will not stuck up with a research in the beginning. Interestingly, if you have modeled this in any software for a numerical analysis the numerical analysis also give you k. If you know how to derive k you can always check whether the k given by the software or whether the k taken forward by the software to find omega n is correct or wrong.

Therefore, you will know whether the omega n calculated by the software is correct or wrong. If you have no clue how to get k you have to depend on omega n given by the software which will be wrong also. Sometimes you notice if you do not model it properly your omega n will become I mean phenomenally wrong, your whole interpretation will start from omega n onwards. Therefore, the whole design or analysis can go divert into the wrong, it all happened because you do not know how to derive k from the first principles, you always dependent on making the model efficiently, but not to work out k by hand.

The procedure is common it is not (Refer Time: 34:27) specific, it is unique and it is common to all the problems in the same manner. Once you follow this it is very easy for me to derive k, whatever may be the geometry what I want to derive k for that is what the point of insistence is here. So, we will do quickly one exercise problem for you let us you see how many of you can you do it.

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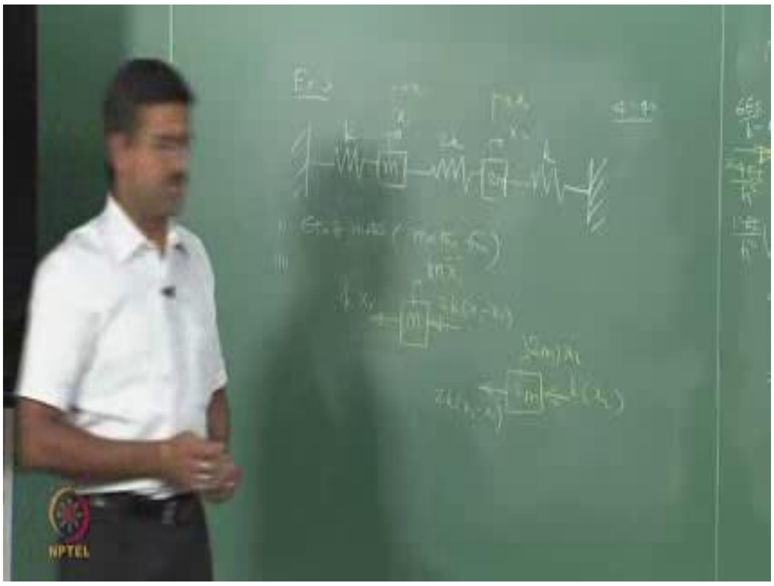

Already we have done this, but just I am giving you this problem for you to solve. What I want is the equations of motion. It is going to be 2 by 2 in the matrix form that is the first question I want. The second question is supposed, if you change this as x 1 this as x 2 will the equation of motion remain same? That is the second question. I want answers from the both questions in 5 minutes.

I can start helping you just to mark. So, k of x 1, 2 k of x 1 minus x 2, 2 k of x 2 minus x 1, k of x 2; of course this is m and this is 2 m and this will have a force of m x 1 double dot this will have a force of 2 m x 2 double dot. Now you have the Newton's law applied here you can easily write the equation of motion and check. I want to tell you a very important point which generally will cause confusion. Please note here this is very important, this is a very classical trick in writing equation of motion. Please notice the directions here, notice the directions here. Generally if you open any literature or text book which writes equation of motion they write like this.

Now, there is no confusion at this point and at this point there is no confusion. The confusion is only here, see the arrows here, and see the arrows here. Apparently when you look these two you will always get instantaneous feeling that this is wrong and that is right, where this is symmetric. If you look closely both of them having the same answer because this is 2 k x 1 minus x 2 towards left and this is towards again left, but x 2 minus x 1. If you put a minus sign here it will become automatically like this.

So, do not get confused and do not follow kindly this, do not follow this. Follow only this, I will explain how this as come again. I pushed the mass towards the right the spring will be bringing it back, so k x 1 and push the mass towards right this spring will push the mass back so 2 k right. The first coordinate of where are you applying write the second coordinate where it is connected. Similarly, look at this mass 2 m more (Refer Time: 38:42) this will move towards left because it is storing 2 k of x 2 minus x 1. And when you push this spring will push back there is no relative moment here so only k of x 2.

It is very very easy to understand, very easy. So, do not be carried away by this and do not try to think yourself I will do problems in both ways, you will land up in mess. Follow only one it is easy. I think from this you can easily write the m and k matrices easily. And I want to hear answer on this second part of the problem. If we change x 1 to x 2 or you swap the coordinates let the mass remain same no problem, but I want to swap the coordinates; will equation of motion change?

So, please try to answer this as an exercise problem yourself and try to tell me what your idea is or what is your observation on these kinds of problems? Also try to solve this problem using energy method and see whether equations of motion are getting comparative. Any doubts, any questions here?

Thank you very much.