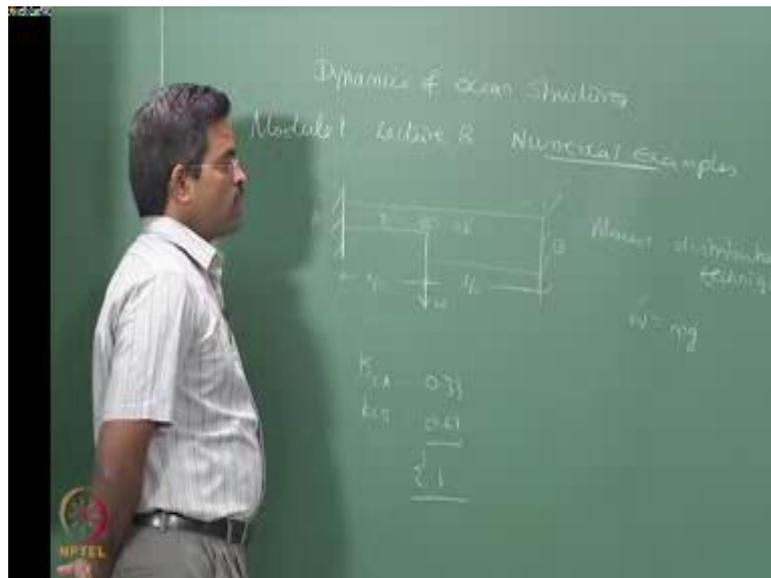


Dynamics of Ocean Structures
Prof. Srinivasan Chandrasekaran
Department of Ocean Engineering
Indian Institute of Technology, Madras

Lecture – 13
Numerical Examples

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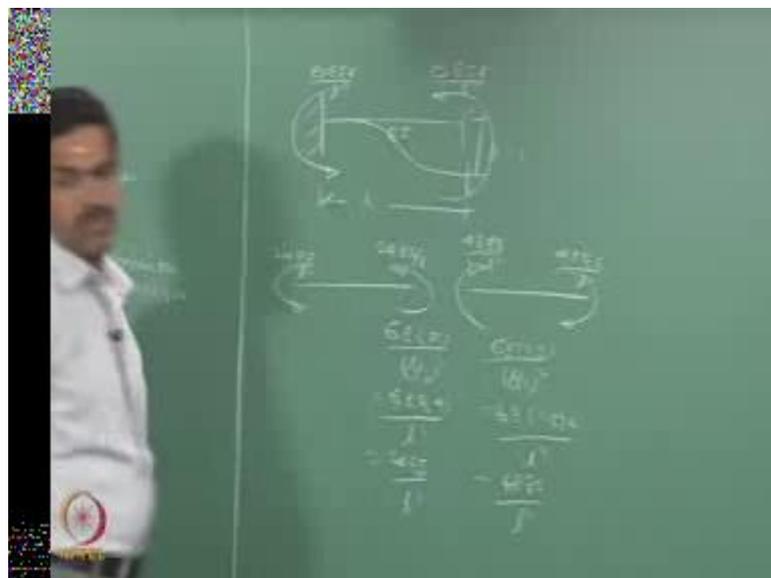


So, we have been working out the example. Now, it is Lecture 13 on Module 1 on Dynamics of Ocean Structure Course. In this module, we were discussing about the fundamental structure dynamic, we are solving some numerical examples. Let me continue with this problem of a fixed beam which we did partially yesterday, let say a fixed beam with variable moment of inertia. So, this is i and this is $2i$ and let say length of the segment is l by $2e$ and let say this is point c and we are going to use the moment distribution method or the moment distribution technique to actually find the equivalent stiffness of the section. So, we already found out the distribution factors. I think this was 0.33 and this was 0.67 . So, the sum should be always 1 , however may be the number members meeting at a joint.

So, now we want to find out the force which is responsible to cause unit deflection at this point because this is the point where my w is hanging. So, I know already that w is equal

to $m g$. So, if I know w , I can always find $m g$ and for a single degree idealize system. If I know k equivalent of this, I can easily find the natural frequency of vibration caused to this beam because of the suspended load w . When you have got both the ends fixed in suspending a load, it is expected that the member will bend. Therefore, looking for a bending stiffness which is in the earlier example, we looked about the axial stiffness $a e$ by l . In this case, we will look at $e i$ by l , but it is not going to be $e i$ by l because there are 2 is and two segments and support conditions are different. Therefore, we have to find out k equivalent in this particular problem.

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So, we will try to revive what we did yesterday. If I got a support, if the support settles by δ , we already said if this is δ we already said this will invoke a moment which is from the first principle of mechanics. It is $6 e i$ by δ square, where this is l and of course, remember having the property as $e i$ and it settles by δ l . So, the significance of marking the arrow direction of the moment is associated to how you are displacing the joint. We discussed this yesterday.

So, the same amount will get transferred here $6 e i$ δ by l square. Since we are interested in finding the equivalent stiffness of the member, we always say I want to find the force for unit deflection. Therefore, δ is going to be unity in the whole exercise.

So, I am trying to write down these values for both these pieces separately here. Let say this is 1 piece and this is another piece. So, you know this is going to settle down because that is what I am trying to do. The displacement should be given the direction of the load. Therefore, I will have a moment of this order and this order essentially is going to be $6 e i \Delta$ by l^2 . So, if I write this $6 e i$ because i is here and l^2 i say it is 1 by 2 , the whole square which will give me $6 e i$ by l^2 which will $24 e i$ by l^2 square. So, I write here $24 e i$ by l^2 square, same $24 e i$ by l^2 square.

Now, the right hand segment that is $c b$, again the left hand support of this part is settling down. So, therefore the arrow direction is going to be lifting it up and similarly clockwise here. So, this again is going to be $6 e i \Delta$ by l^2 . So, I write the value here $6 e$ of $2 i$ by 1 by 2 the whole square which is $6 e$ of $2 i$ of 4 by l^2 square. So, 48 and this is going to be $48 e i$ by l^2 square. This is also $48 e i$ by l^2 square. I want to know a moment distribution for this particular member $a c b$ in a conventional form which we know.

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Member	AC	C	B	BC
Dist. factor		0.37	0.63	
Fixed End Moment (FEM)	-24	-4	+2	+4
Balance (C/B)		-3.96	+7.92	
Final moment	-27.96	-31.92	+31.92	+39.96

$\chi = 24$

So, let us say member $a b$, sorry $a c c a c b$ and $b c$ this is joint c , this is joint a , this is joint b . So, this is joint, this is member, this is a conventional table what we generally make for a moment distribution technique. So, then I write the distribution factors.

Distribution factors always worked at only for a joint which has got more than two members at least. So, now a and b does not have more than two members. Therefore, there is no distribution factor for a and b for c there are two members. Therefore, this is 0.33 and this is 0.67 which we already worked out yesterday as k by $\sum k$. Now, we also find a value called fixed end moments. It is not finite element method. This is fixed end moments. Let us expand this. It is fixed end moments briefly abbreviated as let say f e m. So, let us have a sign convention. We will make sign convention clockwise as positive.

So, you all know that this is c, this is a and this is b. So, I can call this as m_{ca} , this as m_{ac} , this as m_{cb} and this as m_{bc} . The first letter corresponds the joint, the second letter corresponds the next joint that is how the nomenclature written start from here, this anti clockwise, so that minus i only write the value. I will have a multiplier of e_i by l square outside because it is going to be common for all. I will only write the number similarly here again anticlockwise. So, minus 24 similarly c_b , it is of course clockwise and so, plus 48 and b_c again clockwise plus 48. So, the minus and plus corresponds to the sign convection. What we have assumed can assume the other way also does not matter whether the clockwise is negative or positive does not matter, but you should follow a sign convention through and through constantly. So, you will know the problem how to solve.

Now, obviously you will see that at the joint c there are unbalanced moments. It is not balanced. So, I have to balance this. Let see principle of moment distribution. First you balance the unbalanced moment and then, you distribute it. Therefore, it is called moment distribution method. Now, how do you balance this? This is very easy you must know the procedure, but anyway for people, who do not know this, let us quickly explain them, appreciate that we are trying to follow the principle of structural mechanics applied on to dynamics to find out k equivalence. We are not interested in finding the moment distribution $b_m d s f d$ for this problem. No.

We are interested to know the natural frequency of the system that is vibrating. That is very important. So, I want to know the equivalent stiffness k . So, I am using δ is equal to 1 here. So, I am trying to get the force responsible for this. So, now, the positive

number is higher. Therefore, the balance will be negative positive number is higher. Therefore, the balance is negative all the time. Now, if I say this is x , this is x value will be actually equal to the difference of this which is 24. No sign here because sign is already written here. No sign here 24, the difference of these two algebraic difference of these two multiplied by the corresponding d f how much is this. So, I will write it here minus 7 point be louder.

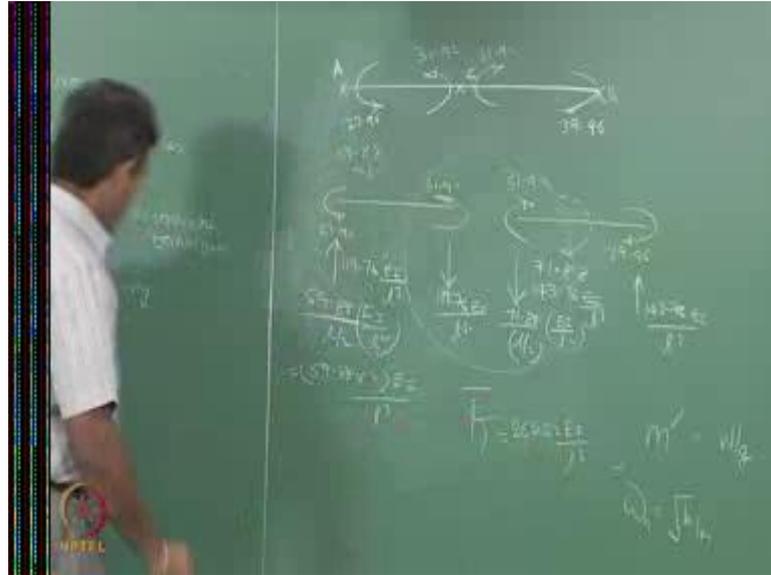
Student: 92.

Obviously, one can find this value also is nothing, but difference of these two again 24 multiplied by 0.67, 16.08 because this sum should make it as the difference of these two in terms of its sign also. So, it is 24. Now, 17 plus 724, right once you do this, you try to carry over. This I call this is carry over. Now, the carry over is nothing, but the same sign half of this. So, this 3.96 and this is 8.04. So, this call one cycle, one balance, one carry over is one cycle. Let us make a total now. So, while totaling you will not total this. This is only factor not to be totaled. You will total this and this. So, it is minus 31.92 and you will total this plus 48 and minus 16. So, it is plus 31.92 and of course, plus 48 minus 8. So, plus 39.06, right and this is going to be minus 27.96.

Now, this has been balanced. Therefore, I can write this as final moments. They are not final moments because we got some values here. The final moment is because the joint is balanced. Now, if you have more than one joint in one cycle, all the joints may not be balanced. So, keep on doing this cycle until they are balanced, but moment distribution has a limitation. You can modify this method within 3 cycles. You can balance it. We are not looking into the description here. That is all dynamic class mechanics class. We will forget about that. My job is again taking back these forces to find the stiffness for unit deflection. That is the problem here. That is what I have. Let us not deviate from that at all. If you have any doubt on further joints are there, how will you distribute? Please read a classical mechanics book and you will understand.

Now, my job stops here that yes, I have got the forces. Let me check whether the answers are 96, right. I will remove this. Let us draw these values graphically in the same problem here.

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So, this is a, this is b and this point is c. So, we already know clockwise is positive. So, I get anti-clockwise at e c. So, it is negative. Therefore, it is anticlockwise which is 27.96. So, convention is you must write the value at the head of the arrow that is the convention. If you had not written, you please follow them. The convection is you must write the magnitude at the head of the arrow. That is the right way of writing. You cannot write anywhere. You want it written here. There is no need to put plus minus here because the sign conversion says clockwise is positive. So, the value is negative. Therefore, it is anticlockwise. If you have doubt in whether clockwise and anticlockwise direction, please see your watch if you are wearing one. Now, you see the other one is again negative. Let us say this is again anticlockwise because it is 31.92 and this is clockwise 31.92. So, it is balanced again positive 39.96. I want to get the forces. Now, I have the moments. So, with this I can plot the minimum diagram, but I want to plot to share force. Actually I want the forces. It is very interesting.

Let us draw the segments separately and say this is 27.96 and this is 31.92. So, now you see this particular segment of the beam has an unbalanced moment of this sum which is 59.88, right. So, when I got an unbalanced moment of anticlockwise 59.88, it should be balanced by a couple which is opposite. Now, this couple should be clockwise which will balance this and couple is nothing, but force in the distance. I know the distance is 1 by 2.

I want to find the force. So, this is simple $59.88 \times l^2$ and $e \cdot i$ by l^2 is already there. You see here is already there because these numbers do not make any meaning except that multiplier of $e \cdot i$ by l^2 . So, I should say this as $59.88 \times 2 \cdot e \cdot i$ by l^3 is that 119.76.

Student: 119.76

So, I will write down $119.76 \cdot e \cdot i$ by l^3 . This will also be same $119.76 \cdot e \cdot i$ by l^3 . Let us do the second part. So, this is clockwise of 31.92. This is clockwise of 39.96. So, it has got unbalanced value of clockwise of. So, this will get balanced by a couple of this order, yes clockwise is unbalanced. Therefore, the couple will be unbalanced. This value is going to be $71.88 \times l^2$ of $e \cdot i$ by l^2 . Now, commonly people make mistake here they say that this value of beam is having value i . This value of beam has $2 \cdot y$ should i multiply $2 \cdot y$ here. Please do not get confused. This is a multiplier drawn from the table. This $2 \cdot y$ and i checker has been already included in these values. Please see the calculations back again. So, no $2 \cdot y$ here the multipliers just borrowed from the table. That is all. So, this is going to be how much? $143.76 \cdot e \cdot i$ by l^3 $143.76 \cdot e \cdot i$ by l^3 .

Now, I am bothered about this particular point. Why this is the point where I give unit deflection? So, the moment is say the deflection is unity. The force at this point will be equal to the stiffness directly. So, my k value will be directly equal to sum of this. You will see automatically the force is happening in the same direction as that of the displacement. Suppose by mistake if you g_1 , these arrows up you have d_1 somewhere wrong. So, much is the total 119.76 plus 143.76 and so, my $e \cdot i$ by l^2 how much.

Student: 263.5.

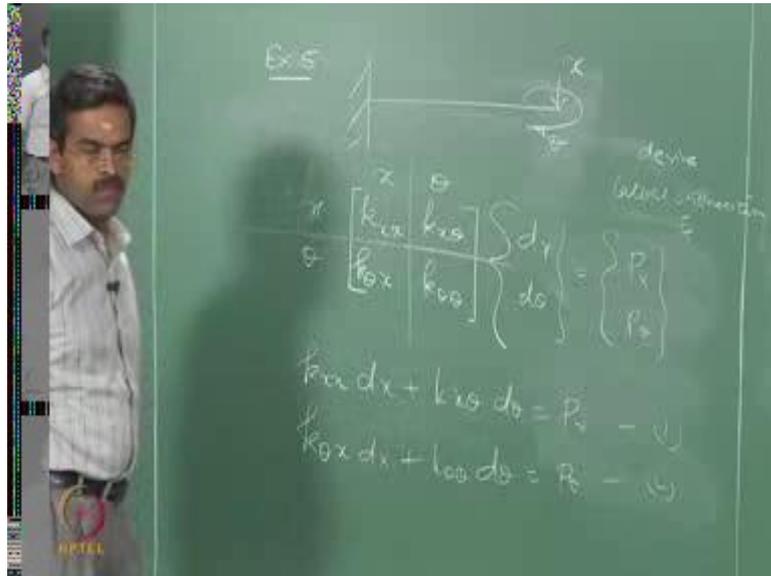
Yeah I can put k bar k bar indicates equivalent stiffness. So, I know m which is nothing, but w by g . I can find ω n square root. We can also check the units of k .

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Let us say $E I$ by 1 cube newton per mm square or newton per meter square. What are units you want to fix I meter to the power 4 meter cube. This will automatically give you to newton per meter which is a classical unit for stiffness. So, mass is already in k g. You will find ω in radians per second automatically which is an equivalent stiffness for this problem. So, it is very easy demonstration of using a classical moment distribution method to find out frequency of vibration of this beam. Now, in this there is an idealization that we already said that mass will be lumped at a point, where you want to measure the frequency or the degree of freedom, but it is not necessary always that mass should be lumped at the point always where the degree of freedom is measured. There are examples I will show you in the next class, where you will violate this and see what happens to a matrix. Any doubt here we will take it understood. We will move to the next problem.

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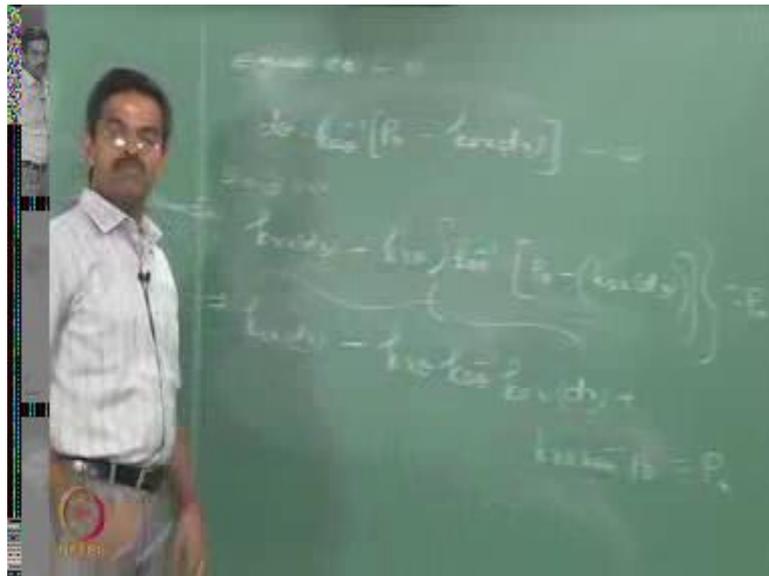


We will have another classical problem here. I have got 2 degrees of freedom pot. Let us say a cantilever beam, where I have 1 degree of displacement and degree of freedom as downward. I call them as x degree of freedom; the other is tip rotation which is theta. Now, interestingly if you had only 1 degree, you could say that stiffness would be for unit deflection can be 3 a v 1 cube something standard equations are there. If it is only rotation, you can still find if there are both available here what would be the equivalent stiffness of this member. So, for this there are two methods. One easiest method is derive lateral stiffness matrix or lateral stiffness term which is a k bar. We have to derive this. We will derive this quickly. How it can be derived? It is very easy to derive. Remember we can easily derive this.

Let us say we all know a standard equation in matrix algebra is k multiplied by displacement. It will give me the force because per meter newton I am using the same algorithm here k matrix. So, x theta is a capital K. Capital k in sense it is a full matrix. The elements are small k row first column, next row first column next row first column next row first column. Next in a given matrix 1 2 3 1 2 3 if you want to read this member it will be k 2 2, if you want to read this member k 3 3 and so on. Similarly, this 1 k 1 3 row first column next that is how we generally read the matrix multiplied by displacement x and displacement theta will give me p x and p theta. That is force in x

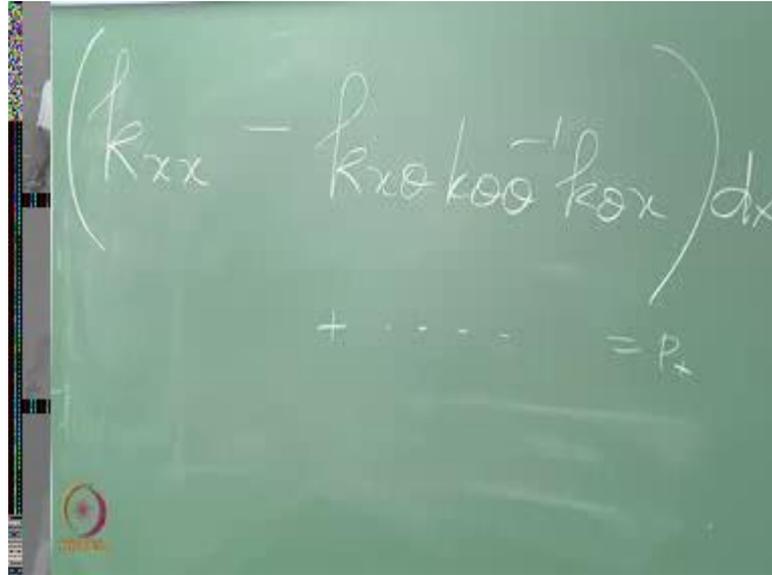
and theta degree. Now, let us expand this $k \times x \, d \, x$ plus $k \times \theta \, d \, \theta$ is $p \times$ equation 1
 $k \, \theta \, x \, d \, x$ plus $k \, \theta \, \theta \, d \, \theta$ is $p \, \theta$ equation 2. Equation 1 has $d \, \theta$. I can
 evaluate $d \, \theta$ from equation 2 substitute in 1.

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So, evaluate $d \, \theta$ from equation 2. Let us do that. $d \, \theta$ will be now equal to $p \, \theta$
 minus $k \, \theta \, x \, d \, x$ of $k \, \theta \, \theta$ pre multiplied inverse. That is the order. So, substitute
 3 in 1, this is 3 in 1. It is 1. So, $k \, x \, x \, d \, x$ plus $k \, x \, \theta$ of $k \, \theta \, \theta$ inverse of $p \, \theta$
 minus $k \, \theta \, x \, d \, x$ is $p \, x$. Let us equate this further $k \, x \, x \, d \, x$ minus $k \, x \, \theta$ $k \, \theta$
 θ inverse $k \, \theta \, x$ of $d \, x$. The first term this, this and this first term of course plus $k \, x$
 θ $k \, \theta \, \theta$ inverse $p \, \theta$ is $p \, x$, ok. Now, these two terms are related to $d \, x$. This
 term is related to $d \, \theta$ which I eliminated already. $d \, \theta$ is not there here.

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$$(K_{xx} - K_{x\theta} K_{\theta\theta}^{-1} K_{\theta x}) dx + \dots = P_x$$

So, let us write the first term, k_{xx} minus $k_{x\theta} k_{\theta\theta}^{-1} k_{\theta x}$ of dx plus is equal to p_x is very easy to remember this equation. I will tell you how. See I will write it here. Just for your understanding I will not look at, I will write it here. It is very easy to remember the equation. Equation may look longer. You can easily remember this. The order what I am telling you because in mathematics or in dynamics you have to remember lot of shortcuts. It is easy for you to. It is a life savior.

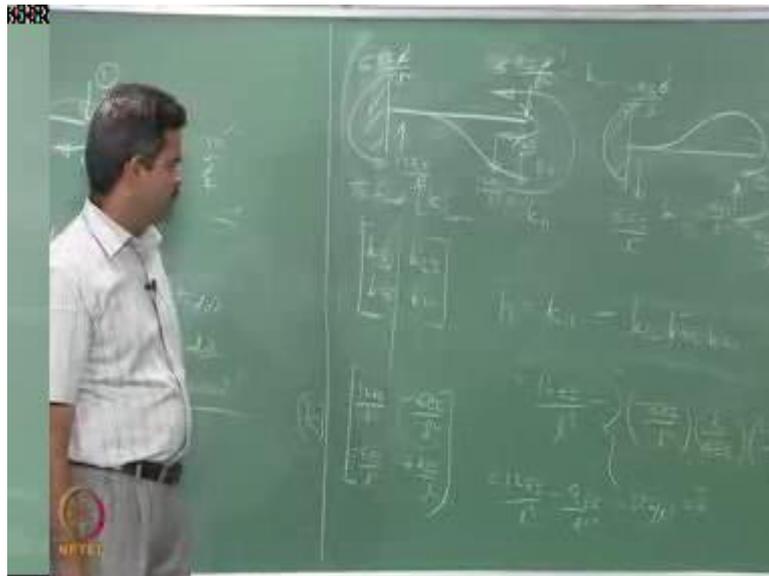
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Now, I want to write $k \times x$ minus $k \times \theta$ θ^{-1} $k \times \theta$ of $d \times$. So, it is very easy $k \times x$ $k \theta \theta \theta \theta$. $K \times$ can be easily remembered like this. There is a compact ability. $K \times x$ will match with $k \times x$ this entire equation is called k bar which is called lateral stiffness matrix in engineering. I will demonstrate the advantage of this. With the problem I will demonstrate this advantage with a problem. You will appreciate that this matrix will be very handy. If you can derive this quickly for a given problem, this is called k bar where k bar is called lateral stiffness matrix.

single value k . That is why it is called sometimes equivalent stiffness. Matrix will be a single value. Now, the whole matrix will convert into a single value. You will see how it is. So, I want to find k_{11} , k_{12} etcetera.

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Let us see what it is. So, for a given problem if this is 1 and this is 2, so to find k matrix which will be a 2 by 2 matrix why say degrees of freedom are 2 will be a square matrix of 2 by 2. So, the matrix will look like this, k_{11} k_{12} k_{21} k_{22} row, first column, second row, first column. Second we write like this.

Now, I want to know this individual coefficient of this matrix. So, what should I do to derive individual coefficients? I must always apply unit deflection at the specific direction and get the force that is k . If I give unit deflection at the first degree keeping the other degree restrain, that is the beam or the member moves only along 1, whereas 2. No movement. Why? You may ask me that is the constrained available in the definition itself. So, I must give displacement only along 1. Let us forget about 2 and find the forces. So, what I will get? I will get k_{11} and k_{21} . That means, for unit deflection along 1, I get forces in 1 and 2. Therefore, stiffness matrix always derived column wise. It is always derived column wise. On the other hand, flexibility matrix will be reverse flow well be derived row wise because they inverse to each other. So, I will get the first

column if I give unit displacement along 1. If I give unit rotation along 2, I will get the second column.

On the other hand, I will get all the 4 coefficients. If I get all the coefficients, individually plug them in this equation; get the single value which is equivalent stiffness matrix of a member which has got 2 degrees of freedom, but with 1 k. So, $1 \text{ m } 1 \text{ k } 1$ omega it is easy to solve. See without before solving the problem, try to understand what you are trying to do in the problem otherwise this will look only as a demonstration and you will never be able to apply this idea in any of your problems in your research. So, do not look at these problems as simple magical solutions applied on the board. No, you must understand how they are proceeding with the problem, why we are doing it if I have this stiffness matrix of 2 by 2 which is 2 degree freedom system. I will have to solve this problem to find omega by matrix method.

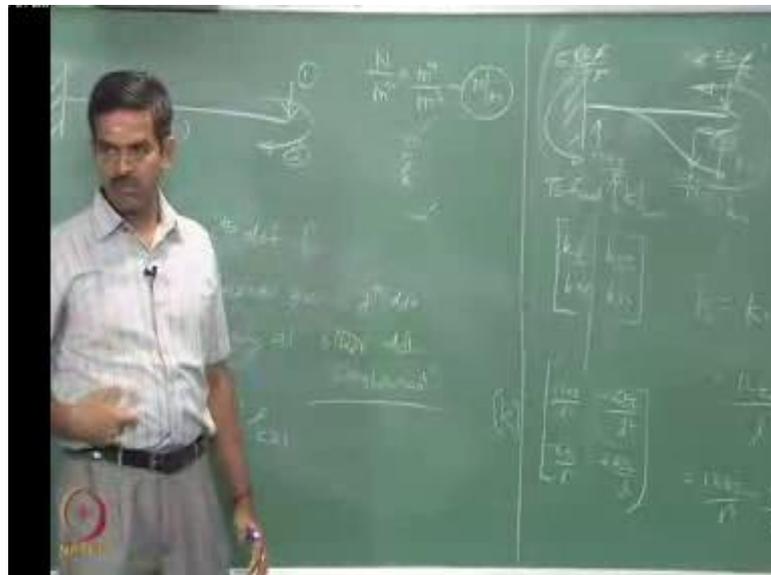
Now, I am trying to make it easy. I am converting this to equivalent stiffness. If I had two spring series, two springs parallel I know how to do that, but in this case the degrees of freedom are different in nature, I cannot apply those standard logic when k equivalent. So, I am using this method. So, let us derive this. So, I must give unit deflection here delta as 1. So, this will have a value which we just now saw. This will also have a value going to be $24 \text{ e } i \text{ delta by } l^3 \text{ delta is } 1$ in this case, sorry $6 \text{ e } i \text{ by } l^3 \text{ delta of } l^2 \text{ delta is } 1$ in this case. Similarly, this is also going to be $6 \text{ e } i \text{ delta by } l^3 \text{ delta}$. So, 6 and 6 are 12. Therefore, I will have a couple which is going to be $12 \text{ e } i \text{ by } l^3$. Let us look only this node, this force in the direction of 1. Therefore, this will be equal to $k \text{ } 1 \text{ } 1$. This force is the direction of 2, but opposite in nature, whereas 2 is clockwise in the original problem, whereas I have got a value of anticlockwise. So, I should say this equal to $k \text{ } 2 \text{ } 1$ with the negative sign. It means my $k \text{ } 1 \text{ } 1$ is $12 \text{ e } i \text{ by } l^3$ my $k \text{ } 2 \text{ } 1$ is minus $6 \text{ e } i \text{ by } l^3$, ok. I am deriving column wise.

Let me move to the second one. I want to give unit rotation. So, hold the stick in the unit rotation with move like this. So, this will invoke a moment which is equal to $4 \text{ e } i \text{ theta by } l$ classical mechanics and this will be equal to $2 \text{ e } i \text{ theta by } l$. Now, this has got unclock unbalanced couple of $6 \text{ e } i \text{ delta by } l$ or $6 \text{ e } i \text{ theta by } l$. So, this will have a couple which is $6 \text{ e } i \text{ by } l^2$ and this also $6 \text{ e } i \text{ by } l^2$. Forget about this part. Look here.

There are two forces. One is this and another is this one is this upward, but the degree of freedom one is downward.

Therefore, this will be equal to $k_1 \delta y^2$ because I have given unit displacement in the second degree and this value will be equal to k_2 . So, I get the second column. So, $k_1 \delta y^2$ is going to be $6 \text{ e i by 1 square}$, but it is in the upward, whereas the original degree of freedom is in downward. Therefore, minus $6 \text{ e i by 1 square}$ and $k_2 \delta y^2$ is clockwise. My δy^2 is also clockwise. So, plus 4 e i by 1 , this is my k matrix. I have all the values $k_1 \delta y^2$ first row, second column, first row, second column $k_1 \delta y^2$ I have $k_2 \delta y^2$. I have an inverse of that k_2^{-1} , I have substitute them and get k_{bar} . Let us do k_{bar} . I will rub this. I will do it. Here k_{bar} is going to be $k_1 k_2^{-1} - k_1 k_2^{-2} \text{ inverse } k_2^{-1}$. So, let us substitute which is 12 e i by 1 cube minus $k_1 k_2^{-1}$, first row, second column. So, minus $6 \text{ e i by 1 square}$ k_2^{-2} inverse is 1 by 4 e i and k_2^{-1} , second row, first column minus $6 \text{ e i by 1 square}$, 12 e i by 1 cube minus 9 e i by 1 cube . Am I right?

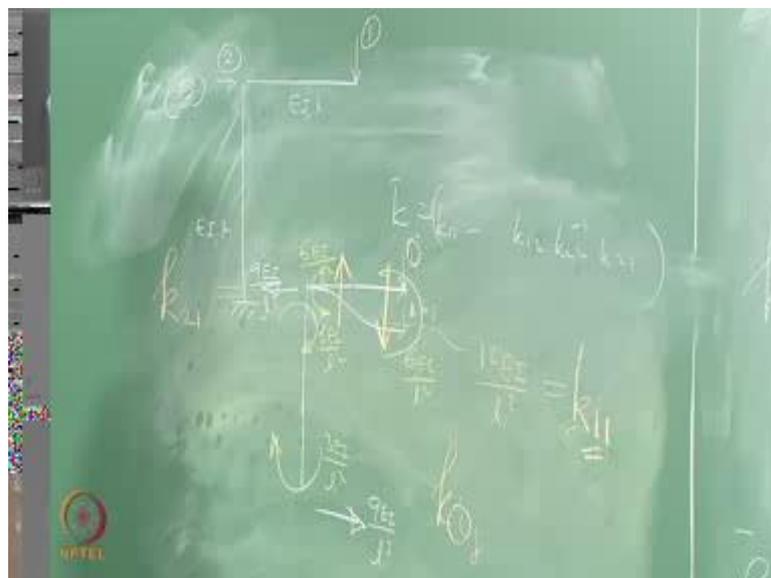
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So, 3 e i by 1 is k_{bar} you can also check the unit e i by 1 cube newton per square meter 4 meter cube newton per meter which is as same as units of k_{bar} , number 1. Number 2 for a problem of 2 degree k should be a actually matrix of $2 \text{ by } 2$, where as you are converting the whole matrix of $2 \text{ by } 2$ into single value which is nothing, but 3 e i by 1

cube. Now, I have got a single value and that is why it is called equivalent stiffness matrix use. So, this method to find out that now if know k bar, if know m , I can easily find out ω n. So, it is demonstrated with an example like this. We will also do another example to demonstrate this in a framed problem because now jacket structures are framed structures will take a base in a jacket problem and we will see how this can be demonstrated for a frame problem.

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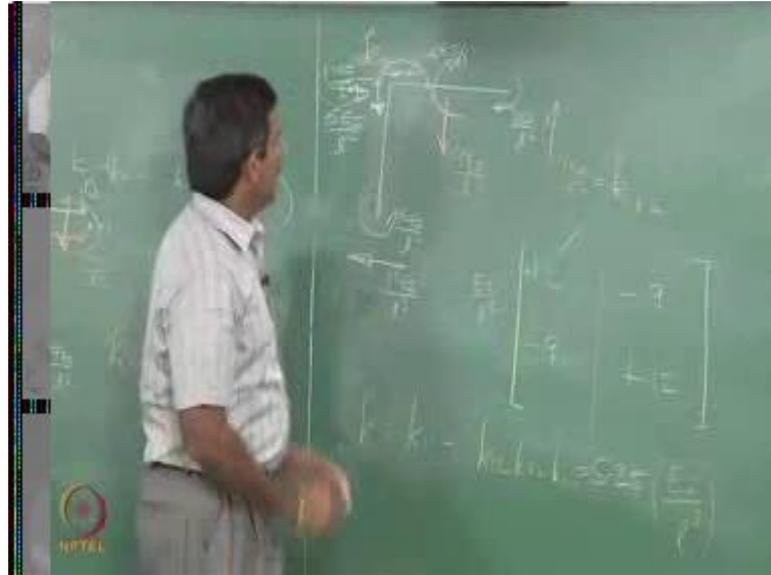
So, this was a. I will demonstrate with one more problem. Let say I have to frame like this e i l and e i l. The frame has got 2 degrees of freedom; one is a tip displacement which is 1 and other is lateral displacement which is 2. Now, let us first appreciate this problem how this is different. There are many differences. Let us see the classical differences. First the classical difference is a beam problem is a frame problem. The second classical difference is both the degrees of freedom are measured with a same point there, but it is not. So, here, but still for this problem also I can find k bar which is k_{11} minus $k_{12} k_{22}^{-1} k_{21}$. I can easily find this. So, I must get first the full key matrix for this. So, to get k matrix, I must apply unit deflection at 1 degree, get the forces 1 and 2, I must give unit displacement at 2 degree to get the forces 2 and 1, get the matrix substitute here, find k bar which is in equivalent single value.

Now, if you know the mass of the system, I can get ω_n quickly. So, there are some cases where you will be requested to or required to find out the fundamental mode of vibration and frequency for a given problem before you proceed with the design of the system. So, these are all equivalent methods by which you can easily find ω_n within minutes.

These are classical matrix methods used by different leading software to compare the answers software and also, to check the answers. It is not that they do only one method. Coding is written for different methods. If any one method fails, the other method will be enabled to automatically sub-routine to find the solution. They check this one method by which every software checks whether the ω_n or k is correct. It is very easy. Matrix method I am using the same equation, same algorithm for solving this problem. Let us pick up and apply the same algorithm here and see how I can derive the stiffness matrix. So, let us give unit displacement in this degree. So, the frame will deflect like this. So, I require a moment which is $6 e_i \Delta$ by l^2 . I am writing it here. There is no space. Also, have the same and this will now anti-clockwise $6 e_i$ by l^2 and $3 e_i$.

So, now this is what couple of $9 e_i$ by l^2 . So, it is got to be balanced which will be $9 e_i$ by l^3 and this has got couple of $12 e_i$ by l^2 . This has got to be balanced by a clockwise couple and this value will be equal to $12 e_i$ by l^3 . So, obviously this value is going to be k_{11} because k_{11} , this is the force in the first degree because of displacement given in the first degree. Now, the force in the second degree, it is lateral. So, I have to look at this value. You may ask me, sir this is also lateral, but this is at this node this is at the other node. So, at this node, this value which will be equal to k , the node is 2, but the displacement is given in 1. Therefore, it is 2. Force is second degree because of displacement given in first degree. This is force in first degree displacement given in first degree k_{ij} force in first degree or i th degree because displacement given in j th degree keeping all other degrees constrained and that displacement is unity. That is why I get stiffness. Therefore, it is unity in all equation. Δ goes away. There is no Δ here. You see the Δ is unity. I have removed it.

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Similarly, let see a displacement of 1. So, this will invoke 6×6 and 6×1 and 1×6 and 1×1 square and this anticlockwise or clockwise 6×6 and 3×6 and 6×3 and 3×3 square. Here people generally get a fundamental doubt at this point. People generally think it is a free hang or overhang. So, why is there a moment? So, moment should be 0 actually because it is a free hang or over hang in matrix method of structural analysis. All members are always considered as stiff ends at both. In matrix method of structural analysis is stiffness method. Every member you take member 1, member 2, member n, it is always considered by default. Both the ends of the members are fixed and then, you have to release them. That is how it is 3×3 square.

The second doubt what people get is sir when we transferred 6 has become 3 here, but when we transfer 6 has become 6 here. Why? If the displacement is transferred, it is full. So, this has got to be clearly these are two doubts. What generally will get and the major doubt what people get is how to mark this arrow direction. That doubt has got to be explained in many numbers of times you must know and of course very silly doubt is how you will mark the arrow direction of the couple. That you must understand physically. If it is clockwise anti-clockwise moment, clockwise couple and so on, you must remember strongly how the rotation takes place.

Now, this in anti, I mean clockwise of 9. So, I will get couple of this type which is anticlockwise which is $9 e_i$ by l^3 $9 e_i$ by l^3 and here this is again 12. So, I will get this direction of this type because this is anticlockwise. This couple be clockwise which will be $12 e_i$ by l^3 $12 e_i$ by l^3 . So, this will be actually equal to k displacement given in two forces in 1. So, k_{12} , this value will be actually equal to displacement given in 2 force also in $2 k_{22}$. So, we have got both the values. Let me write the (Refer time: 47:00) stiffness matrix here k matrix k_{11} $12 i$ by l^3 can always take $12 e_i$ l^3 out 12. Let say plus 12. Why? Positive this force is acting down displacement of degree is also down and the second one is here. So, this force is towards the left, where as the degree is towards the right and therefore minus 9.

Look at the second figure. I am looking for the second column. So, second displacement. So, I am looking here k_{12} , the force is up the displacement is down. So, minus and second degree here force is towards right displacement towards right, so plus 12. So, I have to substitute k_{bar} is k_{11} minus $k_{12} k_{22}^{-1} k_{21}$. I have all the values k_{11} I have $k_{12} k_{22}^{-1}$. I can easily find k_{bar} . Can you give me what is the value of k_{bar} ?

Student: (Refer Time: 48:26).

How much?

Student: 21 by 4 (Refer Time: 48:28).

21 by 4, yeah 5.25, right. It is very easy to find. You need compatibility. This will give you newton per meter. I have converted the problem of 2 degree to a single value. You know m and I can find ω_n easily. So, two examples a and b demonstrated for finding of the equivalent lateral stiffness matrix which is k_{bar} which are very important advantage in application of dynamics for finding out fundamental frequencies. We were idealizing the problem into an equivalent single degree problem. This will always be required in many cases. Any doubts we are also opening up the exams may be in another couple of days and my request to all the viewers is you must register for the exams. There are many questions people are asking. I am not able to solve the tutorial, will I be

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Thank you.