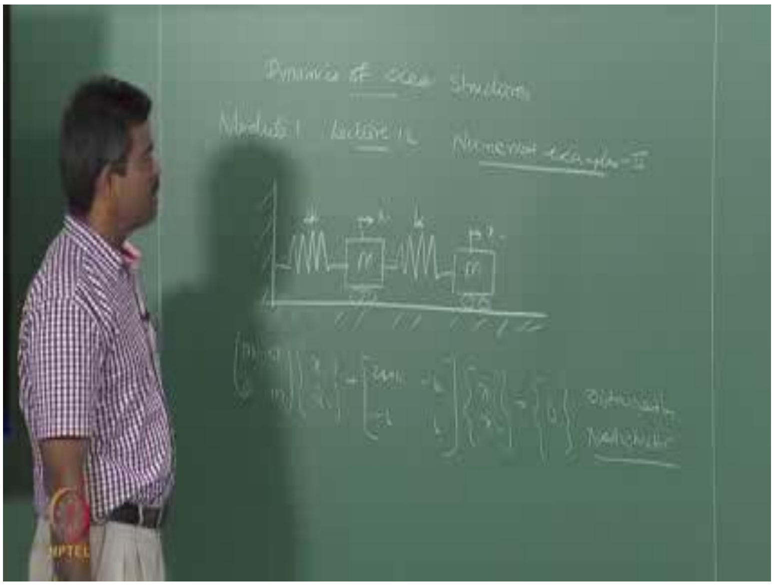
Dynamics of Ocean Structures
Prof. Srinivasan Chandrasekaran
Department of Ocean Engineering
Indian Institute of Technology, Madras

Lecture – 12
Numerical Example-II

(Refer Slide Time: 00:15)



So, let us talk about lecture twelve in module one of dynamics of ocean systems. We have solved this example yesterday. We derived this equation emotion using Newton’s method, we actually prepared a three (Refer Time: 00:29) diagram, we substituted the external forces, acting this mass and this mass separately. We arranged the equation terms and we got this matrix arrangement, and we also discussed that this very easy format to remember, but still one should understand how they can be derived from the fundamental understanding; I will go by the same problem.

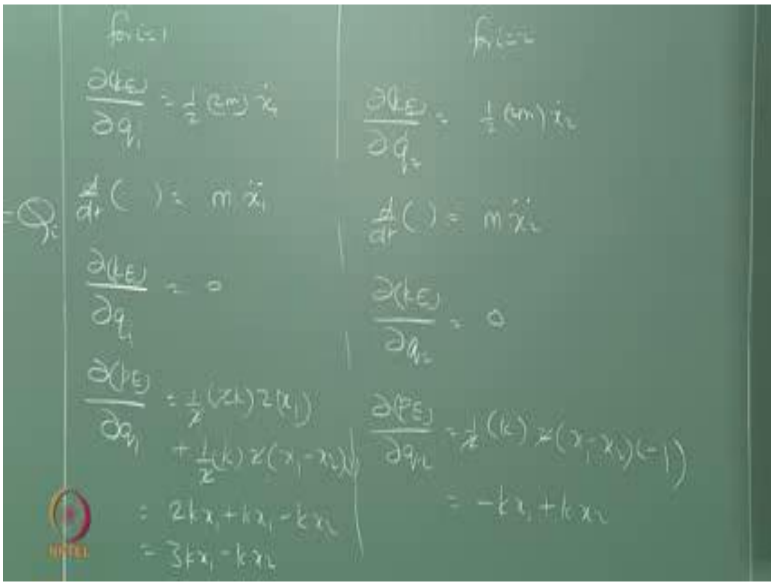
(Refer Slide Time: 00:50)

I will take the second method by which we can even write the equation of emotion; let us say the energy method. The energy method says d by d t of partial derivative of kinetic energy with respect to dou q I dot minus partial derivative of kinetic energy with dou q I plus partial derivative of potential energy with dou q I plus partial derivative of dissipation energy with dou q I dot should be said to q I, where I is the number of degrees of freedom and q I is the force vector, and k e represents kinetic energy for a spring mass system, it is half m velocity square, p represents potential energy, for this particular example it is k x square, and d e represents dissipation energy, in this example of co series naught, but it is half c x dot square. This example is not applicable, because there is no dash powder damping attached to the model.

This equation is very simple, what you got to do is, you have to first find the expressions for kinetic energy, potential energy, and dissipation energy separately, and find the partial derivative. One may ask me a question why partial, this of course, applicable to multi degree difference system model. For example, you will have take this particular spring it is connecting both displacements x 1 and x two. So, when we add in the equation both terms x 1 x 2 will be involved. So, both degrees are connected or integrated by 1 particular member. So, you look at one degree of freedom at a time, so partial derivative with respect to that particular displacement function, and you take a derivative of that, and you equate this and you do this so many number of times as I, where I the number of degrees of freedom.

So, we want to try to apply this equation as such to this problem. First thing what I will understand is I will be two in this case, because there are two degrees of freedom. So, I will write two equations now; one equation respect to q 1 or x one, other equation with respect to q 2 or s two, let us write both the equations. So, first let us write the equations for kinetic energy. So, look at the spring, it is not connected to any whereas, except x 1. So, half m x 1 dot square, plus half m x 2 dot square, now potential energy half 2 k that is this spring, connecting only x 1. So, x 1 square plus half k; that is this spring, connecting these two. Now as the same norm follows we can say x 1 minus x 2 to the whole square, or even if I write x 2 minus x 1 since here squaring does not matter. Now dissipation energy is not there your c is the not present in this case I do not write equation for d e. So, these are the two expressions I have which has got variables as x 1 and x 2. I take a partial derivative on this. So, let us try to this separately.

(Refer Slide Time: 04:59).

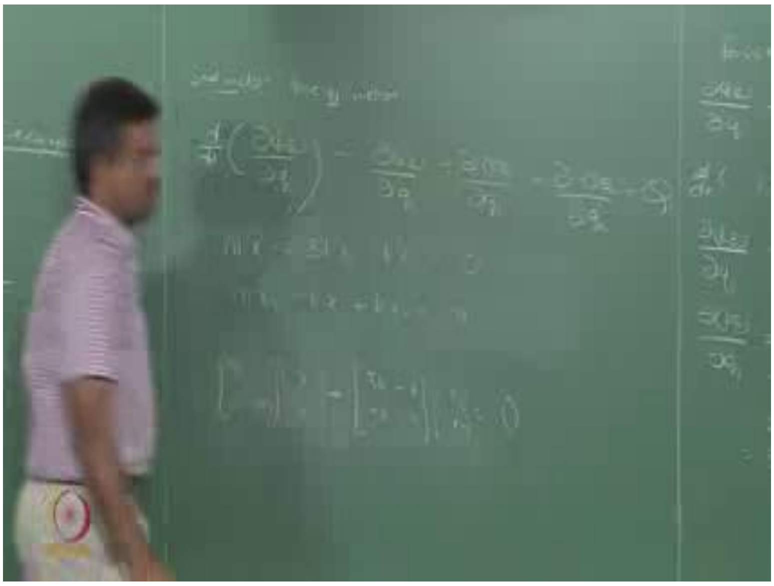


So, for I equal to 1 for I equals 2, let us do it separately. So, I must first find dou kinetic energy by dou q I dot is required. So, I see this equation. So, I instead of say I, I should say q 1 dot; q is the displacement here x 1 or q one. So, it is going to be half of 2 m of x

1 dot. So, I should say d v d t of this function; that is required here. So, I should say m x 1 double dot. Then I must require dou kinetic energy that is partial derivative of this with respect to q I or q 1. In kinetic energy I do not have a term related to displacement, all terms are related to velocity, so zero. Then I should say potential energy with respect to q 1, potential energy is here and q 1 is there in both the equations. So, let us do it here, which will be half of 2 k of 2 x 1 plus half k of twice of x 1 x 2 of 1. So, let us simplify this, which was 2 k x 1 plus k x 1 minus k x 2 which becomes c k x 1 minus k x 2.

Let us do it for x 2; that is I 2. So, kinetic energy with respect to q 2 dot, which is half m x 2 dot; that is the term here, now that becomes zero, I am taking a partial derivative. So, d by d t of this term which will give me 2 m, so m x 2 dot. So, kinetic energy with respect to again zero, because there is no term related to q 2. Potential energy with respect to q 2, I will come to this equation of course, there is no q 2 terms here involved only in this case. So, half of k of twice of x 1 minus x 2 of minus 1, which becomes minus k x 1 plus k x 2. So, I retain this part I will remove this, I have to rearrange this in this form two equation I will write now.

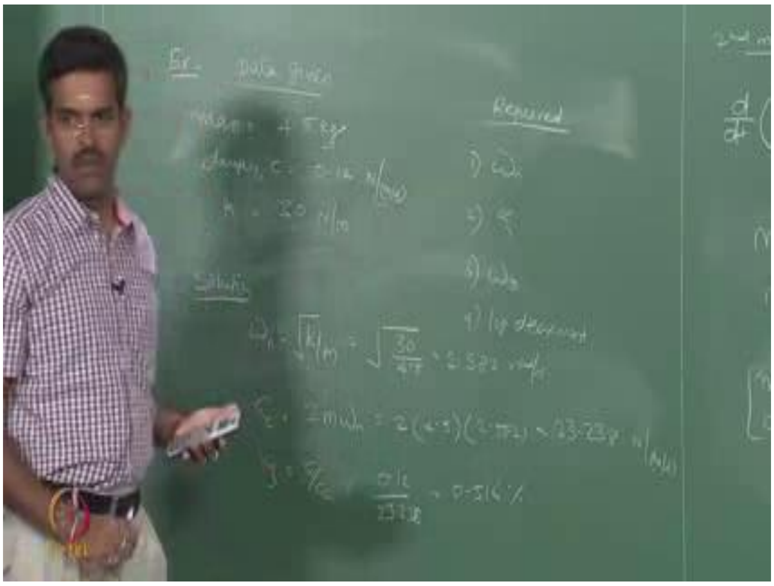
(Refer Slide Time: 08:35)



So, let us write the equations for to one first. So, this term I am having it here. So, I should say m x 1 double dot, this term is zero. The second term is plus 3 k x 1 minus k x

2. And of course, the force applied at the first degree externally, is zero in the problem, there is no force applied. So, zero. Now I write the second equation where I is equal to 2. So, look at there, m x 2 double dot minus k x 1 plus k x 2 up to zero. I can now rewrite this in a matrix form m 0 0 0 m of x 1 double dot x 2 double dot plus 3 k minus k minus k k of x 1 x 2 is 0, which is same as the equation of emotion what we have in Newton’s method one can do it like this also, let us do one more problem. Now let us say example two.

(Refer Slide Time: 09:50)

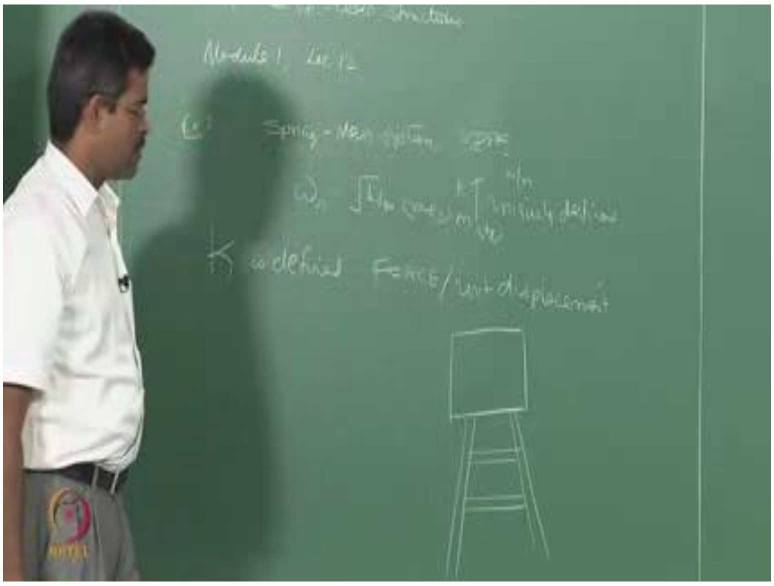


One was both the cases solved, using deriving equation of motion. Example two; I want to find the fundamental characteristics of dynamics for a given spring mass system, there is the data given to me, the data says data given to me is this. Let us say mass in SI units, is 4.5 k g damping coefficient is 0.12 Newton per meter per second viscous damping, and stiffness of the spring also given as 30 Newton per meter. These are all SI units, this is data given. What is required, what is asked? They wanted to find out the natural frequency of the system, wanted to find out the damping ratio, they wanted to find out the damped vibration frequency of the system, they wanted to find out log algorithmic decrement of the given response, decrement because damping is present, and so it is a damped free vibration model.

So, we already know omega n, solution is simple, standard equations is there, omega n is square root of k by m. So, let us substitute k and m in the respective unit's and try to get the values, which is square root of 30 by 4.5 2.581 variance per second. I can quickly find c c which is critical damping, why, I want to find zeta, zeta is the ratio of c verses c c, c is given here, I can find c c therefore I can zeta. From the equation what we discussed yesterday, it is 2 m omega n, we already know the values. So, 2 4.5 and omega n is just now known 2.582 23.238 Newton per meter per second.

Now, I can find zeta which is the ratio of c over c c which is 0.12 by 23.238 0.516 percent. So, you have seen two examples. We will see example three now.

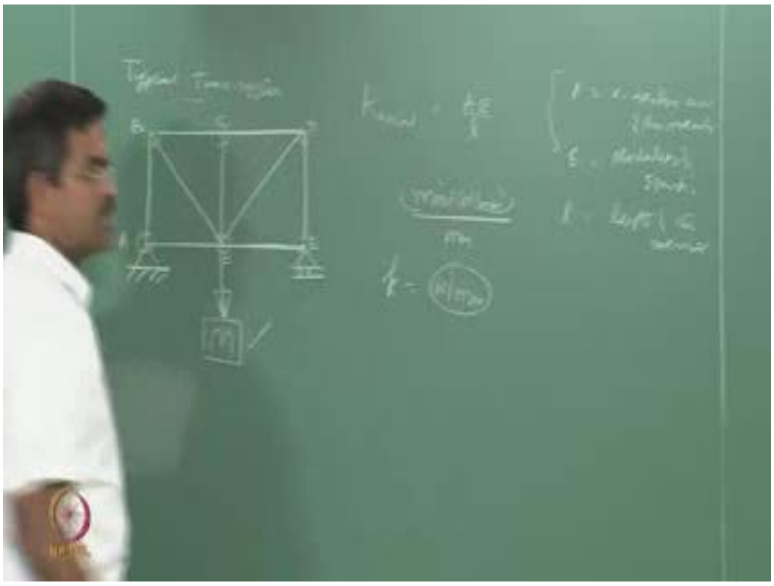
(Refer Slide Time: 13:04)



Let us say in a simple spring mass system, if it is single degree freedom system model, we can easily find the natural frequency of the system, using this expression which is k by m. It means k and m should be uniquely defined. If you are able to do that for a given system you can easily find the frequency of vibration, where as k is in Newton per meter and if m is in k g this will be in radius per second, you can easily find, but there are some systems for which k cannot be or m cannot be easily quantified. We will take an example of that kind, and see then in that case how do you know k. So, let us understand stiffness is defined as force per unit displacement.

So, the algorithm is very simple here, if I really wanted to derive stiffness matrix or stiffness coefficient of a given matrix, give unit displacement to the system, at the points where you want to, and try to actually find the force in each member right. I apply the same concept to for a problem and see how we can do this. So, we all know that, let us say if an offshore platform, may be even a jacket is a template structure. We all know structural engineer or the person who understands take the material, if for example, this is going to be a deck here, the deck needs a support system which will transfer the load from the top side to that of the ground, or to that of the support system. Let us talk about this particular system which is typical truss system.

(Refer Slide Time: 15:08)



So, it is going to be a complicated truss system, but I will take a simple truss system, and try to demonstrate how to get the natural frequency of this system, where k and m is not readily available in the system. So, I will take a very simple example of this kind. Let us see I am marking as a simple truss system like this. If you ask me a question what is a relevance of the this truss system to the that of this, this a complicated truss system where we cannot analyze it now, we will take lot of time, but the same algorithm and principle can be used for the system as well, we will take a simple example so; obviously, there will be a support system.

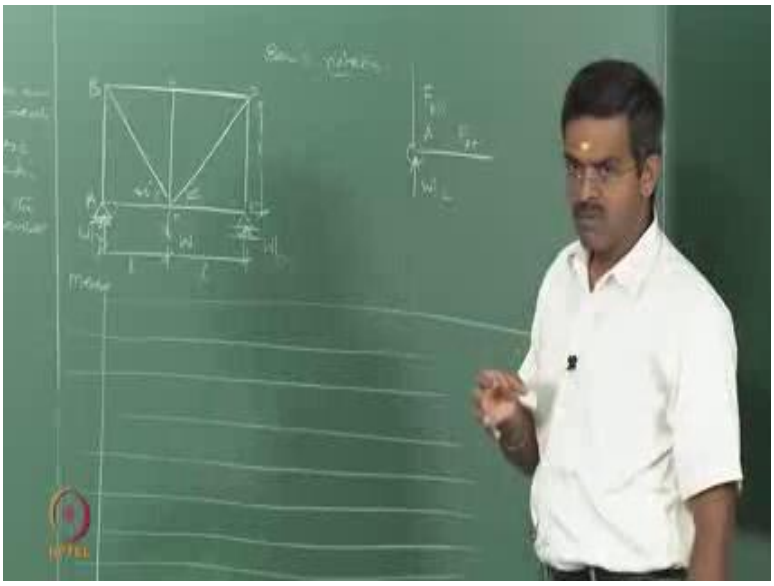
My support is given like this, we name the joints let us say a b c d e and f, these are all called joints or sometimes called as nodes. Let us say the mass of entire system is suspended at this point, and is known to you. You can easily find this mass, because if it is a given system, if you cross section dimensions of each member, if you have the density of the materiel, if you know the volume of the material, you can find the c g (Refer Time: 16:42) every material, you can take a summary and you can find y bar and x bar, you know where m is active.

So, w is m g you will actually know w. So, divide by g you will get m. So, m is known to me, m may not be exactly at this point, m may be somewhere here which is c g of the entire system. So, m is known to me, but it is different members of different access stiffness. The access stiffness of every member, is given by simply a e by l, where a is a cross section area of the material or the member, is modulus of elasticity of the materiel on the material, and l is the length of the member. So, if I work out the nodes unit's here, let us say m m square Newton per m m square m m. So, ultimately the stiffness will lead to this unit, which is as same as force for deflection. Interestingly there would have been only one member here, I could take the a e by l value of that member and I can find easily k, I know m, I can find the natural frequency of vibration of that member or the structure, but here a structure assembly of different member. Now a may be different we do not know, e sometimes may be different, but generally it is not different, but l is certainly different for example, if it is a square system of 45 degree l of these two may be same, but; obviously, the l of this is different.

Therefore, axial stiffness imposed or possessed by each member is different, then what would be that net k of the entire system, which will contribute to the numerator here to find omega. So, it is an argument, if all the members are parallel, we can say k equivalent. If all members are series, we can say k I equivalent. Now here there are members which are neither parallel nor series. So, we cannot find this is a k equivalent, from a standard expression. I want to know k equivalence for this, then only I can find omega n. So, it is a very interesting applied mechanics problem on dynamics, as a simple example. So, I will use unit load method which we already studied in, structural mechanics principle, but I will apply this for dynamics that is a very interesting point.

So, I will take up this particular issue or this problem, and say I will find out the forces in each member.

(Refer Slide Time: 19:35)

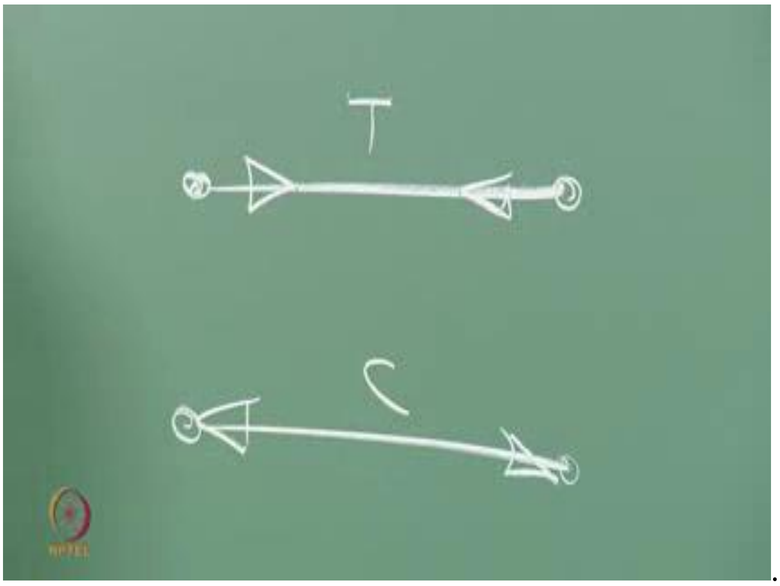


So, if this is we will say, the truss given to me, let us say this is angles 45 degree, support system is known to me. I take this as w was a known value for me. I call this as l; obviously, this also l, because it is a 45 degree, I will open up a table now, member nomenclature, it should also have an order of marking in a nomenclature, this is called bows notation. You must be recollecting this in structural mechanics, this is called bows notation bows notation is a algorithm given, for marking the legend or nomenclature of nodes. So, I am following an order of clock wise. It is not advisable to say a b c e k f no you have to have an order. So, first for the given force, let us find the external reaction. Fortunately in this case the truss system is determinate, if it is not determinate there are methods to solve, but let us know if it symmetric therefore, this is going to be w by 2 and w by 2, because it is symmetric it is determined also we can easily find.

So, let us pick up any node which should not have more than two unknowns. Now the question comes here is what are the nodes. All the axial forces in each member, is an unknown. Now you are deviating to a thinking of structural mechanics, because talking of dynamics you are not interested in forces we are interested is stiffness, which is rather equivalent stiffness then why we are talking about forces, why because stiffness will become a force when I give unit displacement. So, I will give a unit displacement at this point and try to find the forces I get stiffness. So, I have to find the forces. So, what are the unknowns' internal forces in each member becomes an unknown now. So, I can use or solve this using method of joints, and method of sections.

Method of joints is very powerful tool easy. The method of joints rules simply says pick up any node or a joint where there are not more than two unknowns. It means not more than two member should meet, whose force are not known. Now the question is, what are the nature of forces in these members. Truss is an axial system therefore, the forces in these member will be either axial tensile or axial compressive. So, let us take up node a, this is node a, I have a external force w by 2, I have a nomenclature I write force, this member, force a b, one can write force b, a also does not matter. Since I have taken a node a, I am writing a b. Similarly this will; obviously, force a m. So, I am going to make an algebraic sum of these forces, algebraic sum means there should be sign convention I have a nature of this forces, I can assume anything, let me say I am assuming this as compressive, this also as compressive.

(Refer Slide Time: 23:34)



.

Now, one must really know I have already told you this in a truss member, if these are the nodes in a truss member, if these are the nodes, if the member is pulling both the nodes it is tensile, the member is pushing both the nodes it is compressive, what is the influence of the member on the node, will give you the category and nature of the force; obviously, we also said earlier that kindly do not mark like this. This is having no meaning; that is the member is pulling on node or pushing on node in (Refer Time: 24:07) this is not correct. This is vector algebra, I will talk about this. So, I have assumed the nature.

(Refer Slide Time: 24:12).

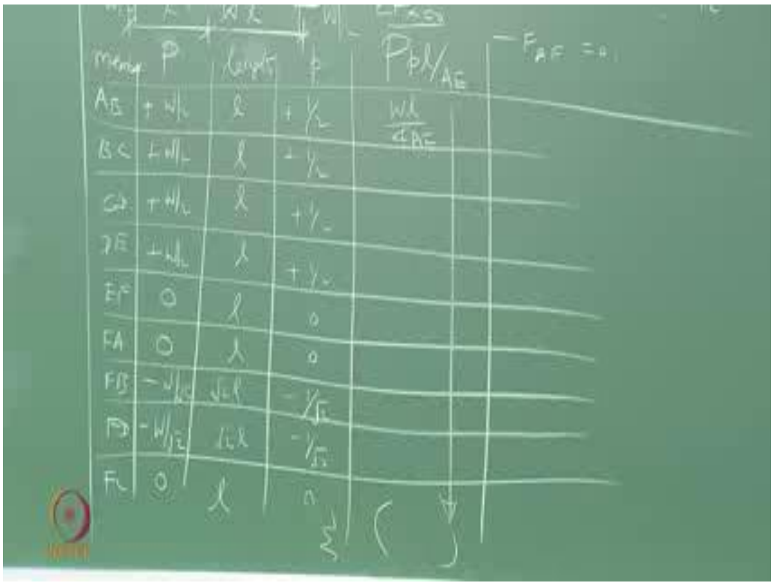
So, I have taken this; obviously, this is understood that it a put compression here, it is going to compression here also, if you put a compression here, you are going to be compression here also. So, I say compression is positive I quickly say that f a b is acting downward and I have sign convention. So, any force upward any force right is positive n, this is down ward. So, minus f a b plus w by 2 is 0; that is sigma f I 0 algebraic sum of all force in y I direction or y axis is zero. So, I apply minus f a b plus w by 2 is 0, which gives me f a b as plus w by 2, this plus indicates that the assumed direction of the force is right the plus does not indicate that the force is compressive.

Please understand this very carefully I already said compression is positive, but this positive does not mean that f a b is positive. This positive means that the sign convention assumed by you is correct. So, it is correct. So, I mark it here and I say this is w by 2, then I also write sigma f x 0 algebraic sum of all force in x axis at this node. So, minus because there is right hand side is positive f a 0 implies that force on f a f member is 0. So, this is a null member, it has no force then one can immediately ask a question, why this member should exist, where is in null member for the given system of combination of forces. This a null member if we remove this w or suspend w somewhere here or here this may not be a null member, number one, number two arrangement of members is not only take care of forces, but to establish stability in the given system, we not move further on mechanics we will stop here.

So, you cannot remove this member what we are interested, is to know is this member is a null member. It means it does not carry any force for the given arrangement of forces. So, from symmetry, because f is acting at the center from symmetry, I can expect this will also be of the same nature, which is w by 2 and this is also a null number this is my symmetry. Now I can now go to node f node f has got these two members solved, but there are three more members. I should also always go to a node where there are not more than two, so I cannot work at here. I cannot work at c, because three are unknowns. Now, let us see b or d b has got three members, but this force is known to me, there are only two. So, I can pick up b solve b and get the value. Can you quickly give me the value of m f b or m b f and m b c that is f b f and f b c work out, and tell me what is the value. So, this is going to be w by root 2.

Similarly it is expected that this will also be w by root 2 and this is going to be compressive of. Now, I can pick up joint f or node f there are five members; one two three four five. I can read the members f a f b f c f d f e bows notation read clock wise f a f b f c f d f e bows notation clock wise now, out of which f a f b f d f e are known to me, there is only one unknown here I can easily solve. So, what is this value? So, it appears that these two will resolve and cancel these two will resolve and equate. So, this also becomes a null for this arrangement, I wanted to tabulate this values right, member forces.

(Refer Slide Time: 28:27)



So, I start from here let us a b b c c d then d e e f and f a, 1 2 3 4 5 6 member then f b and f d f b and f d and f c. So, 1 2 3 4 5 6 7 8 9, 1 2 3 4 5 6 7 8 9, then I know this. So, compression is positive tension is negative let us look at a b compression positive w by 2. Similarly b c just fills up this, for clarity I will just rub this and write it in the top this is l and l. So, these are the forces P, and let us write on the length of the members a b l b c l c d l l. These are all l and this is root 2 l. Now the internal forces in each member, because of w applied here is given here instead of w. If I apply unit force one that is called unit load method apply one, then I will say it is a p and is not actually work out, wherever w is the replaced with one. Trying to find out (Refer Time: 31:00) principle p p l by a. So, you can, for example, this will be w l by 4 a e and so on, fill up the column Try to find the sum of this, this column, sum them here, so this value. So, let us write it here.

(Refer Slide Time: 31:37)

So, delta is p p l by a e sum 2.0.

Student: 44.24.

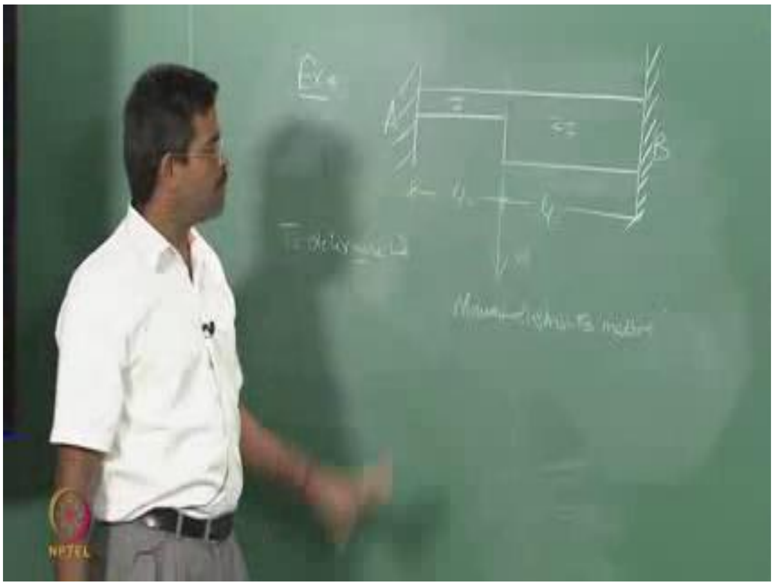
414.

Student: (Refer Time: 31:52).

So, I am interested in not finding the forces, but I am interested in finding out the stiffness, equivalent stiffness. So, I should say equivalent stiffness is the force, where the displacement is unity. So, I am interested in finding the force, which because this displacement is one times of a e by l of 1 by 2.414, find this two. This is nothing, but my k equivalent, which is 0.414 a e by l Newton per meter, if provided all unit's are sufficing this k e is known to me, and w is m g therefore, m is w by g, we know this, we know this omega n is root of k equivalent by m I know this.

So, I know at what frequency this system will vibrate, if the structure is simplified as a single degree freedom system model, as a spring mass system. Why single degree? Mass is lumped at only at 1.0 single degree. So, it is a very simple application of basic structural mechanics on dynamics to find frequency of vibration. So, this method can be solved for any applied, for any complicated truss system also, provided we are able to solve the truss system to find internal forces using united method or there are other methods also, we will not touch upon that, but. So, demonstration we must know this. Any doubt here, remove this. Is it clear how to get omega n.

(Refer Slide Time: 33:44)



Now, you will take example four. Here I have got slightly a tricky problem, let us say I got a fixed name, but with varying moment of inertia, depth is varying actually. We know inertia is b d k by 12. So, depth varies I varies. So, let us say this is my I, this is 2 y twice of this, and this is l by 2 and n by 2. This joint is a this is b, there is no joint a I just drawn a dotted line, just to show that there is difference there is no joint here, continuous beam yeah. I think you have seen a stepped beam like this, no. (Refer Time: 34:52) or you have at least for sure would (Refer Time: 34:56) like this, hundred percent sure would have seen. So, varying I. Now, I want to only equivalence stiffness of this, why I want to know at what vibration frequency is going to vibrate. Let us say I idealize this at this particular location, as a lumped mass, I want to know at what frequency it will vibrate so; obviously, k equivalence is not known to me, w is not known to me, because m can be computed I can easily find w from the class section of dimensions and geometry, but k cannot be easily found out.

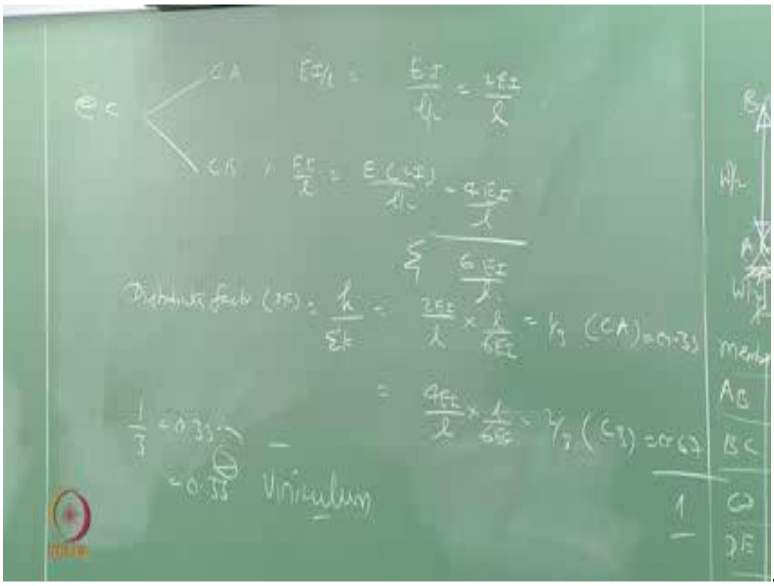
So, I want to know k for this. So, to determine omega n for the given problem, it is a very classical example in top deck system of a offshore structure, because where I got top side crane facility, derricks flares boom etcetera located; obviously, there will be deeper beams. So, you can encounter a support system where, the moment of inertia of the cross section may vary, one. two you can also encounter column section where, the strength of the material will vary, because young model varying from three fifty till as highest twelve hundred is being used in offshore structural members. Therefore, strength can vary, not only the cross section right. So, it is a very common example, classical example. So, I want to know the equivalence stiffness of the system, as idealize single freedom system model. Why we are doing all these things, because by hang one should know how to compute simple values of omega n, there are other ways of also doing it you can mathematically model this, give this properties, and excite it of free vibration you will get omega n automatically from a software also, but you must know whether the answer given by the software is right or wrong.

You may not work out the answer accurately to hundred percent matching to the software for all 15 digits, but you must know the principle digit's of the answer is right. It should not vary widely. So, you have to have hand understanding that yes, can we model this is in simply bit form can we find omega n in simply it form. So, we looking at that starting with this we will move on to the accurate levels later, first let us start with this knowing. Best thing is you do for the accurate level and never bother about the starting level and you always depend on software whatever answer you get, you will always say the beam is vibrating at a frequency of 22 hertz, where as the real frequency will not be 22 it will be 22, we do not know actually and we make a mistake in design, and the system will fail.

So, as I picked up and dependency on structural mechanics like unit load method (Refer Time: 37:38) k equivalence here. For this I will depend on moment distribution method, this also one of the classical method of apply to static (Refer Time: 37:46) structures I will pick up that method, and try to solve this problem using moment distribution method. Generally if you see moment distribution method, is applied to find out the end moments, and the moment at this portion, and the respective shear as a mechanics problems, but I will use this as a tool, to find omega n for a dynamics problems, as we did in the last example, let us see how we do it.

So, most of them may not be aware or would have forgotten, or would not remember, or would not know correctly how this method is applied, because you know the method name, but generally when you look at these methods, people generally dependent on text books, and exactly follow the same steps what the author has written in the book, and exactly try to solve the same problem what author has solved, you are comfortable, when some other problem is given to you, you are totally lost, because again you turn back to the author, author will be able solve the problem, but you never to be able to solve the problem, because your fundamental in the mode of explanation given by the author is not conducive, you have not understood the problem actually. So, let us explain this method very quickly in a simple form first; apply to this problem, and then we will move on to find out k equivalence of this problem. Remember k equivalence is the force for unit deflection, I must be obtain unit deflection at this point why at this point, this is the point where mass is lumped; that is the argument here.

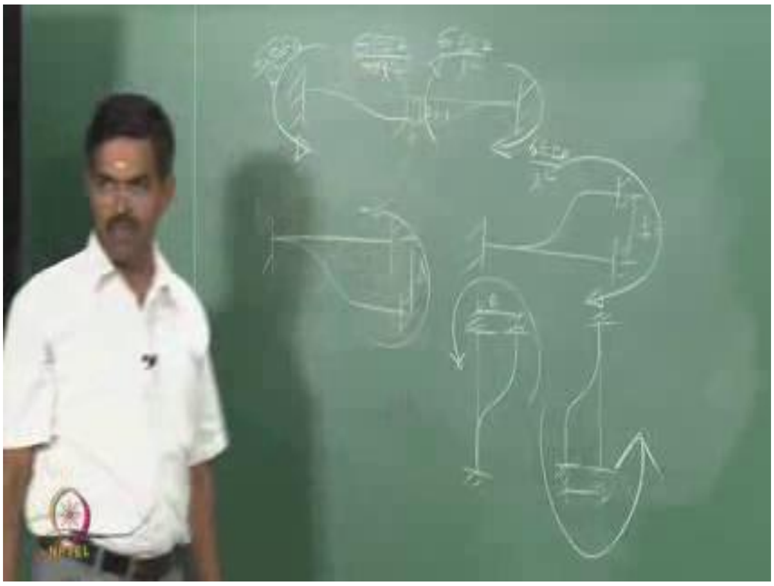
(Refer Slide Time: 39:26)



So, I will call this is joint c, let us intersection c. So, let us say at c ever I have a two members; one is c a, one is c b. So, let me find out the stiffness, bending stiffness is a e by l that is general formula. There are varieties of stiffness axial stiffness a e by l, bending stiffness, a e by l, torsional stiffness, there are many a e by l bending stiffness, why bending stiffness, when I try to pull this beam down the beam will bend, when I try to pull this truss down, the member will expand not bend, depending upon the nature allowed on the member. You must use the model correctly. So, that is the moment distribution that is why unit load method. I cannot apply unit load method here because it is not going expand, it is going to bend. So, e I by l is general equation, but I must say in my case it is e I by l by 2, which is 2 e a by l, because l is l by 2 in this case. Similarly for c b again it is a e by l, but I must say it is e 2 I by l by 2 which is 4 e I by l.

So, I must say sum of this at this joint is nothing, but 6 c I by l. I must find distribution factor d f, which is k by sum of k, which is nothing, but 2 e I by l by 6 e I by l which goes as 1 by 3; that is for the member c a, which goes as for the member c 3. So, it is 0.33 and 0.67. So, the sum should be always one, I think we all know this 1 by 3 is 0.33333, you can always write like this in mathematics, this is called viniculum. Viniculum means this is repetitive, this number is repetitive infinite times; that is called viniculam in maths. So, there is a viniculum number here 0.33, I pick up that number now this all the 0.66 viniculum, but I said 0.67, because sum should be 1, I am rounding it off.

(Refer Slide Time: 42:07)



So, I have the beam, I am applying at the center, a deflection, which is delta. I am saying delta should be one, why I am not interested in force, I am interested is stiffness, stiffness is force for unit deflection, so I said delta as one. The moment I say delta is one from the principle of structural mechanism we know that, now it will apply a moment of this kind, also will apply a moment of this kind. Now one have a very serious questions here; sir two questions, why this is marked down, why this arrows are anti clock wise and clock wise, so very interesting. I have a beam like this. Now this support is settled by delta, may be any value, when the support is settled by delta, it is always understood, and that the structure should reach it is recent trend. So, you have to bring this back to original. Suppose if the structure is like this, a structure rises like this which is also a delta. Now you have to bring back to the structure (Refer Time: 43:34). Suppose you have a column which is originally like this, the column goes to the right which is delta; you have to bring it back to normal. Suppose if the structure is like this the column is moving to the left by delta you have to bring it back to normal.

So, arrow directions are not by clock wise and anti clock wise, it is all dependent on where it is settling bring it back. So, where it is settling brings it back, where it is settling bring it back. So, these arrows are very clear now, why this given downward deflection, because I am applying mass downward. If I apply mass upward I can give an upward deflection also in that case the beam would have deflected like this, and you have marked arrows like this; does not matter. So, from mechanics we know that if delta is displacement, the moment caused, because it is delta will be 6 c I delta by l square ;that is a general rule. Now, one can ask me a question when I give a deflection how to generating a moment. Moment is actually to be generated only at the fixed end, but all I am getting a moment here, this is not explained comfortably in many of the text book, and it is very interesting.

Imagine your hand, keep your hand straight, there is a movie, let us not give the name of the movie, there is a movie, where people you should exercise that stretch your hand, hold the bucket of water, and keep it is horizontal, I mean straight, try to keep it straight. The moment you give a displacement, it will give a pain here, you can try to do that, lift a weight, you will not feel the weight here, pain will come here actually right. It will try to move to the nearest support, it will cause a moment actually, swinging this moment.

So, this moment is because of the deflection. Same effect will also go back here, which is 6 c delta of e I square. This also 6 c delta e I square, this is also 6 c of e I square. Unfortunately l are different, l by 2 l by 2 are also different; therefore, these values are not going to be same. These are generic equations if you give displacement of delta and if the base l and this member is I, while in our case it is not.

We will continue this in a next class to you, I will explain what would be the moments actually caused at each section, then how do you do a moment distribution for them, and from the moment distribution table, how do you actually get a equivalence stiffness, and then from the equivalence stiffness and mass how do you get the frequency of vibration of this beam; that is our focus.

So, we will continue this problem in next class, because I will not be able to do it now, there are only few minutes left over. So, you have to understand the basics of mechanics. You must have learnt there are many text books available for this. There are other text book written by good Indian authors I have given references in N P T E L website, please read them, and remember very carefully dynamics is a course, where what we discuss here is only, may be an eye opener for majority of the topics. If you want to really learn majority of topics parallely, you must open the reference material and read them parallely, then only you have doubts and you can ask it accordingly.