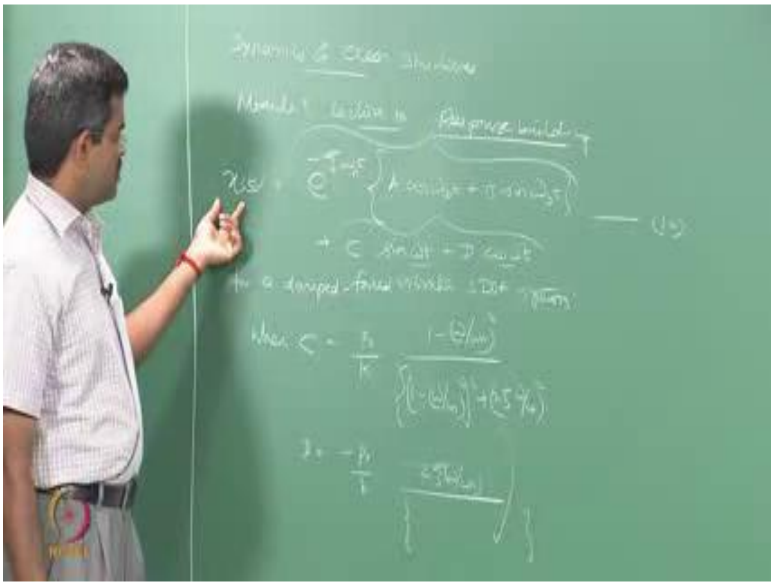
Dynamics of Ocean Structures

Prof. Srinivasan Chandrasekaran
Department of Ocean Engineering
Indian Institute of Technology, Madras

Lecture – 10
Response Build-up

(Refer Slide Time: 00:16)



So, in the last lecture we discussed about the response of a damped forced vibration, single degree freedom system model, where this was the equation of the response x of t is the combination of the complementary function, and the particular integral, where we have the transient response part and the steady state response part, because this is a function of forced frequency, whereas, this is the function of the natural frequency of the system. We know that omega d the damped vibration frequency is a function of the natural frequency of the system, which is system characteristic, and of course, this is depends on the initial conditions.

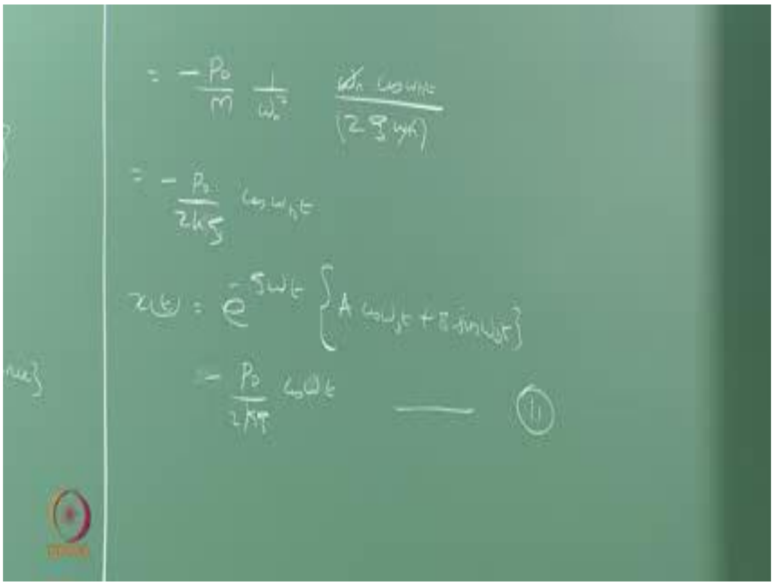
Whereas, c and d does not depend on initial condition, where we know that c is p naught by k, and d is minus p 0 by k, the same (Refer Time: 1:30). So, it has not depend on initial condition of x 0 and x naught 0; therefore, this response will always exist in a given response equation. Whereas, this may depend on the initial conditions of x and x dot and (Refer Time: 1:50) vanish depending upon the availability of the boundary conditions, the initial condition given to the response. So, we call this steady state response and transient response. Now, let us pick up this equation, maybe this is equation number ten or eleven I do not know, let us say ten. So, let us substitute this value and see what happens to my equation in x of t, when the natural frequency is as the same value that of the execution frequency, because this is solved when omega not equal to omega n.

(Refer Slide Time: 02:23)

Now, let omega n is seen as that of the x addition frequency. We call this as resonance response. The objective of this derivation is to understand that if I introduce damping, even at resonance the buildup will not infinite (Refer Time: 2:48) we have to see that, because we already saw in a damped in an undamped forced vibration, the response was building up by phi value at every cycle, and it is becoming unbounded at the resonance frequency. Whereas, in this case we have the objective is to check whether, it is still the same case in case of resonance response; that is the objective of this derivation. So, we know the complimentary function will not change, which is going to be as same as e minus zeta omega n t of a cos omega d t plus b sin omega d t which will not change.

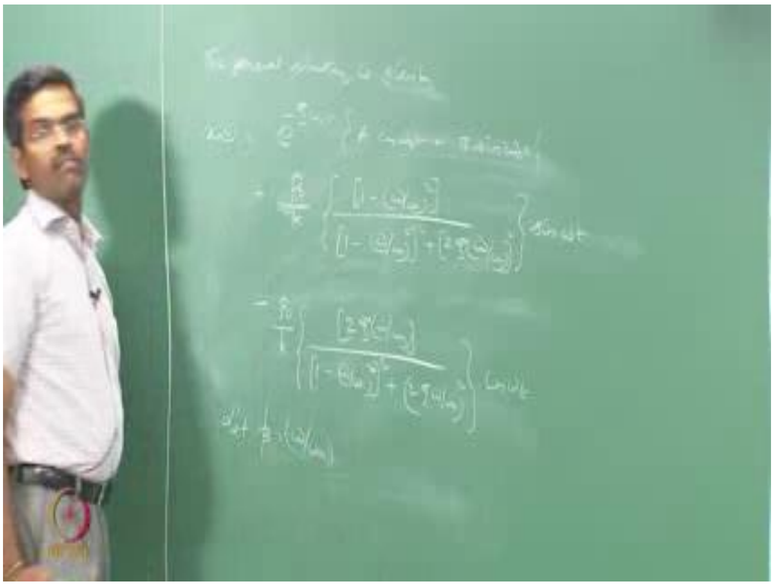
Whereas, the partial integral what we have evaluated now, which will be p 0 sin omega t by there is a p 0 by m here, because we divide the equation motion entirely by m d square plus c by d m d b m of d plus omega n square, thus the particular integral for this particular problem. Now, we know the rule, for example, if the expedition function is a science model or a trigonometric function, I may say d square is minus omega n square or omega square. So, substituting d square as minus omega square as per the rule, and also recollecting omega n as the same as omega; that is the resonance part we are looking at that, that is what we are looking at therefore, these 2 will go off my p I will simply become p 0 by m sin omega t by c by m of d. So, we can also say p by m sin omega t by 2 zeta omega n of d, because we know c by m is 2 zeta omega n then we can say p 0 by m let us say d square of 2 zeta omega n d of sin omega t which becomes minus p 0 by m because omega d square is minus omega square.

(Refer Slide Time: 05:33)



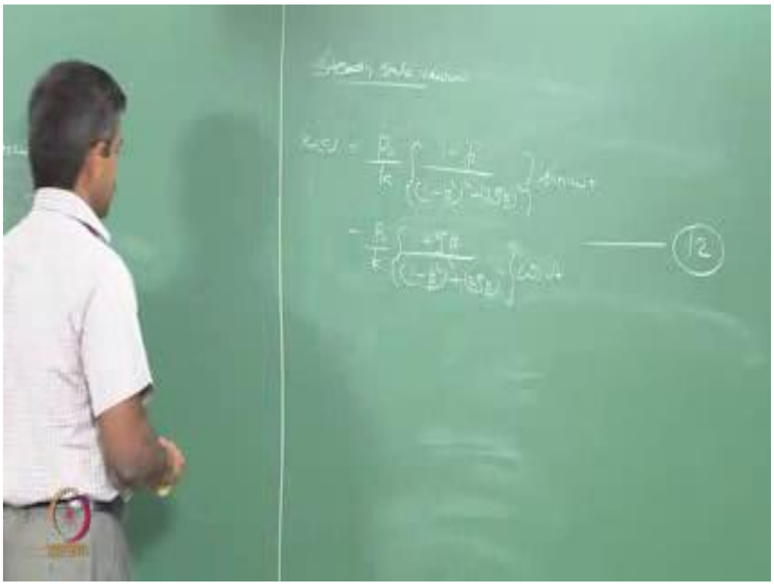
So, I get minus p naught by 2 k, because omega n square is k by m this omega and omega n goes away cos omega n t; that is my particular interval. So, my entire solution or the complete solution will become e minus zeta omega t of a cos omega d t plus d sin omega d t minus p naught by 2 k zeta cos omega t. Here I write omega n or omega does not make a difference, because on looking at the function at omega equals omega n. So, does not make any difference. So, let me call this equation number eleven, which is the response time history for a damped forced vibration at resonance. Now let us go back to the general expression, I will get back here again, and try to derive the dynamic amplification factor. Let us see what how do we do that.

(Refer Slide Time: 07:14)



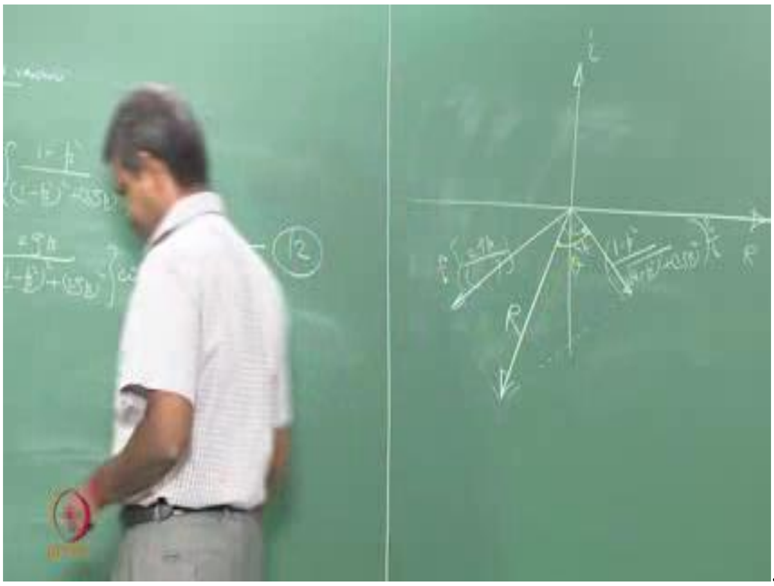
So, the general solution is given by this is a specific solution, where x of t is derived at omega equals omega. The general solution is what we wrote earlier I am writing the same equation back again, is given by x of t e to the power of minus zeta omega n t of a cos omega d t plus b sin omega d t plus p 0 by k 1 minus omega by omega n the whole square; that is the general solution. Now, let us talk only about the steady state response part of it. To make this equation in a combined and closed form, we will express 2 rations. Let beta with the ratio of the frequencies.

(Refer Slide Time: 09:15)



The steady state response is given by, let us say x of t. I am looking only at the steady state part of it p 0 by k 1 minus beta square by 1 minus beta square square plus 2 zeta beta the whole square of sin omega t. Here I think this as a ratio as beta minus p naught by k 2 zeta beta by cos n t. Let me call this equation number twelve. Now, I can express this equation twelve graphically, because there are 2 components here sin component and cosine component, slight carefully this is cos, using what is called as an argon diagram. So, we can be drawn an argon diagram, and you can write this, express this graphically. There is an advantage of writing this I will show you how.

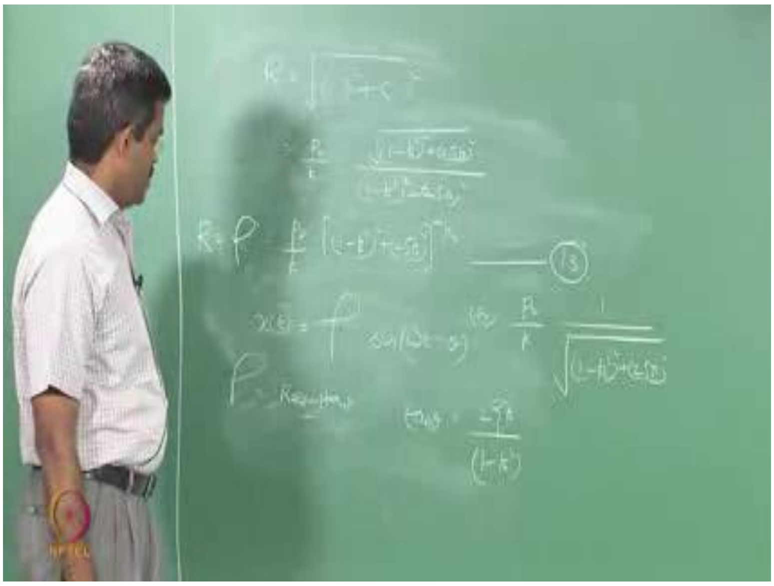
(Refer Slide Time: 10:47)

.

Let us say there are four quadrants of this diagram, this axis represent the imaginary part, this represent the real part. So, our 2 values; 1 and two, I would say that they are offset from the vertical axis by an angle omega t, and I complete the parallelogram to get me resultant which is r. So, let us say that this arm, when I swing away from the angle I must get the sin component which is positive. So, I must say that this value should be 1 minus beta square by root of 1 minus beta square square plus 2 zeta beta rho square. There is a multiple of p 0 by k is also there and whole square of p 0 by k.

Whereas, we all know that this angle is also going to be cos omega, this also going to omega t, because angle between the values are ninety, but as per as the cosine component is concerned it is negative. Therefore this value is going to be 2 zeta beta by the same denominator multiplied by p naught by k. I can now easily find the resultant of this, by squaring and taking a root of the sum of squares. So, let us see what is the resultant of this, and let us say the resultant is making an angle of theta by this. So, let us say this angle is theta.

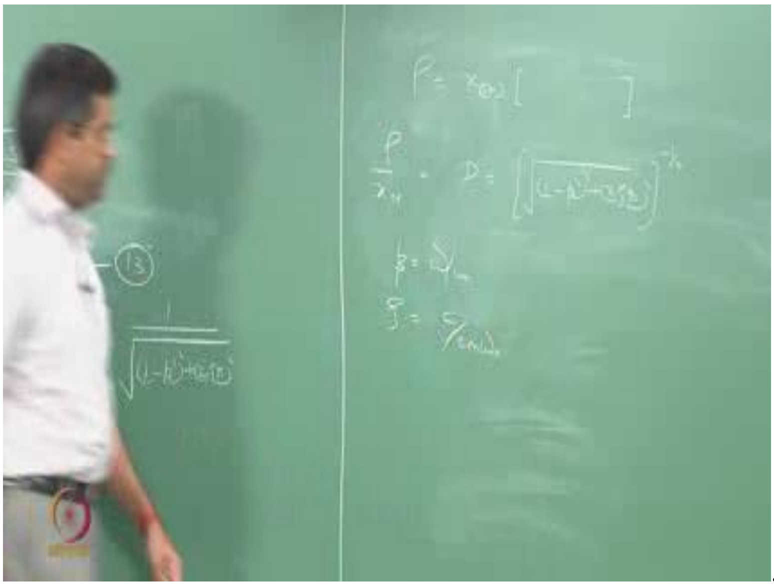
(Refer Slide Time: 12:54)



The resultant of this, a sum of squares of this, if you sum of the squares of this p 0 by k, you get the numerator as root of 1 minus beta square plus 2 zeta beta the square of course, the squares of this divide by sum of the squares of this. So, it is 1 minus beta square the whole square plus 2 zeta beta the whole square, which will amount to p 0 by k 1 minus beta square plus 2 zeta beta minus 1. Now I can express x of t which was the original equation twelve, the steady state response part, making use of this expression now, which can be rho sin omega t minus phi minus theta the rho is a resultant. This is given by this equation, this is equation number 13.

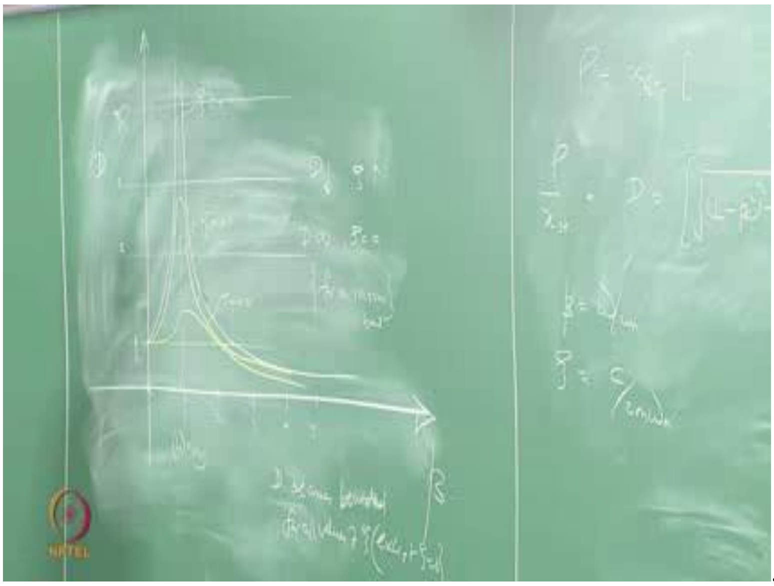
So, r this is also equal to rho given by this equation p naught by k minus minus r, or you can write it like this the other way it is 1 and the same it is p naught by k 1 by square root of 1 minus beta square square plus 2 zeta beta whole square, whatever way you want to write you can. So, you can remember this easily r. Now from the expression there or from the argon diagram there, I can also write tan theta, the theta is the angle between the resultant and the arm here, is this value by this value which is 2 zeta beta by 1 minus beta square, the denominator gets canceled.

(Refer Slide Time: 15:33)



Now I can say x of t by rho, because x rho is x static of x rho is s static of that is what it is a static of some value. So, therefore, rho by x static will give me the dynamic magnification factor which will be root of 1 minus beta square square plus 2 zeta beta whole square. Now that will go to the denominator, so 1 by root of this, this value. Now, we know beta is omega by omega n that is what beta is and zeta is c by 2 n omega n. We can plot this equation for different values of beta and zeta.

(Refer Slide Time: 16:58)

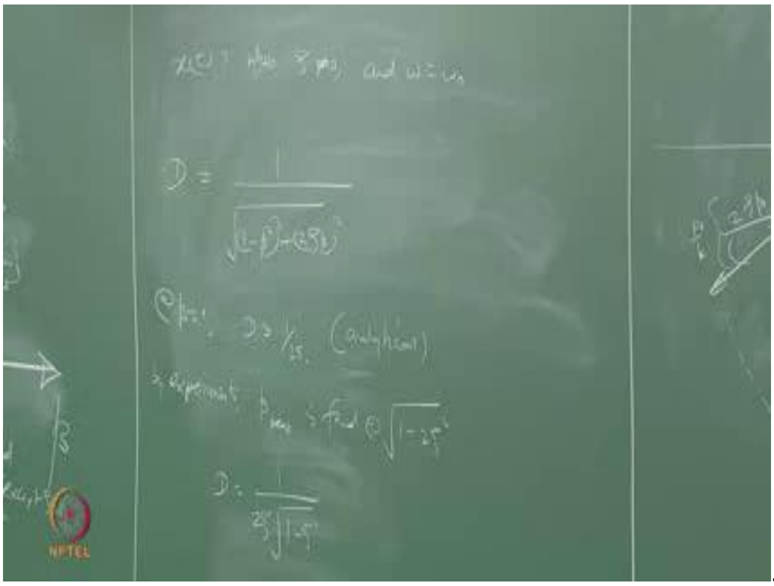


.

I will call this as value of beta, and of course, this I will plot the dynamic amplification factor d at beta equals 0, let us say beta equals 0, at beta equals 0, d will be 1, it starts from one. And let us say beta at 1 2 3 4 5; beta 1 means omega equals omega n, it is a resonating band. So, approximately at this band let us say at this particular point, if zeta is not present if damping is does not exist. We have already seen that the response will become unbounded, this slowly shoot up will go to infinity unbounded, and then once the band is shifted, it will slowly come down.

This is for zeta equals 0, there is no damping. now for any other value of zeta for any other value of zeta in d equation you will see that, the response will never be infinity, there will be some finite values let us take zeta at 0.220 percent, 0.2 the values somewhere here you substitute, you will see start from here it goes here, and comes down like this way, let us say this is zeta 0.2. Similarly zeta at 0.5 and so on, for different values of zeta the dynamic amplifier keeps on decreasing. It means the observation is d decreases with zeta increases, at zeta equals 0 d it becomes infinity; however, this happens only for a narrow band, very narrow, d becomes bounded for all values of zeta, except zeta equals 0, everywhere there is an upper limits of d.

(Refer Slide Time: 20:22)


.

Now the argument was, what happens in x of t when zeta is not equal to 0, and omega equals omega n; that is the argument what we are looking at, at resonance what happens. So, at resonance when beta is equal to 1 for zeta not becoming 0 the dynamic amplification value is bounded. Let us see what is the upper limit of the value. So, let us take the equation of d back again and check what happens. So, d is 1 by root of, let us say 1 minus beta square of square plus 2 zeta beta the whole square, at v equals 1, d actually closely becomes 1 by 2 zeta this is analytical this is analytical from the equation; that is what happens here.

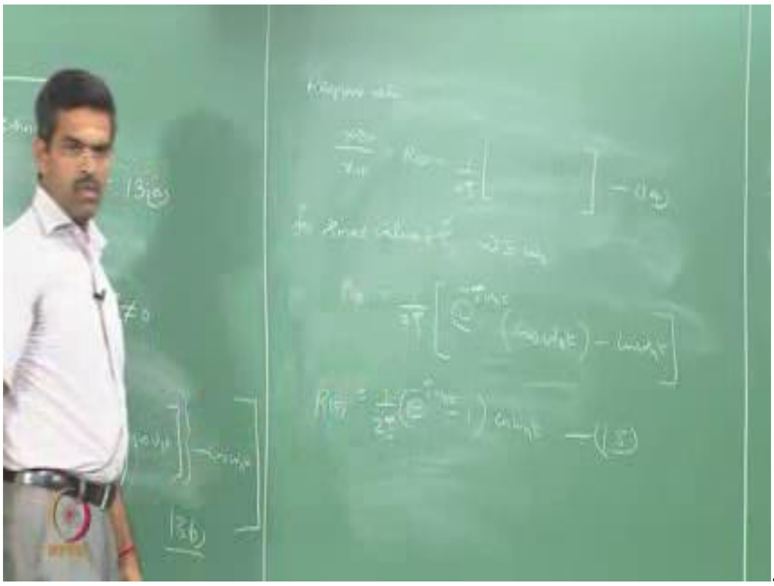
Now, by experiments people have conducted experiments, and they have found that beta peak; that is response at beta equals 1 at peak, is founded 1 minus 2 zeta square. Substitute these values back in this equation; you will get d as 1 by 2 zeta root of 1 minus zeta square. For very small value of zeta this equation will give you the same expression as analytical. This equation will give you the same answer as that of analytical for small values of zeta, so they match; therefore, x of t at resonance.

(Refer Slide Time: 22:50)

Therefore x of t at beta equal 1 is e to the power of minus zeta omega n t a cos omega d t plus b sin omega d t plus minus p 0 by k 1 by 2 zeta cos omega t or omega n t does not matter, because I am saying beta is 1 this already we have derived that is equation eleven or ten, I do not remember check that, this is my response. So, what it mean is x of t which is the response of a single degree freedom system, is a closed form solution for beta equals 1 provided zeta is not 0. They have a close form solution. Now let us say the system starts at rest, let x 0 and x dot 0 be 0.

Let us see what happens to x of t. I call this equation let us say thirteen a, let us check thirteen a and see what is my x of t, applying these conditions, because if we apply this condition a and b will go away, you can evaluate. So, just evaluate a and b and tell me what is the final answer for x of t applying this conditions, for this equation. So, I will write it down here, it is going to be p 0 by k 1 by 2 zeta, e to the power of minus zeta omega n t cos omega d t plus zeta by root of 1 minus zeta square sin omega d t minus cos omega n t 1 by 2 zeta cos omega n t is already there. Let us quickly look at the response ratio call this equation thirteen b. Thirteen b is nothing, but the value of a substitute for a and b from the initial condition.

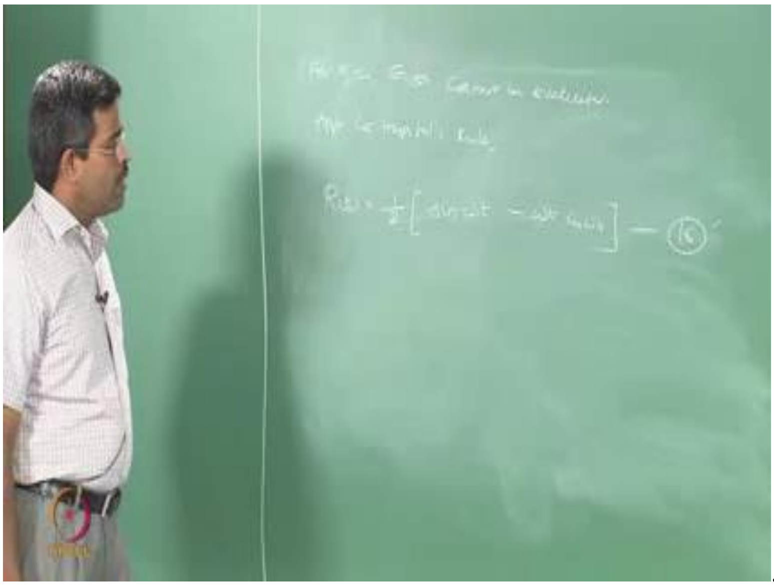
(Refer Slide Time: 26:10)



Now, let us find the response ratio, which is x of t by x static, because I have got p 0 by k here, which is x static, which will give me r of t as 1 by 2 zeta r the whole value, equation fourteen, same value will come back again. Now in this equation let us say for small value of zeta, is damping ratio. You will see that omega will be practically equal to omega d, because omega d is omega 1 minus zeta square, so for small values it is going to be equal. Therefore, my r of t for small values of zeta, for small values of zeta, so this will practically be neglected, this is very small value. It will become 1 by 2 zeta 1 by 2 zeta, because p 0 by k goes here that is my response ratio e to the power of zeta omega n t, there is a d k; that is why it is minus, exponential d k of cos omega n t, omega d and omega n will be almost equal of cos omega n t taking this minus cos omega n t is that, this one, this term is there already.

So, I can say 1 by 2 zeta e to the power of minus zeta omega n t minus 1 of cos omega n t; the equation fifteen, this is my response ratio. Now interestingly I want to capture, the behavior of response ratio, at zeta equals 0, undamped system what happens to it, because we know that the moment you provide damping, there is going to be decay. So, in the previous case what we did was, we said damping is not there, and we captured omega equals omega n the response, and we said it is unbounded. I was keeping on increasing phi for every cycle; that is the previous case. In this case if we substitute zeta as 0 this equation cannot be valued, because it goes to infinity is it not. So, I have to apply le hospitals rule, I will apply this and try to modify this equation.

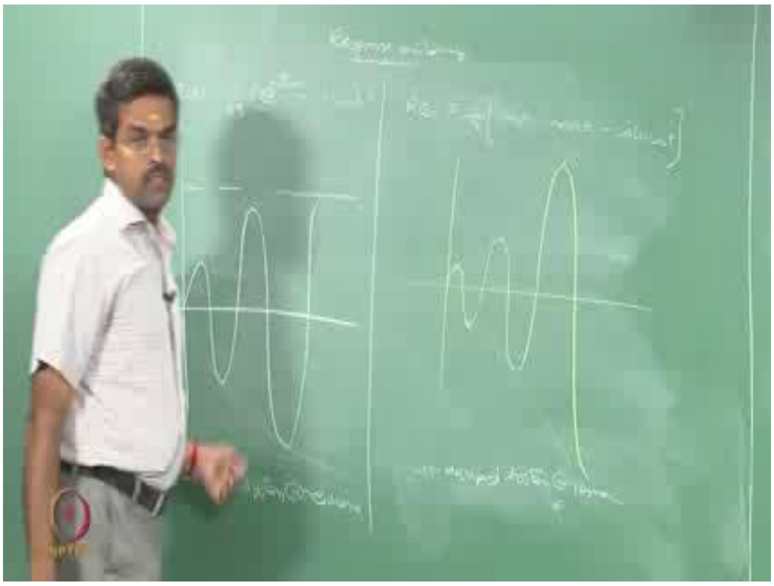
(Refer Slide Time: 29:10)



.

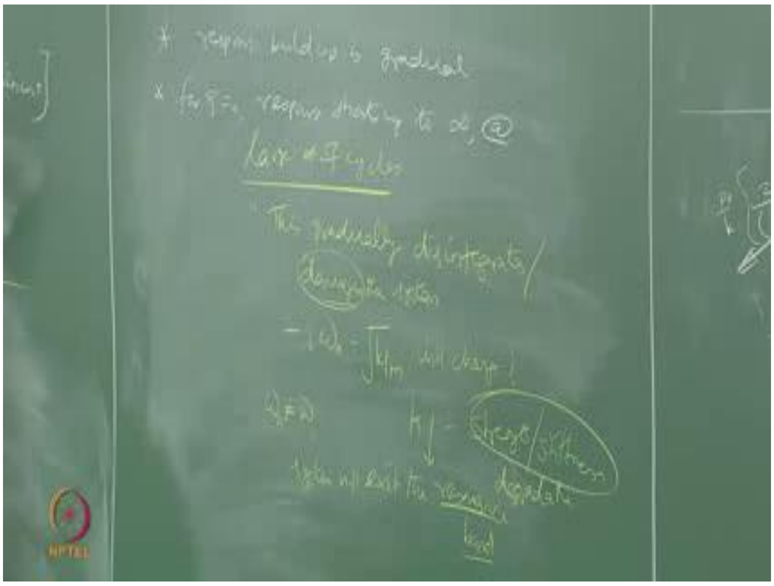
So, for zeta equals 0, equation 15 cannot be evaluated. So, apply le hospitals rule, and evaluate this. So, if you do that r of t will now become 1 by two, sorry sin omega t minus omega t cos omega t equation 16. Now I want to compare both the equations which equations I will compare. I will compare equation 16. I will compare 15. Let us say I will compare 2 equations now, to really check the response build up. Now I want to compare the response build up.

(Refer Slide Time: 30:26)



One equation I have is this which is r of t is 1 by 2 zeta e minus zeta omega t minus 1 of cos omega n t. The other equation we have is, half of omega n t cos omega n t minus sin omega n t. So, the response buildup for this we know look like this, and we already said by every cycle it is increasing by phi whereas, in this case, it will increase and becomes steady. So, the upper bound is 1 by 2 zeta. So, this is the damped system at resonance. This is undamped system at resonance. So, one can write quick observations about this, comparing both these plots. The first observation one could write is, in both the cases the response build up is gradual for zeta equals 0 undamped system, response shoots up to practically infinity at large number of cycles, is very important.

(Refer Slide Time: 32:40)



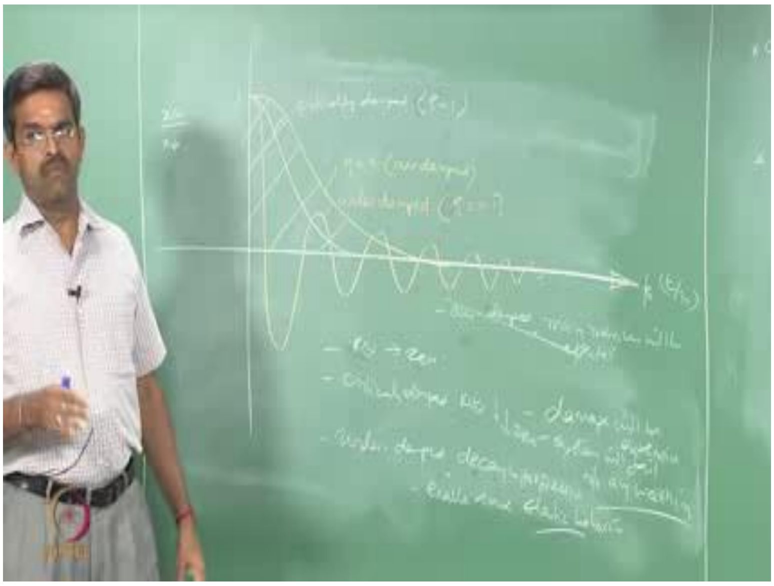
The shoot up of response to infinity is not instantaneous, it gradually picks up, and what it means is this gradually disintegrates or damages the system. We can infer one more important fact in this. Once the system is set to be damaged, omega n of the system, because both are at resonance. Both are at resonance, then equation omega n or omega does not matter. Omega n which is the function of root k by m will change, why, because k decreases, due to strength or stiffness, degradation; that is what we address as damage. Once stiffness or the strength degrades, omega either decreases; therefore, omega will no more be equal to the excitation frequency, the system will be will exit the so called resonance band. It will not say in the band, it will come out of the resonance band. That is a very interesting characteristic of the system itself, which is used to the design of offshore structures.

(Refer Slide Time: 35:26)

.

We can write one more observation, we shall remove this and write. At zeta not equals 0; that is for a damped system, even at resonance. Assume that the resonance band is wide enough to further cause damage to the structure. So, even at resonance, the response is bounded at 1 by 2 zeta. It means even for a small value of zeta, the binding is highly controlled, because is denominator; that is why even 2 percent zeta will work. You can also quickly see this plot to understand the comparison between under damped and over damped systems.

(Refer Slide Time: 36:25)



Let us plot the ratio instead of beta I can also plot t by t n, because both have them are 2 phi by t omega b omega n I can plot like this also. Let this be x t by x static starts from one we know that the response ratio is one. If the system is critically damped what is meant by critically damped. Critically damped means zeta equals 1; that is what critically damped is. If the system is under damped, let us say zeta is 0. 1. The system is over damped, this point bulges out (Refer Time: 38:02). You must notice some observations from this figure; one, in all the three cases critically damped, under damped, and over damped, response is set to 0. It means analytically all the solutions will lead to re centering capability.

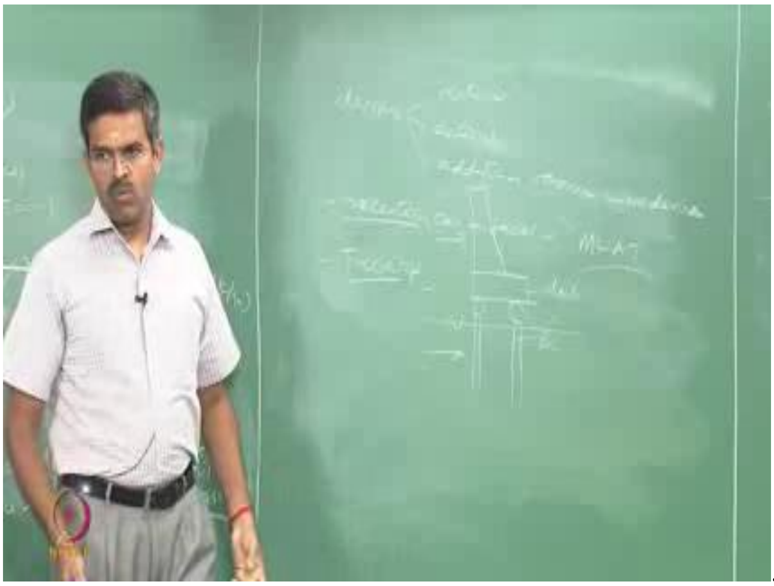
In case of critically damped system, the response ratio is instantaneously bought to 0. So, the damage will be extensive, and system will fail without any warning; that is important, because the damage is instantaneous. In under damped system, the decay is progressive, decay of the response not the system, decay is progressive, because we are looking at r of t is progressive, and it will enable some elastic behavior in the system, enables some elastic behavior to the system. The moment I say it enables elastic behavior, stiffness of the system changes.

The moment stiffness changes omega n changes, and the whole argument of omega equals omega n at resonance, is not valid. In case of over damped, large amount of members will be affected. To understand this you must convert the time (Refer Time: 40:35) response to the frequency demand response, and a spectrum. You will know the energy of the spectrum. So, you will always see this is like lot of energy. So, the damage caused to the large members will be higher.

So, therefore, in general in structural engineering people never prefer over damped system. Critically damped is of course, hypothetical. People generally prefer undamped systems only, because of the validity that, even for a small value of zeta, even at resonance the response will get bounded. So, there is no problem, and we generally go for 2 to 5 percent maximum in offshore structures.

So, that is about the discussion on response behavior, of under damped and damped at resonance, response behavior of critically damped, under and over damped, not necessarily at resonance, but in general this is the understanding what we physically gain, which will help us to design the systems, because these equations and these understanding should be able to now emerge to design the system in a structural form. One may ask me a question how this will help me in suggesting a new geometric form for offshore structure. I can quote one simple example in a minute, and show you how it can be done.

(Refer Slide Time: 42:11)


.

Let us talk only about damping, there can be different sources; one can be material damping, one can be external source of damping, one can be an additional response control devices, as dampers. Suppose you understand that the hydrodynamic damping or the material damping or structures inherent damping is lesser. You can introduce an additional control device in the system, so that you can bring down the instantaneous response in less number of cycles. So, re-centering can be faster. The objective is not the response control; the objective is re centering can be faster.

A classical example of this is a multiple articulators, classical example of this. Whereas, the buoyancy chamber is shifted in such a manner, that the re centering is brought faster. So, buoyancy chambers introduce additional damping, because of hydro dynamic effects, sloshing etcetera, because the variables submergence effect this cause additional damping and that brings the objectives. So, this is a design objective. So, all these inferences can be converted to an understanding of design.

Similarly in case of triceratops, this is a new geometric form it is coming up for Ultra De Walters. People introduce these are buoyant legs, this is the deck, and people introduce Bolgenos in between them. So, whatever response the buoyant legs will have. Of course, the water level is somewhere here, the Bolgenos are above the water level, whatever response the buoyant legs will have, they will be absorbed by the joint, and it will have to be transferred to the deck. If the deck has some aerodynamic response it will be again absorbed with the joy now come back to the legs. So, introducing additional damping characteristic, fundamentally to bring the re centering capability more. So, the design is understood from dynamic behavior as a fundamental equation like this.

Thank you.