Advanced Marine Structure

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Lecture - 17
Plastic Design -II

In the last lecture we discussed about the necessity or the idea which prompt and the researches to shift the design principle from the elastic to that of our plastic one, where the researches or there the designers wanted to make use of the addition resisting of the material, which is available beyond the first hill point. First hill formation is generally because of the presence of research in especially in particular material like steel on my on steel, so one can think of using or utilizing the reserves strength which will enable more or less complete utility value of the material. Because I can stretch the material to maximum load carrying capacity, there are 2 reasons why the designers wanted to shift parallels, transept parallels from elastic design quality Medlow design were the following;

1, the load carrying capacity can be here in enhanced from held value to ultimate value and the same time the ductility feature of the material can be used fully. So that it is guaranty that the material will not fail until the strain reaches, the strain the ultimate the value or the till the ratio is expected to full fill the reserve strength of the material, so this 2 were inherently prompted the designers to shift the design principles from elastic design mechanism to that of a plastic design principles. Provided this system can be applied only for material and structures. Material should have enough ductility and the structure should be startlingly in terminate, as the higher of higher order of into terminuses in the system, the benefit is far and far. Because essentially theductility factor or the ductility capacity will try to redistribute the movements from high listed section to the next high listed sections.

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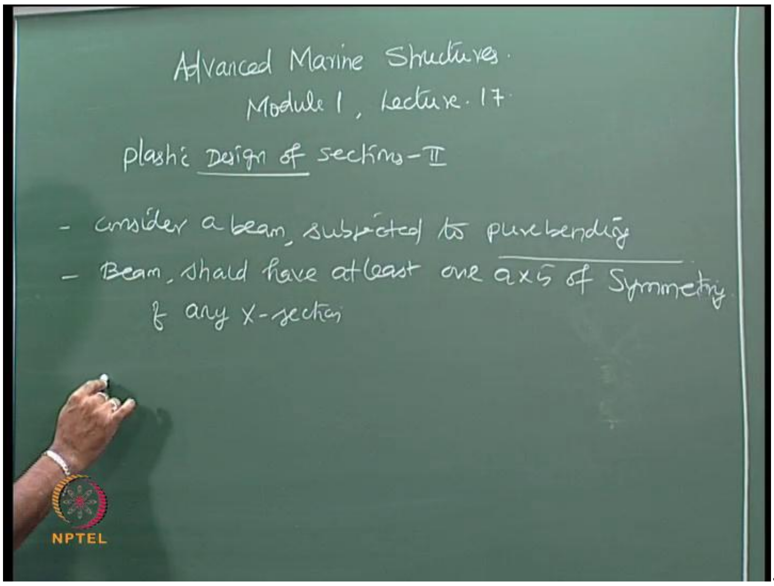

So we will continue to see what would be themethodology, may be we can estimate the movement carrying capacity of a simple sector or what I will do iswill considera beam, subjected topure bending. The beamshould have at least 1 maxof symmetry. It can be of any cross section, of any cross section, that is notimportant.

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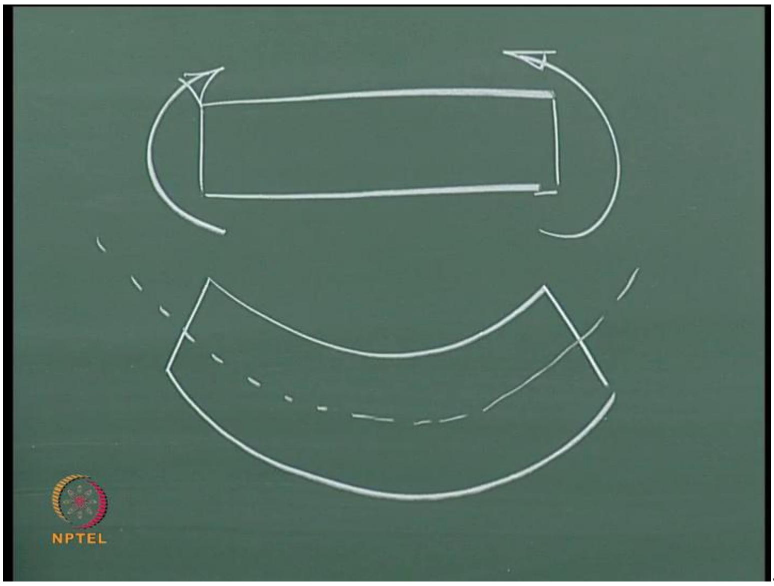

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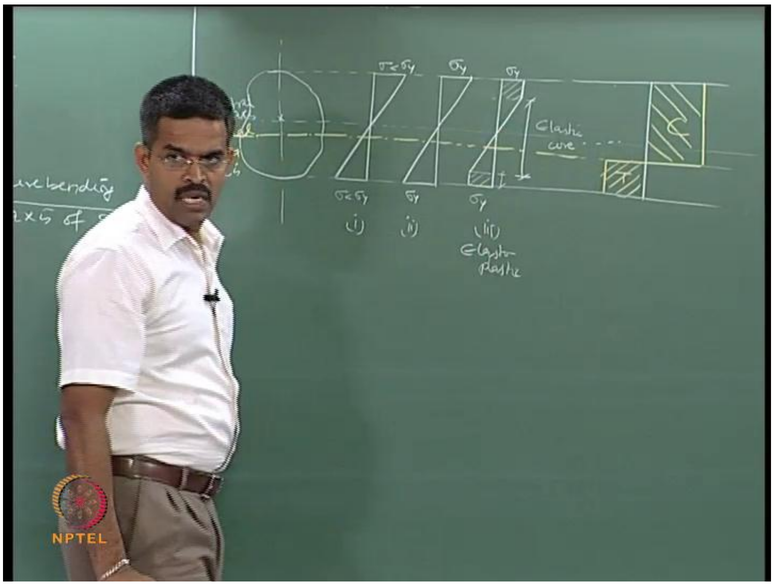

The cross section should at least have 1axis of symmetry. So let us say I have a beam, I subject thisbeam to pure bending. So the beam will start bendingand of course, there will be an axiswhich is very important to discuss, will talk about axis later. So I amapplying pure bendingon movement at the ends of the beam, for the beam is bent and keeps on increasing the movement and the beam will keep on bending. Let us say the beam has any specific cross section of any shape.

These are the extreme fibers of the beam, now the beam will have 2 axis actually, one is called as neutral axiswhich a called as neutral axiswere the strain remain 0, the other one is a new axis which we call as equal area axis. I am drawing a low line; hereI call this as equal area axis. So initially the strain in the strain forever and correspondingly this this in extreme fiber will remain lesser then the yield value.

The first case, as a further keep on increasing movement at the ends the extreme fibers will reachthe yieldvalue that may bethe next stage. Now if you further increase in the movement and the ends of the beam then you will see that some section of the cross section will be yield, soyou will find that till this point and till this point you willfind the strain, oh sorry stress, will be equal to sigma y and this will remain still elastic, I can callthis as what I call as an elastic core.

So the section will startremaining elasto plastic, because there is an elastic core presents there’s a plastic core also present in the section. As it keep on further increasing in the movement, the stage willcome ultimatelywere the entire sectionwill get plasticized. So there is very interesting thing which happens here, when the entire section get plasticized, the stresswill move in this format. Whereif we call this is a compression, this as tension.

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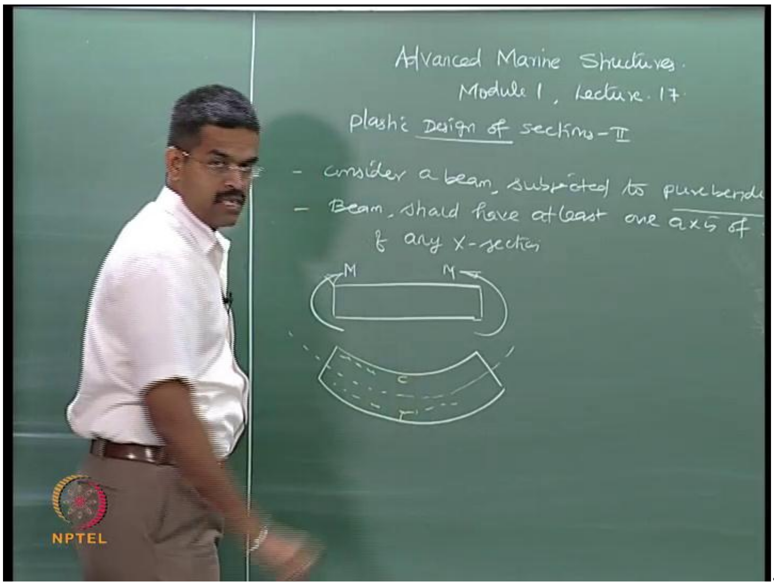

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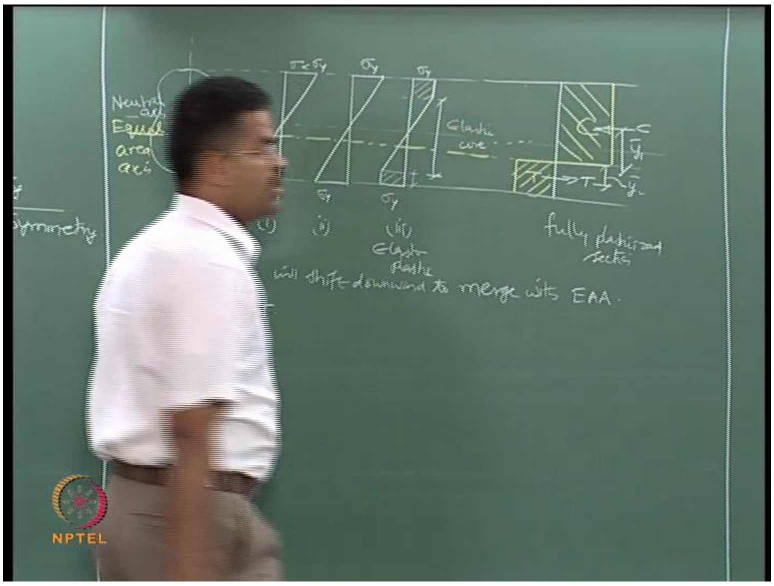

Because you see here, when I am applying the movement it the end bottom fiber will longestand the top fibers will compress. So I can say this is compression, this is tension.So I am marking compression and tension in the extreme top and bottom fibers.

So interestingly the neutral axis will start shifting downward and will merge with the new axis called equal area axis. So the neutral axiswillshiftdownwardsto mergewith equal area axis, so I can say now this is fullyplasticizedsection, fully plastic.

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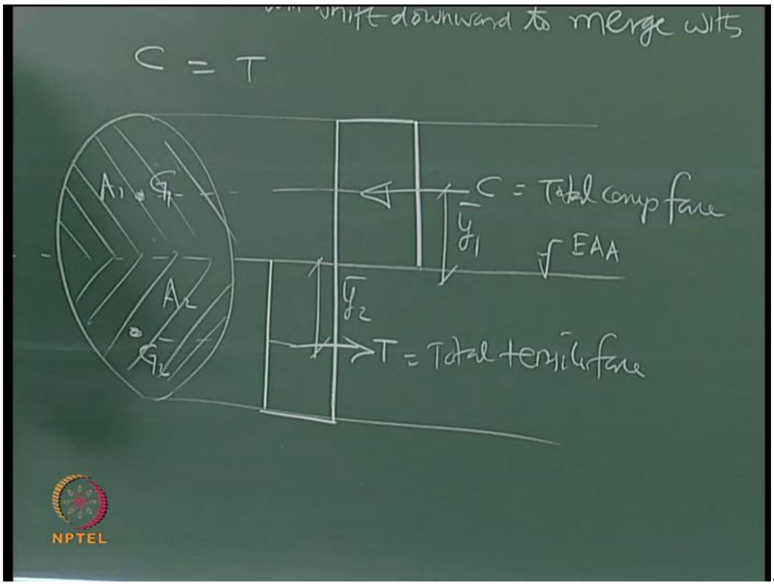

This is elasto plastic, of course this are all completely elastic. Since this is equal area axis, obviously C should be equal to T, that is why is called as equal area axis so the figure may not show that because this section is not uniform. Let us say this is my compressive force C, this is my tensile force T and this act as a CG, Let us take y bar 1 this acts as aCG asy bar 2. I can redraw this figure slightly in a different manner saying that; Let us see the sectionas gotfully plasticized.

I can call this is my total compressive force, this is my total tensile force, can call this as my y bar 1, thisas my bar 2, whereC is the totalcompressive force, where T is the totaltensile forceand this is nothing but, myequal areaaxis. So I can callthis as G1, this as g 2, where G1 and G 2 are the centered points of the compression tension area respectively. So I can call thisas A1and this as A2.

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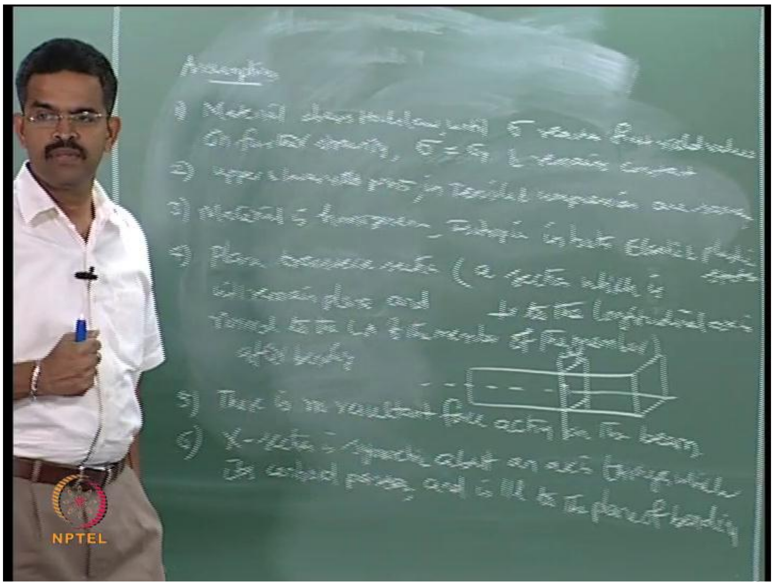

And so, while loading the section from the extreme fiber reach elastic on ayield value to the entire plastic section, we make certain assumptions. Material obeys Hooks law untilthe stress reaches, I should sayfirst yield value.So, on further straining, stress remains constant and sigma y and remains constant. Now upper and loweryield points in tension and compression fibers are same.

Material is homogenous and isotropicin bothelastic and plastic states. Plane transfer section that is a sectionwhich is normallongitudinal axisof the member so what do you understand by this, I have a member, this may longitudinal axis member, I cut a section which is perpendicular to the longitudinal axis of the member.

If the member has a breath Bthe section is also have a breath B which I am drawing. So by let us say, so this section is normal to longitudinal axis and this remains plane. Ok this is plane, will remain planeand normalto the longitudinal axis of the member after bending also.There is no resultant force acting on the member. The cross section is symmetricabout an axis through whichits centroid passes.

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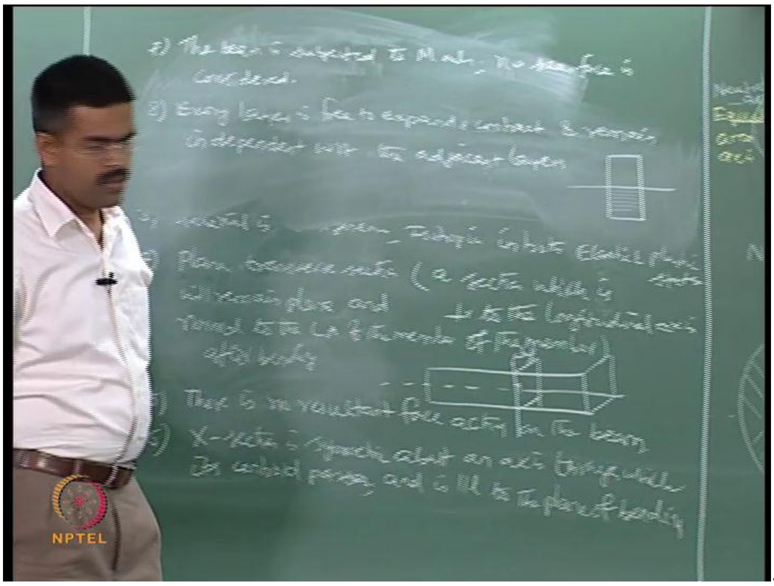

And is parallelto the plane of bending, when it, what does it mean. Suppose I am trying to locate a centroid of the section about which the section remains symmetric and that point lying on the plane which is parallel to plane of the bending. Ok, no shear force axis a section only bending movement is considered. The beam is subjected to M only, no shear forceis considered.

Most importantly every layeris free to expand and contractand remain independentwith respect to the adjacent layer. What does it mean, In a given prosecution ifI say this is an equal area axis of above which the centroid is located is section remains symmetric.I take any fiberin each fiber along the cross section or having freedom to independently expand and contract.

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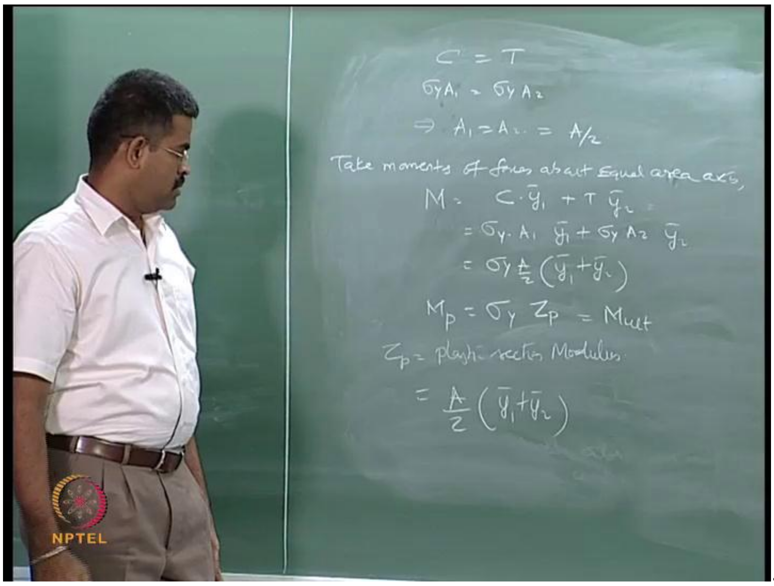

There is no fiction between layers. Having said this I will retain this figure, let us now derive the movement carrying capacity of this beam, which is the plastic movement carrying capacity. So we already said Cis equal to T, the total compression should be equal total compression force in the cross section and we already knowC is equal to sigmaone y into A 1and this is also equal to sigmay into A2. Because I say A 1 area of compression as a C from this figure and A 2is area of tension, C from this figure they should remain same. WhyI am using sigma y, because it’s an assumption that once a stress reaches the yield value there after the stress remains constant at sigma y.

So, this implies thatA 1 is equal A 2, which implies thatthis is nothing but, A by 2 because A 1 plus A 2 is total A. Let me take in take movement aboutmovements of forcesabout equal areaaxis, about this axis they will form a clock ways couple ;I should say thatthat movement should beequal to C into y bar 1plus T intoy bar 2 ,which is sigma yinto A 1 into y bar1 plus sigma y A 2 y bar 2.

I should say sigma yA by 2y bar 1plus y bar 2, so this gives a similar comparison to me sayingsigma yand ZPisM P is also equal toM ultimate. When I use introduce a new symbol Z P, Z P is calledplastic section modulus, y is given bya by 2 y bar 1 plus y bar. Now how do you define the plastic section modulus?

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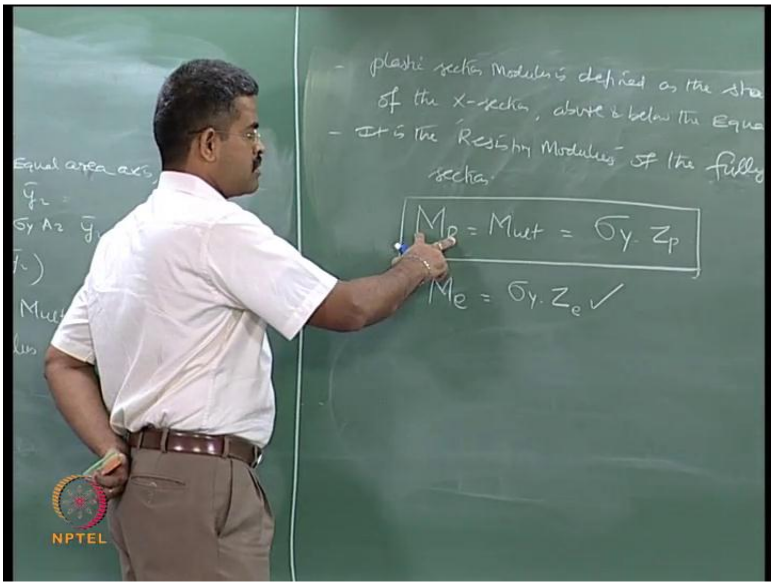

Can see here plastic section modulus has 2 components, one is the area other is y so product, so I am taking movement of the area about an axis which is separated up and below or top and bottom or compression tension by y bar 1 y bar2. So I should say that plastic section modulus is defined asas this static movement of the cross section above and below the equal area axis.It is also called asthe ‘Resisting Modulus’of the fullyplasticized section.

Therefore plastic movement of resistance which is also equal to that ultimate movementcarrying capacity of the beam is nothing but, sigma y into Z P that becomes very interesting a simple derivative. I can compare this with elasticmovement carrying capacity which we all know is nothing but, sigma y into Z, to make it very clear I may even write Z e here does not make any different. This is called simply this section modulus which we all know. So this weknow, this is a new one which have derived for now. I can also find ratio between these2 movement carrying capacity there is M P by M E, I can find that which I will call as a shape factor. I will introduce that.

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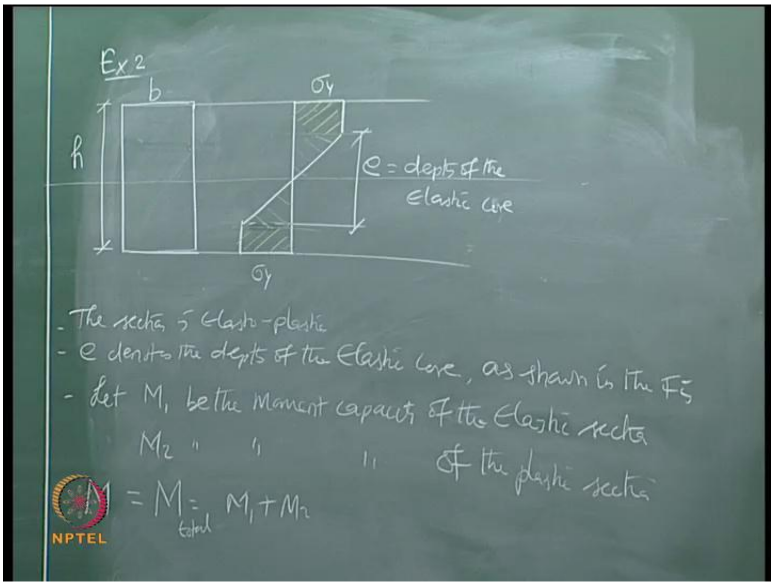

Now we have taken a general cross section where area is A and we found out y bar 1 y bar 2, we said A by 2 etcetera. Now for standard cross sections can we find these values and way to find out what is actually Z e. Can you find out, will take a standard cross section, may be start with the rectangular cross sectionthen will work out some other cross section and see how can a really find out the plastic movement carrying capacity of a given cross section readily. Ok, I will take an exampleof a rectangular cross section;I will take a rectangular cross section.

Let us say the breath of the cross section is b and depth of the cross section is h. I am drawing an elasto plastic section which has an elastic core depthas e, e isdepth of theelastic coreand of course the stress hereremainssigma y.Sothe section is elasto plastic, e denotesthe depth of the elastic coreas shown in the figure. Now this section as got 2 movement carrying capacity, 1 is a capacity of the elastic section aloneand this capacity of the plastic section. There are 2. Let me call.

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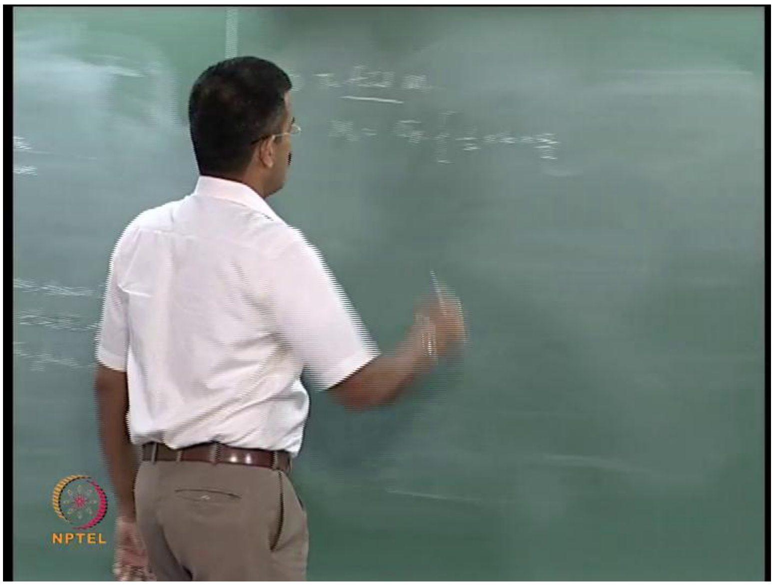

Let M 1 be the movement capacity of the elastic sectionand M 2 be the movement capacityof the plastic sectionand we all now agree the total movement carrying capacity section M total will be a some of M 1 and M 2. Let me now estimate M 1 separately M 2 separately, which will get M total which will be the total movement capacity of the elasto plastic section, which I can call simply asM. Nowto find M 1, that is the movement carrying capacity elastic section.

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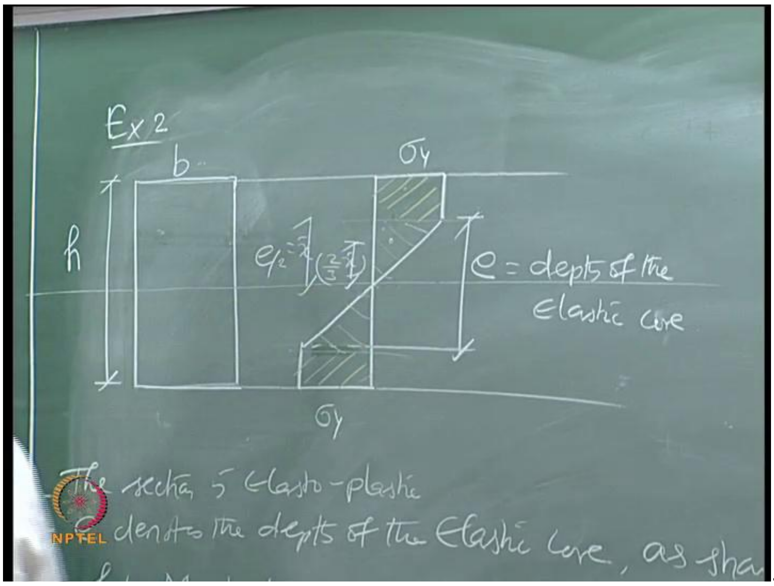

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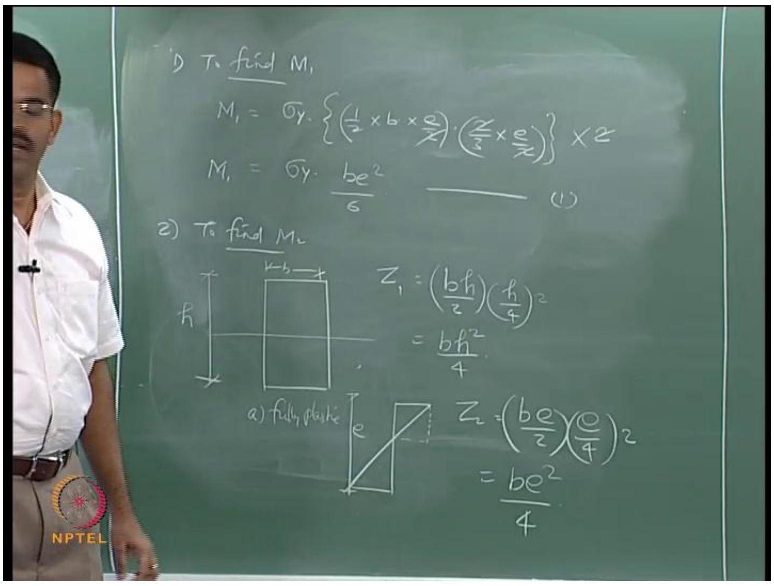

So M is going to be simply stressmultiplied by the first movement of the area, which is nothing but, half baseheight because this is e by 2and the CG of the area will be somewhere here, which is going to be a call this as e by 2 this plan and this is for examplex bar and this value will be equal 2, 2 third of x bar. I am writing it of by 2 which is going to be 2 third of e by 2and such sections are there one above the axis one below the axis.

Ok can I multiply by 2 because this is symmetric. So even simplify this, I will get sigma y b e square is that all right and calling as M. Let me call this is equation 1. Now I want to findM 2, this I well do slightly in a tricky manner, what I will do is if I section is fully plastic; if a section is fully plasticthen I will get the Z value, the z value as b h by 2.

This is h, you see from the figure, this is h and of course, this is b, b h by 2 and h by four of twice that is a fully plastic section. It isok, which will becomeb h square by 4 that fully plastic. I can this as Z 1 which is fully plastic. But, the section is not a fully plastic it is partially elastic also; it means I know I have a section whose stressdistribution is also elastic, which has the elastic core equals e.

Now what we do, I replace this part by dotted line, can a do that? It’s going to be equal now 1 is C, 1 is d equal. So I find only this part and see what happens, so I can call that Z2, which will be b into e by 2 b into e by 2because this is e into e by 4 half pies.

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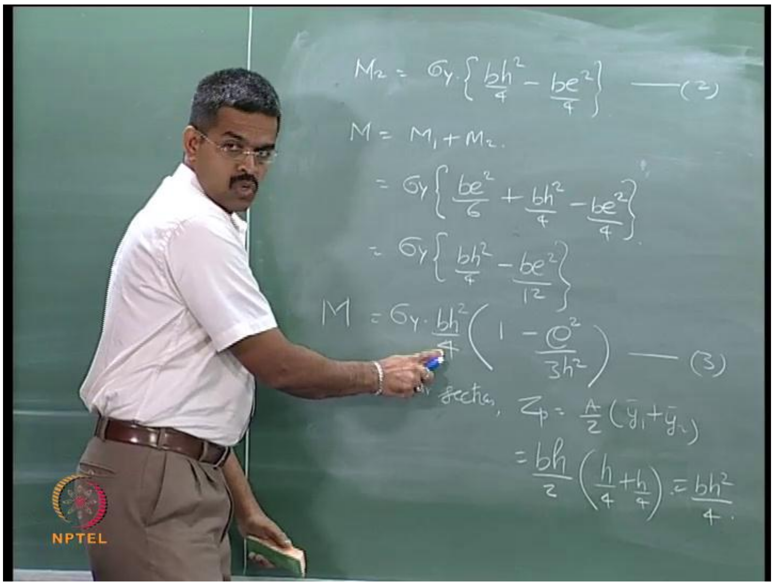

Which will become b e square by 4is that ok. But, my real section is combination of this 2. So I can know say, let the more thisM2 is the moment of that plastic alone which is the let resistant of total plastic minus elastic, is it know, that's what M 2 is M 2 could be sigma y half b h square by 4 minus b e square by 4. Do you agree? So the total movement carrying capacity M is a nothing but, M 1 plus M 2which issigma y half, b e square by 6plus b h square by 4minus b e square by 4. Can you quickly simplify this, so I can say sigma y half b h square by 4minus b e square by 12, is itok, which I say sigma y half b h square by 4 one minuse square by 3 h square. Can a write like thiswhich is my M, is it not?

I will call this equation number, this off course 2 so call this3. 1 is here, now for a rectangular section which we are discussingZ p is given by a simple equation which is a by 2 half y bar 1 plus y bar 2, is it not. Which can be simply b h by 2 that's my rectangular section, b and h are the cross action section properties of the section. A 1 or y bar 1 will nothing but, h by 4 plus h by 4which can give me b h square by 4.

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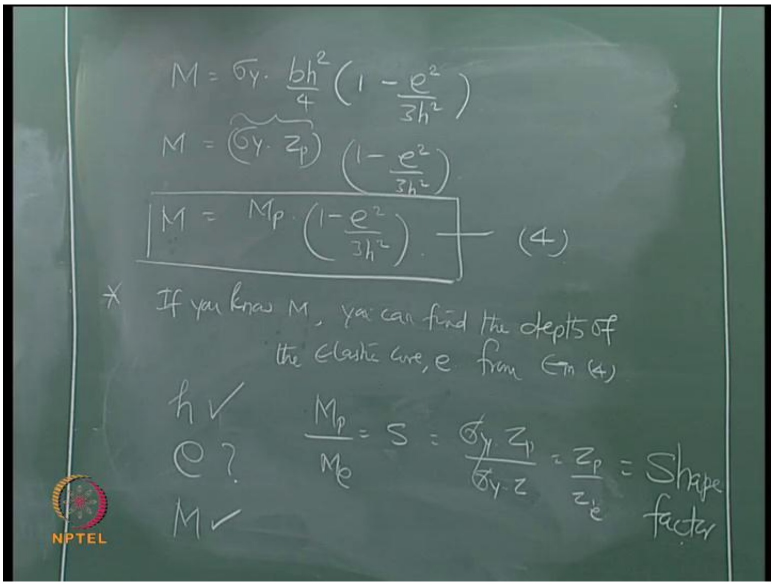

Is itok? I have b h square here also, now I can replace the equation asM is now rewritten as sigma y b h square by 4, 1 minus e square by 3 h square which can now written as sigma y Z p of 1 minus. With it, we already saw this in previous step that I can replace this as M p, that is plastic capacity the section 1 minus e square by 3 h square with is M, that is a very interesting outcome of the derivation.

What is the inference which is derived from equation number 4, if you know the movementapplied on to the section, if you know the bending moment coming on to the section you can easily estimate the depth of the elastic core efrom equation 4. You may wonder how. Look at equation 4.

The variables areM , M P, e and h out of which given cross section h is known to me ,for a given cross section depth as section is known to me for a rectangular. e is what you are determining, agreed? M is known to you, you know M the movement coming on the cross section. Now M P by M is the simply the shape factor, so if you know the elastic moment in the cross section which is nothing but, simply sigma y Z P by sigma y Z which is nothing but, sigma P sorry Z P by z elasticwhich is what I call as Shape factor.

So I know M P alsoI can trace and find out what could be the depth of the elastic core for a given movement in the cross section. Any doubt here? So equation 4 can help you to estimate the depth of elastic core for any moment in the cross section M, Let us see what happens so the equation when the elastic core does not exist, it means fully plastic.

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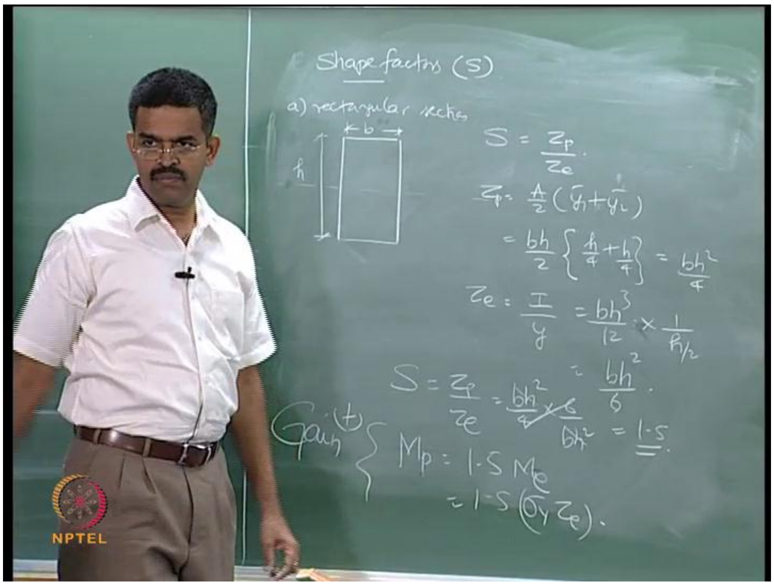

Ok it is fully plastic. So before that let us try to find the shape factorsfor different cross section sections which were interested, let us say obviously we take the rectangular section for the begging. The cross section is known to mewhich isdesignated as h as the depth and b as the breath.

We already know shape factor is ratio of plastic section by elastic section. Modules Z P we already know is a by 2 y bar 1 plusy bar 2, which is nothing but, b h by 2h by 4 plus h by 4, which is b h square by 4. Whereas as the elastic section modules as for the rectangular section is nothing but, I by y max, which is b h cube by 12 into 1 by y max in my h by 2, which gives me b h square by 6. I can find shape factor as Z P by Z ewhich is b h square by 4 into 6 by b h square.

Which will give me1.5. What does it mean, the plastic movement carrying capacity of rectangular section is 50 percent more than the tuff elastic which is 1.5 times of sigma y half Z e is that right, so my moment carrying capacity of a rectangular section of a beam is 50 percent more than that of an elastic section. So shape factor will give me the induction what would be the additional capacity of a section, when you do plastic design the section is a remaining same, I am not increasing width and depth of the section, b and h remains same, just by shifting my design philosophy from elastic to plastic I get additional load carrying capacity in the section which is a gain, is it no, this for rectangle.

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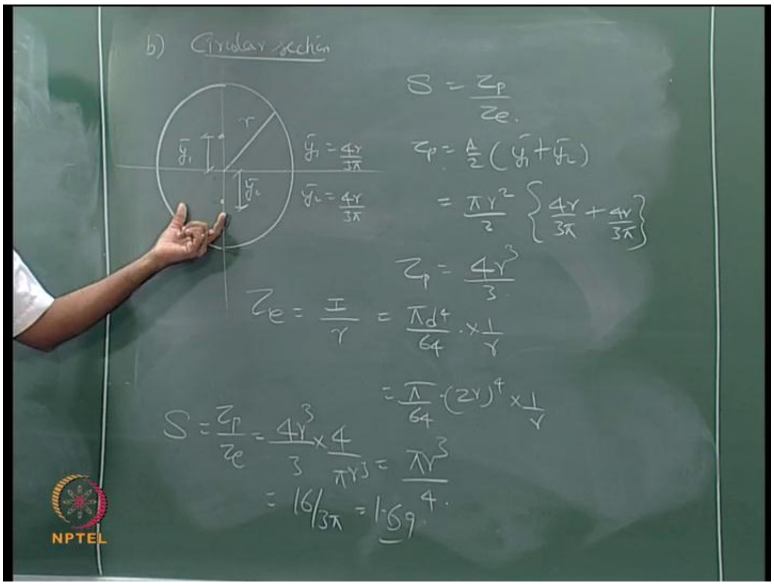

Now look at circular sections. Let us say the radius of the section is r, we already know the CG of the upper half which I call as y bar 1 will be nothing but, 4 r by 3 pie, is it not?Which will as same as the low section, also which will be y bar 2, y bar 2 are also equal to 4 r by 3 pie. So I know the shift factor is given by a simple expression.

Z P by Z e and Z P is given by a by 2 of y bar 1 plus y bar 2, which is going to be pie r square by 2, pie r square is the area of this section, y bar 1, 4 r by 3 pie plus 4 r by 3 pieso which will give me 4 r cube by 3which will be Z P. Ok.Z elastic I already know nothing but, I by y max, y max in my case this r, I is let us a pie d per 4 by 64 by 1 by r. I can say pie by 64 of 2r to the power 4 1 by r. See tell me what we simplify this.

What is that pie r cube by, no no please check. So can we quickly tell me what is yes? 4 r cube by 3 into4 by pie r cube is it16 by 3 pie? What is value? What is this value is it 1.69? 1.69, let make it is 1.69, so if have a circular section the plastic moment carrying capacity M P is a about70 percent more than elastic.

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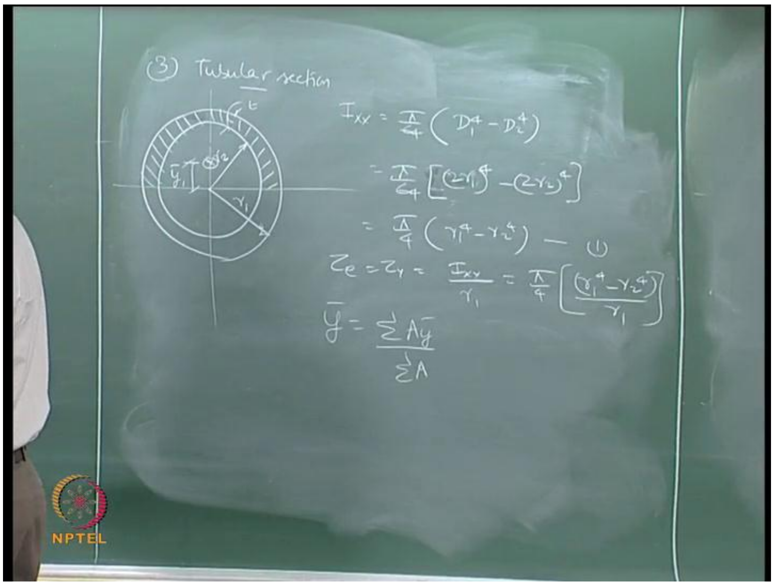

Now let us quickly compare rectangular that of a circular section which is solidsolid. The moment carrying capacity of the solid circular section is larger than rectangular. It is because of this reason in marine structures people you circular instead of rectangular because movement carrying capacity is higher the plastic moment carrying capacity is higher but, commonly without use solid section, we use angular that is tubes, now a let us see what is a shape factor for tubular section.

So I call this is r 2, this is r 1, when off coursethis as t. Let us quickly see what is a movement of inertia of this sectionwhich is pie by 64, D 1 to the power of 4, D 2 the power of 4which is pie by 64, 2 r 1 to the power 4minus 2 r 2to the power of 4.So we get pie by 4 half r 1 minus r. Is itI calls equation number 1. See we want to findZ elastic orZ y, which is the section modulus I should say is nothing but, I x x by y max, y max in my casesr 1, ya so which will be pie by 4 r 14 r 24 by r 1. That is my Z e, now I am interested in finding y bar 1 which is CG of this section, only this section. So what we naturally do is I will take simple equation of a y bar by sigma a.

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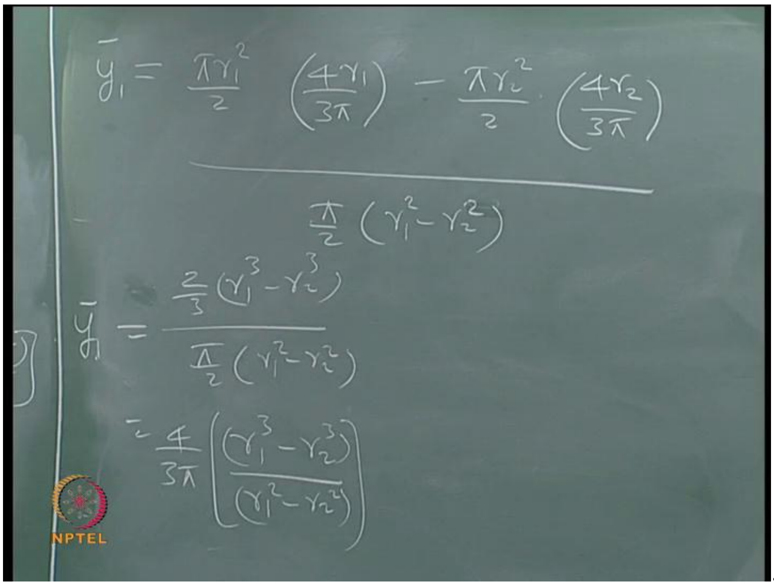

If you want to fine y bar, I should say sigma a y bar by sigma a and find y bar, first principle. Let as do that here, so let us say pie r 1 square by 24 r 1 by 3 pieminuspie r 2 square by 2 4 r 2 by 3 pie.That is sigma a y bar, is it not sum of sigma a y bar. I put minus because angular section do very sigma a, which is pie by 2 r 1 square minus r 2 square, that is my area of this section. I am only finding your pie bar one y bar 2 is below that we see let simplify this get me what the value is which will simply to not to be 2 by 3 r 1 cube minus r 2 cube byPie by 2 of r 1 square minus r 2 square, do you agree?

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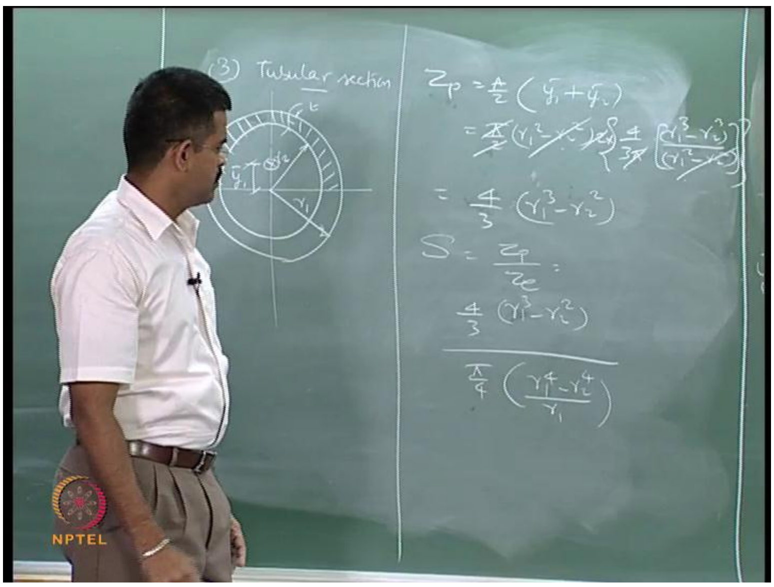

That is my y bar, which can be written as4 by 3 pie half r 1 cube minus r 2cube by r 1 square minus r 2 square. Is it ok, I just simplified this? Now already know is Z P, Z P is nothing but, A by 2 half y bar 1 plus y bar 2.A that is nothing but, pie 2 half r 1 square minus r 2 square that is half of the area with this4 by 3 pie half r 1 cube minus r 2 cube by r 1 square minus r 2 squarethe whole twicethat. Let me put it to 2, multiplying 2 here, so this goes away, his also goes away,this also goes away; what is a left over, 4 by 3 r 1 cube minus r 2 square, is it not, that my Z P.

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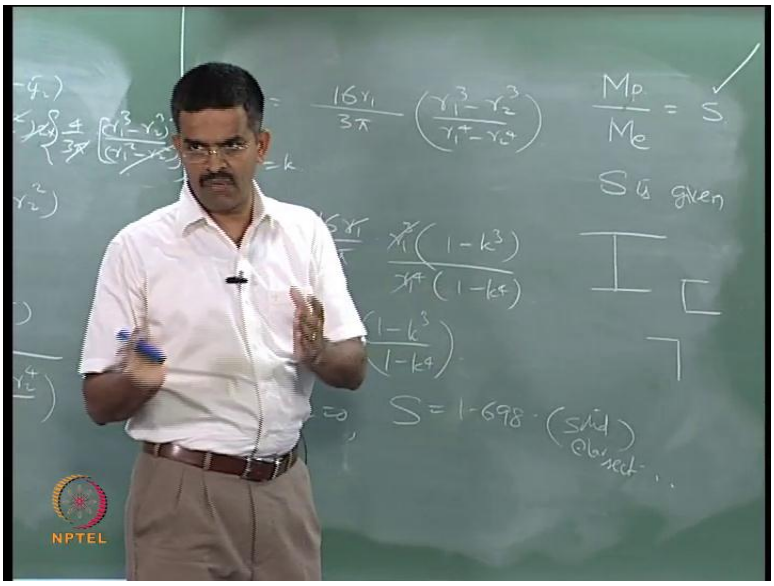

Iam looking for shape factor which is Z P by Z ewhich is 4 by 3 half r 1 cube minus r 2 cube divided by what is a D, we already have equation 1 ya pie bar 4 half, ya can you give me this, is itok? So I can say, 16 r 1 by 3 pie simplify 16 r 1 by 3 pier 1 cube minus r 2cube.By r 1 4 minus r 24 that, so let me sayr 2 by r 1 be k, so S now becomes 16 r 1 by 3 by 3 pie r 1 half, which is 16 by 3 pie half l minus k cube by 1 minus k 4. Is it fine, so for k equals 1.

That is r 2 or r 1 same that solve it to you, you will find; not this, for r 2 equals 0 that is inner radius,this only 1 radius, you will get this S1.698 which is same assolid circular. So what all we know is if you know the shape factorfor any given fraction, if I can find out shape factor for any given cross section,

I can always find out what is the enhancement in my plastic moment carrying capacity in comparison to that of elasticmoment carrying capacity. What you wanted to know is only the shape factor. Many other iteration cores for most other iteration for most of the section like standard table, the shape factor is given. For example, I section, chance, angles; shape factor is known so is given theta like a hand book. So I can easily find out select section whose shape factor maximum so that I will get the maximum gain in a movement carrying capacity of the section. So that's what we are interested conveying today.

That how 1 can find out the variation in anelastic core, in a given section if you know the moment at any section number 1, number 2 how plastic moment carrying capacity can be easily determine for a given cross section if you know shape factor of a given cross section is a given data in most of the hand books of steel cores. You have also understood that circular sections have more capacity of load carrying capacity in terms of plastic moment compared to the rectangular for which circular sections are more commonly used in many structures, essentially tubular sections but, they have more capacity.

Thanks.