Advanced Marine Structures
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Lecture - 4
Deterministic fatigue analysis

In the last lecture, we discussed about the time history. Once I have the stress range time history available to me, based on which you can use what we call as the rain flow counting technique, to find out the fatigue damage of each stress group and successfully the cumulative fatigue damage,okay?In this lecture we will talk about deterministic fatigue damage, using some spectral approach.

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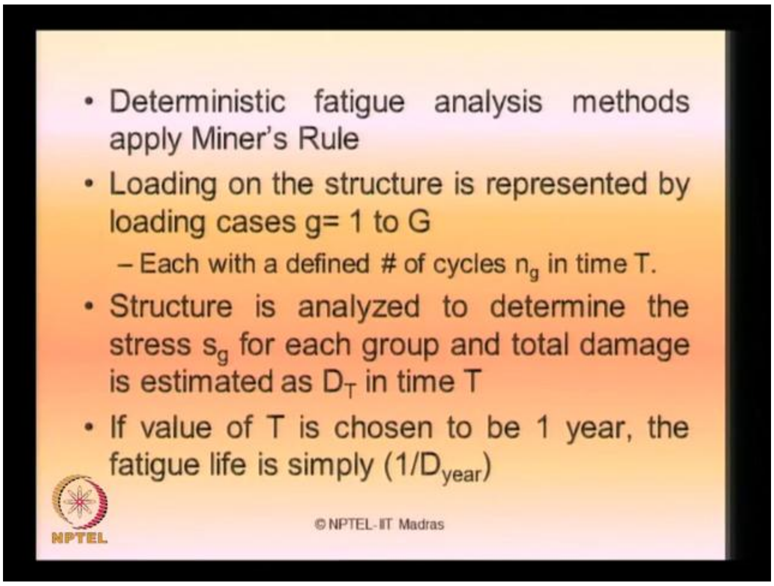

Deterministic fatigue analysis methods actually apply the minus rule, loading on the structure is represented by the loading cases varying from 1 to G, as we saw in the last example. Each cycle will be defined with n g which is called number of cycles in time T the structure is analyzed to determine the stress group which is g of each groupand the total damage is estimated in a given time T.If the value of time t chosen to be 1 year, then the fatigue life is simply 1 by the damage per year.

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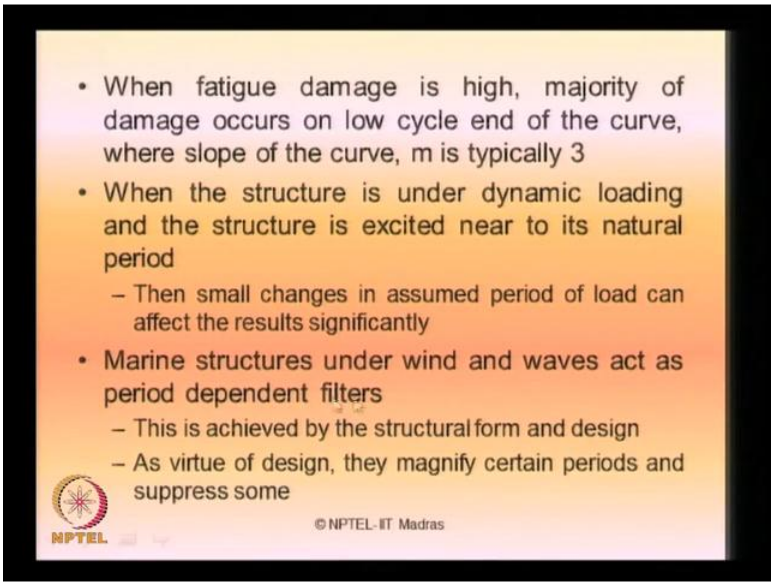

When the fatigue damage is high, majority of the damage occurs on low cycle end of the curve, where the slop of the curve is typically 3 on marine structure materials. When the structure is under dynamic loading, the structure is then excited near to its natural frequency, in that case the small changes, in assumed period of the load can affect the result significantly, because it will start resonating.So, all your examples all your case studies what you did, when the structure is subjected to dynamic loading and the frequency matches with the natural frequency of the system, then your method of estimating thefatigue damage based on stress cycle count, will be wrong.

So, marine structures under wind and waves act as a period defendant filters,that is a very important propertybecause this property actually comes from the virtual design of a marine system, because we design a floating structure, we design a deep water duct platforms. So, they filter certain frequencies, they allow only certain frequencies. As a virtue of the design they magnify certain periods and suppress some of them,okay?So, this is essentially assumed by the structural form and design what you follow in marine structures.

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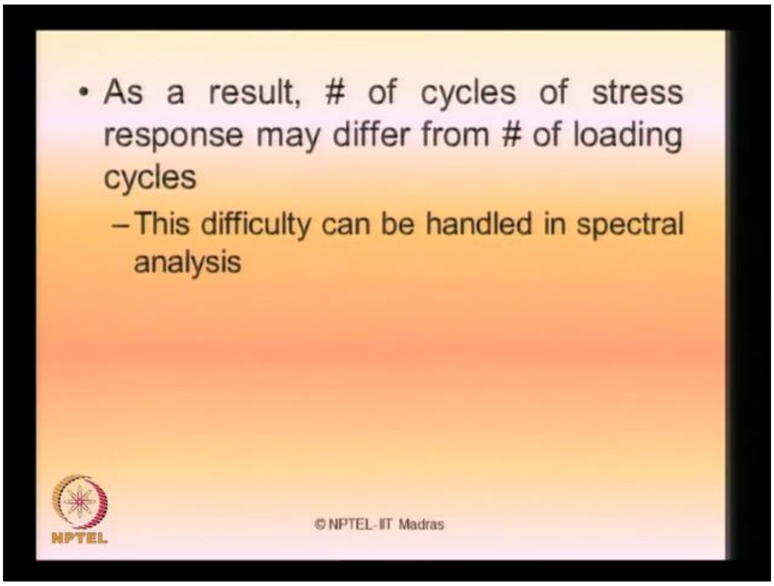

As a result, the number of cycles of stress response may differ from that of number of loading cycles. That is very important because you are suppressing the loading system on certain frequency domains or certain period domains.So, this difficulty can be handled in what we call, spectral analysis, because once you started playing with the selection of time domain where your structure is operating and at that range you want to locate the fatigue damage, then obviously you cannot do it with the time series of stress cycle ranges, you should look for the spectral analysis.

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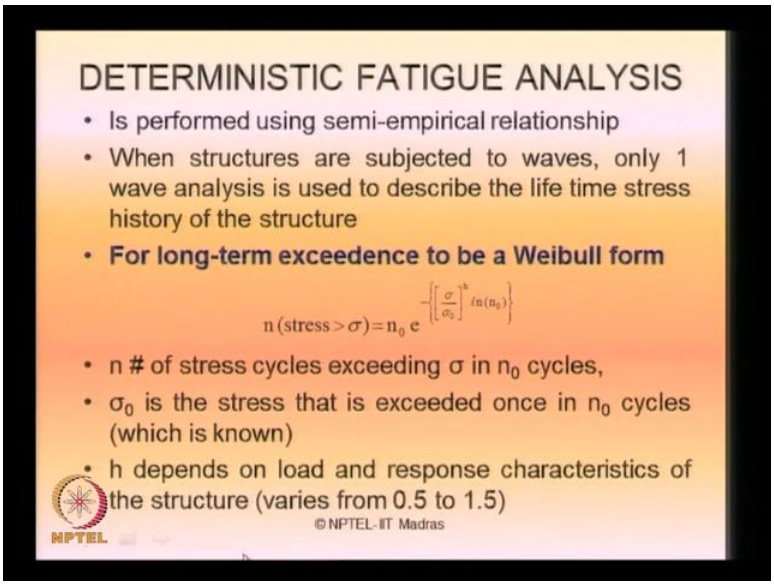

So, let us talk about deterministic fatigue analysis. It is performed using semi empirical relationship, when the structures are subjected to waves, only one wave analysis used to describe the life time stress history of the structure, that is assumed. For long term exceedence to be a Weibull form… If you understand that the long term exceedence is of a Weibull distribution, then the stress which exceeds the range sigma, the number of that will be given by this expression, as you see in the screen now. Where n is the number of stress cycles, exceeding sigma in n naught cycles.

Sigma naught, what you see here, in this equation is the stress that is exceeded once in n naught cycles which is known to you for a given problem. And H, what you see here depends on the load and responsecharacteristics of the structure. Essentially it differs or the value lies between 0.5 to 1.5,so this is an empirical relationship, which is used when the long termexceedence of your stress range is our Weibull distribution.

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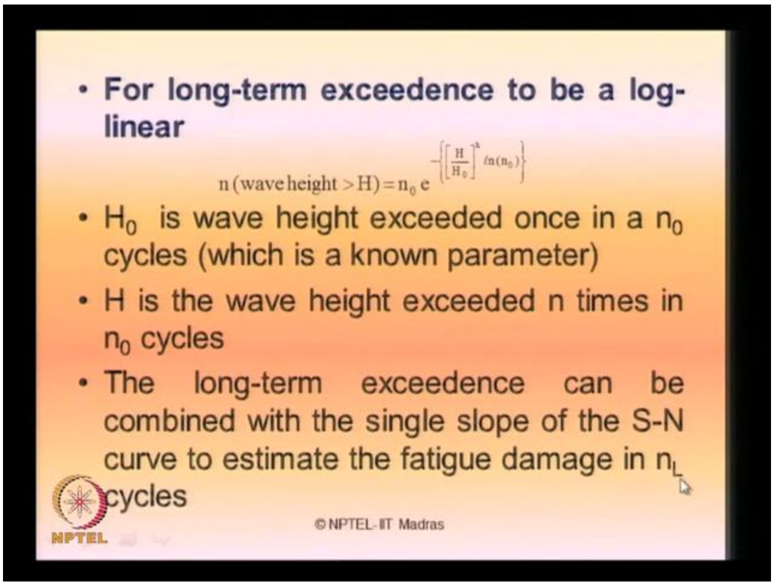

If it is of a different form, let us say it is of a log log linear distribution. These are the two common types of distribution, which is considered for long term exceedence, in marine structures. In that case the equation what you see here, is related to not this stress, but related to the wave height, directly. The H naught what you see here, is the wave height, which is exceeded once in naught cycles, which is again a known parameter. And of course,H what you see here, see here is the wave height, which exceeded n times in n naught cycles. That n is here, which you are finding out and n naught is already known.

The long term exceedence can be combined, with the single slope of SN curve to estimate the fatigue damage in n L cycles.If your SN curve is having a single slop, m is an unique number.

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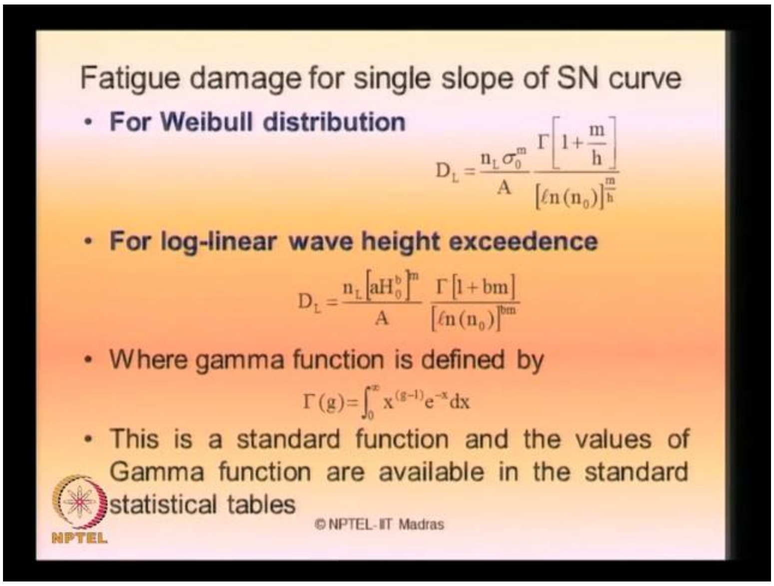

So, if you want to estimate the fatigue damage for single slop of SN curve, where m is fixed, where m is fixed then if it is a Weibull distribution,I can use this equation DL is given by n L sigma naught to the power m and this is a gamma function. The gamma function is evaluated using this equation.And this is a natural logarithm of n naught m by H, where H is defined already, if it is a log linear wave by it exceedence,then use this equation for fatigue damage.

If the slop of the SN curve is a single value, there also you got the gamma function, in this case use the stress rangewhereas, here use the H value, it is the wave height.The gamma function in both these expressions, is given by a simple equation like this. This is a standard function and the values of gamma function available in the standard statistical tables. For our understanding, let us try to plot or let us try to list some of the gamma function values, which are available in all standard statistics books.

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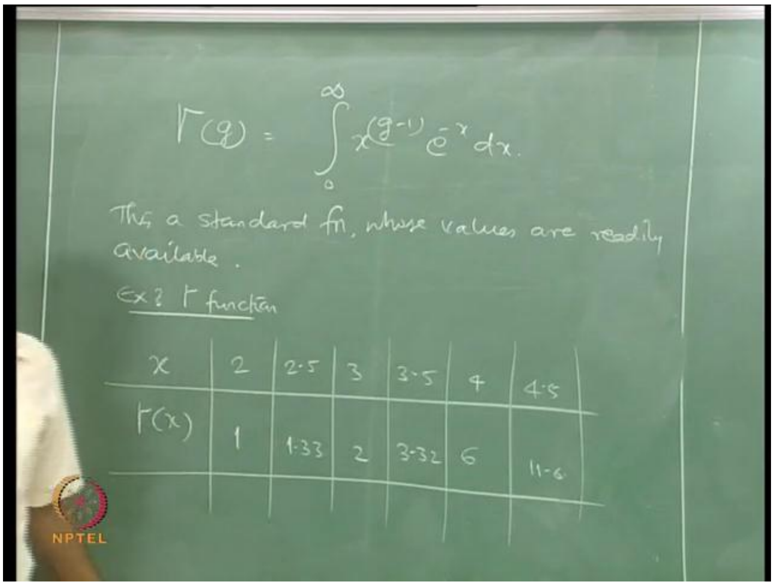

Let us see how this can be evaluated.Gamma function, which is actually given by an expression0 to infinityx of g minus 1 e minus x d x. For various values of x,I should say is a standard function,whose valuesare readily available in the table, butstill let us try to write down this value. So,I should say an example of the gamma function,so let us say for different values of x,I am looking for gamma x2,2.5,3,3.5,4, and 4.5,the gamma functions, the value of these functions for different expressions available here or1,1.33,2,3.32,theseare all the values of the gamma function respective values of the argument,6,11.6.

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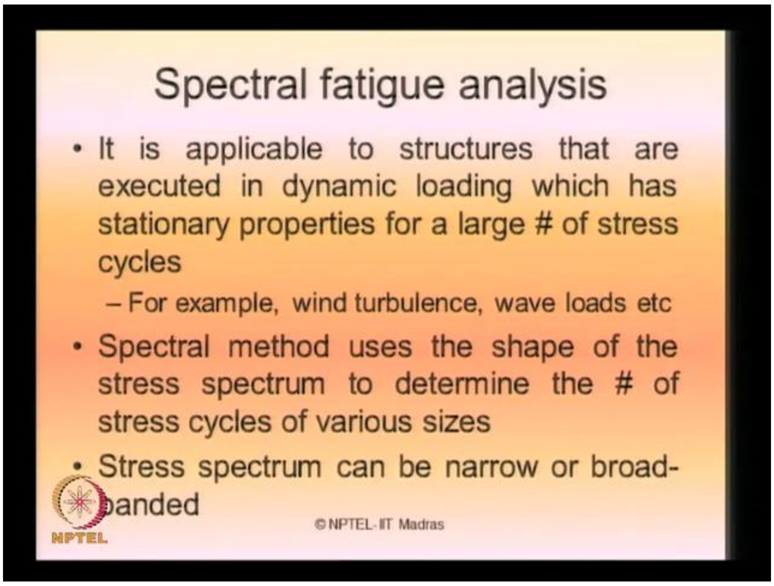

Now, let us look at the spectral fatigue analysis, it is applicable to structures that has executed the dynamic loading, which has stationary properties for larger number of stress cycles.For example, if you are looking for wind turbulence, wave load effects etcetera, which are executed in dynamic loading format, then I must use what we call spectral fatigue analysis. The spectral method uses the shape of thestress spectrum, to determine the number of stress cycles of various sizes because the last example in the rain flow counting method, the shape of the space spectrum was not counted.

We only look at the peaks and valleys of the stress cycle and counted the cycles, number of cycles in the stress range and found out the fatigue damage estimates.This is not applicable, when the structure is excited in a dynamic loading condition and unfortunately, the frequency content of the loading matches with that of the natural frequency of the system.So,I must use the spectral method, which should impact use the shape of the stress spectrum itself, to determine the number of stress cycles of various sizes, sizes means stress ranges.Now, the stress spectrum can be narrow banded or broad banded. Now, how a narrow band stress spectrum will look like, and how a broad band will look like?

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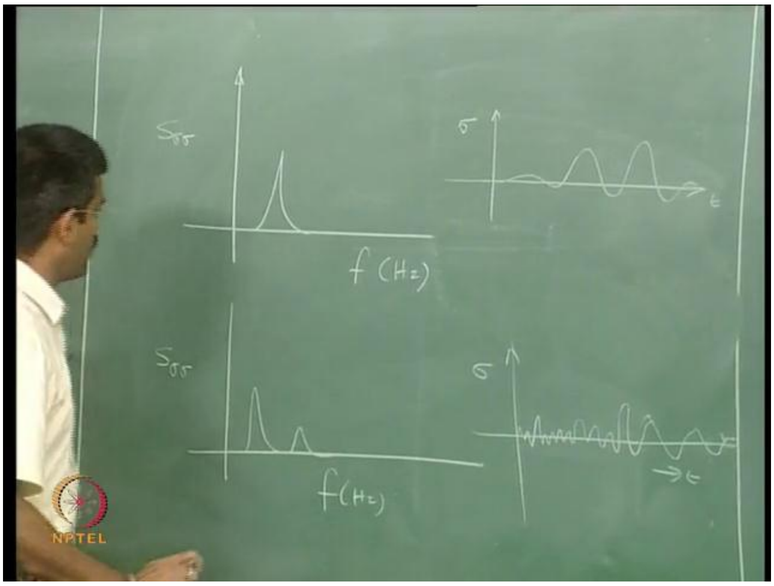

Let us say, this becomes the narrow band spectrum,where this is my frequencyin hertz and this becomes my Ssigma, and this is my narrow band spectrumand the corresponding time is t can look like thisand so on. For a broad band,f in hertz S sigma sigma, it have multiple peaks, in that case look at the time history, this will varyvery randomly. In fact,so obviously you cannot easily use a peak and valley countingfor this, right? Because a variation is very highly random,so this is what I call asnarrow band spectrum and time history, this is broad bandspectrum and time history.

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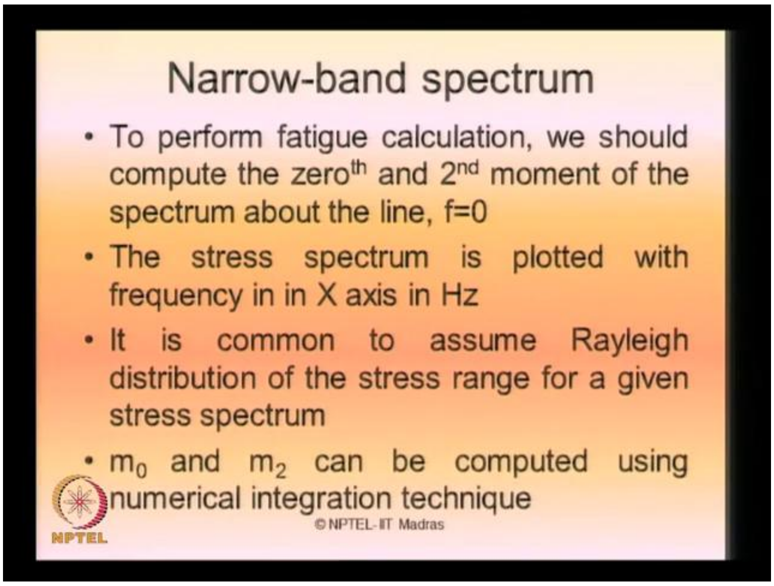

Now, let us say my stress cycle range,it is a narrow band spectrum as you see here. If it is a narrow band spectrum to perform fatigue calculation, we must now compute zero th and second momentum of the spectrum, about the line where f is equal to 0.I must compute the zero th and second moment of the spectrum, which I call as m naught and m 2,I will come to that. The space, thestress spectrumis now plotted, with the frequency in x axis in hertz. As you see here it is common to assume a Rayleigh distribution of the stress range for a given spectrum.m0 and m 2 can be computed using numerical integration technique.

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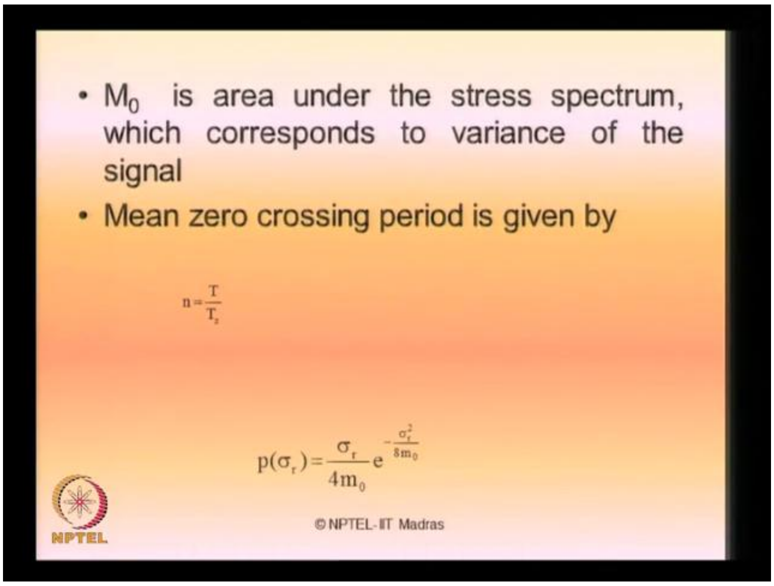

m naught is area under stress spectrum, which corresponds to variance of the signal.

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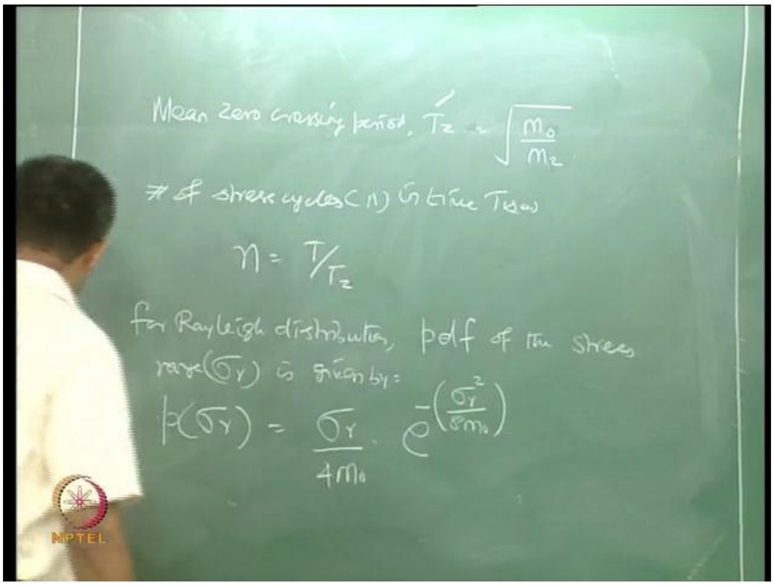

The mean zero crossing period is given by the mean 0 crossing period,which I call as T zwill simply given by square root ofm 2 by m naught, sorry inverse of this m naught by m 2.And the number of stress cycles,which I call as nin time T secondsis given by simply T by T z.T z is already known to me, then the probability density function for Rayleigh distribution, of the stress range,the probability density functionof the stress range,is given bysigma r,which is sigma r by4 m naught e to the power of minus sigma r square by 8 m naught.

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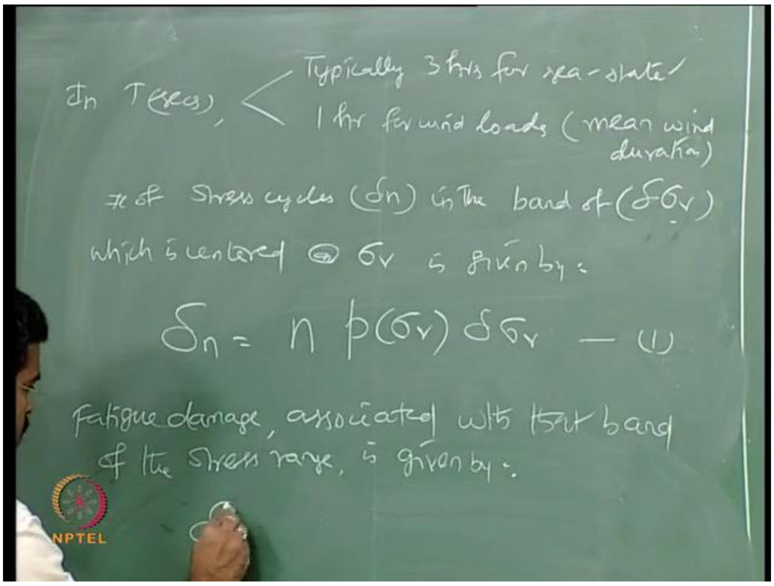

Therefore, in T seconds,this T seconds can be typically3 hours, for a sea state. In marine structurescan be 1 hour forwind loads. This is for sea state in terms of waves can be 3hours, it can be 1 hour for mean wind duration. Therefore, in T secondsthe number of cycles or the number of stress cycles,which I called as delta nin the band of delta sigma r,which is centeredatsigma r is given bydelta nis n. The probability density function, what you already had in the previous case, deltasigma now, this is equation number one.

Now, the fatigue damageassociatedwith that band of stress cyclebecause even in the rain flow counting also, we tried to find out the damage for each range.So, here instead of saying range, we are saying it is band. That is nothing butdelta, sigma r is the stress range,that is a small band of stress range,if it is narrow band we are discussing this. So, fatigue damage associated with that band of the stress rangeis given bydelta D,I am using delta for the small band, so which is nothing but delta n by n. Delta already you know, we can substitute here.

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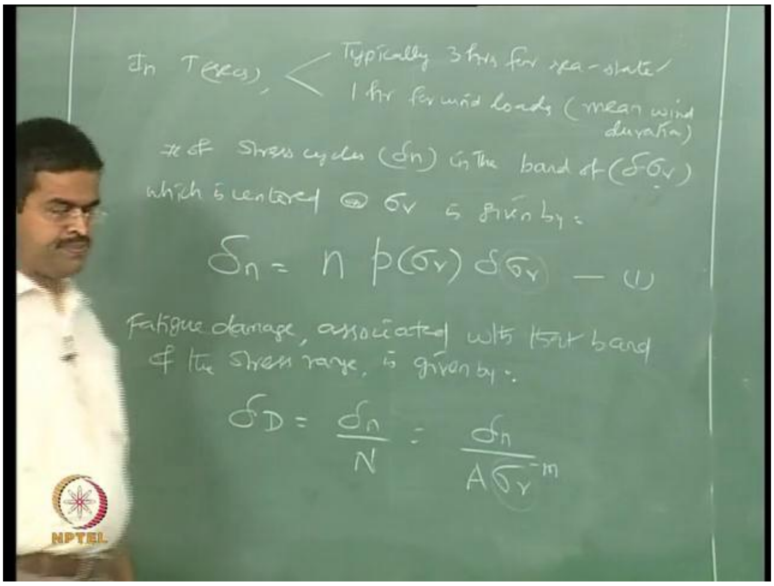

Now, we already know n also, delta n by…n is the standard equation for the SN curve which is nothing butA sigma r minus m.I am using this range,which is the range of this band,equation two.Now,I have already know the expression for delta n, which is a function of probability density function of sigma r. Let us substitute back all of them and see that, delta D can be now simply n of probability density function is sigma r by 4 m naught e to the power of minus sigma r square by 8 m naught of delta sigma r by A sigma r minus m, called it is equation number three.

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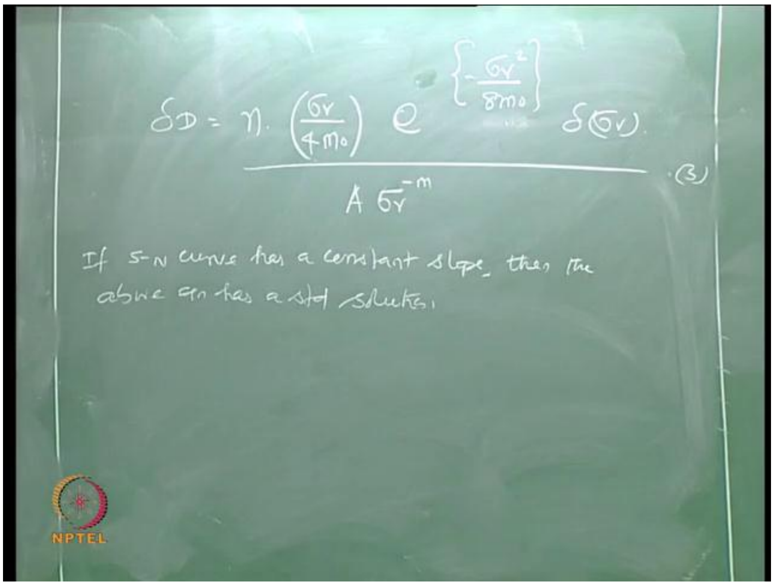

Now, ifthe SN curve has a constant slop,then the above equation has a standard solution,for a constant slop. Let us write down that equation here.

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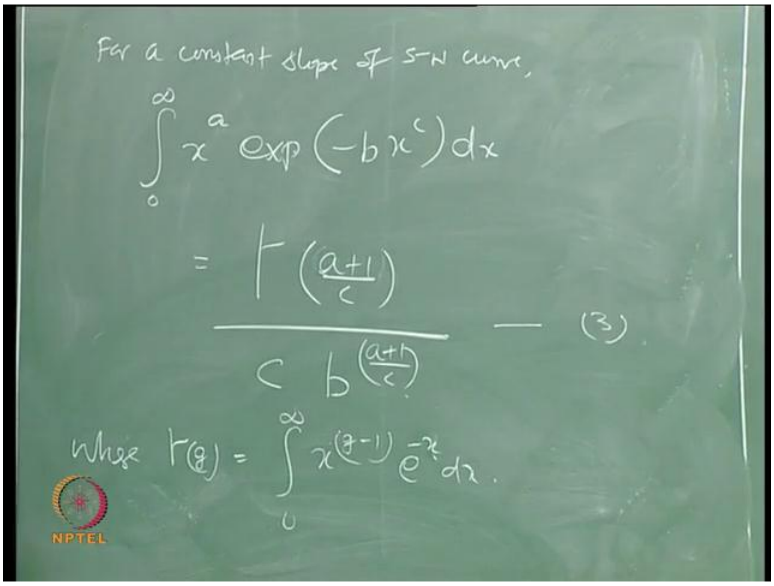

For a constant slopof SN curve, the solution turns out to be a standard gamma function, which is written here,0 to infinityexponentialminus b x to the power of c d x,which can be simplygamma function ofa plus 1 by cdivided bycb a plus 1 by c.I call this equation number three,where we already know where gamma function of any value g is given by 0 to infinity x ofg minus 1 e minus xd x.This is standard equation, where we are evaluating the gamma function.Now, let us compare this equation with our standard form and see what happens?

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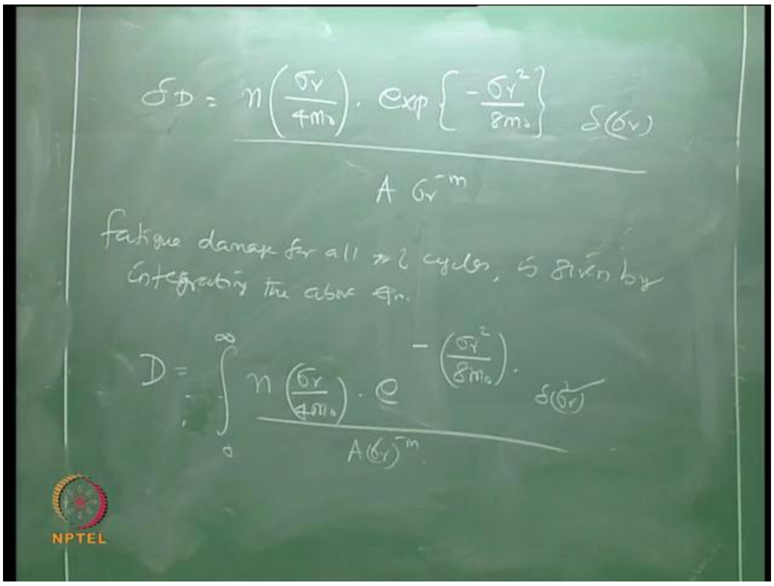

Let us say delta Dalready I know is nsigma r by 4 m naughtexponentialminussigma square by 8 m naught deltasigma rby A sigma r minus m.This is the damage for a small narrow band of sigma. I am looking for the cumulative damage, is it not?That is what we are also doing the last example. So, the fatigue damagefor all cyclesis given by integrating the above equation.Let us integrate this, so I must get D, which should be integral of 0 to infinity,the same equation simply I write here.n sigma r by 4 m naughte to the power of minussigma r square by 8 m naughtdelta sigma r1 bybyA sigma r minus m. Now, the r given to sigma r, let us try to simplify this equation.

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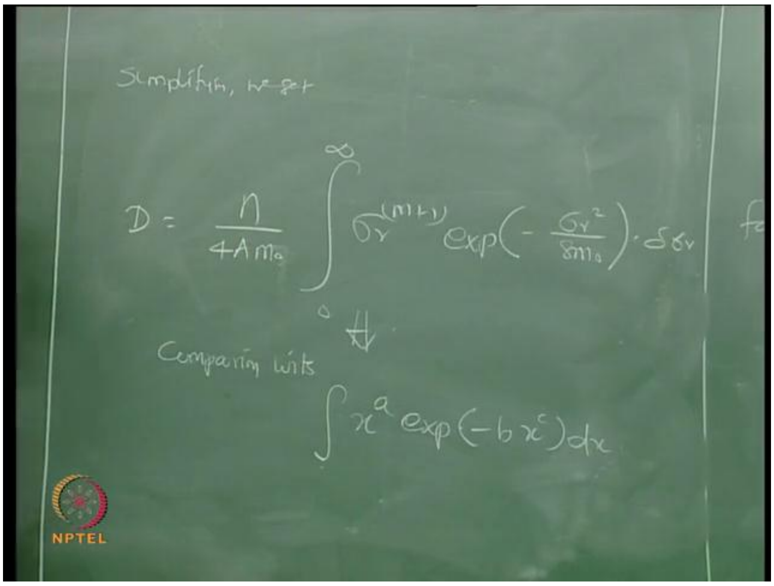

Simplifying,we get,I am rewriting this D can be said as n by4 m naught,n by 4 m naught I take it out integral of0 to infinity. So,I have sigma r here,I have sigma r of minus m here,I can say sigma rof m plus 1.Then exponentialminus sigma r square by 8 m naughtdelta sigma r. Now, let us compare this with the standard equationcomparing withsorrynotA m naught, can be written as 4 m naught,4A m naught, is thatokay? Sigma m plus 1 exponential sigma r square by 8 m naught.

Now,I compare this with thethe expressionx power a exponentialminus b x c d x.The argument is, if it is x power a it is d x, if it is sigma r it has delta sigma r,the argument is similar.I can compare these two, when I comparebecause this also integration from a limit, integration from a limit.

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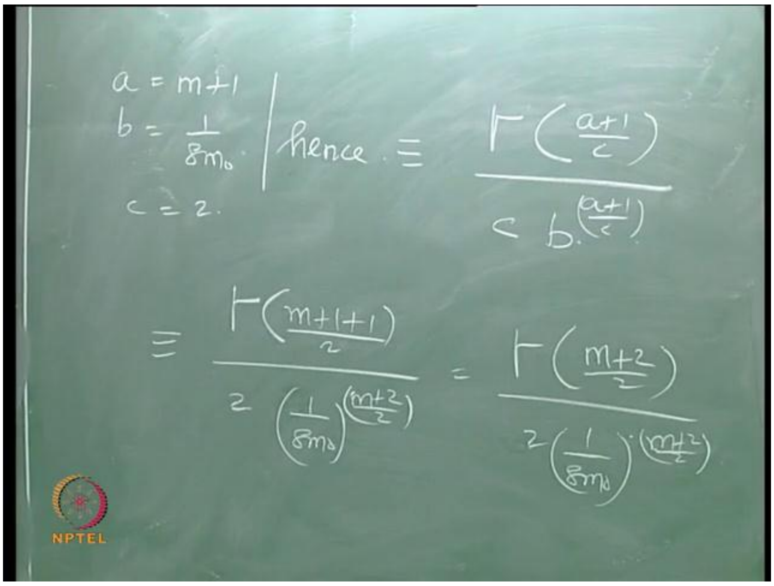

I can straight away say a is m plus 1, is it not? a is m plus 1 and b which is 1 by 8 m naught because there is a minus minus, sigma r c x, c can be different but b is 1 by 8 m naught, 1 by 8 m naught and c is 2.So,I can now evaluatethis expression,it is a standard gamma function, that gamma functions has given to you. That wasgamma of a plus 1 by cby c b raised to the power of a plus 1 by c, is thatokay?Is thatokay?

Now,I can write this value as,gamma function of a is 1 plus 1, m plus 1by c is 2 by c is 2 b is 1 by 8 m naughtraised to the power of m plus 2 by 2, is itokay? I get this as m plus 2 by 2, which can begamma of m plus 2 by 2 by…This can be evaluated like this, hencethe total fatigue damagefor n number of cycles.

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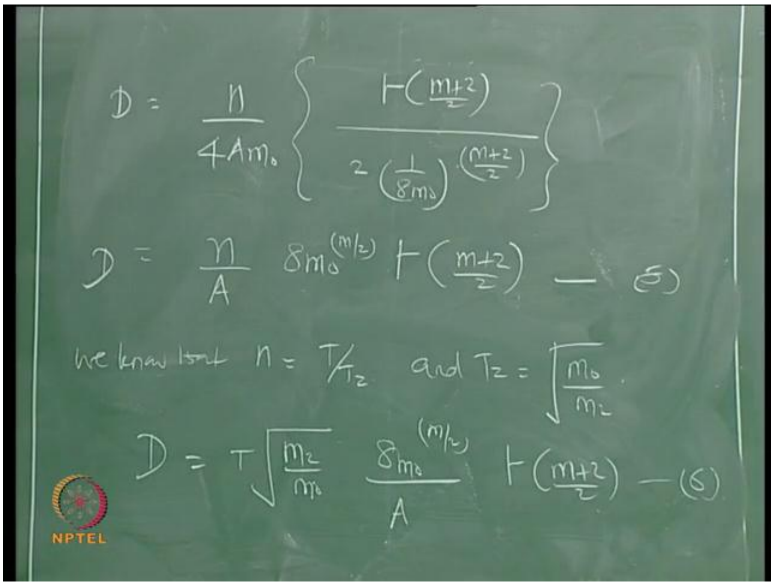

D can be now said as, n by 4 A m naught gamma of m plus 2 by 2 divided by 2 of 1 by 8 m naught of m plus 2 by 2, simplifying because I have got m naught here, m naught here, 2, 8, 4 etcetera.Simplifying them,I have got a power also, simplifying them can sayn by A because this 4 and this 4 goes away.I will get 8 m naught m by 2of gamma functionm plus 2 by 2, that is my total damage,is equation number five.Now,I already know,we already know that n, what is n? Turn back and tell me what is n?n is number of cycle, which we can said T by T z, is it not?And what is T z?Square root ofm naught by m 2, good.So,I have T z here, substitute here,I have n,I have n here substitute here,I get D asT m 2 by m naught because this T is it a denominator,8 m 0 raised to the power of m by 2 by A gamma function ofm plus 2 by 2, is that okay?

This becomes equation number six.For a given slope, standard slope,these all for constant slope. For a constant slope of m may be 3, you already know it is going to be 2.5, look at the table.For gamma 2.5, we already know the value of this function. So, if you know the spectral moments of m naught m 2 etcetera, m is the slope, m naught m 2 spectral moments and you already know the value of T, where you are evaluating the D. And A is again the constant of the SNcurve, you can find the cumulative damage for a narrow spectrum of the stress cycle, as you see from this expression number six.

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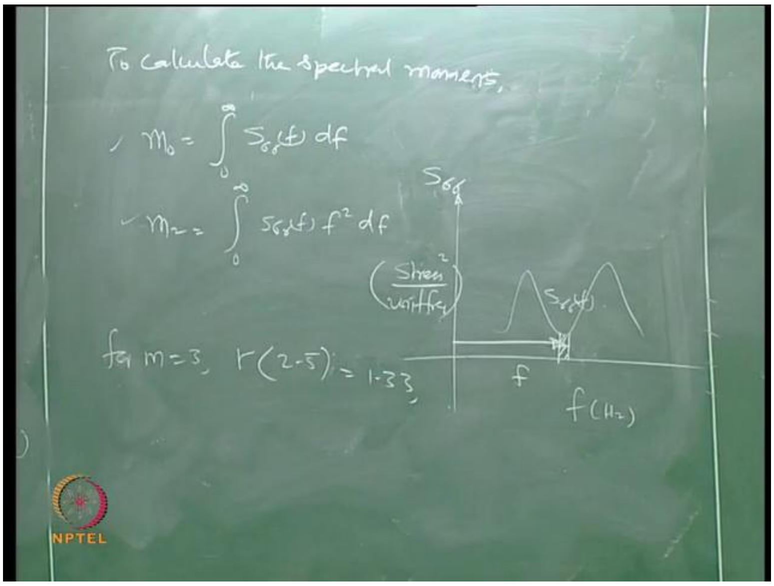

Now, to calculate m,a spectral moments, the general expression is, to calculate the spectral moments m naught is given by 0 to infinity S sigma sigma f d f and m 2 is given by 0 to infinity S sigma sigma f f square d f, where I try to plot a spectrum, the frequency is in hertz and this becomes my stress.

Let us say S sigma sigma, the ordinate is nothing butstress square by unit frequency.So, if I get a specific spectrum like this,you want to look at a specific value f is the distance by a plot here, this is f and this is x sigma sigma f.So, m 0 and m 2 are given by these two equations, and gamma function is already known to you, for m is equal to 3 for m is equal to 3gamma of 2.5 from the table is 1.33.

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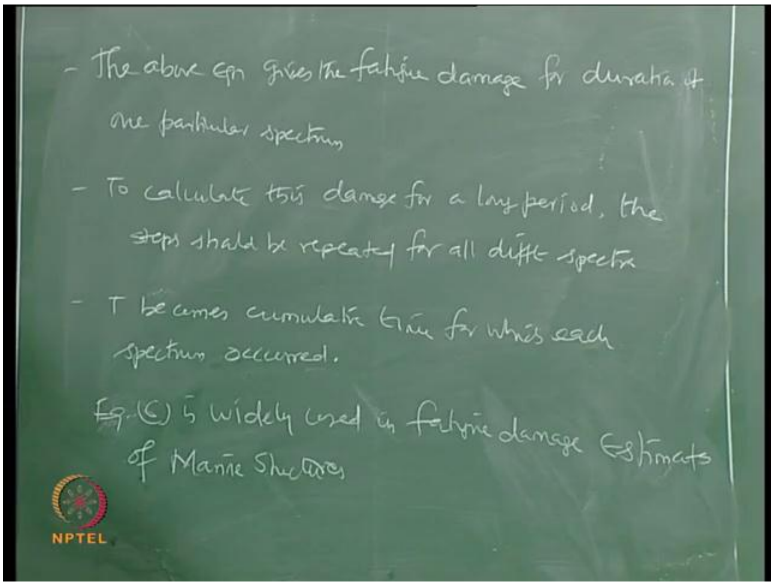

You know the gamma function here, so the above equation, the above equation what you see here, gives the fatigue damage for duration of one particular spectrum.If you really want to calculate this for a long period,for a longer period, thenthis should be repeated.The stepsshould be repeated for all different spectrabecause you have got a specific spectra S e title, for all different spectra you will repeat this, that can occur in the marine structure. In that caseT then becomes cumulative,timefor which each spectrumoccurred.

So,T is no more related to one specific value,it becomes a cumulative value. So, equation what was a spectral damage equation, lastly we had? The equation number, equation sixis widely used, infatigue damages estimatesof marine structures,if it is a narrow band.Now, let us quickly compare this with the constant amplitude loading because in this case stress cycle is vary,that is why we are plotting the stress cycle spectrum.

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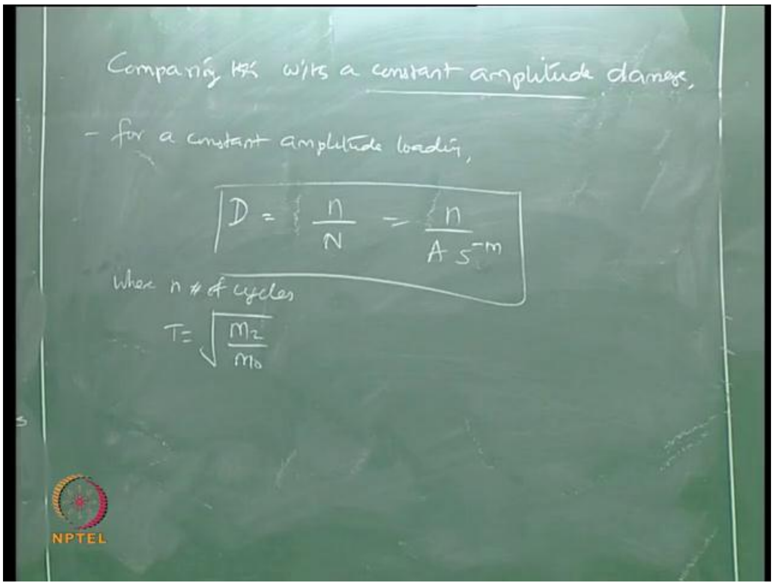

If it is a constant amplitude damage,comparing thisconstant amplitude damage we know that,for a constantamplitudeloadingDthat is the damage is given by n by N,is it not? We already know this equation which is nothing butn by AS minus m, where n is the number of cyclesand T is of course, square root of m 2 by m naught.I want to compare this method of estimating damagefor a constant amplitude with that of equation seven,that is what I am, six,that is what I am trying to do. So, instead of having a single, simple, amplitude of the stress,I will have a effective stress amplitude.

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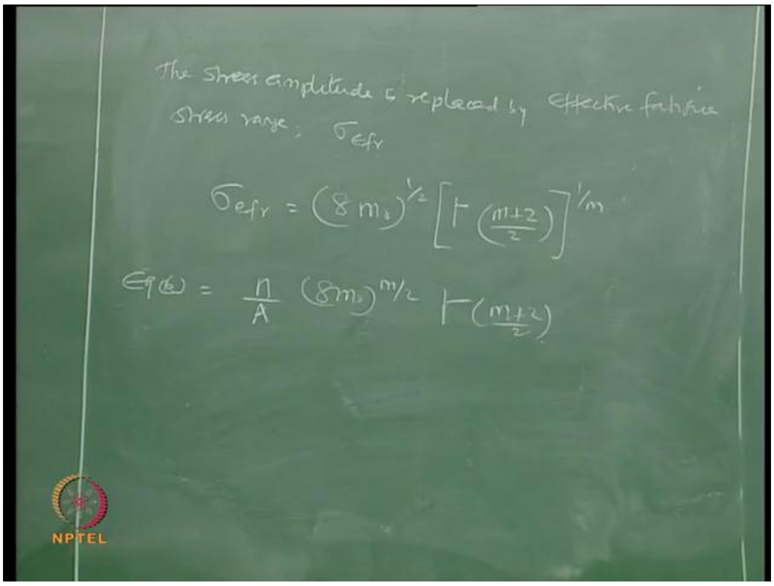

So, the stress amplitude is replaced byeffective fatigue stress range, which is sigma e f r. Effective fatigue stress range sigma e f r,is given by 8 m naught to the power half,that is the equation. What I am writing from the equation six is, it not gamma m plus 2 by 2raised to the power of 1 by m, is thatokay?Look at equation six,equation six I am writing for our understanding, equation sixwasn by A8 m naughtraised to the power of m by 2 gammam plus 2 by 2, is it not?I am comparing this,I am trying to find out, what is the equivalency of effective fatigue stress range compared to the standard damage?I get this equation, equation number seven.So,that is what we call aseffective fatigue stress range.

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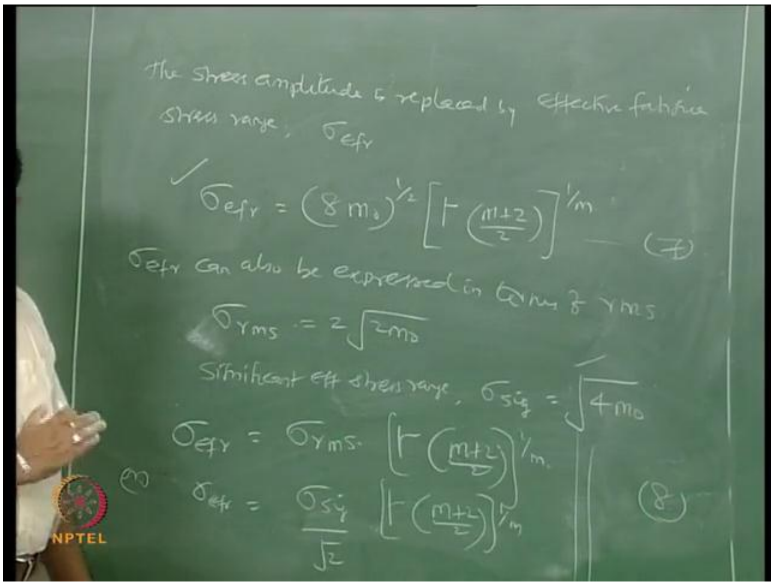

You can also rewrite this effective fatigue stress rangecan also beexpressed in terms ofr m s values.So, sigma of r m s, that is root mean square of the stress range is given by,2 of root of 2 m naught.So, significanteffective stress rangesigma significantcan be4 m naught.Therefore,I can write sigma e f ras sigmar m s 8 can be 4 into 2, that is 2 root m 2half is already there.So,I can say this is sigma r m s, simplym plus 2 by 2 of 1 by mor sigma effective r can also be said as sigma significant,which I know here, which is 4 m naught, which can now be gamma function ofm plus 2 by 2 to the power of 1 by mby root 2,is it okay?

Because 8is2 root,I already have 2 here, so I divided by root 2. So,I can writesigma effective fatigue stress range asthis in terms of r m s or this in terms of significant because from a given stress spectrum, you will be getting these values.Sometimes you will get these values in the literature, you can find effective stress range. Once you know the effective stress range,I can easily find the fatigue damage as comparable to a single constant amplitude damage like this.Now, let us quickly look at thesummary of this.So, for our understanding, let us plot a table for different values of m. m is nothing butthe slop of the SN curve,where I saysigma e f r by sigma r m s. Sigma e f r by sigma significant for different values of m.

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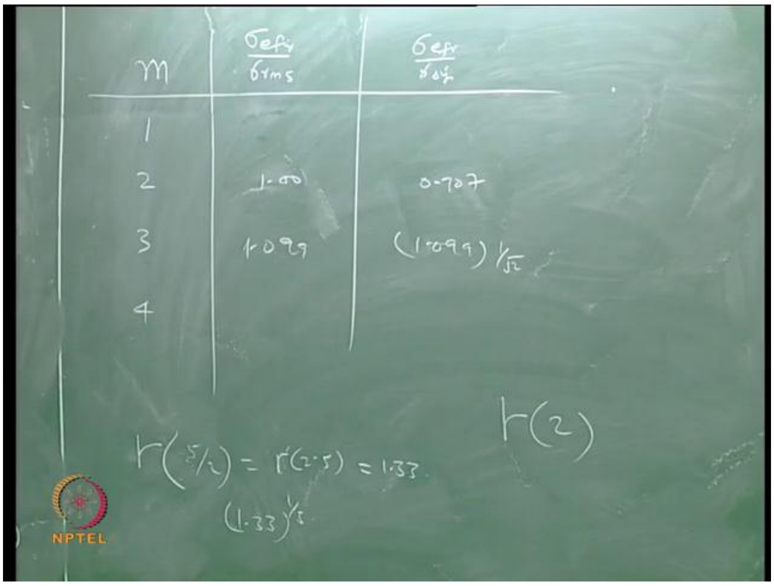

So, for different values of m,I must know how to evaluate the gamma function for values of m gamma function equation will be given to you and you know the table also.So, let us plot for… The slop of the quick cannot be 0,slop is having some value,1,2,3,4. Let us say for example, can you give me the value of sigma e f r for 2 because 1 you may not have the value. So, if you have substitute m is equal to 2 here,I get gamma function as at 2.5, which is 1.33.1.33 to the power of 1 by 2 that root of 1.33. So, sigma f r with sigma r m s will be that value, how much is that? Root of 1.33,so this value is going to be 1.99 by root 2and so on so forth.I can easily find out these values, let us quickly write the summary.

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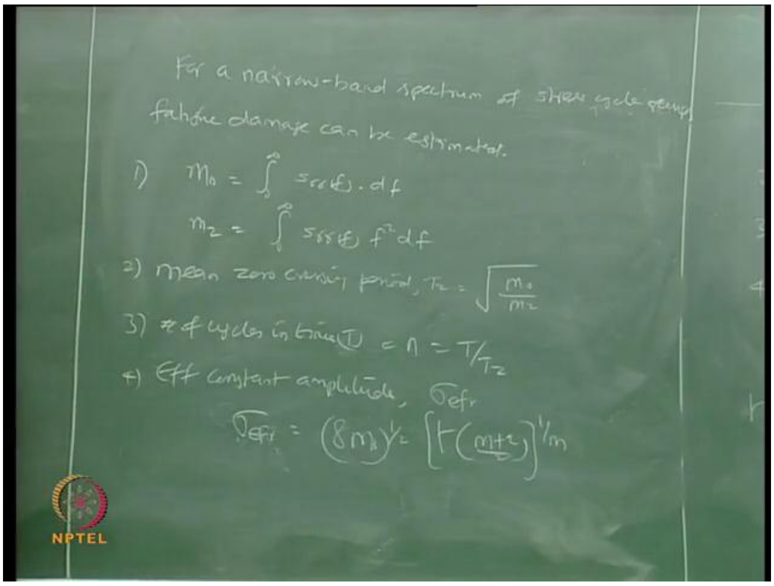

For a narrow band spectrumof stress cycle range, fora narrow band spectrumof stress cycle rangefatigue damagecan be estimatedas below.First find out m naught and m 2 because you need the spectral moments.m naught is nothing but0 to infinityS sigma sigma f d f. m 2S sigma sigma ff square d f. Then findthe mean 0 crossing period,which is T z, which is nothing butthe ratio of m 0 by m 2, is thatokay? Is it right?T z,m 0 by m 2.The number of cycles in time T,which is given by nT by T z.Once I know this,I findeffective constant amplitude,which I call sigma e f r. Sigma e f ris given by,8 m naughtto the power of 1 by 2 gamma function of m plus 2 by 2 to the power of1 by m.

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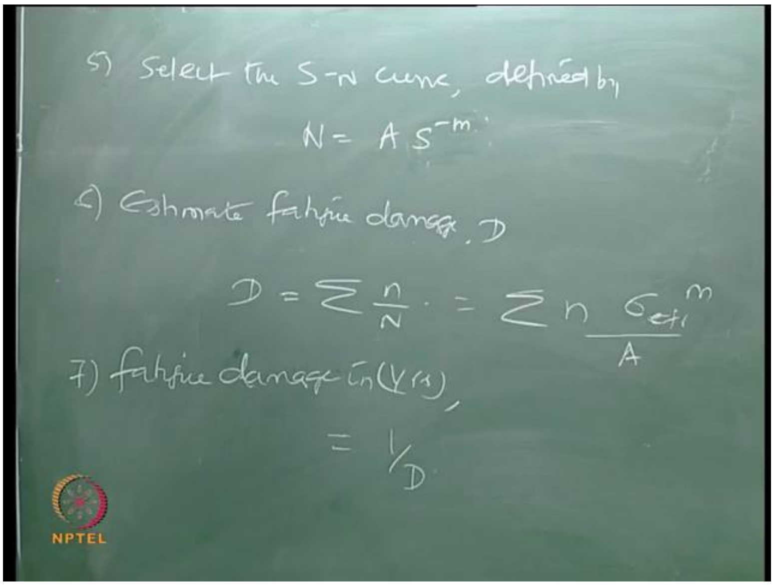

Now, select the S N curve, select the S N curve defined by N is equal to AS minus m.Now, estimate the fatigue damageD, which is given by sum of n by N,which is nothing butsum of n sigma e f r,that is my stress because instead of S,I am going to use effective stress range, which is going to be mdivided by Abecause actually n is AS power minus m, it goes off becomes m and S is sigma e f r.Sigma e f r is already known to me, so if I know m naughtfor a given spectrum, which is given by this equation and if you know the values, slop of the curve m, which is approximately 3,I can find sigma e f r.

Sigma e f r is known to me,I can find the damage. In the next class, we will talk about the broad band spectrum because we discussed about narrow band spectrum here for a stress cycle range. If it is narrow band,I can easily handle this7 steps,I will get the fatigue damage in years. If I have got a broad band,I will apply some correction and convert that into a narrow band,that is what people have been doing, to find out the fatigue estimation for broad band spectrums.Now, in this lecture we understood that why at all we go for a spectral fatigue damage, why not time series?

In the last lecture we discussed about the time series estimates of fatigue damage, simply by rain flow counting technique, where we understand only the peaks and valleys. In this case I cannot handlebecause if it a dynamic loading as in case of wave or wind, the response on the structure excite under this band of near resonance frequency, will not clearly give you a correct picture of the damages. So,I must look for a stress cycles spectrum. It can be narrow it can be broad, if it is narrow I know how to handle it, if it is broad how to handle it? Next lecture we will discuss this.

Thank you.