Advanced Marine Structures
Prof. Dr. Srinivasan Chandrasekaran
Department of Ocean Engineering
Indian Institute of Technology, Madras

Lecture - 6
Ultimate limit state and reliability approach II

So, in the last lecture we explained you how the ultimate limit state, in terms of design mechanism. How the partial safety factors can be connected to a reliability approach, where R and S are seen as variables, in reliability approach, random variables. Whereas, in design they are not directly seen is random variable, but their effect of their variations or uncertainties are accounted, in terms of using or substituting for the partial safety factors of load affects and the resistance; that is gamma S I and gamma R. So, one should be able to estimate these values using reliability theory. Therefore, one can be sure deterministically that what would be probability of failure, if you are using a specific characteristic value of the load and that of the strength, and using specific safety factors for load and resistance respectively.

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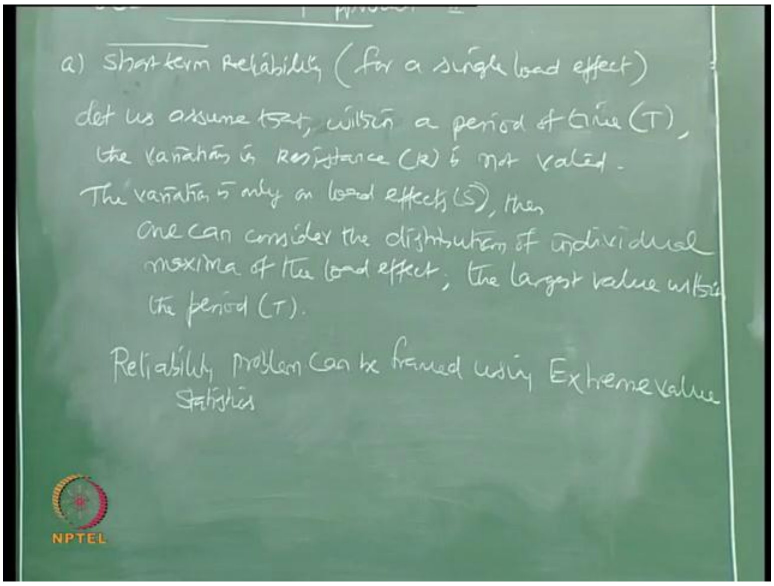

Now, let us say we want to look for short term reliability. To make it easy let us consider this for a single load effect. First of all why do the call this as a short term reliability? Let us understand or let us assume that within a period of time, let us say T, the variation in resistance is not valid. What does it mean is, R does not vary much, not significant, which can be true because using the same material for a period of let us say, structural life of 15 years and the material has got a high quality control, in terms of its manufacturing processes.

Therefore, the variation in terms of its yield strength, in terms of modulus of elasticity etcetera can be ignored because the variation practically will not be there, right. Though the variation will be there because of the strength of determination, degradation, etcetera, but let us assume that for a specific short term period, that T we will fix, may be 1 year, may be 6 month, may be 10 years for specific period, if R does not vary. The variation is only on the load effects S, then one can consider the distribution of individual maxima of the load effect, which could be the largest value within the period T. We can considered the largest value, what we call maxima. In that case the reliability problem can be framed using what we call, extreme value statistics.

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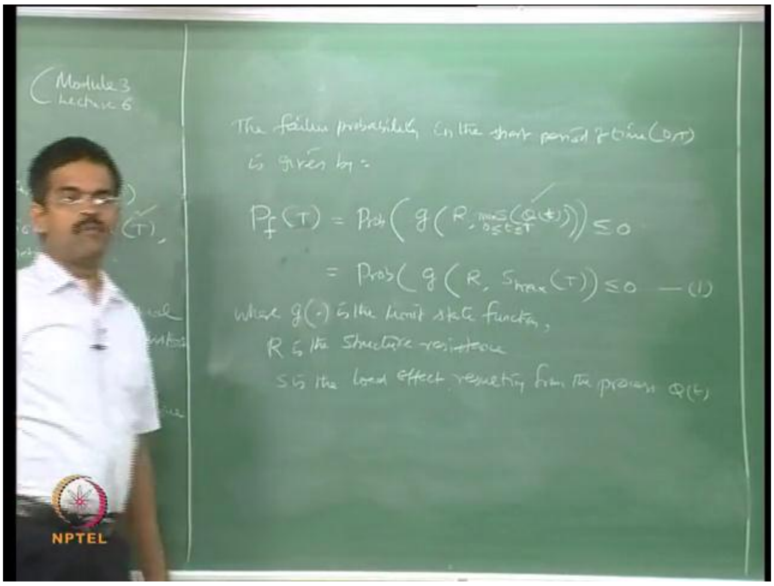

So, the failure probability in the short period of time that is 0 and T is given by P f, which is now meant for a specific time, let us say P f of T, where as T is a period where I am looking at. One can say however the time period of T is very large, still I can call this as short term because considering the design period of 100 years, considering the service life of 25 years. Any value you take can be considered as a short period or short term. So, probability of g of, that is the failure function, which can be a function of R and S max or let say S varying from 0 less than T, less than T, T is between 0 and T only. We can now look for maximum of S amongst Q of T, which can be less than and equal to 0, let us say

Q of T, but T is a variable here. This can be also rewritten as the failure function, that is probability of g of R coma S max, I am saying this an S max value, within a period T, which can be less than or equal to 0. I will call this equation number one.

So, what I am trying to say here is, if load is not considered sorry the resistance is not considered to be varied within a specific time T, consider only the maxima of the load effect and form a failure function, and find the failure. That is what we call has short term reliability, where g is the limit state function, is written has I think, you can write it legibly here g of some function, some parameters. In my case R and S are the parameters, R is the resistance and S is the load effect resulting from the process Q of T, that is the load process. I am looking for the maximum value in a given load process within a period 0 to capital T. I can also call that is S max, that is what the language of this failure function means. Alternatively there is another approach for this.

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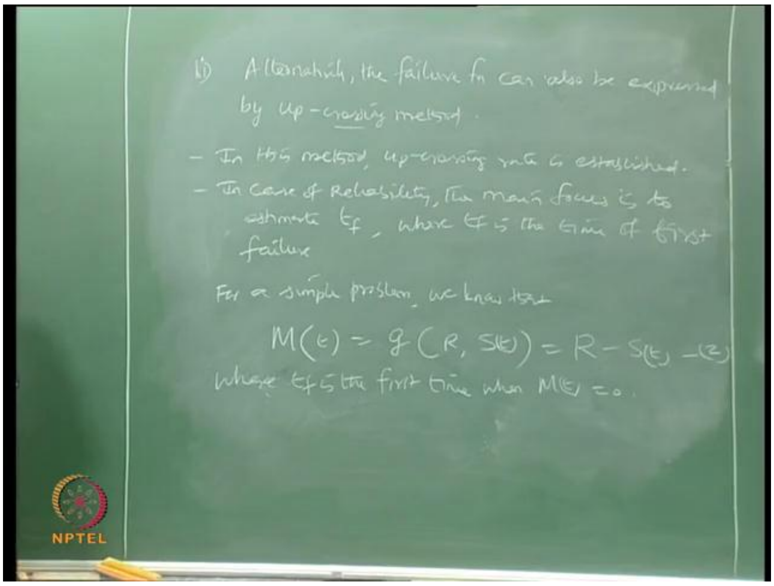

This is the second approach for this. Alternatively the failure function can also be expressed by up crossing method, let us see what is this up crossing method. In this method up crossing rate is established. So, in case reliability the main focus is to estimate t suffix f, where t suffix S f is the time of first failure. So, for a simple problem, we already know that is the safety margin M of t, which is function of R coma S of t. I think you will agree that why I am not writing R as function of t, we already said that R is not varying, which can be simply R minus that is margin of safety, is it right? The equation number two, R minus S of t, where t f is the first time when M of t is 0, it is a failure.

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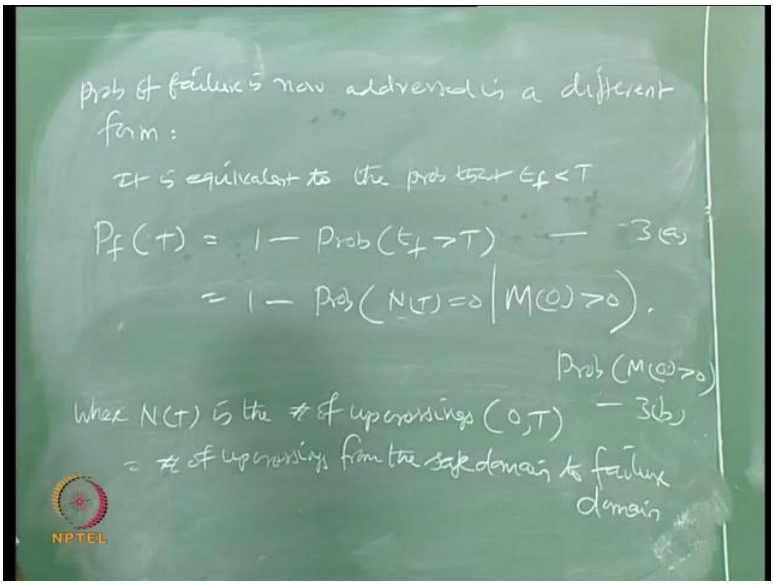

So, probability of failure is now addressed in a different form, as given below. So, instead of looking at R exceeding S or S exceeding R, I am now going to look at; what is the time when M of t becoming 0 and what is the time at which this occurs for the first time. I am just rewriting the problem into failure function and writing this probability of failure in terms of estimating t f, that is called up crossing rate approach. So, it is equivalent to the probability that t f is less than T. It means within your short term time t, there is going to be a failure, that is what probability of failure is. If within your capital time T if the failure does not occur, the probability of failure is 0, 100 percent reliable, is it not?

So, I am looking at the problem in a different prospective, saying that what is the Probability of t f less than T can occur? What is t f? t is the time at which the first failure has occurred. It is very clear, right? So, I am looking for a short term reliability, where I am trying to find out the up crossing rate of this function, which is R minus S or margin of safety. What is the first time at which it is going to occur and what is the probability that this will occur within t, that is my problem now. So, I can write the probability failure now as T because again it is a function of time, which is 1 minus probability of t f greater than T, is that correct? Because t f less than T is a failure, which can be said as 1 minus probability of N of T, set to 0 given M. M is the margin of safety, M of 0 greater than 0, multiplied by probability of M of 0 greater than 0, I called this equation number the three. I call this as three a and three b. There is multiplication here, I do not have a space, so I have written like this.

Now, the question is what we understand by N of T, where N of T is the number of up crossings, within the interval 0 to T. Since, I am saying N of, 0 to t, that is the number of up crossings from the safe domain to failure domain. I mean how many times your margin of safety is not satisfied? What is the margin of safety? R minus S of t, if R minus S of t is set to 0, it is on the border, right. So, you must first know the condition, what is the margin of safety. Given that condition, how many times this has occurred, that could be a probability of failure.

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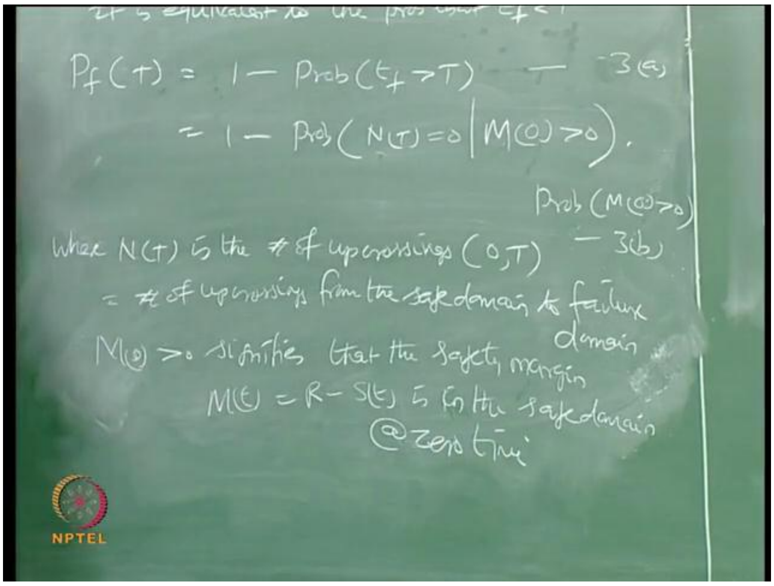

Now, in this case M 0 greater than 0 signifies, M 0 greater than 0 signifies that the safety margin… What do you mean by m greater than 0? R is greater than S, is it safe or unsafe? Safe, so M greater than 0 indicates safety margin, where M of t is R minus S of t is in the safe domain at 0 time.

That is why I have used 0 here, that is the reason why you have used 0 here. At zero'th time this will be applicable, it is safe. Now, from 0 to capital T, what is the probability or how many number of times this has been violated? That is my probability of failure.

Now, in this example or in this derivation calculating P f of t is complex task, so, complex task.

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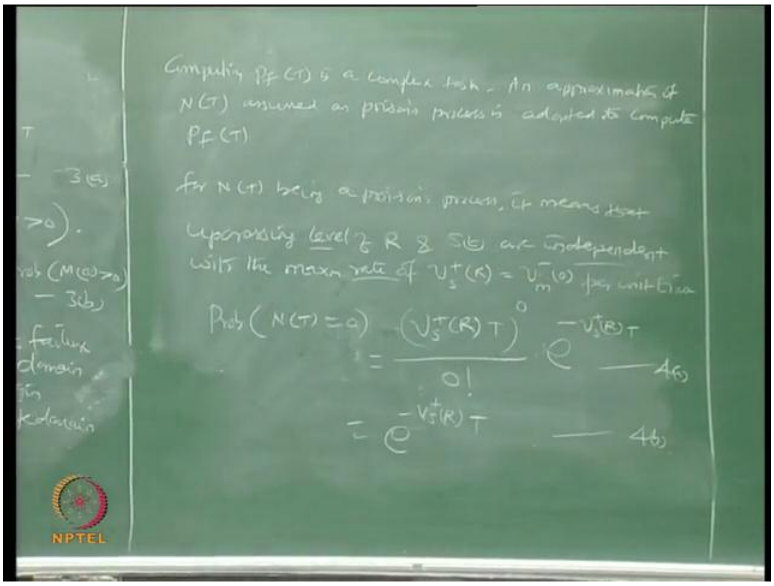

Therefore, an approximation is made, an approximation of N of T assumed as Poison process, is adopted to compute P f of T. What does it mean? When is say N of T poison process it means that, for N of T being a Poison's process it means that up crossing level of R, up crossing level of R and S of t are independent. They happen independently, with the maximum rate of V s plus R is equal to v m minus 0.

So, where as V s plus R is the maximum rate of up crossing, it is indicated as V s plus R and this happens per unit time, that is why it is called as rate of up crossing. Now, probability of N of T exceeding 0, up crossing or let us say equal to 0 because up crossing means it is 0. You have a signal for example, you want to find how many times signal has crossed 0, you look for only the forward slopes, look for only the forward slopes and so on, how many times it has crossed in a given T. So, for identifying the forward slopes you must look at this value of equal to 0.

So, probability of N of T set to be 0 can be now given by, now using the maximum rate of up crossing, I am using V s R plus now. V s R this is R, of T to the power 0 dived by 0 factorial, that is a general expression. e to the power of minus V s R of T, which can be simply e R of T, you can call this equation number four a and four b.

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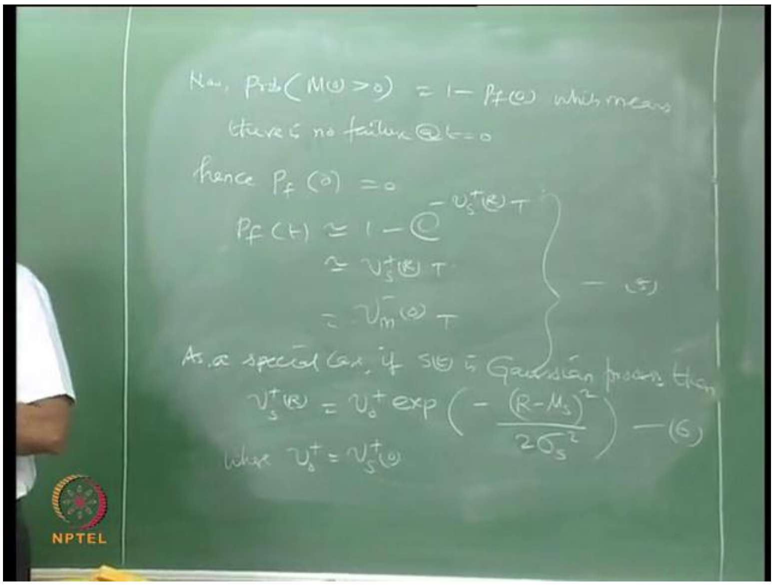

Now, the probability of M 0 greater than 0 that is safety margin at zero eth time is nothing but 1 minus probability of failure at 0, is it not? Which means there is no failure at t equal to 0, agreed? Hence, probability of failure at 0 can be taken as 0, because there is no failure at 0. Then probability of failure at t can be approximately given by 1 minus, I am using this e to the power of minus V s plus R of T, which can be approximately V s plus R of T, it is approximate.

Which can also be said as, we already said this here, V m minus of 0 of T, call this equation number five. Now, as a special case, if S of t is the Gaussian process then V of s plus R can be said as V of 0 exponential R minus mu s the whole square by 2 sigma s square, it is an exponential value. I call this equation number six, where V 0 plus this value is as same as V s plus of 0. So, this is the second approach by which I can find out the probability of failure using up crossing rate, which is V s plus R 0 for a short term reliability, where R is not varying significantly, Whereas load effect s is very significantly within a period of 0 to capital T. I am looking for T f, which is the first time at which is this failure has occur, n number of times this failure has occurred or margin of safety is violated, thinking that at 0 it is absolutely safe, at 0 it is absolutely safe. Clear?

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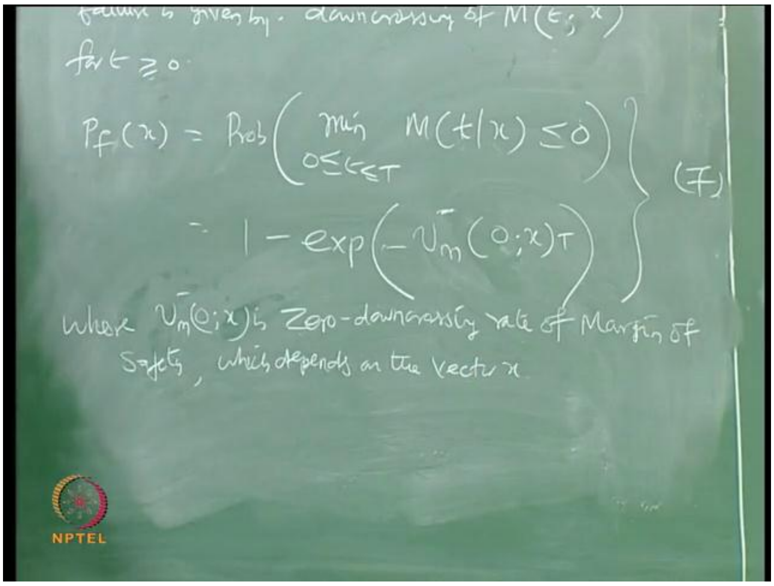

Now, the given values of R, let us say instead of R let us say generally the given values of random variables x, that is x 1, x 2, etcetera, which represents the uncertainties. The condition probability of failure is given by the down crossing of M of t. So far we have been seeing the up crossing of M of t and x for t greater than 0. So, this can be given by the probability of failure of x, which is a random variable, varies from x 1, x 2, x 3. Please make a correction, here the random variable is a transpose of this, it is a vector. Can be given by the probability of, minimum value of this, earlier we looked at the maximum value, if it is s effect. Now, I am looking for the margin of safety, so you should look for the minimum effect, marginally it is unsafe or safe, margin of safety.

So, minimum value of 0 less than t, less than capital T; same argument, M of t given x for given set of x I am looking t, less than equal to 0. What does it mean less than equal to 0? R is less than S, so it is unsafe. That is the failure, which can be said as 1 minus exponential of minus V m minus of 0, I am looking the down crossing therefore, I am putting the minus here, down crossing is negative which is happening. Arguments are 0 and x for given t, I can called this as equation number…What is the number? Seven, 0 of t, let me check this expression, where V m minus of 0 x is a zero down crossing rate of margin of safety, which depends on the vector x and the vector x is a function of R and S, margin of safety is nothing but R minus S.

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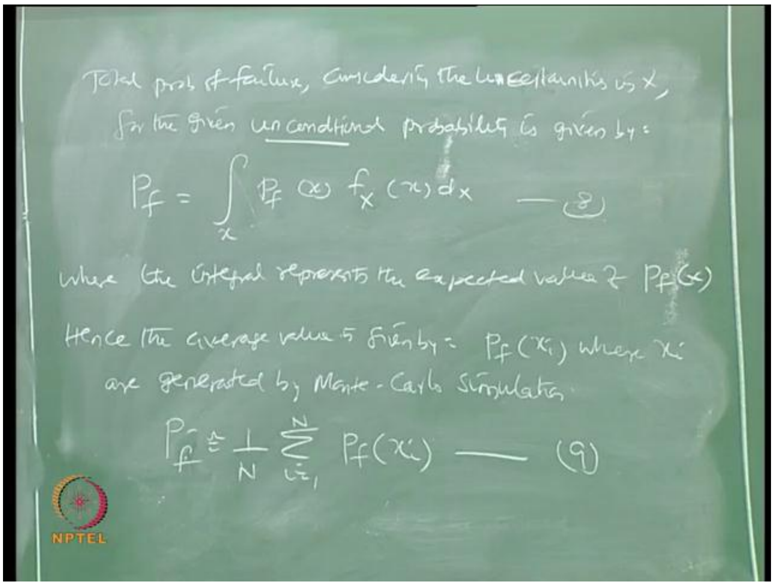

Now, the total probability of failure, considering the uncertainties in x for the given conditional probability, can be said as probability of failure. Now, let us make it as unconditional for the given, unconditional, this is unconditional. Is given by probability of failure, which can be integral over the domain x, that the random variable set P of f x, f of X x dx, this is capital X, equation number eight. Where the integral represents the expected value of the variable x or the probability of failure because it is not the variable, is the probably failure of x.

Hence, the average value is given by p of x I, where x i are generated by Monte-Carlo simulation. Therefore, we can say P of f in this case is going to be an average of, N values of, equation number nine. Of course, this is an approximate value, cannot be accurate. So, this is the short term effect of reliability for a specific time period t, where R is not varying significantly compare to that of load effect S.

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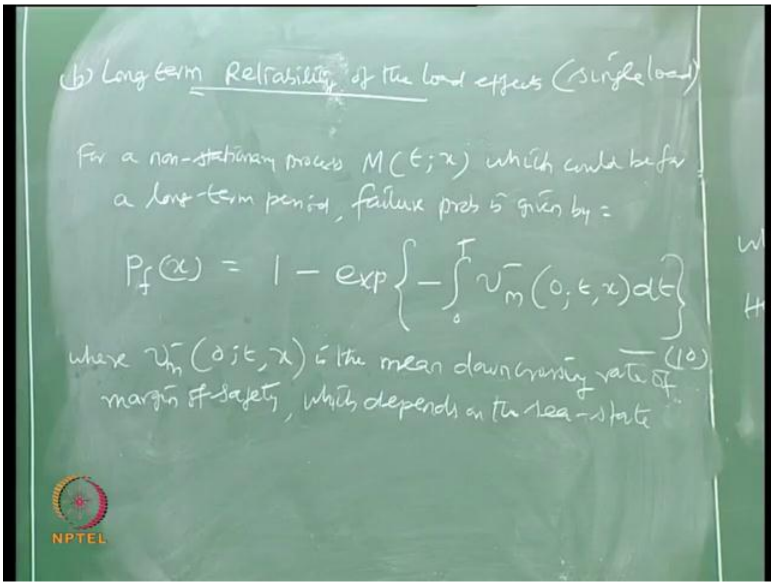

Now, let us look at the long time variability. Not variability, actually it is long term reliability of the load effects, long term reliability. We are talking about the single load here, earlier also we discussed about the single load only. Now, let us say for a non stationary process, which is let say M t of x, some process, which could be for a long time period. Failure probability is given by that is my domain V m minus, M is function here what I am arguing. Therefore, it is the down crossing rate and the variables are 0, t and x for a time period d t, it is the variable.

So, I can call this as equation number ten, where V m minus of 0 t and x is the mean down crossing rate of margin of safety, which of course depends on on the sea state, which… Sea state that changes with time. Why I am saying only sea state? We are knowing, already we are saying that the variation in R is not significant with that of S.

Variation is included here because I am not binding this for a specific value of t, I am taking any time t. Look at the previous expressions, it was maximum or minimum of the failure function between 0, small t and capital T. So, we are looking only in interval 0 to t, but here it is open domain, any time t. But even in the time t we are saying the process non stationary, it is a margin of safety process ,which is acting, which is having actors as R and S. But the variation in R is not significant compare to that of S because S is very significantly because of the sea state variation with time.

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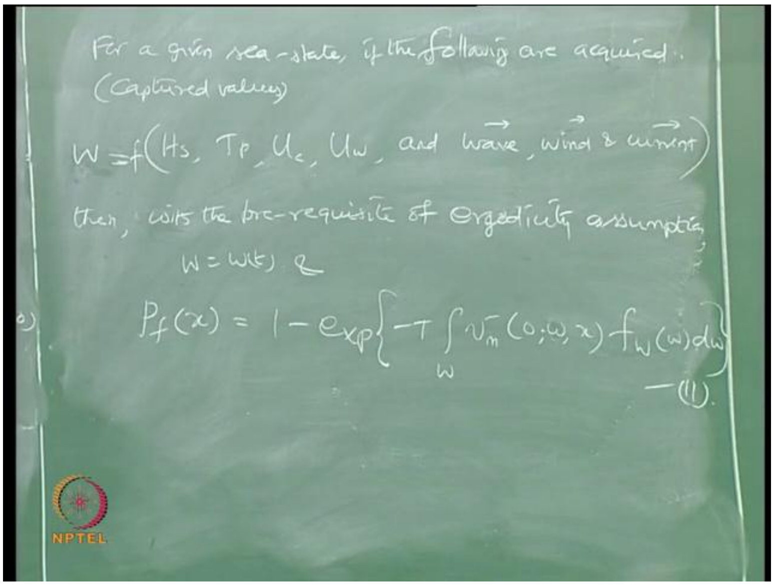

Suppose if we know for a given sea state… Suppose for a given sea state I can acquire the following details. For a given sea state if the following are acquired, what are they which you called them as captured value. What are they? H s; significant wave height, T p; spectral wave period, U c; current velocity, U w; mean winds speed and also the predominant direction of wave, wind and current. If I am able to collect them or captured them and put these values in a subset called W, capital W, then with the pre requisite of, with the pre requisite of ergodicity assumption.

Ergodicity, what is an ergodic process? I want you to look back in the standard statistics and understand, otherwise they may be available in some other parallel course given in NPTEL, in some other literature. In fact I can quote one of the courses what I did on the dynamics. We discussed about the random process and dynamics, where we discussed about ergodicity, stationarity, everything, please look at that you will understand what we talk about here.

So, we have an captured set of values and we also say there is a pre requisite, that these value are ergodic, it is an assumption. Then W can be written as W of t and probability of failure of the variable x can be now said as, 1 minus exponential of minus T for a domain of W because my W is a captured value now. Instead of saying 0 to t, i have captured this values for specific period and there ergodic in nature, right? Therefore, I am rewriting this expression ten in a different form; V m minus of 0, t, x. Instead t I should say w because that is my argument now here, f of w w, d w. Is that clear? I can call this equation number eleven. Now, my domain is now transferred to a captured value W, where I know all these variable parameters, which are responsible form a high level of uncertainty in S.

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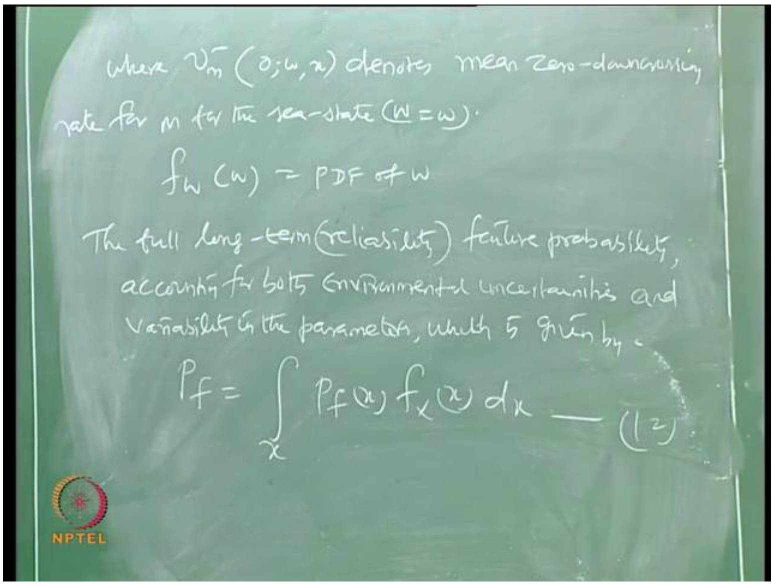

Where again V m minus of 0, w, x denotes mean zero-down crossing rate. Why down crossing? I am using minus here, up crossing means I would use plus. For down crossing rate for m, for the sea state w, it is not for t. I should say vary specifically sea state W equals w because w are the set of captured values. Now, f of w, the one which is shown here is the probability density function of this W. Therefore, the full long term reliability or I should say long term failure probability; one and the same. Long term failure probability accounting for both environmental uncertainties… How can you say these are accounted for environmental uncertainties because W is a captured value, which we have for specific variations, including the directions and there probability density function also they have.

So, it is accounting for environmental uncertainties and variability in the parameters, which will be given by probability of integral over domain x, probability of x, which is f of X of x d x, equation number twelve. So, in this lecture we have understood what is called short term reliability and what is called long term reliability. In short term reliability there are two approaches we discussed. The famous and common approaches up crossing rate of the margin of safety, where we have arrived the close from solution for a single load effect of S. For a long term effect we have used a different methodology of approach, saying that the process is non stationary. Instead of finding out t f, is the first time at least the margin of safety is violated, I am looking for a domain now, where t is now transferred to capture domain of w; small w ,where I know all these parameters captured for a given sea state.

Now, account for this variability and the parametric variation for a given sew state, I am not saying for a given t, I have transformed the t domain to a captured value domain w. That is how we have said by variation is on the domain w. So, expressions I think eight, equation eight and twelve will give you the probability of failure for a short time reliability and a long term reliability respectively, in this discussion. Now, the question comes; having understood the different levels of uncertainties, having understood a short term and long term effect, having understood the connectivity between reliability and ultimate limit states, which we have seen in the last lecture.

What are the different levels of reliabilities? What factors effecting each level of reliability? We must discuss this. Once we understand this, we will take it forward and understand what you mean by methods of reliability; like FOSA method, AOFS method etcetera. There we will end the discuss on this module on reliability of marine structures. Any questions here? You go through this and try to understand, the references for this specific discussion is in Arvind Naves and Toargier More; advanced marine structure where we have given the reference in the NPTEL, please look at this and study them. Otherwise you can refer to any standard book on structural reliability, they are discussed. It is a general problem formulation, only certain aspects like this will be particular to marine structures, so this can be followed easily.

Thank you very much.