Advanced Marine structures

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Lecture - 5
Ultimate Limit state and Reliability approach I

So, in the last lecture we discussed about something on acceptable level of risk. We said in marine structures or in offshore structure engineering, essentially that it cannot be 0, there is some accepted level, which is formulated by international regulator agencies like EPA, nuclear reaction, power commissions, etcetera.There are international authorities, which frame what would be the minimum acceptable level of risk based on which you can operate your platforms or marine structural systems.

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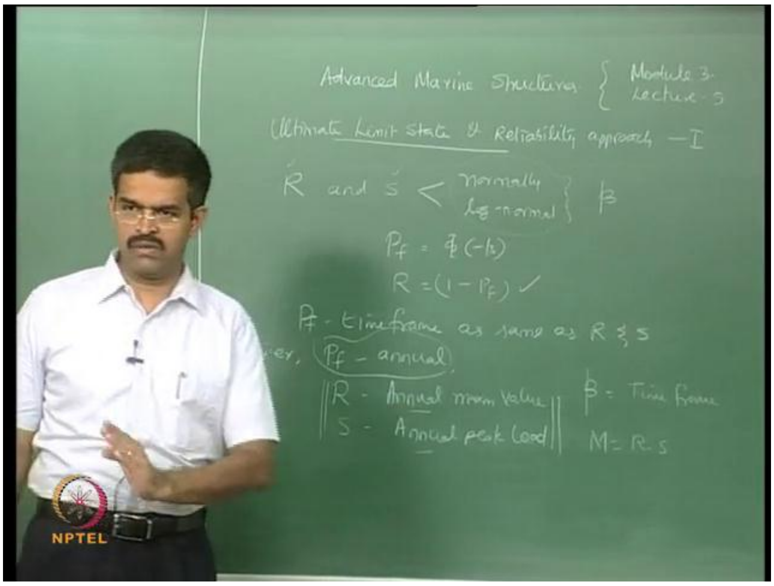

We have also said that, if theresistanceand the load effect are normally distributed, the variables or if they are log normal,then there are simple close form expression available to compute, what we call thereliability index.And we already said the probability of failure can be simplythe function of reliability index and I said cable which we gave you in the last lecture based on which you can easily use this table to compute the probability of failure. And the reliability can be simply said as 1 minus probability of failure and thereendsa solution for this problem, if the resistanceand the load effects are distributed as per our notion or the joint probability density functions can be evaluated in a closed form as you see here.

So, the emphasizing point here is the probability of failurewill be on the same time frameas that of your R and S. R and S Suppose for example, if probability of failure is annual, then R should be the annualmaximum valueand S should be correspondingly load effect ofannualpeak loadand so on.So, depending upon the period which is associated to the probability of failure, which you are looking at accordingly are the variables, which will govern the probability of failure will be capturing the same time. On the other hand betathe reliability index has thesame time frame as that of the variables R and S,that is what we conclude in the last lecture and said that reliability index cannot be apply for infinite time.

There is a time frame within which you can apply the reliability index becausethis time frame is essentially depending upon the time frame of your variablesR and S. And we also said margin of safety M can be simply R minus S. If this is exactly 0,let us say it is just on the virtue of failure. If R is greater than S, it is safe, if R is less than S it is unsafe or failure mode, R is resistance and S is the load effects,that is why s is standingthere as load effects.

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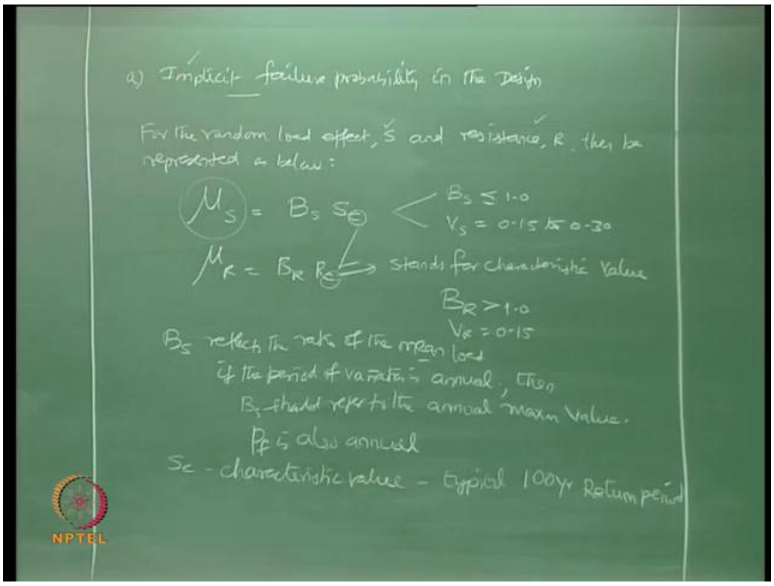

Now, what now we will seem,what would be theimplicit failure probability, which is applicable in the design? So, we like to see there are two variableshere, one is the resistance or the strength of the material, other is of course, the load effects coming on the material or on the member or on the structure out of these two the probability of failure or reliability is govern by which of these variable more seriously? We would like to look at them.So, what would be the effect of this implicitly, not explicitly,in built failure phenomena, which can significantly influence while design in probabilistic terms.

That is what I am looking at in this topic. Let us see that how we will, by through an example we can easily illustrate, which of these two or more important?Does not mean that the other is less important both of them are variables both of them have uncertainties, which are irreversible reversible, we have seen all of them in the last few lectures, all of them are equally important. But among these two of one has to look at the superiority level, let us see how it is effecting the design.For the random load effect S and resistance R, theycan be representedas belowbecause we need there statistical characteristics of these variables. What are the statistical characteristics of these variables?

Mean and standered deviation and variance, let us see then. Let see mu S that is for the load effect, which can be taken as BS in to let say SC, where in this case BS is less than or equal to 1.0 and the variable sorry the variance is laying between the value of 0.15 to 0.30. Similarly, mu R, BR in to RC, in both these cases the subscript C stands forcharacteristic value.We understand, what is a characteristic value of the strength and what is the characteristic value of the resistance? We know this, already we have define n number of times, we must be knowing this now. And BR is greater than 1 and VR is, let us say 0.15, that is a variance of the resistance. If the resistance R and the load effect Scan be represented by these two parameters in this order,you may wonder that why BRS more than 1 and B is less than 1, can you give me, why it is so? (( ))Yes, we are to look for the…(( ))

More that is the general aspiration to B, so that iswhat we have saying this is less than 1 and more than 1.Similarly, this can be 0.95, this is 1 by 0.95. So,it is more than 1, that is what we are saying here.For example, right? So,if these two are there, let us substitute this in maybeta index problem and see what happens to be reliability index? Now, specificallybeta S reflectsthe ratioof the mean load.

Why mean load, because I am looking at themean of S. Suppose, you looking for an annual variation, ifthe periodofvariationis annual, thenbeta S should refer tothe annualmaximum value. And therefore, one can say P fis alsoannual. Now, the characteristic values as you see here may be SC refers to the characteristic valueof the load effect, which is typically a 100 year return period, is it not that is what you are looking at?

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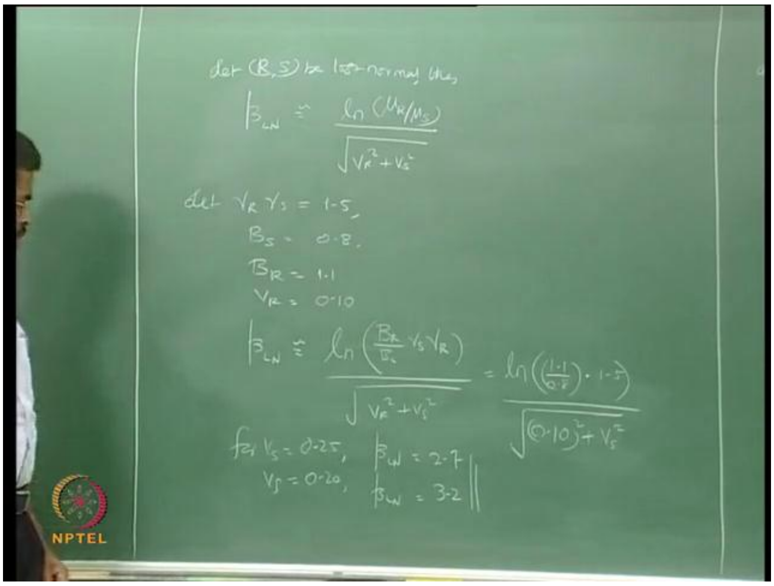

Let us assume that, let R and S belog normal, thenbeta l n, which we saw the expression yesterday can be approximately equal tosquare root of VR squareplus VSsquare. Now, letgamma R and gamma S, what are these? These are partial safety factorsfor resistance and for the load effects B1.5, that is a typical value. What we have in most of the international coats, let us take BS it should be less than 1,I take this as 0.8. Let us take BR as 1.1 and we already know VR is 0.15.We also have another expression, which is derived from this by substituting for mu R mu S and VRVS, which can bel n of BR by BS gamma S gamma R by square root of VR squareplus VS square is also an approximate value.

Let us substitute this so BR already I have as 1.1 by 0.8 multiply by 1.5 divided by square root of… Please make a correction here VR in this case is 0.1 not 0.15, please make a correction here. So,VR is 0.10. So,0.10 squareplus VS square.Now,I have a band of VS varying from 0.15 to 0.3. Let have say for VS equals 0.25 by betal n. Can you find out this value? This comes to 2.7 and for VS point 2,that is in the same band beta l n comes to 3.2. So, for a very small variationin the variance of the load effects from 0.25 to 0.2, where when VS is decreasing beta l n is increasing.So, we can simply right,so you can also use the table and tell me what are the corresponding probability of failure?

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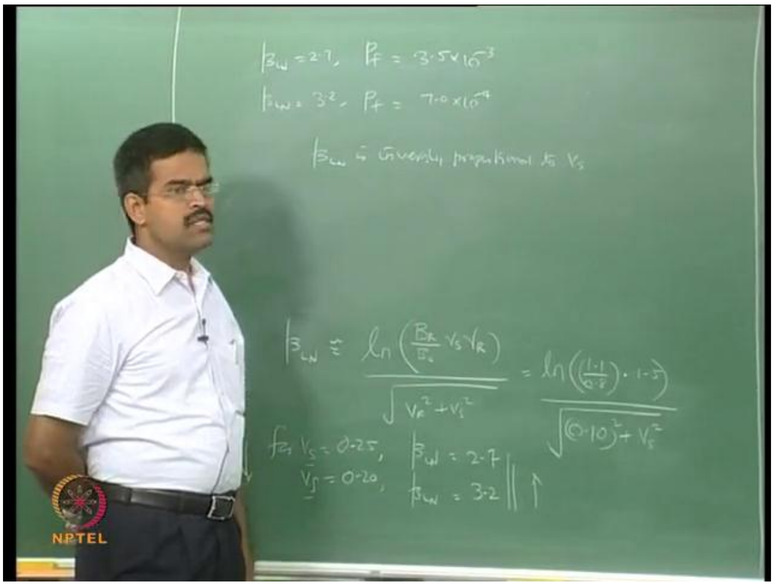

For beta l n of 2.7,what is the probability of failurefor beta l nof3.2,what is the probability of failure?

Student: 3.5 into 10 upon minus 4

3.5 into 10 upon minus 4and for 3.2?

Student: Minus 3, sir.

Minus 3and for 3.2?

Student: Minus 3, sir.

Let us say 7,10 upon minus 4.Now, you can see a very drastic difference and probability of failure for a very small variation of VS.So, what we can say is beta l nisinversely proportionalto VS.So, what does it mean? The variance of the load effect significantly influence the probability of failure and it is inversely proportional to the beta l n.It is very interesting, it all depends upon how accurately you model the random variables in the load effects and find the parameters of mu, variance,etcetera. If you make any mistake or any data omission, in calculating the first or a second moment values,let say mean standard deviation, variance of the load effects, forget about the resistance here we are fixing it.

It can significantly give a difference, since the meaning here reliability of probability of failure. This is an implicit effect of the probability of failure in the design.It is nothing buta statistical variable of the load effect, which can affect may design since very seriously, because it is very much different.Or even reliability index is very much different,right?It is a very important, let say the illustration through an example, that how these parameters cansynthesize, the probability of failure in the design?Now, let us talk abouthow reliability frame work can be use full in ultimate limit states?

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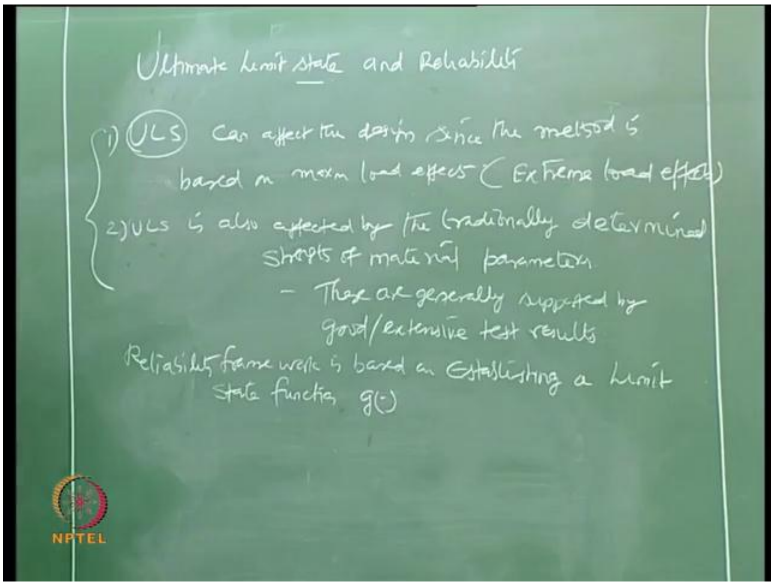

One can say the ultimate limit stateand reliability, let us sayultimate limit state canaffector influencethe design.Since,the methodis based onmaximum loads,maximum load effects orI can even call this asthe extreme load effects.So,that is one statementhow ULS can be influenced by the load effects, because it is based on extreme load effect,that is why we call as collapse load,where asULSis alsoaffectedor influenceby the traditionallydeterminedstrengths of material parameterslike yield strength Young’s modulus, which all related to strength of the material or resistance factors.

Now, these are generally supported bygoodand extensivetest results,generally you do not find the resistance parameters adopt you conduct this by on different samples. Take a proper mean, so all these will have a specific implication on yourULS, that is why in this equation.If you see generally the varianceof the resistance is closely fixed is not given a band,whereas a variance of load effect is given a band because there are high degree of uncertainty, in the load effects compared that of the material strength because material strength is calculated more or less in a very systematic manner, with the experiments in the laboratories.

And there is an extensive quality control in manufacturing of this material. Therefore, there is no much variation, though there is a variance here.We are not saying it is100 percent right, butit is limited to a very narrow band of only 0.12,whereas here and this as sensitized the beta l n. as we saw in the last example by small variation of 0.25 to 0.2 beta l n. probability of failure in directly or affected significantly.So,VSor variance of load effectssensitizes the probability of failure significantly with respect to or in comparison, to that of the resistance or strength of the materials.

So,one can clearly come to an conclusion saying that, thank God to the manufacturing process in the quality control, implemented by people our variation uncertainties on the resistance factors of the material is for narrow compared to thatof in the load effects. Now, one can ask a question, why I cannot compromise or let us say find out this to higher accuracy. As I said they are cost by randomness in nature, they are irreducible.I cannot control on them, that is the reason why the variance in the load effects are higher,it is not because that we do not know, we do not have a mathematical process of narrowing down these variance,it is not that it is cost with the randomness in nature, which is unexpectedly or cannot be modeled correctly is because of that reason, we say that this is having a largerband compared to that of the strength of material.

Now,ULS is a design process, which is affected by two factors. One is load effect now, in this case we say ultimate limit state, we have talk about extreme loads.ULS is also bother about the tradition determine parameters, now let us look in to these two unreliability frame work and see how this can affect the reliability frame work on ULS? Therefore,based on the based on these two, the reliability frame workis based onestablishinga limit state function,that is what wehave seen yesterday,which isg, is it not? This what we have seen yesterday. This limit state function can be time variant,that is what we saw in the last lecture.

How y of t can cross the out crossing of the limit boundary to find out the probability of failure, we have seen two different curves in the last lecture, we have found out how the limit state function can be established. So, essentially reliability is establishing the probability of accidence of limit state boundaries, is that clear? You form a boundary and you are looking at what is that probability there is boundary will be out crossed? So, that outcross means failure,not crossing means safe, is it clear?

So,I am looking at the out crossing of the boundary which is the established by a limit state condition and reliability is not 100 percent accurate, only because of this reason establishing the limit state function itself is a difficult task, one.Two determining the joint providencethe function or integrating this over a domain of the limit boundary, itself is complex,which u have seen in the last lecture.Having said this, let as now takesingle value of R and S and see how this affects myULS, then we will take multiple value of R and S.

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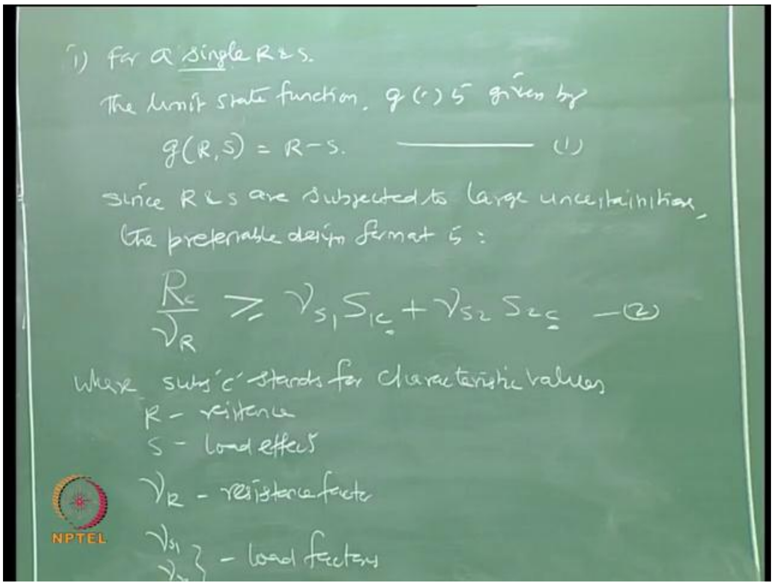

Let us sayfor a single R and S,that is resistance and load effectsthe limit state functionis given by g of R and S.Can be simplyR minus S calls equation number 1.Now, we already know that sinceR and Sare subjected to largeuncertainties,the preferable design formatis simply using a safety factor on this; that is what we have done in the design. So, we should say the characteristics straight of the material by gamma is not not that is the safety factor of the material should be higher thanor greater than the safety factors. What we usefor load 1Cplus safety factor, what we useforload 2C.

This C is stands for the characteristics value and they are can be two loads or three loads like thisand gamma S1, gamma S2, are respective factors for different kinds of loads because as we all understand given an API given in ABS. The load safety factors are not same for all kinds of loads,somewhere it is 0.95, somewhere it is 0.9 and so on, right? So,I call this equation, number 2.So, in this equation number 2,I should saywhere the subscript Cstands forcharacteristic values,R stands for resistanceand S stands for the load effects,mu R is,I should say resistance factorand mu RS1and mu S2or I should say load factors.

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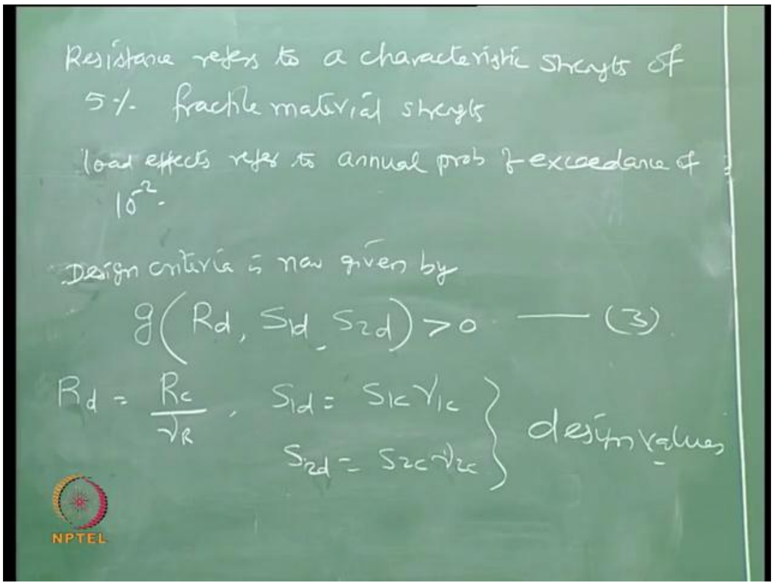

Having said this, we also know thatresistancerefers to a characteristics strengthof5 percent.Fractilematerial strengththe variation in a material strength is only 5 percent or within 5 percent,95 percent does notvarywhereasthe load effects refer toannualprobability ofexceedanceof 10powerminus 2.I think this, we were discuss in the first module, if you remember?

It is understood that the characteristic value of load means that the probability of exceedance is 1 and 100, that is what it is it means for a character strength of load. Now, the design criteriais now given by,let saylimit function of R d S1 dS2, dshould be greater than 0 should not fail. There is a successful designequation 3,where d subscript all stands for design. We can also saywhereR d isRC, when you are, that is my design value is it not that is what it take itdesign and S1 dcan beS1Cmu 1CS2 dcan beS2Cmu 2C, which are all called asdesign values.

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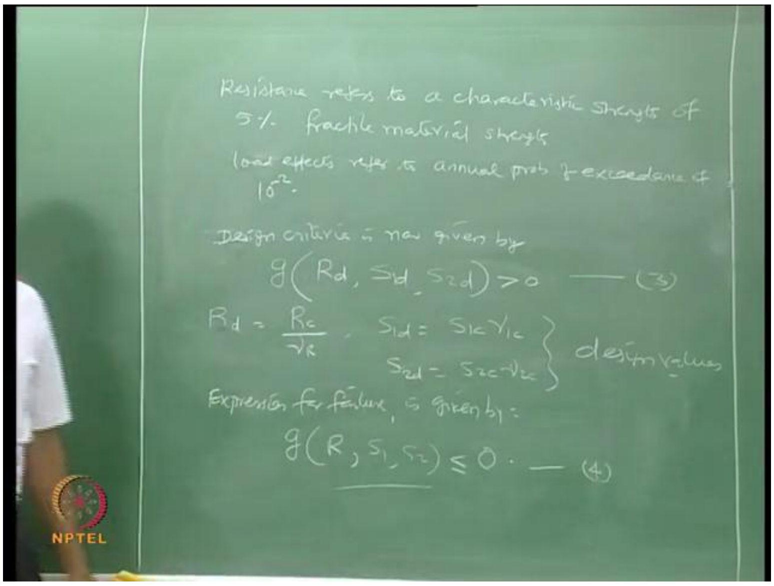

Now, the expression for failureis given byg ofRequal to 0,even if it is equal to 0is said to be fail because it is a verge of failing, right?R minus S is the margin of safety, if it is set to be 0, it is fail,it is on the boundary.If R greater than S is safe,R less than S is unsafe that is why we say it is less than or equal to 0 is my limit function.This is for a single value of R and S, where we can easily find out this my favor multiple R and S.

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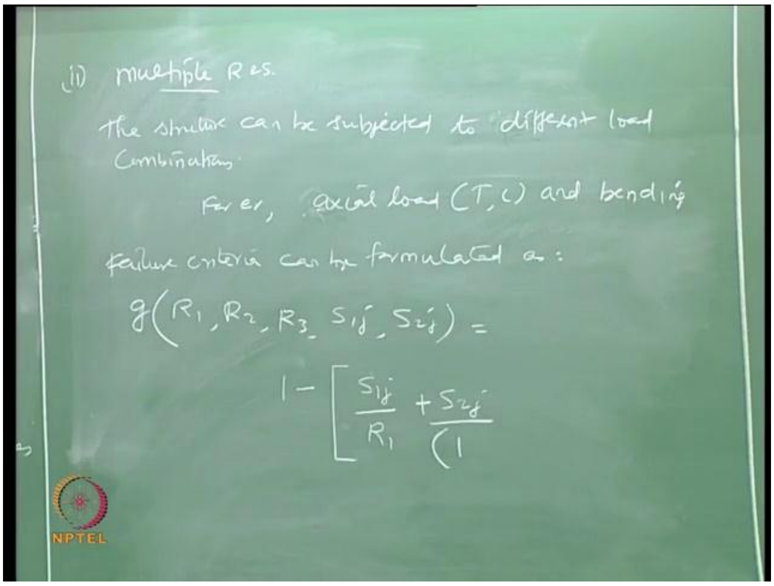

Then how do you go about it?What do you mean by multiple R and S?The structure can be subjected todifferentload combinations, for example, there can be effect of axial load,may be tension or compressionand bending,so it is a multiple loading effect.Multiple R and Smeans it is not that there are different varieties of loads, that is already there in the previous case. Also S1,S2,S3 etcetera, butwe are looking. Now, at the combing effect of these forces or effects on the member,that is what we call as multiple,yes. What do you understand by multiple R?

I got a composite section, the strength of the material can vary between the two sections are composed together to form a single unique material, where we address already said functionally graded materials. Now,can I have a multiple R problem also, depending upon what I am going to recommend for my design. So,that is the meaning of multiple R and S in that case, failure criteriacan be formulatedasg of R1,R2,R3S1,jS2.jI will come about that, what is j can be given by1 minusS1j by R1.

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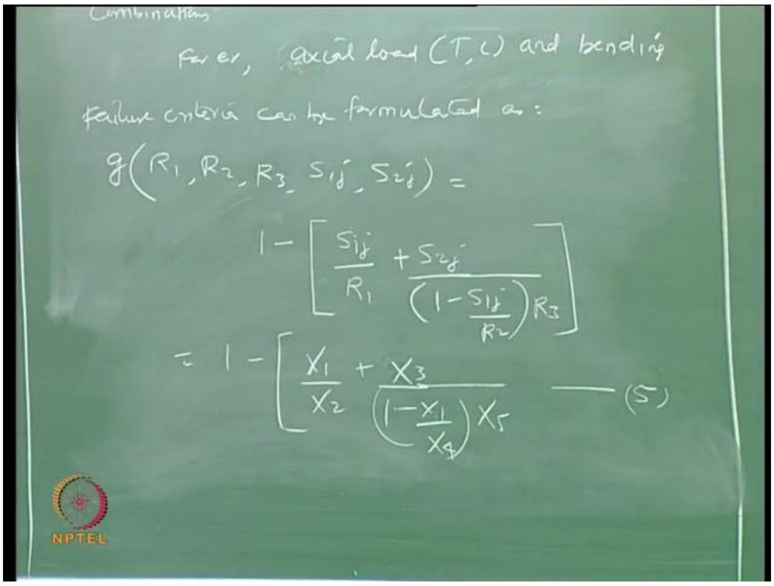

Which can be also said as1 minusx 1 by x 2 plusx 3 by1 minus x 1 by x 4of x 5.Why this is x 1, because it is same as this,same as this the equation number 5.

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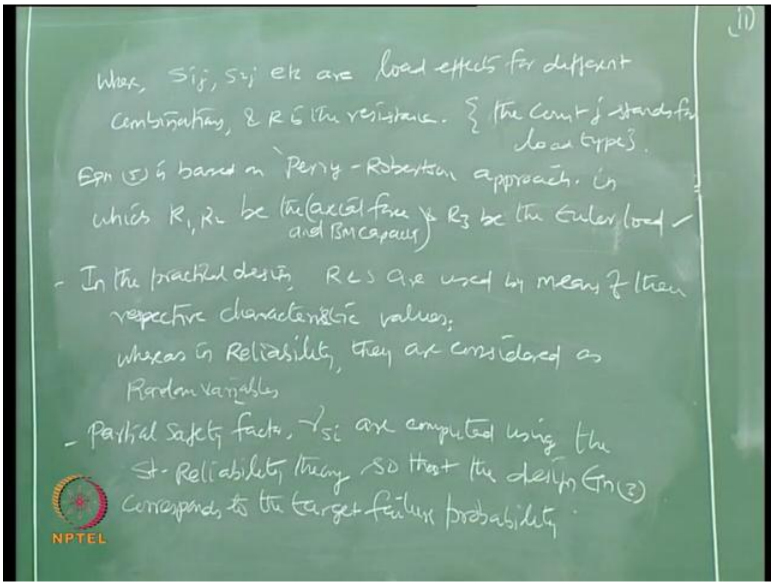

WhereS1jS2jetceteraorload effectsfor differentcombinationsand Ris the resistance, essentiallythe count jstands for different load type.Now, equation 5is based onPerry Robertson approach. That is a very famous approachin reliability analysis where asin which, R1,R2be the axial forceandR3be the bend limit sorrythe Euler load axial forceandbending movement capacitytogetherand R3 be the Eulers load.Now, in the practical designR and Sare used by means ofthere respectivecharacteristic values,where asin reliabilitythey are consideredas random variables. That is, so they are differentiated in the ULS and the reliability frame work.

Now, the partial safety factorsthat is gamma Si12 etcetera of a different loads are computedusing the structural reliability theory.So, thatthe design equation,what are the design equation? We gave in the last one single R and S.I think we said the design equation will be R by safety factor and S by the safety factors S in to multiply by S l equation number 3.

Student: 3

3 there is a derive an equation given in equation 3 corresponds tothetargetfailure probability.So, this is a variation what we use in case of multiple R and S compared,set of single R and S. So, there is a one to one correspondence between ULS, that is ultimate limit state design methodology and that of reliability force correspondence between these two,or similarity is in design applications, we consider themusing characteristic values, whereas if youreally want to consider them in reliability problem to find the probability of failure, we consider them as random variables.So, to amalgamate or the mix up or to have influence of one more of the other, what people have done is estimate the safety factors using reliability theory.

That is where reliability is bridged to ULS, see for able to estimate the partial safety factors using reliability theory and targeting the probability of failure that is equal to 0 or greater than 0. Less than 0 means for sure it is going to fail, estimate the safety factors based on that concept and used that safety factor in a design that is our reliability theory and ULSor bridged, is it clear?That is why they are connected together, right? So, one should have a rational way of estimating these safety factors using reliability theory, where the parameters contribute in to this, that R and S are considered as random variables.

Once it is obtain use it in the design as characteristic values and do the design, so it means that the design used by calculating are employing these valueswill have a define probability of failure, which you already know 10 for minus 3,10 for minus 4 you know 1 in 10,000 structure,1 in 10,000 may fail. That is a probability of failure,if we use this gamma Si in your design, you understand this is how they are arrived, is it clear?That ishow ULS and reliability theory are bridged.We stop it here,we have the next lecture discuss it in detail about this and move on to the levels of reliability theory.

Thank you.