

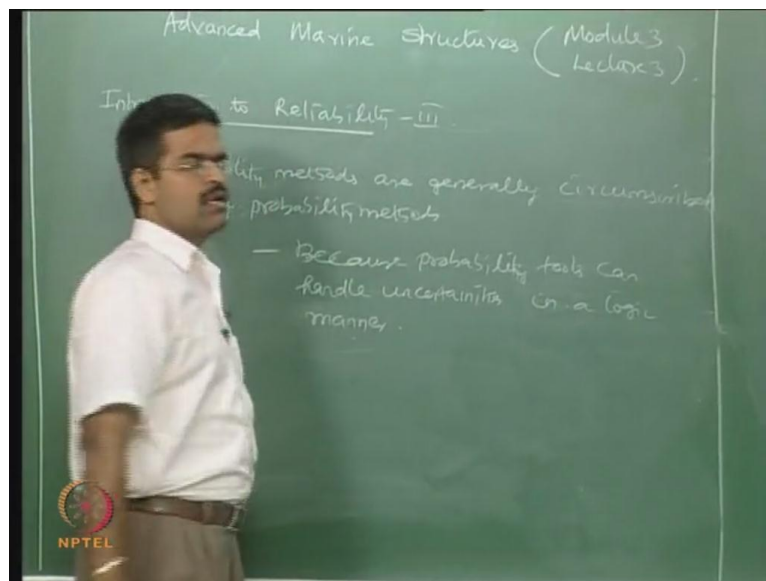
**Advanced Marine Structures**  
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**Lecture - 3**

**Introduction to reliability III**

In the last lecture, we discussed about different types and different kinds of uncertainties. We also said that the second and third kind of uncertainty mostly associated with a mathematical modeling errors and the other one can be easily handled using, what we call as a Bayesian approach. So, what we can make an important statement, in this lecture which is the continuation of the last lecture is that.

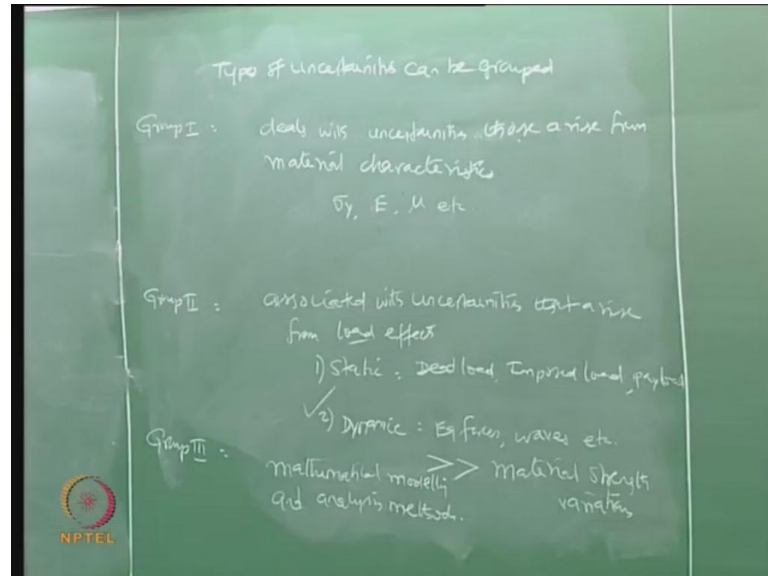
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Reliability studies or reliability methods are generally circumscribed by probability methods. On the other hand, probabilistic tools are used to carry out reliability analysis. We all know the answer for this question, but still try to answer this, for because probabilistic tools or probability tools can handle uncertainties logically. They are capable of handling uncertainties in a very logic manner. Therefore, generally reliability methods are circumscribed using probability tools. Now, we have seen 2 types of uncertainties, one is arising from the randomness in nature which is auditory type. The

other is from the modeling methods or analyst tools which is an epistemic type. Now, there are different groups, that is.

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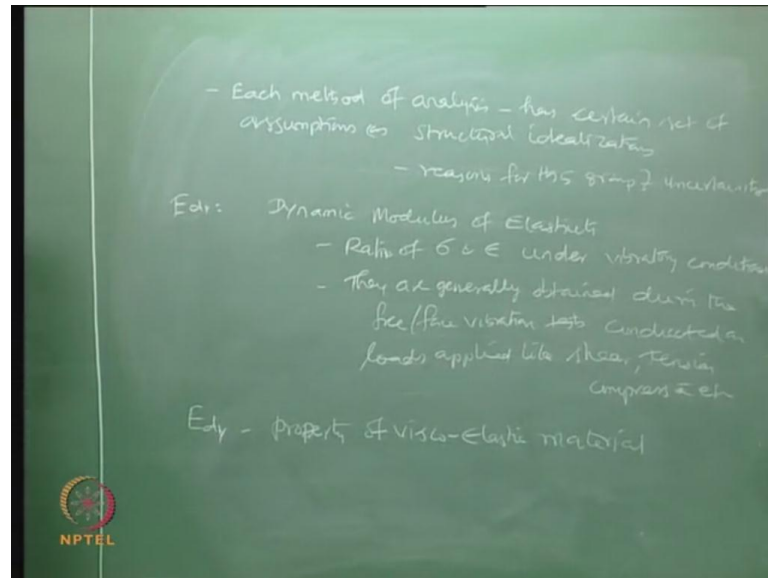


The types of uncertainties can be grouped as group 1, group 2 and group 3. There are 3 groups. group 1: deals with uncertainties associated or arise from, those arise from material characteristics, like young's modulus of the material, modulus of elasticity etcetera  $\sigma_y$ ,  $E$ ,  $\mu$  ratio, which are all purely material characteristics, they are grouped as 1. The second group: are associated with uncertainties that arise from load effects. The movement, I say load effects, I have got 2 kinds, again one is static other is dynamic. I can give example of static, dead load, imposed load. What we call as a live load, pay load on the top side of the platform etcetera, dynamic, earthquake forces, waves etcetera.

Now, the uncertainties associated with the dynamic forces of the load effects cost be dynamic forces, are much more than the top static, that is one comparison between this two. Further, the uncertainties associated with group 2 are much larger, much larger than that of material. So, one should focus more on uncertainties from group 2 is expected, but interestingly group 1 and group 2 are combined. Very interesting and very silent, very valid example is that if he looks at one of the characteristic material, which is young's modulus. If you study young's modulus in static state of loading, it is different. We study young's modulus in dynamic state of loading, it is different, whereas dynamic and static found group 2 of uncertainty variation of  $e$  itself of group one. So, this two are linked; it

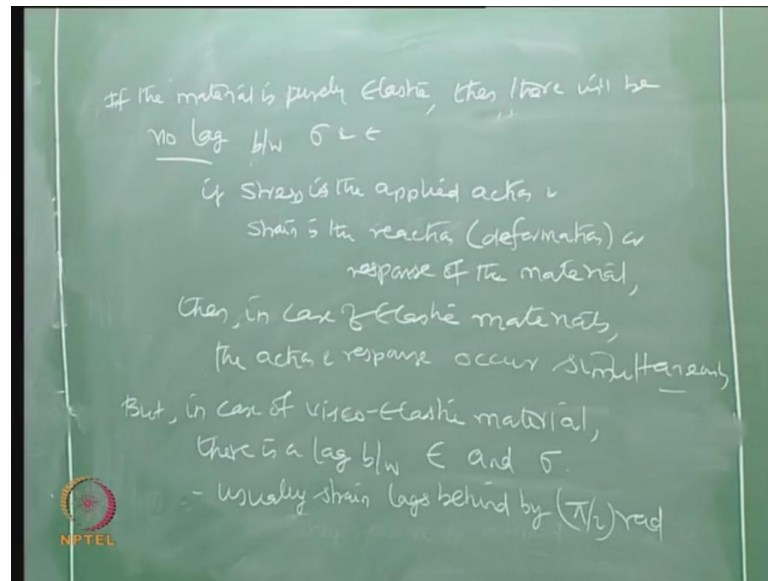
means, in the given type of uncertainty, there are no harlin differences between the groups of uncertainties, they can loop over the other also. That third group of uncertainty, mathematical modeling, they arise from mathematical modeling and analysis methods.

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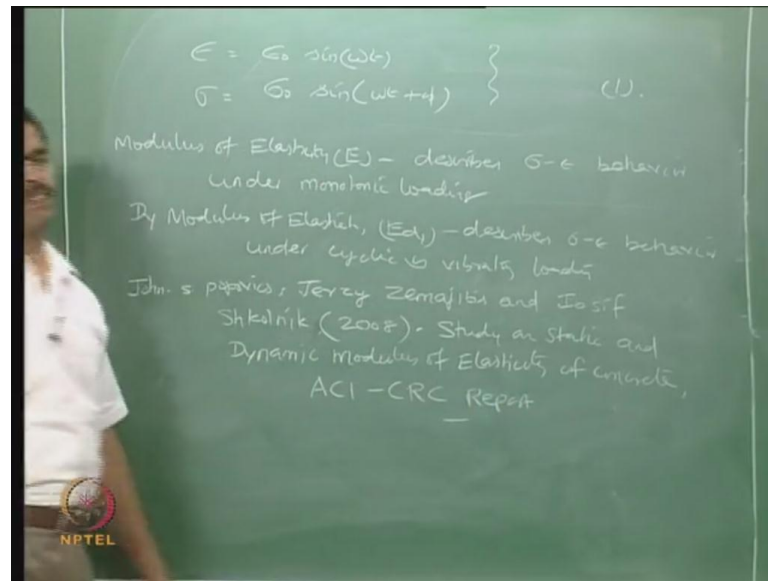
Why because each method of analysis has a certain set of assumptions or structural idealizations, which are reasons for this group of uncertainty. So, let us quickly highlight, what would be the effect of group 2 on 1. Let us talk about young's modulus dynamic or modulus of elasticity or dynamic modulus of elasticity is nothing but the ratio of stress and strain under vibratory conditions. They are generally obtained during the free vibrations or post vibration test, conducted on loads applied like shear, tension, compression etc. Dynamic modulus of elasticity is a property of visco elastic material where static modulus of elasticity is a property of elastic material.

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If the material is purely elastic then there will be no lag between stress and strain, what does it mean? If stress is the applied action and strain is the reaction or deformation or response of the material then. In case of elastic materials, the action and response occur or occur simultaneously. There is no lag, but in case of visco elastic material, there is always a lag between the strain and the stress usually, strain lags behind by  $\pi/2$  radians. So, they are out of phase by 90 degrees. The strain will not occur at the maximum point, where the stresses occurring in the same phase can be shifted by 90 degrees, that is a property in viscous elastic materials.

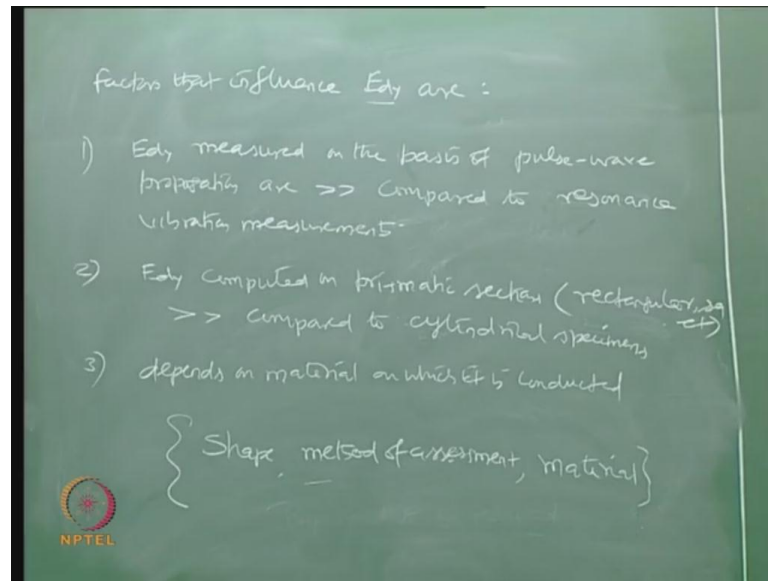
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So, strain can be  $\sin \omega t$ , whereas stress can be  $\sin(\omega t + \phi)$ , it is an advanced state. Equation number 1, more interestingly modulus of elasticity which is simply E describes stress-strain behavior under monotonic loading, where  $E_d$  has dynamic modulus of elasticity indicated by  $E_d$  describes stress-strain behavior under cyclic or vibratory loading. If we look at the research conducted by people in the recent past, people have conducted modulus of elasticity observed on the same material.

On different specimens and different loading patterns, they did not show the same behavior very interesting reference. There is a very interesting report given by Popovics. A talent in 2008, explaining a difference between the characteristics of modulus of elasticity and dynamic modulus of elasticity on a specific material concrete under different. So, he has listed that various factors which influence dynamic modulus of elasticity of concrete as a material are per any material in general of the following.

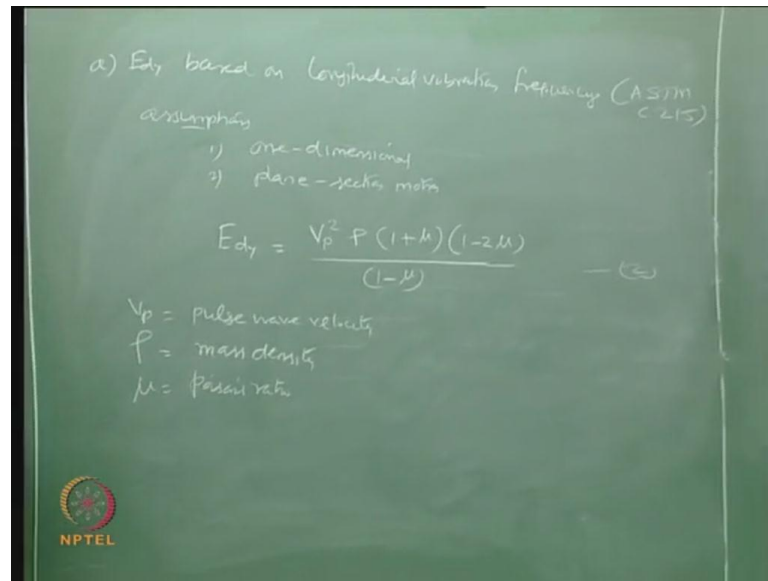
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Dynamic modulus of elasticity measured on the basis of pulse wave propagation are higher compared to Resonance vibration measurements. So, method by which are estimating the dynamic modulus of elasticity is also an important factor, which influences the modulus of elasticity. So, there are 2 techniques by which one can find out, one is a simple force of free vibration on to the resonance condition. Other is what we call pulse wave propagation technique. So, both give different dynamic modulus of elasticity contained on the same specimen on the same material.

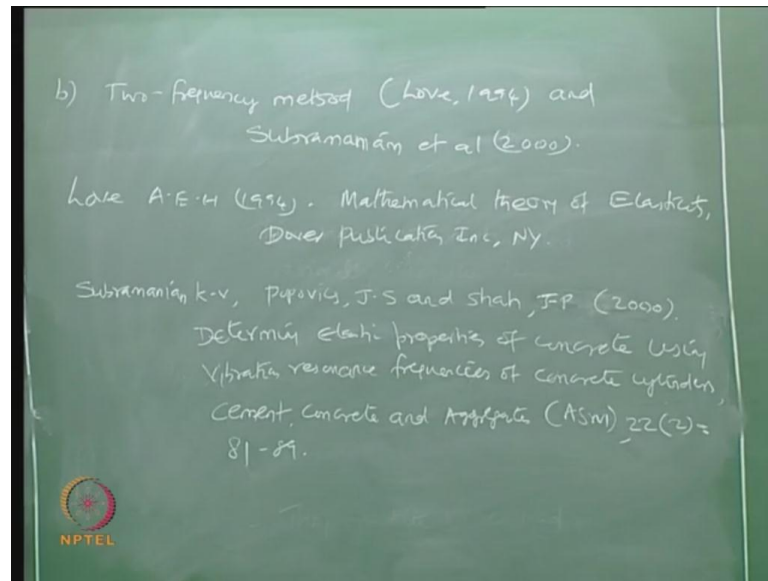
Dynamic modulus of elasticity, computed on prismatic sections like for example, rectangular, square etc are higher compare to that of cylindrical specimens. And they also said that dynamic modulus of elasticity depends on the material on which it is conducted. So, one can say the factor influencing dynamic modulus of elasticity could be shape of the specimen taken for the test, method of assessment. Whether, you are using pulse wave propagation technique or using simple free vibration method on resonance condition etc, 2 of 3 could be of course, the material characteristic itself. So, you will give you some relationship established by different researches on computing dynamic modulus of elasticity.

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So, let us say dynamic based on longitudinal vibration frequency. This is based on ASTM C 215 technique. There are certain assumptions for this experiment conducted. It is one dimensional, the other is it is a plane section motion. This is not distortion being invoked in the material, say as  $E_d$  dynamic can be given by  $V_p^2 \rho$ . The equation number 2, where  $V_p$  is the pulse wave velocity,  $\rho$  is mass density, and  $\mu$  is poisson ratio of the material. So, using this equation, I can find out dynamic modulus of elasticity, if he knows for varying of a various pulse velocity waves.

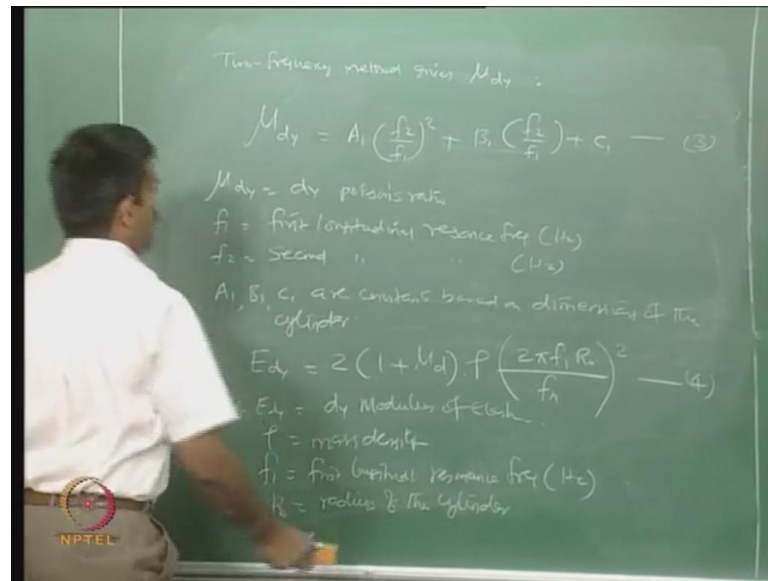
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The other methods using two frequency techniques given by Love 1994 and Subramanian et al 2000. Love A E H 1994 mathematical theory of elasticity, Dovers publication, New York, Subramanian Popovics determining elastic property is of concrete is J P shah. Let me write, determining elastic properties of concrete using vibration resonance frequencies of concrete cylinders, cement concrete, cement concrete and aggregates ASM 22 2 82 89, it is 81 89. According to this technique, two frequencies they have been used, which first identifies the dynamic Poisson's ratio. Then based on new dynamic young's modulus or the modulus of the elasticity dynamic is been estimated.

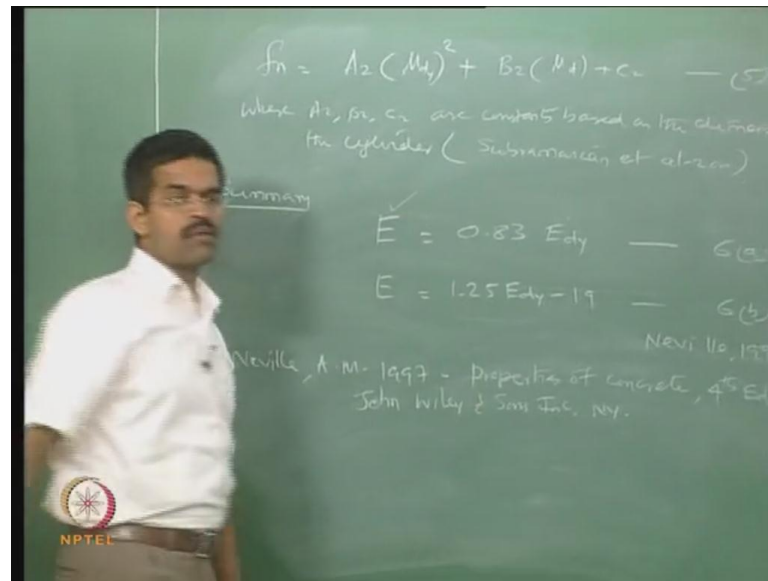


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So, two frequency method gives  $\mu_{dynamic}$  as below. So,  $\mu_{dynamic1}$  is first longitudinal resonance frequency in hertz,  $f_2$  is a second longitudinal resonance frequency in Hertz.  $A_1, B_1, C_1$  are constants based on dimensions of the cylinder. Once I know  $\mu_{dynamic}$ , then I can find  $E_{dynamic}$  using this equation, where  $E_{dynamic}$  is dynamic modulus of elasticity,  $\rho$  is the mass density of the material,  $f_1$  is the first longitudinal resonance frequency in Hertz and  $R_0$  is a radius of the cylindrical specimen.

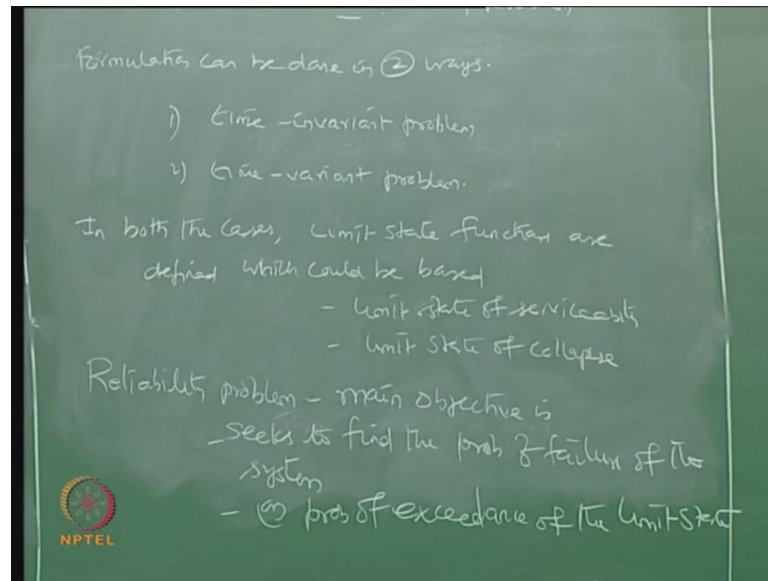
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And  $f_n$  is given by where  $A_2$ ,  $B_2$ ,  $C_2$  are constants based on the dimensions of the cylinder given by Subramanian et al. So, as a summary, it is seen that static modulus of elasticity, a simply modulus of elasticity is about 0.83 of dynamic, that is one equation. The other equation says this can be also equal to minus 19 both are given by Neville 1997. So, one can see that, if you are using static modulus of elasticity or simply modulus of elasticity in your calculations for the design or analysis you are actually underestimating in capacity, because the modulus of elasticity. The material varies depending upon the shape of the material, the material actually being used and the dimensional the cylinder of the material and the loading methodology.

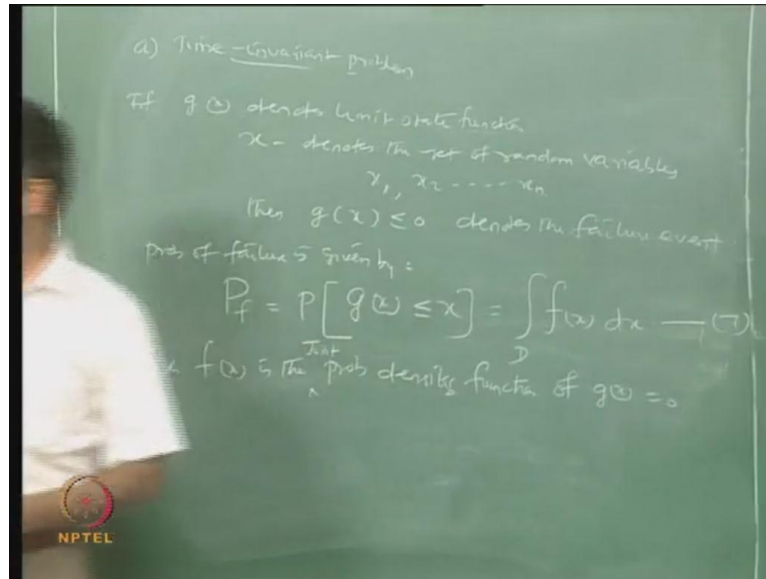
So, once you strictly use dynamic modulus of elasticity in my estimating the parameter of the design, where you are using static values. So, this one important uncertainty which comes from group number 2, which influence on group number 1 on a specific type of uncertainty which is an important parameter in reliability studies. So, it is very important that we must know what a variation is approximately, which compares the modulus of elasticity with that to the dynamic modulus of elasticity, there is a variation having understood this.

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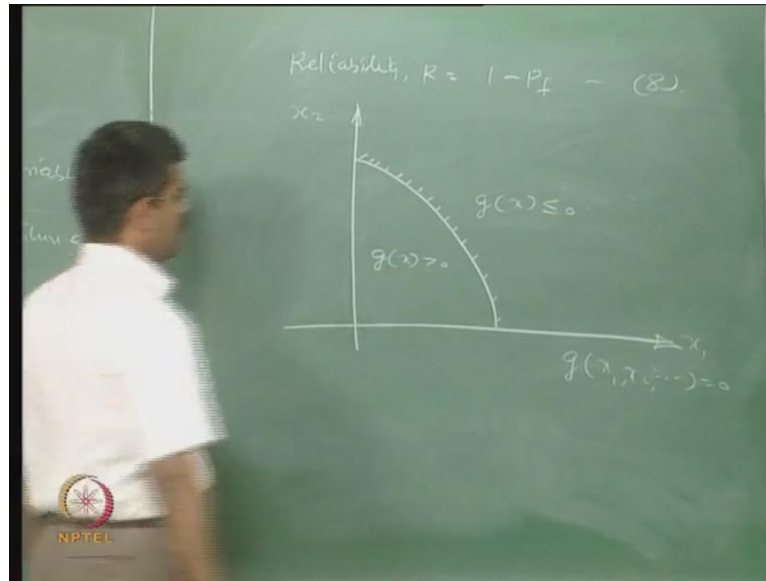
Let us now talk about formulation of a reliability problem. How do you formulate a reliability problem? Formulation can be broadly done in 2 ways. 1: a time invariant problem, 2: a time variant problem. In both the cases, limit state function or define which could be based on either limit state of service ability or limit state of collapse, either way. So, the main objective of reliability problem seeks to find the probability of failure of the system and probability of accident of the limit state.

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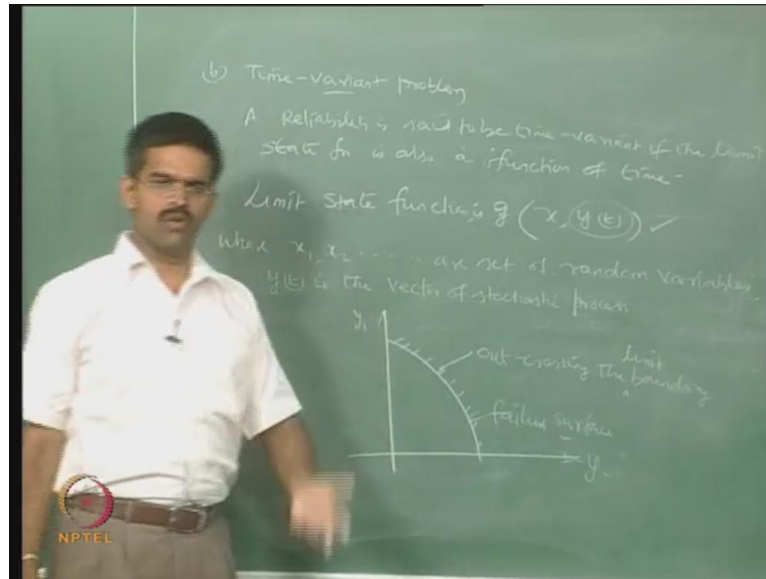
Let us talk about time invariant problem. If  $g(x)$  denotes the limit state function and  $x$  denotes the set of random variables  $x_1, x_2, \dots, x_n$ . Then  $g(x) \leq 0$  denotes the failure limit. The probability of failure is given by  $P_f$  is the probability of  $g(x) \leq 0$  which is nothing, but integral of  $f$  of  $x$  or the domain  $bdx$ . I call the equation number, is it 57, where  $f$  of  $x$  is the probability density function. Since,  $x$  is having combination of  $x_1, x_2, x_3$ , I can say even this as joint probability density function of  $g(x) = 0$ , we can plot this.

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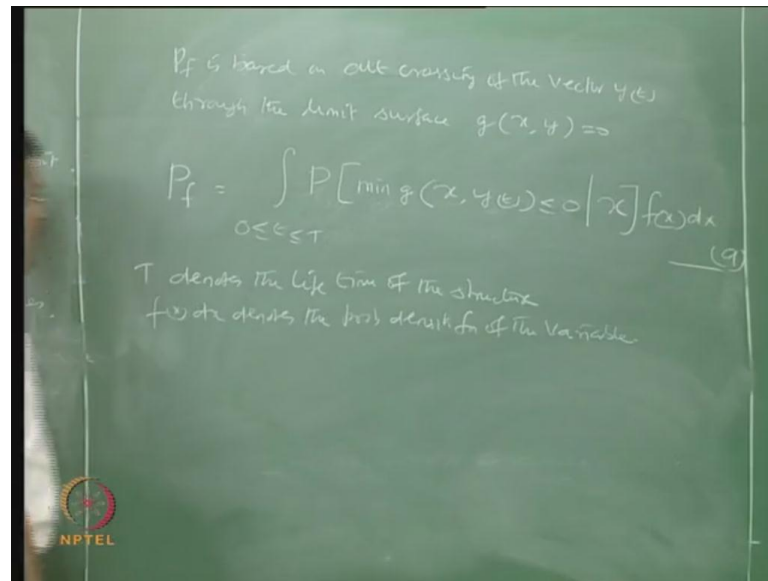
Now, reliability will be given by 1 minus probability of failure. Let say, you can plot this domain, let say  $x_1$  of  $x_1 \times 20$ . So, this  $x_1$  and  $x_2$ , I am plotting in a two dimensional domain, this, my failure domain, anything outside the domain is a failure. So, this is safe domain, so you must integrate the joint probability density function over this domain that is one of the type of uncertainty is very complex that is why it cannot be accurate. Now, let us talk about time invariant problem, sorry time variant problem, now the dependency of time will also need to be established in this kind of reliability problems.

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A reliability problem is said to be time variant, if the limit state function is also a function of time. Now, the limit state function is  $g$  of  $x$  and  $y$  of  $t$  also, that is time variability, time dependency is also there. Where  $x_1, x_2, x_3$  are a set of random variables and  $y$  of  $t$  is the vector of stochastic process. Now, if I try to plot this failure as  $y_2$  and  $y_1$  and this becomes my limit boundary. Failure is expressed as out crossing the boundary or I should say out crossing the limit boundary. So, it is a failure surface, so my failure surface is out cross, I construct the divided as a failure event.

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So, probability of failure is based on out crossing of the vector  $y$  of  $t$ , through the limit surface  $g$  of  $x$  comma  $y$ . So, probability of failure is now given by integral from 0 to  $t$  less than  $t$  of probability of minimum  $g(x, y)$  of less than 0 given  $x$  for any given value of  $x$  into  $f(x) dx$ . Equation number, what is the number here? Here in this expression capital  $t$  denotes the life time of the structure. And of course,  $f(x) dx$  denotes the probability density function of the variable. We stop here; we will discuss this in the next class as an extension of the time variant problem, any questions?

Thanks.