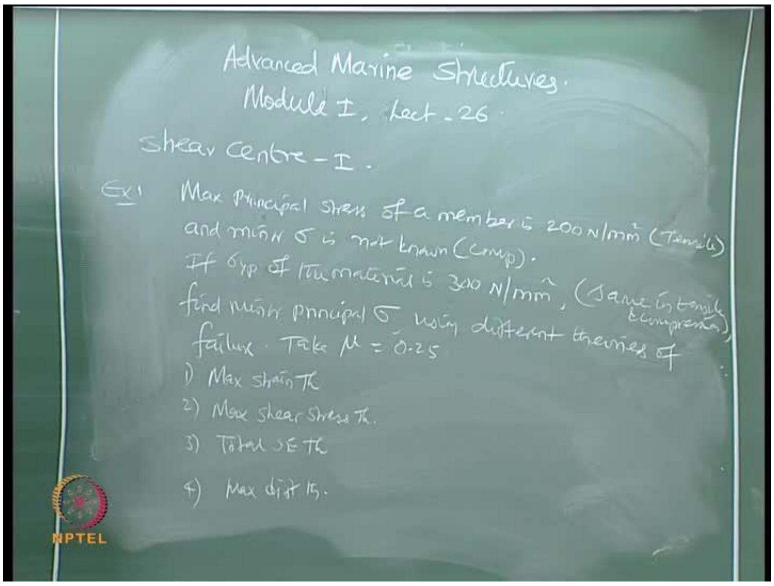
Advanced Marine Structures
Prof. Dr. SrinivasanChandrasekaran
Department of Ocean Engineering
Indian Institute of Technology Madras

Lecture - 26
Shear centre-I

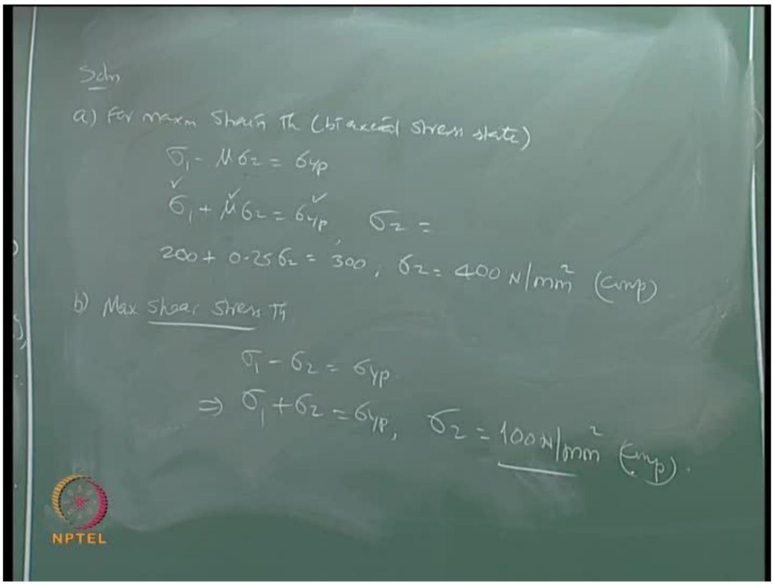
(Refer Slide Time: 00:52)



So we will discuss now one another important topic which is, sheer centre. In the last lecture we discussed about, or last few lectures discussed about plastic analysis and design. And we also discussed about theories of failure as discussed by, as given by 5 different theories. For understanding let usgo quickly to numerical examples, these are examples of different theories. Then we will move on to the sheer centrewhich is an important topic as far as thin asymmetric sections are concerned. We will tell you why it is important and that is how that is predominantly important as a designer for marine structures, let us see that. But before that let us do onedesign example using different theories. Let us say the maximum principle stressof a memberis given as 200 newton per mm square tensile and the minor is maximum, and the minor sigmais notknown, butthe nature is compulsory. If sigma y p of the materialis300 newton per mm square same in tensile and compulsion, find the minor principlestressusing different theoriesof failure. Now takemue for the material as 0.25, so following theory should be used; Maximum strain theory, Maximum sheer stress theory, Total strain energy theory and Maximum distortion.

Read the problem think it for few minutes. Let us see how we will solve this problem. You have the governing equation of all these theories I gave you in the previous lectures please turn them back and be ready with the equations, and read the problem what is given and what is asked how we handle this problem.

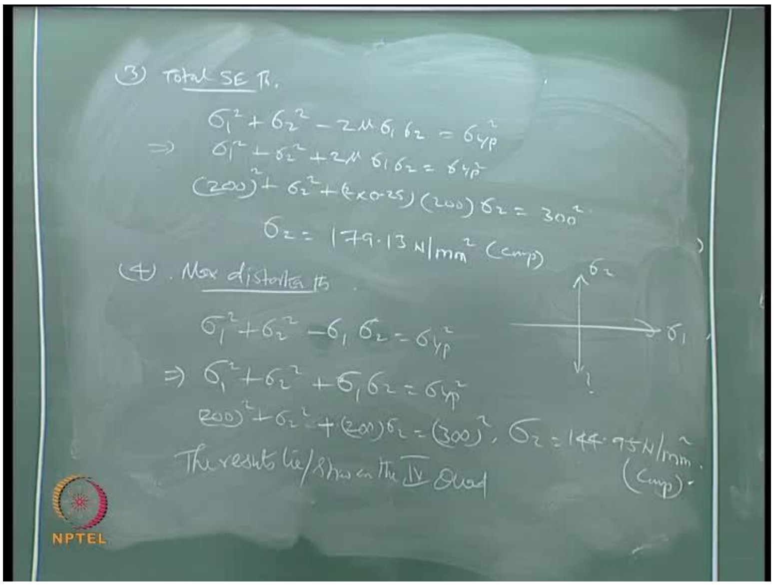
(Refer Slide Time: 03:45)



So let us say for maximum strain theoryin a biaxial stress state, see I have deliberately made these 2 stresses of different nature. Because 1 is tensile other is compulsion, so I am looking for quadrants of 2 and 4; so it can give me a good difference. I deliberately made this as a choice for a problem. So let us see how they are vary, so for the maximum strain theory for the given biaxial stress strain what is the controlling equation? Can you give me the equation? Is this the equation? Sigma 1 minus mue sigma 2 sigma y pand already we know sigma 2 is the compression that is the indication given in the problem. So,I should saysigma 1plus mue sigma 2, now is sigma y p because this is minus of minus and you know sigma 1, you know sigma y p, you know mue, can you find sigma 2?

So, this is 200plus 0.25 of sigma 2, is 3,right? Which gives me, sigma 2 plus 400 that is1.Answer is compression.Maximum sheer stresstheory, According to this theory what is the control equation? Minus sigma 2 issigma y p, is it not? So for my problem sigma 2 being compressive, which gives me sigma 2 as simply 100 compressions, is it not? Take away this.

(Refer Slide Time: 06:34)

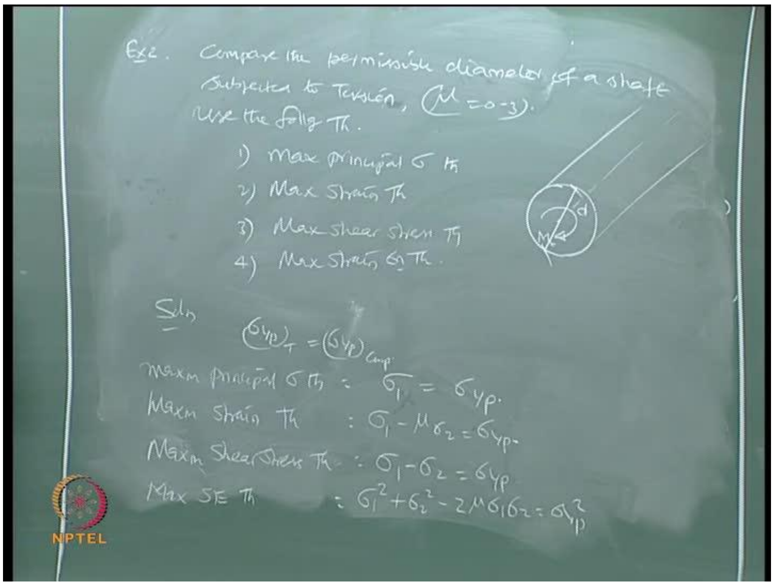


Now total strain energy theory, what is the control equation? Sigma 1 square minus 2 mue sigma 1 square sigma y square p, am I right? You know everything solve the quadratic and get me sigma. So, for sigma 2 being compressive sigma 1 squaresigma 2 square 2 mue sigma 1 sigma 2 is sigma y p square. Now what is sigma 1 value? 200then you find sigma 2;solve the quadratic and get this is 179.13.

Look at the fourth 1, maximum distortion theory. According to this theory the control equation is sigma 1 square plus sigma 2 square.Ya minus sigma 1 sigma 2 goodis y p square, is it not? So, sigma 1 square plussigma 2 squareplus sigma 1 sigma 2 square is sigma y p square. So,200 square plussigma 2 square, so hereplus 200 of sigma 2 is,300 square which gives me sigma 2 as 144.95newton per meter square compression; and if we see the discrepanciessince sigma 2 is compressive all these values are in which quadrant of your stress theory? Sigma 2 is compressive.

Sigma 2 is compressive, sigma 1 is tensile, which quadrant? This is strainsigma 1 sigma 2 sigma 2 is negative, so I am in the fourth quadrant. So, as I expected the variationbetween the valuesare significantlyhigh. So, designer it’s very large. Second example, this is more alarming I will show you an example now.

(Refer Slide Time: 09:57)

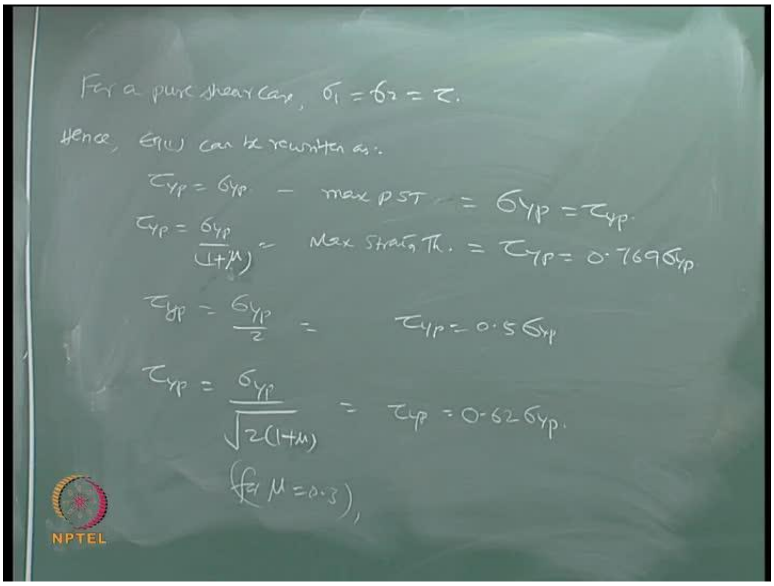


Compare the permissible diameterof a shaft. I have a shaft,I want to check the diameter of the shaft, subjected to torsion.Taking mue as point you use the following theories;1. Maximum principle stress theory, 2. Maximum strain theory, 3.Maximum sheer stress theory, 4. Maximum strain energy theory. So, I have a shaft whose diameter is d. The shaft is subjected to torsion attesting moment M tattesting moment, M t. Let say this diameter of my shaft. I want to estimate the diameter of the shaft, the design problem based on the following things; so all should give me the same value, more or less similar values.

Let us see what happens when we use the different theories. Let us say for exampletake up the maximum principal stress theory.Before that let us saysigma y p in tension is same as sigma y p in compulsion. So the maximum principle stress theory sayssigma 1 is equal to sigma y p, is it not? That enact irrespective of other status stresses.

When the maximum principal stresses reaches yield point failure has started, that is what the theory says. According to maximum strain theorywhat is the equation? Maximum strain theory, is this the equation? Maximum strain theory, then maximum sheer stress theory ?There is stress caused inthis equation? Is it right or wrong? Then according to the maximum strain energy theory, ok?

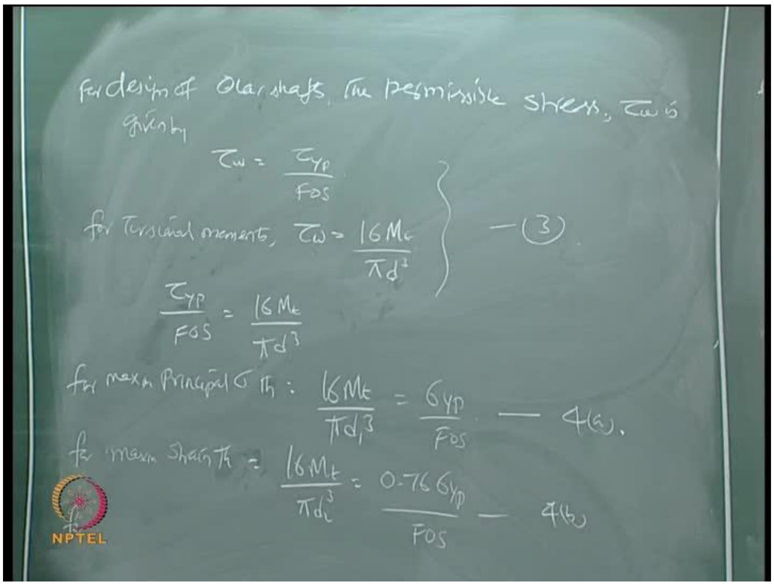
(Refer Slide Time: 14:23)



Now for a pure shear case as in this problem sigma 1 equal sigma 2 will be equal to tow, the shear stress, pure sheer case hence the equations 1 can be rewritten as tow y p is sigma y p this for the maximum principal stress theory, tow y pis sigma y p by 1 minus mue that is for the maximum strain theory. Now, in this case they are of different nature so it will become 1 plus.Therefore different nature it will become 1 plus.

So for maximum sheer stress theorytow y p will be sigmay p by 2, and for this case tow y p will be sigma y pby root of 2 of 1 plus mue, is thatok? Again different nature, when different nature so plus 1; root because I am talking about squares. Now I have the value of mue, I have the value of mue can you just tell me what are these values, equivalently I am substituting for mue because mue already have has 0.3, can you tell me these value? So this is going to be simply sigma y p no change in this that is tow y p, and in this case tow y p will be equal to0.769, and in this case tow y p is equal 0.5, and in this case tow y p is equal to how much? Point, remove this.

(Refer Slide Time: 18:07)

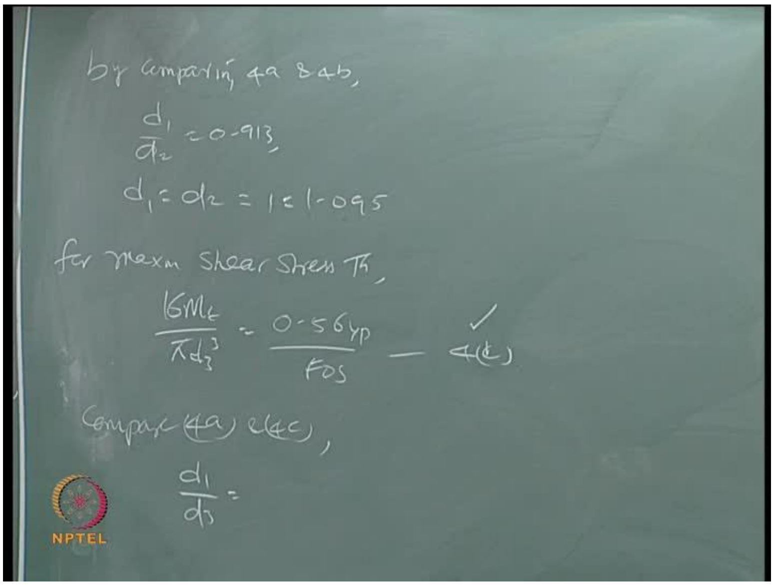


In the case of design of circular shaft, the permissible stresswhich I saytow2 is given by tow y p by some factor, is it not? For torsional momentsis also equal toshear stress by torsion, for torsional moments is also equal to 16 M torsion by pie d 1 2.

How do you get this? How do you get this? What is the control equation by bending?What is the control equation for torsion? What is the control equation for bending? M by a stress by y is e by r control equation for torsion? t by j is stress by y max is it not? So, t is empty, j is polar moment of inertia. For the circular shaftwhat is polar moment of inertia, polar moment of inertia for a circular shaft, what is moment of inertia for a circular shaft?Pie d by 4 by 64, what is polar moment of inertia? 32 half of that, I am talking about y also which is d by 2. I get 16, is that clear. So,now let us find out this equation, I call this as equation number, let me call this equation number let me call this equation number; you missed out some number in between this is 3 that can be 2 or whatever may be. Now I have tow y pby factor of safety16 M t by pie d 1 2 or simply d cube pie d cube. Now this tow y p is different for different theories, for principle stress it is sigma y p directly, for maximum strain it is 0.769, for the other theory 0.5 and the fourth one 0.62. I will keep on substituting and find keep on different diameter and compare; can you give me what is the diameter for the first theory?

So, let us write down the equation first 16 M t for maximum principle stress theory, 16 M t by pie, I call it as d 1 for me to understanding; d 1 cube is sigma y pby factorbecause equation 4 a, is itok? For maximum strain theory16 M tby pie d 2 cube is 0.76 M, I am talking 0.76 sigma y p by factor, 4 b.

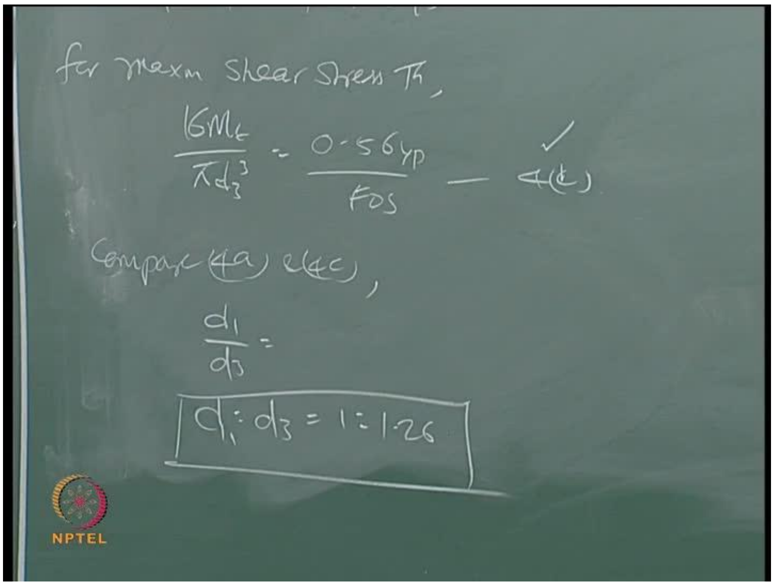
(Refer Slide Time: 22:28)



Forlet us compare these 2 then you compare 4 a and4 b get me the relationship between d 1 and d 2 by comparing4 a and 4 b. See, say if I bring this multiply these2 equations can be equated, is not? So, you have got a ratio of d 1 by d 2, is it not?

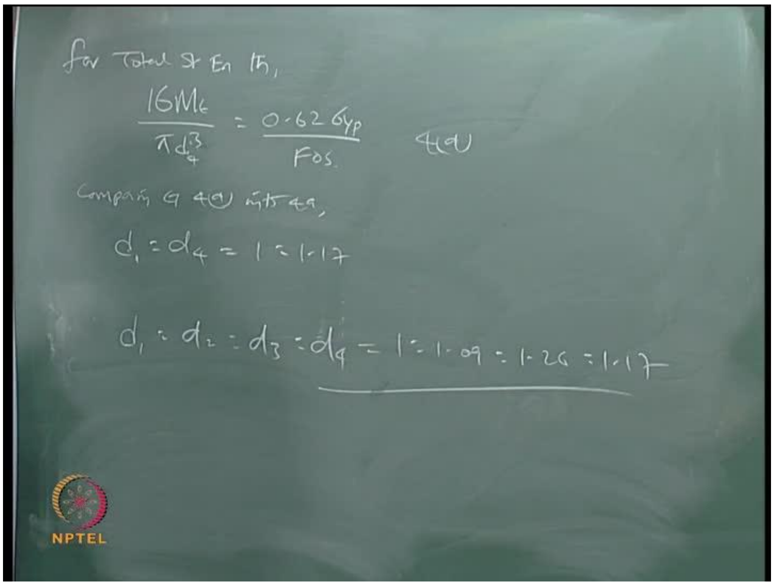
Give me the ratio. So, d 1 by 2 will become0.91 that it is d 1 is to d 2, 1 is to 1.909 is it right? 1.909. The diameter suggested by the maximum strain theory for this problem is about 1 point about 10 percent more than the diameter suggested by the maximum principle that is why what we are meaning. The diameter recommended by the maximum strain theory is about 10 percent more, about 10 percent more than the diameter suggested by the maximum principlestress theory.

(Refer Slide Time: 24:48)



Now let us do for the third case, for the third theory maximum shear stress theory.Now let us say 16 M t by pie d 3 cube and this was 0.5, I call this 4 c. Now compare4a and4 c and comparing this now can you give me the ratio betweend 1 by d 3. So this says d 1 is to d 3is 1 is to the proportion is 1.26, is very high. It is about 26 percent higher when you use this theory for design. I will rub this I will rewrite here.

(Refer Slide Time: 25:15)

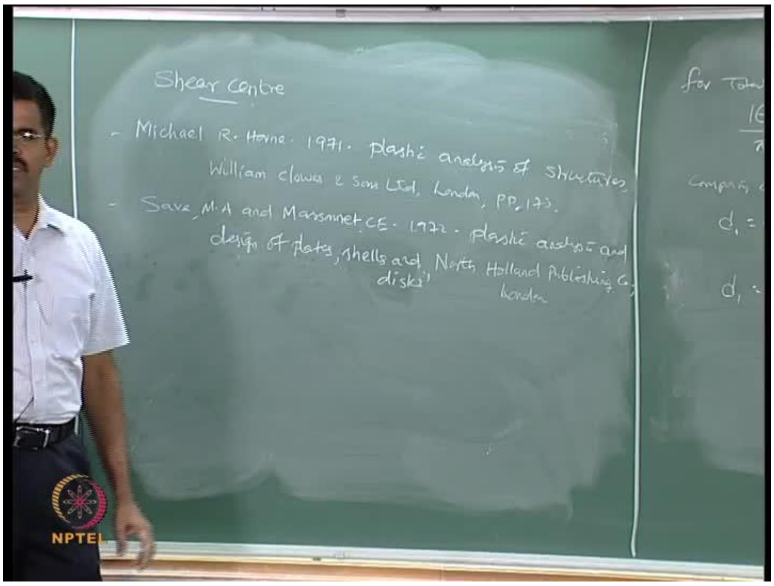


Forstrain energy theoryplease do not mind me shortened here, you can write it for total strain energy theory16 M t by pie d 1 cube or d 4 cube is 0.62 is it? I call this as 4 d.

Now, comparing equation 4 d with 4 a we get d1is to d 4 and 1 is to 17 percent. I should say d 1, d 2,d 3, d 4are in the relationship of 1 is to 1.09, 1.26, and 11.17. So, that is amazing, simple theory simple problem give different dimensions for the design. This is where the design is getting deviated by different design engineers by following the same analysis and design, for example plastic design. So, if we chose any wrongtheory applicable to your problem you will land up in a wrong diameter, simple example.

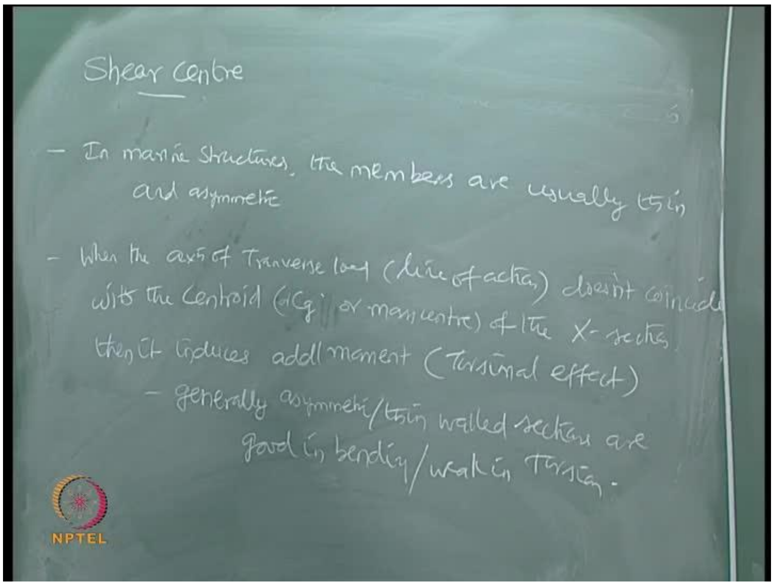
Ok, so you have to be very careful in understanding the failure behavior based on the theories. Let us talk about shear centre, any questions here? Are you understanding the importance of this problem, we have demonstrated how the diameter selection can be chosen can be varying by using different theories on a simple problem like this. So, even there exist uncertainties on the theory suggested by the literature. Therefore, our original argument of limit state design or ultimate living states being probabilistic non deterministic is all justified; because we cannot land up in a single unique answer even the theory suggest different solutions as we see for this example. So, it is not that simple.Your design is always a close form answer in unique number, no. Let us move on to the next topic which we are interested now to discuss which is very closely relevant to marine structures is shear centre.

(Refer Slide Time: 28:10)



I can give a very interesting reference for the plastic analysis of structures, please read this book if you have time; Michael R Horne1971 “Plastic analysis of structures”all the relevance of this theory is all discussed by this author, William Clowes and Sons limited, London,pp173.A good book for plastic analysis of structures. One more book is there,it is slightly of a higher order butstill “Save, M.A and Marsnnet” there is double n , “Plates and shells, North Holland publishing , I think this is design of plates shells and discussif you remembercorrectly.North Holland and publishing city London,these tworeference are very good for plastic analysis and design. You can go through them, of course these examples are not applicable directly to marine structures, but you can still find the members which are designed using in theory and discrepant ofthe different theory are earlier discussed hear so you can read it.

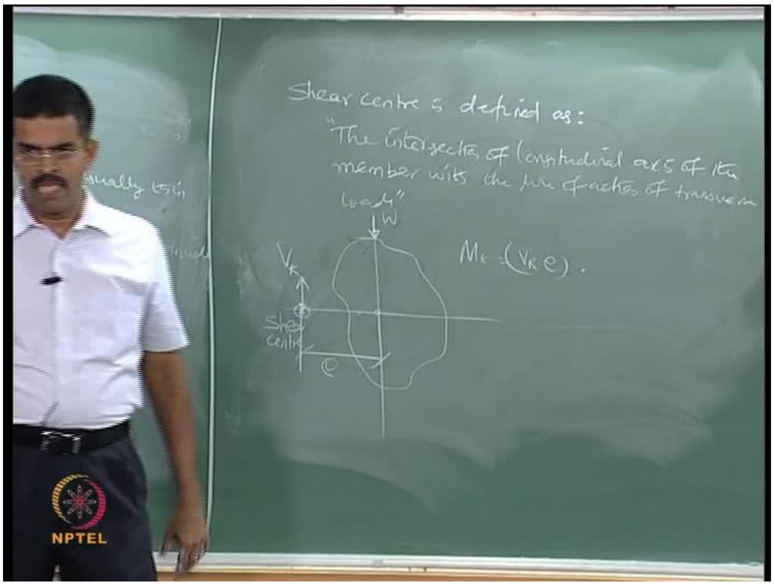
(Refer Slide Time: 31:27)



Now the question comes, what is shear centre,how it is relevant in main structures? Now the most important factor in marine structure, the membersare usuallythin and asymmetric, why thin? Because we are talking about y and c we do not want to increase the payload, we do not want to increase the weight during installation etc we say thin. Thin doesn’t mean that it is very very thin, the thickness of the material in comparison to diameter is very small. Thus d by t ratio is very small, not thin means that we are using a paper it will be like a paper, not like that. The thickness of the member compared to this diameter is very small, because we want large diameter to y and c effects that is different and we want to for storage for blasting there are daily applications seen in the previous lectures of this module.So, we understand why we are talking about large diameter structures. We havea specific choice of material or member which has thin cross sections, means thickness of the material of the member is less. We haveasymmetric cross sections, why, because we are working on different geometric shapes which can effectively disperse the wave loads, ok,so asymmetric.

Now, whentheaxis of transverseloadsaxis means the line of action, I should say the line of action of transverse loaddoes not coincidewiththe centroid orcentroid gravity ormass centre of thecross sectionthen it inducesadditional movement and this movement will createtorsional effect in the section, that is the problem. And generallyasymmetric section, thin sections are good in bending but very weak in torsion. Now the difference between the line of action of transverse force or transverse loads to that of c g is what we call, actually is a shells, ok. We will see this now here in a classical definition of shear centre.

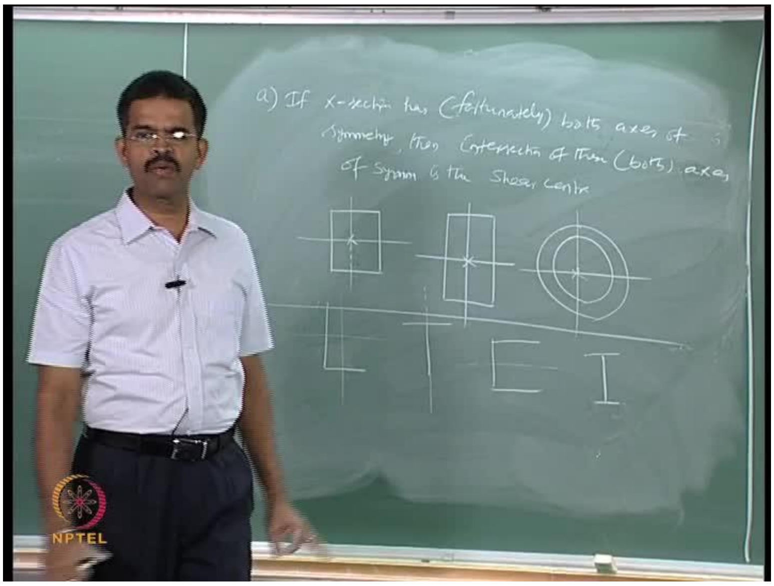
(Refer Slide Time: 35:08)



Shear centre is defined asthe intersection of longitudinal axis of a memberwiththe line of action of transverse loads. Let us have a section, some cross section, asymmetric.This is my c in section.This ismy w, then what 2 is nothing but, the self feeds geometrically mass will axis this point. Whereas this point is what we call as shear centre because this is the point where my line of action of later loads selected. The difference between these 2 is what we call as e, it may lie in the same section, it may lie outside the section also; I will show you. It may lie somewhere here also, the shear centre may lie here also or may lie outside also.

So, if I say this is W and this is the V ray r, because it is a reaction of all the forces,transverse force, then additional movement comes is nothing but V r into e. And of course, we know that V r will be equal to W for static equilibrium, is it not? They should match. Then we also said that W into, so our problem is for a given section what to be the value of e. So, what is the offset of the shearcentre from the centrefor a given section which is asymmetric? Now the question obviously comes if this is symmetric what will happen, if a section has both types of symmetry?

(Refer Slide Time: 38:05)

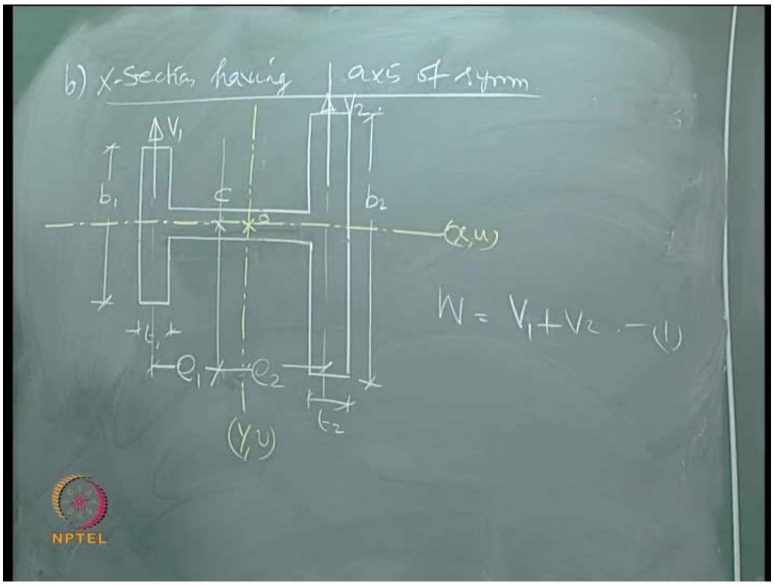


If the cross section has, I should put a word here fortunately bothaxisof symmetryor what I mean to say is the geometric shape of the cross section is symmetric at both the principle axis. The movement you say axis is principle axis, thenintersection of these, these means both, these axis of symmetry isthe shear centre. So, we have no problem at all.

For example,let us say a square, we have 2 axis symmetry, this itself is the shearcentre.This is the geometric centre or the mass centre, no torsion. Rectangle can identify 2 angles axis so no problem, circular no problem, angular no problem. Now the problems are L T, channel, I with unequal planes is it not, so all of them have only 1 axis symmetry. For example, in this case, is there any axis symmetry? In this case, is there any axis of symmetry? No, axis you have own vertical axis symmetry is it not; In this case you have one horizontal axis symmetry.

In this case we have both of the axis if they are equal.If it is not equal then, in this case it is symmetric to both the axis. So if you have sections which is has got both the axis symmetry,fortunately, itis in a geometric shape.We have absolutely no problem.Shearcentre will become coincide with that of geometric centre and the mass centre or centroid, so we have no difficulty of invoking an additional movement which will cause torsion in the cross section, there is no difficulty. So, for sections where there are 2 axis of symmetry we need not have to bother about the shear centre. So, we will talk about b case where sections have 1 axis of symmetry.The shear Centre will lie on that axis but, where it will lie, on this axis, is it here? Here? Where? So we have to locate the shearcentre but, the shear centre will lie on that axis of symmetry itself, one axis.

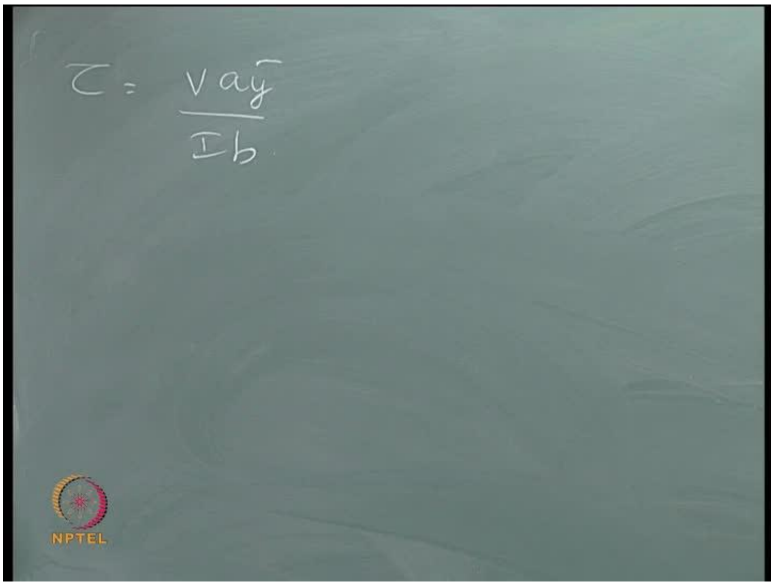
(Refer Slide Time: 41:11)



So, we will take up an example where we have got sections.I should say cross sectionshaving1 axis of symmetry, which is the case what we discussed. How to compute the shear centre for this? So, we draw an example and try to derive this.The shape may be symmetric but, the thicknesses are different. So let us say this is b 1and this is b 2, this is t 1, this is t 2 and we have somewhere the c here and this is my principle axis.I call this asx and u, you call thisy and b, and this is my geometric centroid rope and I call this as e 1 and this as e 2.

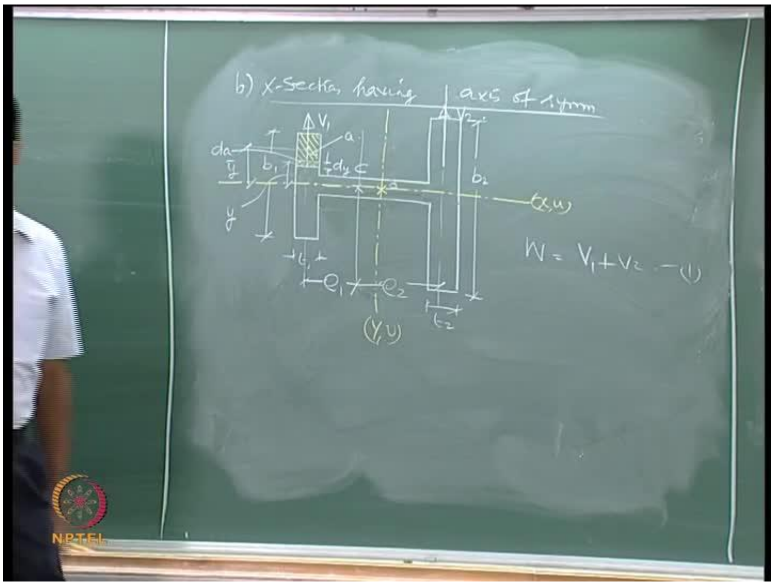
Let us say, the shear resistance of this flangeis V1 and of this flangeis V 2 and I am neglecting the wave. So,I should say that total load acting on the system will be now resisted only by V 1 and V 2, thisis equation number 1.

(Refer Slide Time: 44:58)



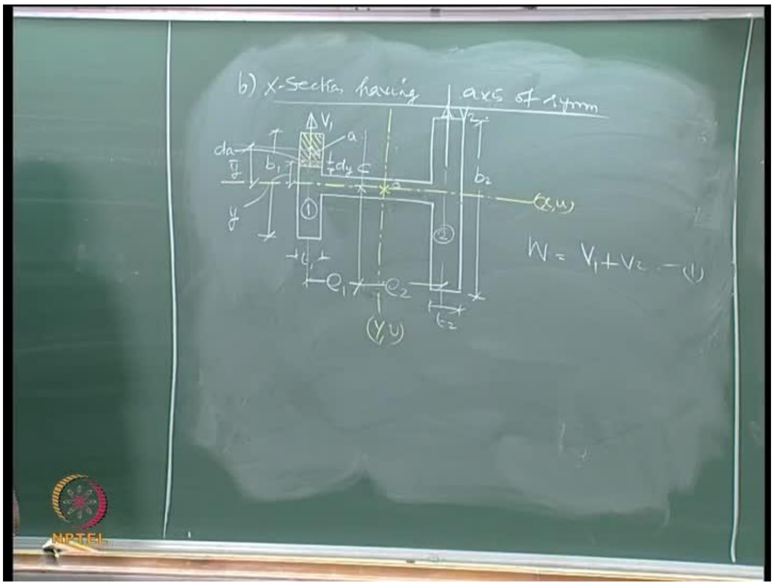
We know shear stress is given by a general equation V a y bar by I b. So, let us take the piece 1 here and mark a strip of area above this. Let us consider this strip which is at a distance y, let us call this as d y.

(Refer Slide Time: 46:07)

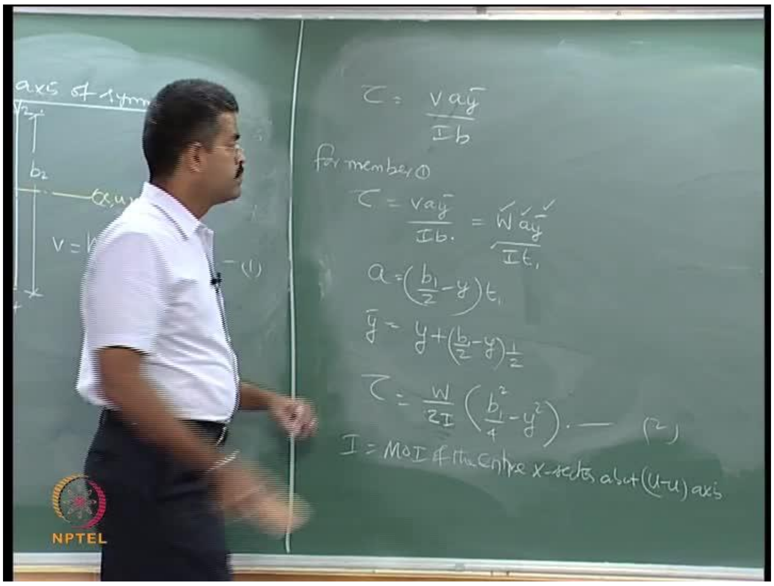


And area about this is what we call as a, and area of this strip alone is what we call as d. So, V is nothing but, the shear force acting at the section at that time, a is the area of above the level of contagion, y bar the centroid of the area respectively line of configuration, I is the movement of inertia of whole section and b is the width of the section under consumption. So, if I say the centroid of this area which I know, this we measured from herethis I should say y bar as per the equation.

(Refer Slide Time: 46:25)



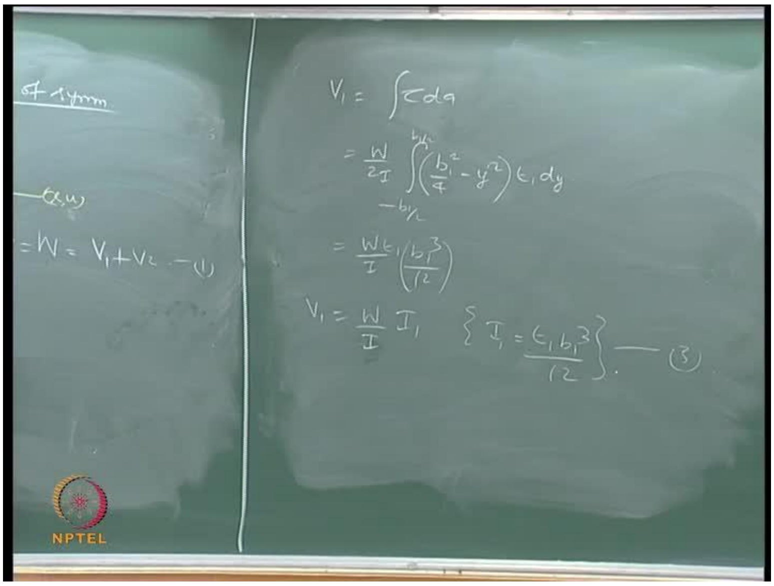
(Refer Slide Time: 47:01)



Let us expand this for member 1; I call this just member for 1. This is my 1, tow is V a y bar by I b. This total V which can be W, I should sayW a y bar by I b and in my case the breath of the section ((recognition) which is t 1, I can say t 1. And what is areaof this piece?It is b one by 2 minus y, eliminating the thickness, into t 1 that is the area is it not? And of course, y bar if the distance of that from here is the axis symmetry is it not? So y bar can be written as, we already seen this is y, I can say y plusb 1 by 2minus y half, half of itb 1 by 2 minus y eliminate the thickness is very small, d y is very small, half half of that.

That is what my distance of the centroid from here. This of the shear centre, is it not? Substitute here and get tow. So, we know V that is w, we know a, we know y bar, I we written as I, t 1 is we written as t 1. So, give me an explanation for tow for member 1.So W by 2 I b 1 square by 4 minus y square, I call it equation number 2, in simply we can find out. In this case of course, I is movement of inertiaof the entire cross section remember that, about in my case going to be u-u axis. Ok, not that part alone whole section. So, I want to find V 1, I know tow, remove this because I want, I am interested in this V 1.

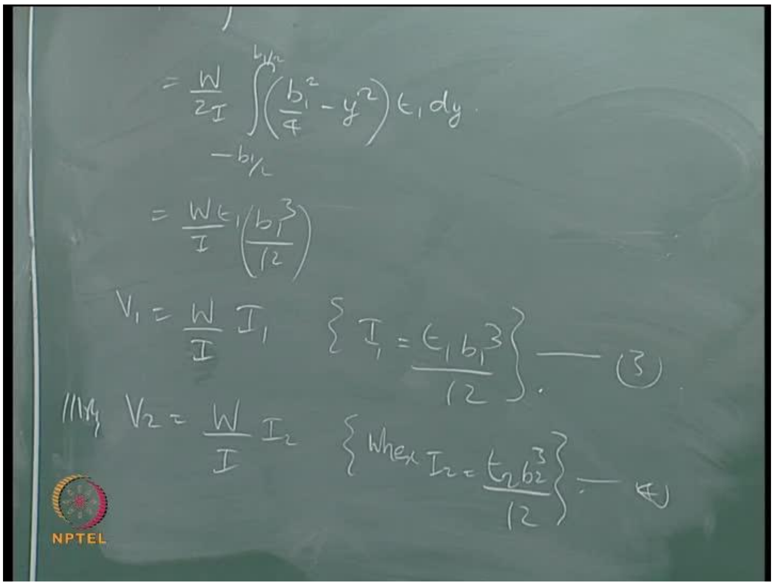
(Refer Slide Time: 51:25)



So, V1is going to be tow d a entire, is it not? So, W by 2 I integral tow value was b1 square by 4minus y square is that right? And d a is this area which is t 1, and we are looking for the whole member so from this point we should say minus b 1 by 2 t 2 plus b

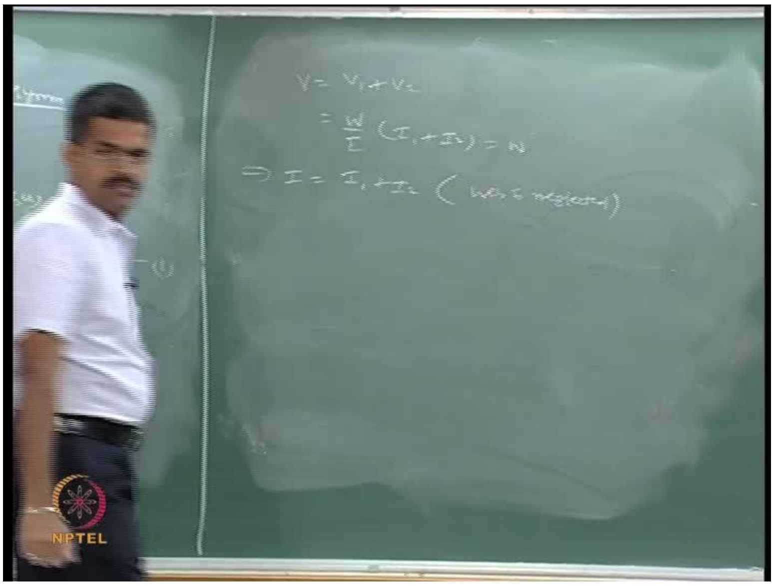
1 by or 0 to by 1 by 2 by or work force integral will get me this quickly.You getW by I, t 1 b1 cube by tow is simplified. I can always say this equation as W by t, sorry W by I into I 1, where I 1 is the movement of inertia of this equation alone about this axis which is t 1b 1 cube by 12 is itok? Call equation 3 I think.

(Refer Slide Time: 54:02)



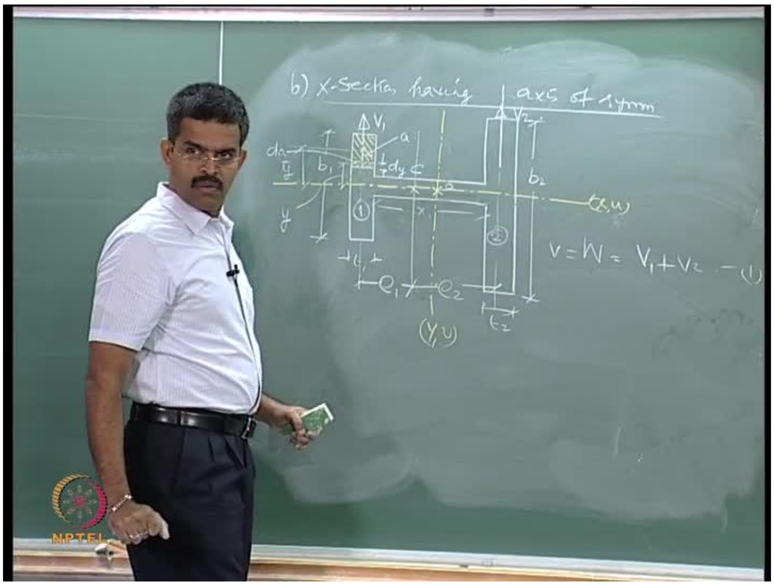
So similarly, I can always say V 2, please do it instead of b 1 you will get b 2, do the same axis again it will be W by I of I 2 where I 2 is t 2 b cube by 12. Now total V, I am neglecting the V by this web, only flanges.

(Refer Slide Time: 54:43)

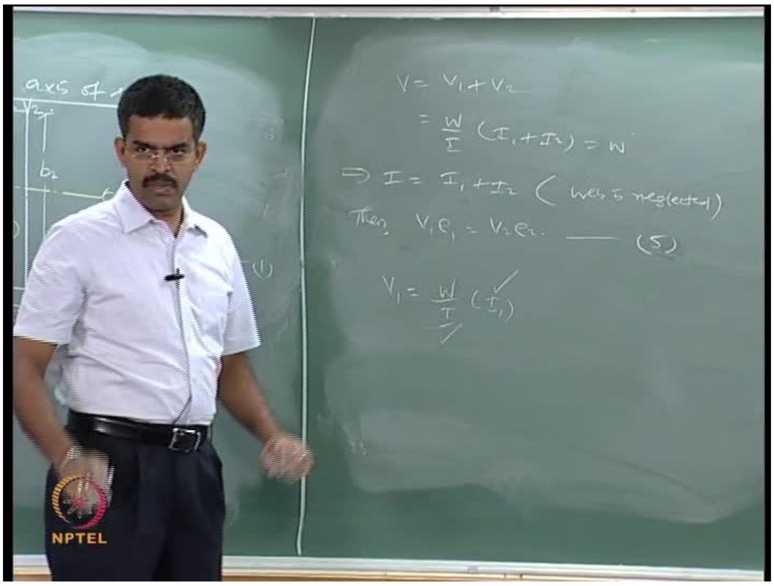


So, total V as we see from equation 1is V 1 plus V 2, which is W by I ofI 1 plus I 2 is thatok, which is also equal to W. So, what does it mean, it implies that the total movement of inertia is only sum of I 1 plus I 2; web is neglected. Remember that because web also has movement of inertia which will be this dimension, e 1 plus e 2 minus t 1 by 2 this dimension. If I know this is x 1 x 1 t q by 12, the thickness is very very small; you can neglect that is why getting this relationship.

(Refer Slide Time: 55:29)

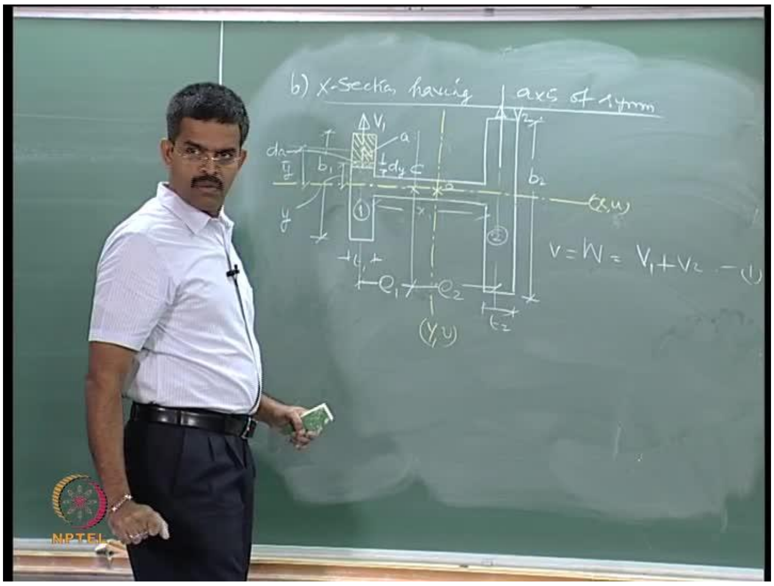


(Refer Slide Time: 55:53)



Then taking movement about this point c V 1 into e 1 is V 2 into e 2. So, you know the relationship between e 1 and e 2.

(Refer Slide Time: 55:29)



For a given section of course, this dimension is known to youtherefore, if e 1 is known we can find e 2; V 1 and V 2 are already known to me. How V 1 is nothing but, what is V 1? W by Iof I 1, I is known to me, W is given to me I can find e 1. Similarly V 2, so I can get the ratio of V 1 and V 2, I can express it and find e 1. So I can locate the shear centre in the given.

This is one example where the sections are symmetric about 1 axis; it is not symmetric about the other one because the thicknesses are different. So in that case how will youlocate the shear centre? So there are 2 things we answered in this lecture, 1 few design examples understanding that how the selection of diameter for a given simple example can varywhen you apply different theory. The second is what is the shear centre, what is its important in geometrical design for marine structures, when the sections have fortunately 2 axis of symmetry then we have no problem at all, shear centre will be going inside the centre of the section, then there will be a problem and inducing movement which is torsion which is generally thin asymmetry section as we select to choose for marine structure are very good in bending but, they are very weak in torsion. So ,whenever a section is subjected to additional is torsion, we must locate the shear centre and check for its shear stress exceeding permissible limits.

So,this is one of the important aspects of design in marine structural members. In the next example, next class, we will take a few more examples case centre and try to solve some more sections and then we are more want to the design check for members and as suggested by different course, ok!.