Advanced Marine Structures
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Lecture - 25
Theories of failure – IV

So, we willcontinue to discuss on the topic on advanced marine structures. In the last lecture we discussed about four theories of failure,we understood the importance of why we have to study the theories of failure because, when you take the simple tension test as the reference or the benchmark test based on which, we compute the yield point stresses which is considered as one of the failure criteria, when the stress reached yield point as far as plastic design is concerned. But simple tension test has parallely other problem, that many of the values simultaneously reach their optimum value like maximum shear, maximum strain energy, principle stress, etcetera.

So, we reallydo not know which will yield or which will lead to the actual category of failure, will it be because of yieldingor will it bebecause of fracture,andwhat is the application of this theory? Because in certain domains,and incertain theories we said in quadrant one and three they do not agree, and quadrant two and four there are serious discrepancies.

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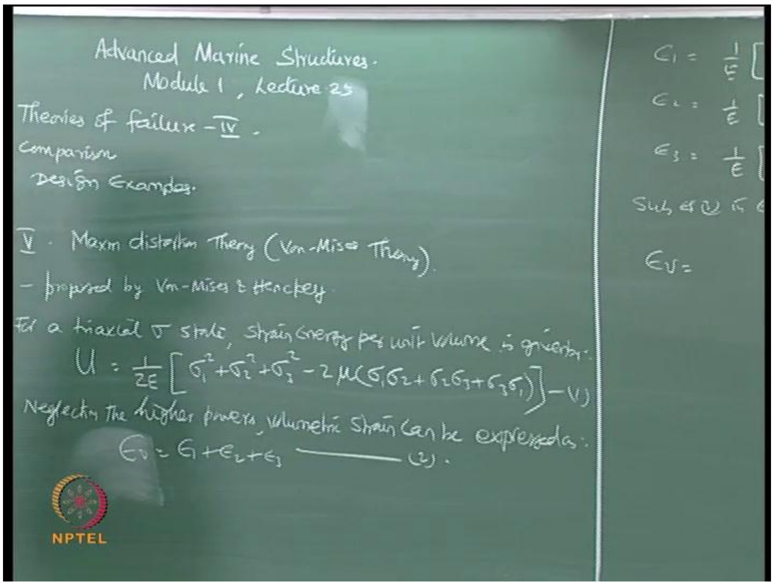

The fifth theory which will discuss todaywill be the maximum distortion theory;it is also called as von Mises theory.This theory was proposed by von Mises and Henckey as we saw yesterday. So, the statement of the theory is already given in the last lecture; we will not repeat it here.We willstraight away go to the equation. We already said for a triaxial stress state, the strain energy per unit volumeabsorbed by the body is given byequation one.We can say neglecting the higher powers.

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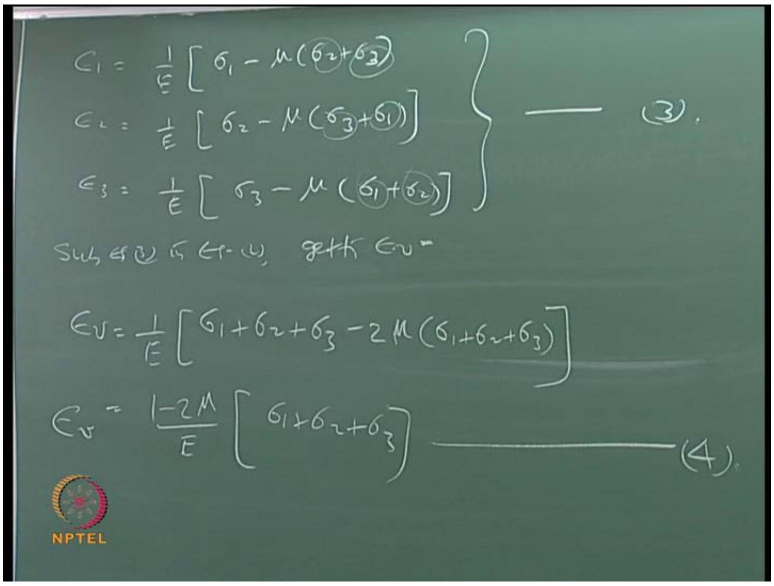

The volumetric strain can be expressed asepsilon v is 1, 2, and 3 where epsilon 1 epsilon,2and epsilon 3;1 by E of sigma 1 minus, sigma 2 plus sigma 3; this is three, 1 by E of, I call this as equation number three. I can substitute three and two and get E v. So, substitutingequation threein equation two,getting the volumetric strain; the volumetric strain simply substitute and summarize I can write it like this1 by Eof sigma 1 sigma, 2 sigma, 3 minus, 2 mu because each of of them have two products. You can see here sigma 2 is twice, sigma 1 is twice, and sigma 3 is twice,2mu of sigma 1 sigma 2 and sigma 3. So, we can also write this as1 minus 2 mu by E of sigma 1 sigma 2 plus sigma 3,call this as equation number four.This is my volumetric strainneglecting the higher powersof the equation one. So, if you look at the equation four we can make a very important statement.

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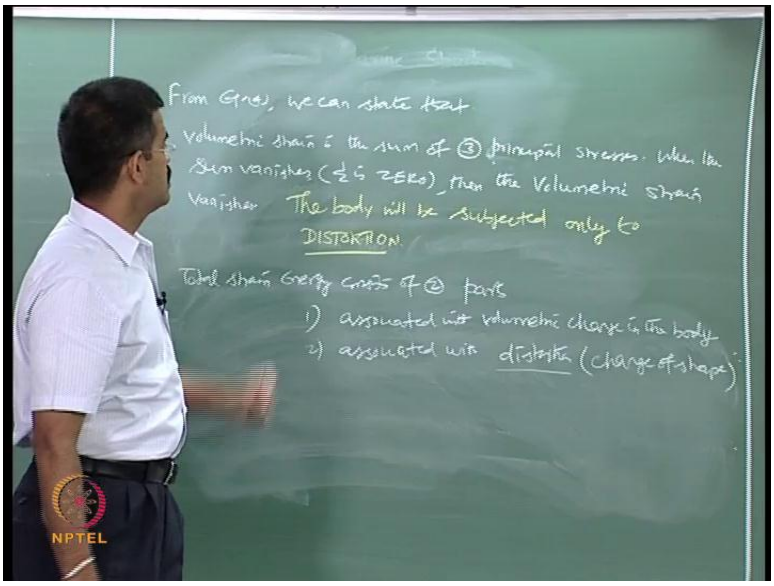

From equation four,we can state thatvolumetric strainis the sum ofthree principle stresses.Whenthe sum vanishes or when the summation in zero, then the volumetric strainvanishesfrom this equation. So, what does it mean?It means that the body will be subjected only to distortion, that is what is, volumetric strain will vanish. Now the total strain energyconsists of two parts,one which is associatedwith volumetric change in the body,the otheris associated withdistortionwhich is otherwise the change of shape.Remember change of shape and changes of volume are not same, only the shape changes, volume may remain same. This is what we call as distortion. So, for zero volumetric change, I want to study the maximum distortion theory, thereforeI should say that the volumetric change the body is set to zero, for zero volumetric change, as you see from the statement, the summation on the principle stresses should be equated to zero. Let us do that.

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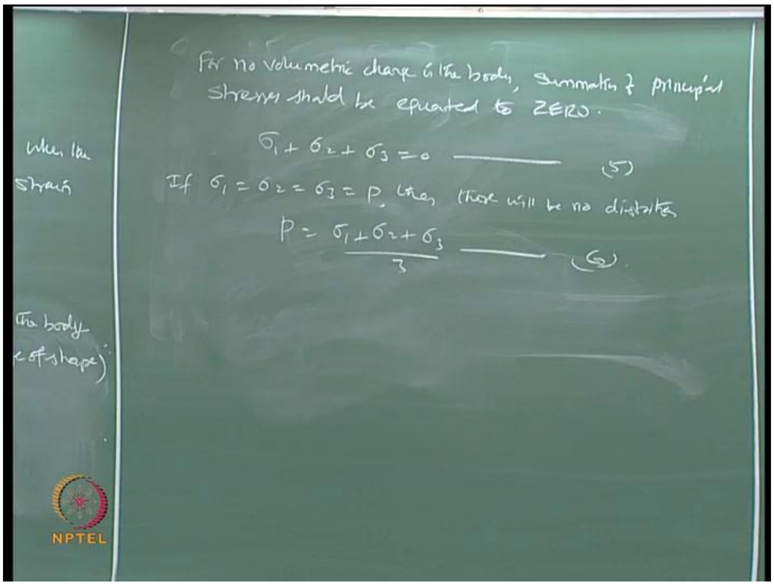

So,forno volumetric changein the body,the summation of principle stressesshould beequated tozero.Then you will get pure distortion. Let us say sigma 1 plus sigma 2 plus sigma 3 is set to 0.Now if sigma 1 equal sigma 2 equal sigma 3 is P,thenthere will be nodistortionbecause the summation is not 0. So, let us say P is now equal to an average of this. If P is set as an average of this, it means we can schematically express this.

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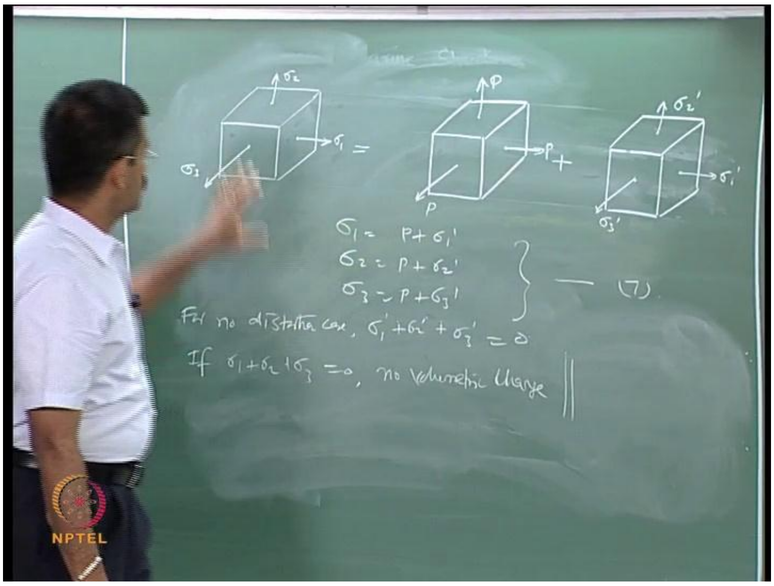

Let us say, thatwill be equal tosummation of two things. What you wanted isa tiraxialstress state; I can say now this will be equal to,we already said they are equal to

Pand there is not going to be any distortion. In that case, we say that the stressesadditionally created herelet us call them as sigma 1 dash2dashand 3 dash respectively.So, we can write simple equations saying that sigma 1 is P plus sigma 1 dash, sigma 2 is P plus sigma 2 dash, sigma 3 is P plus sigma 3 dash.Let us call this equation number 7. For no distortion case, sigma 1 dash sigma 2 dash sigma 3 dashshould be set to 0.If this exists which is not equal to 0, but if sigma 1 plus sigma 2 plus sigma 3 is set 0, no volumetric change.For sigma 1 dash sigma 2 dash sigma 3 dash set to 0, no distortion in that case,two exquisite casesbut the triaxial stress state is the combination of this, what I am saying is instead of 1 2 and 3 I am saying this as P axis. So, this is going to be a combination of these two cases.

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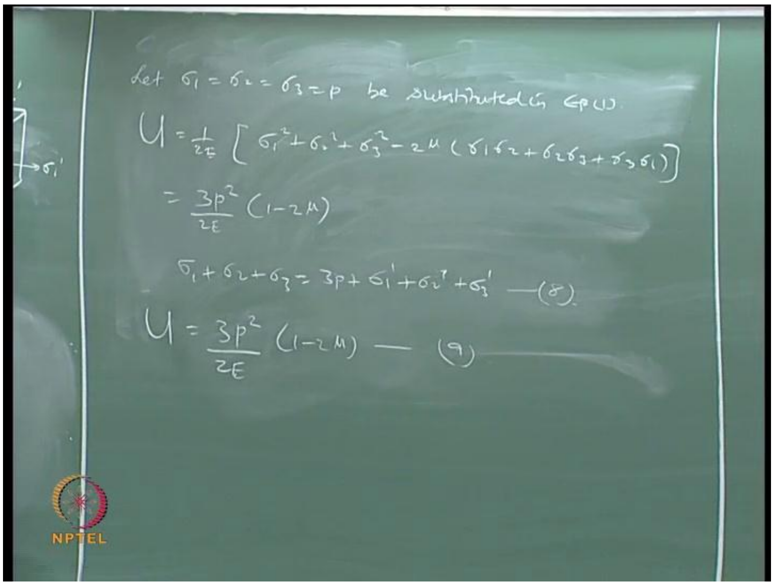

In that situation,let sigma 1 sigma 2 and sigma 3 is Pbe substitutedinequation one; equation one is the total strain energy value.Let us see the equation one. I am rewriting it here U is given by 1 by 2Eof sigma 1 square sigma 2 square sigma 3 square minus 2 mu ofsigma 1; is this the equation 1.So substituting this, I may get3 P square by 2E1 minus 2 mu. Already we can also say from this figure that sigma 1 plus sigma 2 plus sigma 3 is 3P plus sigma 1 dash sigma 2 dash plus sigma 3 dash.I call this equation number eight. So, now I can say U is 3 p square by 2E of 1 minus 2 muequation 9.

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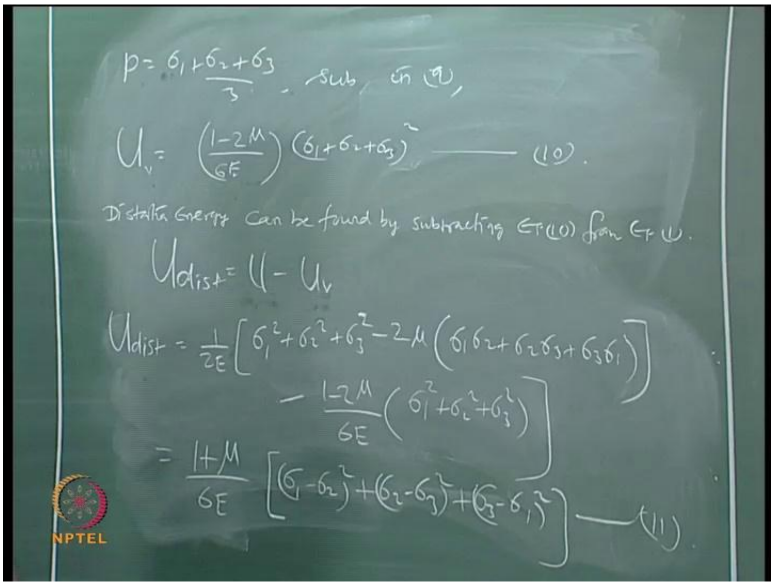

So, we already know P is sigma 1 plus sigma 2 plus sigma 3 by 3. Sosubstitute in nine,it becomes 1 minus 2 mu by E of this 3 and this 3 goes away, simply sigma 1 plus sigma 2 plus sigma 3 whole square, this isequation 10. So, if you really wanted to find the distortion energy, itcan be foundby actually subtractingequation ten frorm equation onebecause equation is a total strain energy which has volumetric change plus distortion both. I subtract this from one, I get the distortion only.So, let us do that. So, U distortionis Uminus U volumetric. So, let us do that U distortion.Get this equation, let us see,We have equation oneis showing U and U v is above. This is actually volumetric.This is going to be 1 minus 2 mu by 6E no, because this is square. This is going to be 6E; please make a change here.

When you substitute it in equation nine, because it is square, it is 6E here,1 minus 2 mu by 6E of the squares of this.So, substitute form U minus U v and getmu distortion, see what equation we are getting. So, it is going to be 1 by 2E of sigma 1 square sigma 2 square sigma 3 square minus 2 mu ofminus1 minus 2 mu by 6E. So, after simplifying 1 plus mu by 6E of sigma 1 minus sigma 2 the whole square, sigma 2 minus sigma 2 the whole square, sigma 3 minus sigma 1 the whole square is equation 11. Sigma 1 minus sigma 2 1 plus mu by6E, you can simply and check. So, for any uniaxial state sigma 2 and sigma 3 is going to be zero.

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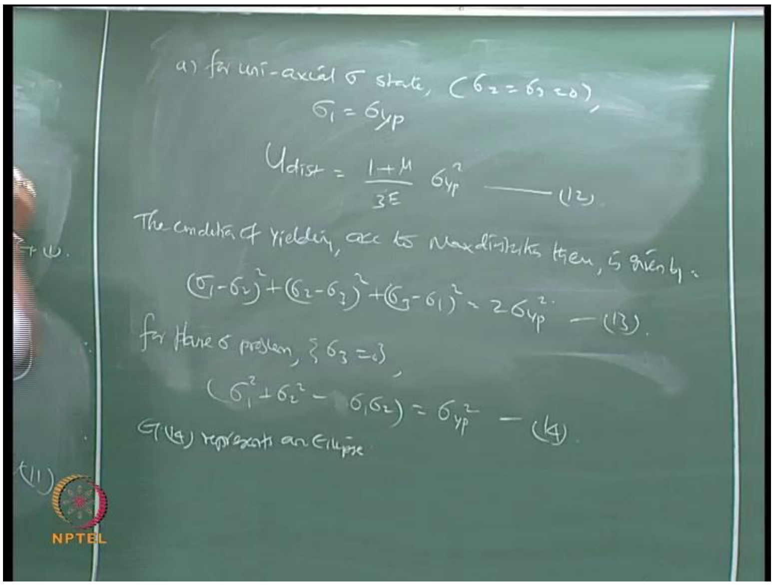

So, U distortion and we all know that sigma 1 is going to beyield value, that is the definition of the theory. So, U distortion is going to be what will be the value?1 plus mu by 6Eofsigma y p square of twice because there is one square here. So, I can say this asequation twelvebecause there are two squares here. One is sigma 1 here also,one sigma 1 here also;remaining all are zero. So, the condition of yieldingaccording to maximum distortion theoryis given by sigma 1 minus sigma 2 the whole square2 sigma y p square, call this equation number thirteen. Sofor a plane stress problem,this equation can be that issigma 3 is 0 by axis stress state.

This equation can be sigma 1 square plus sigma 2 square minus 2 sigma 1 sigma 2 is going to be 2 sigma y p square, the 2 goes away because there are twohere y p square,which is again an equation of an ellipse. That two goes away,this two here in all the cases, the two goes away. It is an equation of an ellipse,say, equation 14represents an ellipse. So, let us drawthe failure boundary as indicated by the maximum distortion theory and see what are the discrepancies in different quadrant as we saw in the earlier theories. Let me draw these theory boundaries.

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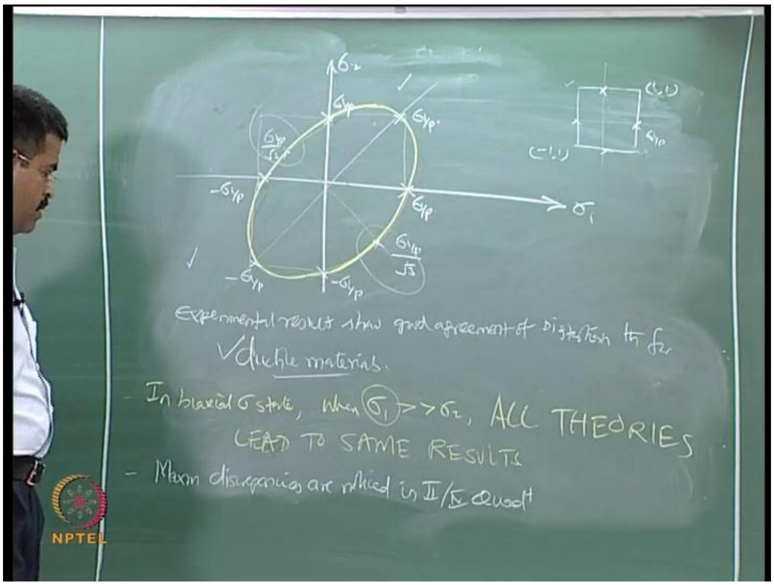

So, these valuesare all sigma y p'sand the minor axis values will be sigma y p by root 3.You can check thatand this value is more or less equal,my figure may not show that. Even this is also y p,this is also sigma y p. So, I can very well see this theory in the firstand third quadrants it is agreeing more or less with the maximum principle stress theorywhereas in quadrants two and four, the maximum principle stress theory says that this stress will be equal to sigma y p because this is the maximumprinciples stress theory.So,in quadrants two and four, this theory says that the maximum stress will be equal to the yield value, whereas in this case it is only 1 by root 3 of that.

So, there has been a good discrepancyin quadrants two and four given by the distortion theory compared to that of the maximum principle stress theory. So, it has been seen experimentally thatexperimental resultsshow a good agreement ofdistortion theoryfor ductile materials.So, this theory is very good for ductile materials. There is a very important statement you want to make here, whenin a biaxial stress state,when sigma 1 is very large compared to sigma 2,when one of the principle stresses is very large compared to the other, all theoriesleadtosame results.What does it mean? All these theories holdgood by concluding foran uniaxial stress state.

The problem starts only when it is biaxial or triaxial which is a fact and reality in case of marine structures.When the value is very large compared to the other, then all theories will yield more or less the same results. The second observation we can make ismaximum discrepanciesare seenin second and fourth quadrants, that is when the stresses are unlike in nature, one in tension, one incomparison.Let us quickly compare all the theories,So, I want to draw superposing all of them in one figure. I think we have now drawn the boundaries separately, I want to draw a figure where I can superpose all the theories, then discuss thecomparison.

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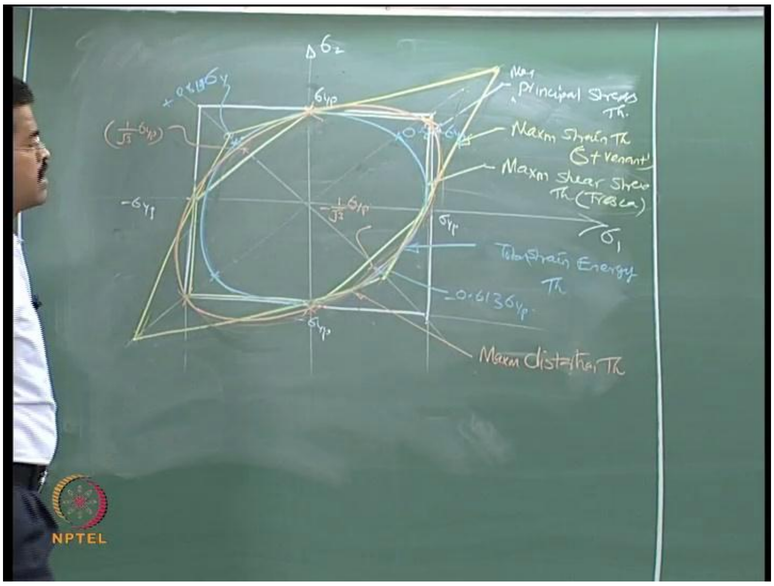

So I am going to draw that figure.This is sigma 1,this is sigma 2,this is sigma y p.these are all positiveminus sigma y pminus sigma y p.Let us draw with, please open back your notes and see how we draw the principle stress theory boundary. I am drawing it here, if I am wrong please correct me. This is my principle stress theory boundary. I can write it here based on the Rankin’s principle. The second theory which we will draw will be the maximum strain theory. So, tell me what is the specialty about the maximum strain theory given by St. Venant’s?St.Venant’s, what is that?What will be the shape of the boundary? It is going to arhombus.

So, here the values can be even more thansigma y p whereas here the values are lesser than sigma y p and it will touch sigma y p here.More or less this is how my boundary will look like. So, let me draw this.See this has to pass through sigma y p. It has to pass through sigma y p. This is mymaximum strain theorygiven bySt. Venant’s.Let us talk about the maximum shear stress theory given by Tresca, irregular hexagon it came, I am drawing it just adjacent to this sothat it is also visible.It is one and the same but I am drawing it adjacent. So, this is mymaximumshear stress theory; it is given byTresca.

Let us draw the total strain energy theory.This ismaximum strain energy theory,not maximum, total strain energy;it is not maximum. There is nothing like maximum strain,total strain energy.So, what are these values?We already have them what are these values? They are major axis;major axis sigma y p, is it. Total strain energy theory, I think it is 0.87 no,why you people are sleeping in the class?You people are sleeping in the class, is it?0.87 sigma y p and what are the minor axis values?0. 6163 y p of course minus. Thisis the total strain energy theory. Now we will plot the maximum distortion theory.

Just now we plotted it here. They all will touch sigma y p in all the corners;all thesigma y p in all the corners. Soone here, and of courseall these points will be touchedand soon and this valueis this plus. What is 1 by root 3? I get an ellipse like this. So, this is mymaximum distortion theoryand this value is 1 over root 3 of sigma y p which I already engraved it here, of course minus and this value is 1 over root 3 of sigma y pwhich is 0.57.Now you can easily writegood amount of inferences form this figureon thesecomparisons of theoriesin single domain.

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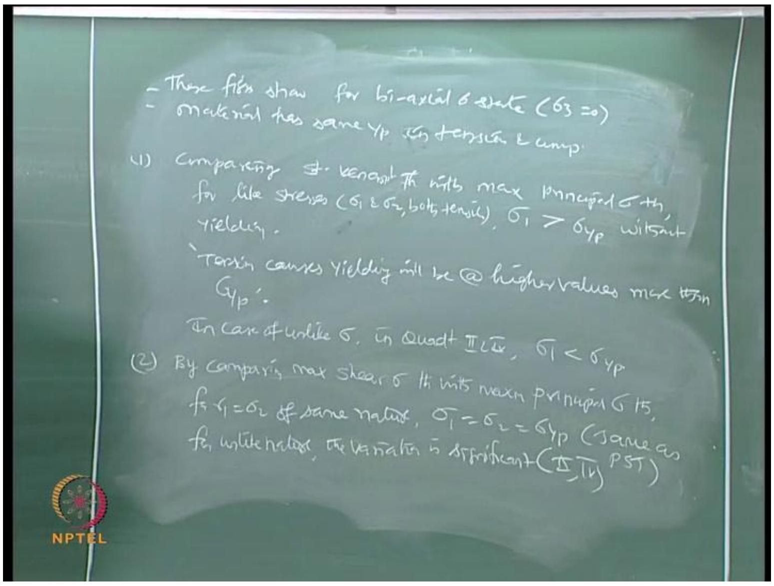

So, these figures show for biaxial stress state,that is sigma 3 set to zero. It has an assumptionmaterial has the same yield pointintension and compression. Let us quickly compare St.Venant'stheory with maximum principle stress theory.By comparing St.Venant's theory which is the maximum strain theory, this one the rhombus with the maximum principles theory which is the white one, it shows forlike stresses,that is sigma 1 and sigma 2,both tensile are compressive. Magnitude may remain same sigma 1 might be equal to sigma 2, but they are of the same nature.So, you should look at the quadrants of one and three same nature,maximum principle stress can even exceed sigma y p withoutyielding.

What does it say is,tensioncausesyieldingwill be at higher valuesmore than sigma y p,that is what the statement is.Tension will be more than sigma y p.Of course in case of unlike stresses that is in quadrantstwo and four, sigma 1 is far lesser than sigma y p, you can see here. This is my rhombus point, it is far lesser than sigma y p.Now I am comparing the maximum shear stress theory that is the green one, the irregular hexagon with the principle stress theory. By comparingthe maximum shear stress theorywith maximumprinciple stress theoryfor stressesof same nature, sigma 1 equal sigma 2 is equal sigma y pwhich is as same as principle stress theory.For unlike nature,the variationis significant,that is in quadrants two and four, see here.They are significant.The third inference, can I rub this, are you able to follow my hand writing.

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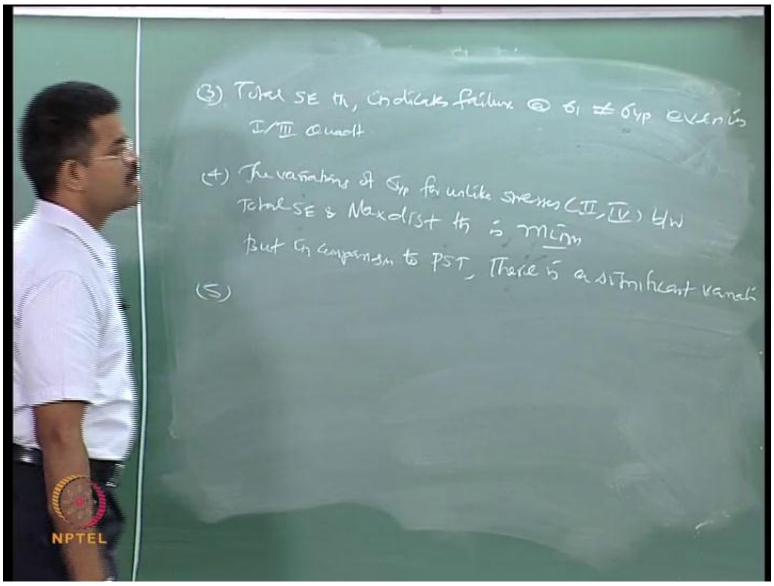

The third inference isthe total strain energytheory indicatesfailureat sigma 1 not equal to sigma y p evenin firstand third quadrants, that is very dangerous. It is only

87percent;0.87 it yields.0.87it yields, what does it mean? When your point at the stress state lies above this boundary or outside this boundary somewhere here, according to the total strain energy theory, which is shown in blue color here, failure hasoccurred. But according to principle stress theory, according to St.Venant'stheory failure has not occurred. Sothat is a discrepancy, one theory says failure occurred, others theory says it has not occurred,it is much below sigma y p.

So, what does it mean? If you send plastic analysis, you take the stress till sigma y p and then only the failure starts, if your assumption is that, this is not holding good or valid as far as this theory is concerned. Because this theory says when the stresses are alike sigma 1 sigma 2 are same nature and magnitude equal, failure will occur even at 87 percent of sigma y p itself. So, that is a dangerous interpretation. Our results of sigma y p occurring causing failure for plastic analysis and design could be over conservative, could be dangerous as far as this theory is concerned.

Now let us see the second and fourth quadrant more in detail. The variationsof sigma y p for unlike stresses, so I am focusing on quadrant two and four. Betweentotal strain energy theory, the blue one,between the total strain energy theoryand, the orange one, maximum distortion theoryis minimum; they closely agree. What does it mean? This information can be interpreted as change of shape in the body will not cause serious deviation in failure. Maximum strain energy includes volumetric as well as distortion, maximum distortion talks only about change of shape. They agree in unlike stresses two and four.

Soif the stresses are unlike in nature, one in compression, one is tension, same magnitude change of shape of the body will not significantly influenced the failure stress or yield stress but in comparison toprinciple stress theory, there isasignificant variation.Why? This theory says at 60 percent of yield, failure has started where as principle stress theory says only at yield, failure will start. What does it mean? This is for ductile material, this is for brittle material.So, that is yielding, this is fracture.Though both of them talk about yield point only, remember this is also sigma y p, this is sigma y p percentage. Both of them are talking about sigma y p multiplied only but this is for ductile, this is for brittle.

This is what the failure theories are, and this is what the discrepancies are.This we must understand very clearly before we really understand what we do for plastic analysis and design. Soin the next class, we will take about it as an example and then we will talk about very important part inmarine structures where we talk about shear center. Shear center is a very important aspect of calculation. It is one of the design aspects to be computed and very important because this induces torsional moment in the membersin marine structures. When this kind of torsional moment will occur what is the shear center, how to compute it for geometric sections we will see. We will also do couple of problems on usingthe design examples of this principle, I mean thisfailure theories, couple of example,then we will move on to shear center in the next class. Do we have any questions or any more important information or interpretationwhich I left by comparing these theories.

The most important and eye-catching information which you must not miss out is the commentor the focus of St.Venant'scompared to principle stressand the focus of total strain energy theory in quadrant one and three compared to principle stress. These are the two important eye -catching, say, predominantly or significance seen variations in your comparison of failure theory.So, the necessity of understanding failure theories, the necessity of appreciating the deviations of the failure theories in unlike stress regions and like stress regionsis important. As a prerequisite for us to really know what we really mean by yielding for plastic analysis and design which we said or which we discussed in the earlier lectures.

So, this lecture terminates the discussions on theories of failure. This is only just a curtain raiser. If you want really you can look into them in biaxial andtriaxial stress state in detail, I think you should refer to some of the fracture mechanics lectures in detail for the materials.So,we have just as a prerequisite, we must understand, we must know this but still for the completion of the viewer's interest, we have discussed this in detail as far as far as possible applicable to plastic design alone. Ofcourse we willapply this into examples and show how these theories actually give me numerically a different meaning and as a designer how I am, let us say, influenced by these results, we will see that in next class.