Advanced Marine Structures

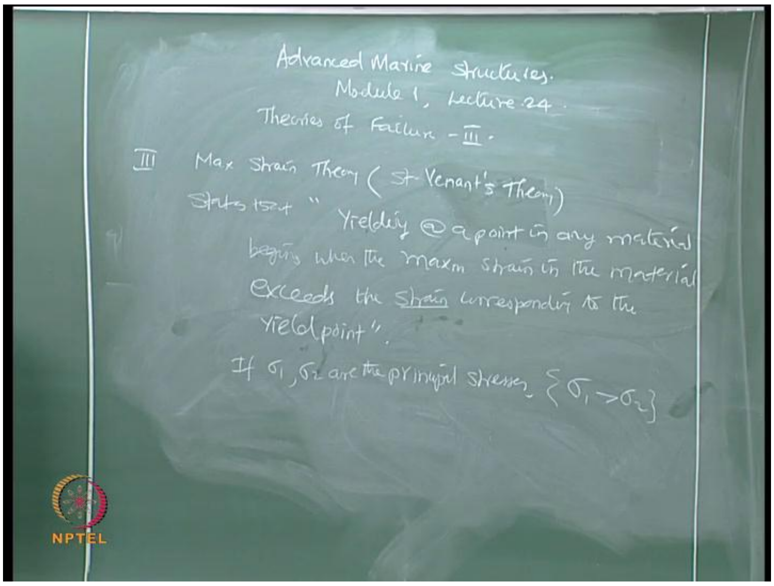
Prof. Dr. Srinivasan Chandrasekaran
Department of Ocean Engineering
Indian Institute of Technology, Madras

Lecture - 24
Theories of failure - III

So, in the last lecture we discussed about 2 theories of failure. One is the maximum principle stress theory which says that, when the maximum principle stress reaches the critical value which can be either in ultimate stress or the yield value failure occurs and failure can be due to excessive deformation which can be yielding phenomena. It can be also by a fracture which happens in a brittle material. So, the theory states very clearly the behavior in the first and second, third quadrants, where the stresses are like in nature, may be either they both are tensile or both are compressively given bi-axial stress state. But when they are unlike then the theory is not able to predict the behavior properly. The other theory says the next theory which we talked about is the maximum shear stress theory which is given by Justin Taraska.

The theory says that, the maximum shear stress value can be that ultimate value reached in at a yield point in a simple tensile test, which is exactly equal to 50 percent of the yield strain or the yield stress the material. So, tau y p can be said as 0.5 of sigma y p then the failure occurs and it is proved experimentally that this theory holds good validation for brittle materials. But the problem with this theory is that in a tri-axial stress state when all the magnitudes are almost equal then the shear stress is very negligible or negligibly very small. In that case, this lead to a brittle failure which is very good for brittle material. So, one should use principle stress theory in such situations.

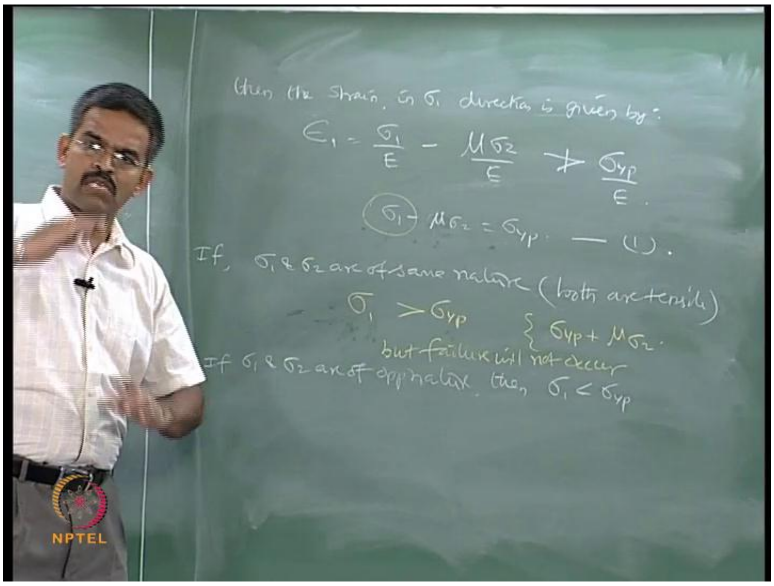
(Refer Slide Time: 02:00)



The third theory which talks about failure is maximum strain theory. This theory was given by St. Venenant and named as St. Venenant's theory, sorry Venant, St. Venant's theory. This theory states that yielding at a point in any material begins when the maximum strain in the material exceeds the strain corresponding to the yield point. You will agree that for every stress state, there is a corresponding strain happening parallel. So, look at the yield point stress and the corresponding strain if the maximum strain in the any fiber of the materials reaches or exceeds this value then the theory says yielding will start occurring.

Now, as I told you in the beginning in a simple tensile test all this phenomena will occur simultaneously when you subject a material or an object to uni-axial stress state. When you subject them to bi-axial or tri axial stress state there is no guarantee which will occur when, because we saw in the last Tresca's theory when the minors are equal then the shear will not be influencing the failure. Therefore, it is ruled out. So, in this state it says that yielding will start occurring at a point only when the maximum strain in the extreme fiber or any fiber in the material reaches that strain which corresponds to the yield strain or yield stress point of that material. So, let us say if sigma 1 and sigma 2 are the principle stresses.

(Refer Slide Time: 05:20)



For a bi-axial stress state, sigma x and sigma y are given to you, you can easily find out sigma 1 and sigma 2. I gave an equation in the last lecture and agreeing that sigma 1 exceeds sigma 2. I mean it is larger by magnitude then the strain in sigma 1 direction is given by the following equation, this should not be greater than sigma Y P by E that is the condition is it not it should be either equal to or cannot be greater can be lesser than that.

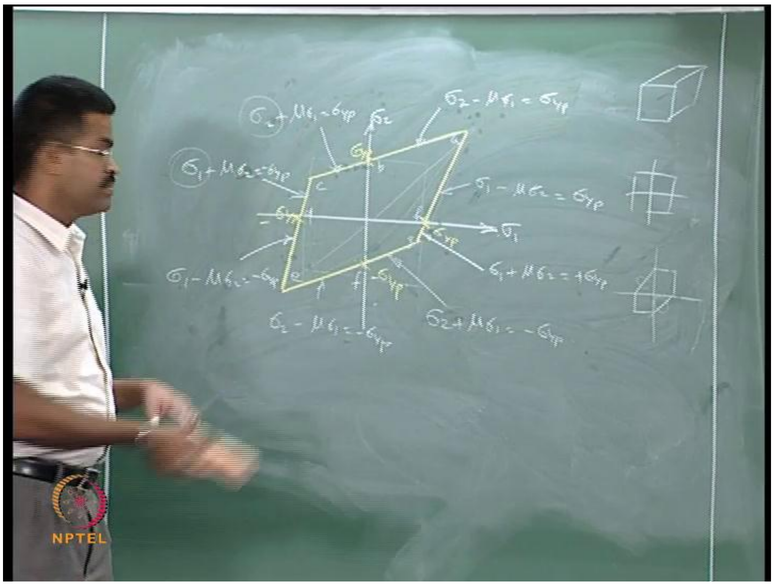
So, this condition says that sigma 1 minus mu sigma 2 should be equal to sigma Y P. So, if I call this equation number one, look at this equation very carefully is very interesting. If sigma 1 and sigma 2 are of same nature, let us say for example, both are tensile. Now, what is the equation mean, this equation tells me that the maximum principle stress can even be more than sigma Y P, because is going to be, is it not.

But, still failure will not occur. Is it not, because I am equating it and finding out the value of sigma 1, so there is a problem possibility that the maximum principle stress according to this theory can even exceed the yield value. Even if it exceeds the yield value failure will not occur. That is very important because, I am only equating it here now with this argument. The argument of principle stress theory says because the principle stress theory says when the maximum value reaches the ultimate of the critical value failure occurs. But, in this case theory allows sigma 1 by magnitude even it more than sigma Y P if sigma 1 and sigma 2 both are of the same nature.

So, what do you mean by this, In Quadrant1 and 3, Quadrant1 and 3 where the stresses both can be either tensile or compressive, the magnitude of stress at failure can be even larger than sigma Y P also, is it not, Is it clear are you able to appreciate difference. But, of course, if sigma 1 and sigma 2 are of opposite nature then we agree that sigma 1 will be much lesser than sigma Y P. Right, so in Quadrant 2 and 4 the value is different. It is lower than sigma Y P in Quadrant1 and 3 the value more than sigma. Is it not, so it is never equal to sigma Y P anywhere in any of the Cordiant, is it clear?

Is it clear? Whereas in Tresca's theory which is a maximum shear stress theory. In quadrant 1 and 3 the stresses are equal to sigma Y P. In maximum principle stress theory all the 4quadrants thus where equal to sigma Y P. In the third theory, in quadrant 1 and 3 the stresses are greater than sigma Y P in quadrant 2 and 4 they are lesser than sigma Y P, it is a different theory here and more interestingly this theory says even though the stresses exceeding v failure does not occur. So, far we have been thinking the both the theories that failure will occur when it reaches sigma Y P is it not. So, there is the deviations, let us try to plot this theory and see what happens.

(Refer Slide Time: 10:09)



So, this theory profile will look like a rhombus because the values are in 1 and 3 more than sigma Y P in 2 1 4 they are lower than sigma Y P. So, let us try to plot 4quadrants, is the positive values. So, this will be a rhombus.

So, this is sigma 1, this is sigma 2 and all these values whatever you see here these are all sigma Y P. So, we can write down the equations for these profiles separately, let us name them as a, b, c, d, e, f, g and h.

So, a b, this a b talks about an equation where sigma 2 minus mu sigma 1 is sigma Y P. Both are tensile, isn't not; both are tensile first coordinate whereas this talk about sigma 1 minus mu sigma 2 is sigma Y P. You may wonder how may I am writing this equation, It is very simple; look at this line whichever stress amongst 1 and 2 touches sigma Y P look at that there is a possibility that the stress can exceed sigma y p that is the guarantee is not.

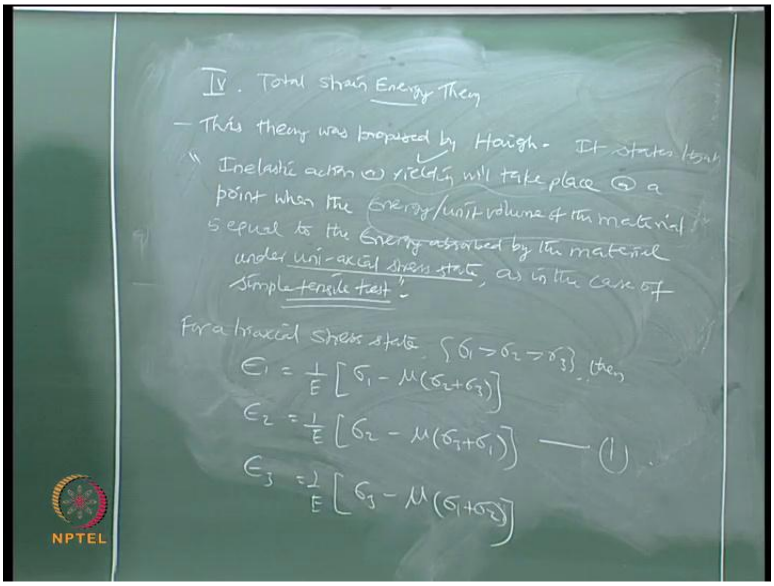
So, in my case sigma 2 is already sigma Y P. Now, I have to show that sigma 2 can exceed sigma Y P, that is why the sigma 2 minus mu Y P. Similarly, in this case sigma 1 is already touching sigma Y P. I have to show that it should exceed sigma Y P, so sigma 1 minus mu sigma 2 sigma Y Pis it not. Let us look at this quadrant, Ya sigma 1 we should say sigma 1minus mu sigma 2 is minus Y P then off course this line sigma 2 minus mu sigma 1 is minus Y P. So, first and second quadrants are defined, let us come to the second and fourth quadrant, talk about this line.

So, this line says if both the stress is of the same nature then the corresponding principle stress value will be lesser than sigma Y P. So, I want to write the equation for here already sigma 2 is sigma Y P. So, I should say sigma 2 plus mu sigma 1 is sigma Y P. Now, sigma 1 is negative in this quadrant, is it not? And is compressive. So, I would substitute minus here when this goes this side it will be opposite. Similarly, here sigma 1plus mu sigma 2 is sigma Y P, so ultimately sigma 2 as well as sigma 1 will be lesser than sigma Y P.

By aggregate value of mu sigma 1 or mu sigma 2, respectively they will be lower than sigma Y P. Let us talk about this line, can you write down the equation for this line that is f g good sigma 2, yeah yeah and obviously for this 1 ya, oh this is positive is it not sigma Y P is positive. Here, is negative your sigma 2 is negative, sigma 1 it is positive is it not, which one.

You mean to say this quadrant, no that top this one, top, this one, ha this will be thank you, this is will be negative sigma 1correct. Now, we can see here that in part 1 and 3 the principle stress values on stretched beyond sigma Y, in quadrants 2 and 4; the principle stress values are compressed less than sigma Y P. This is not been shown in earlier 2 theories because first theory said it is simply a hexagon or it is simply a profile is like this is it not whereas the Tresca's theory said like this. So, in second and fourth the Tresca compressed it.

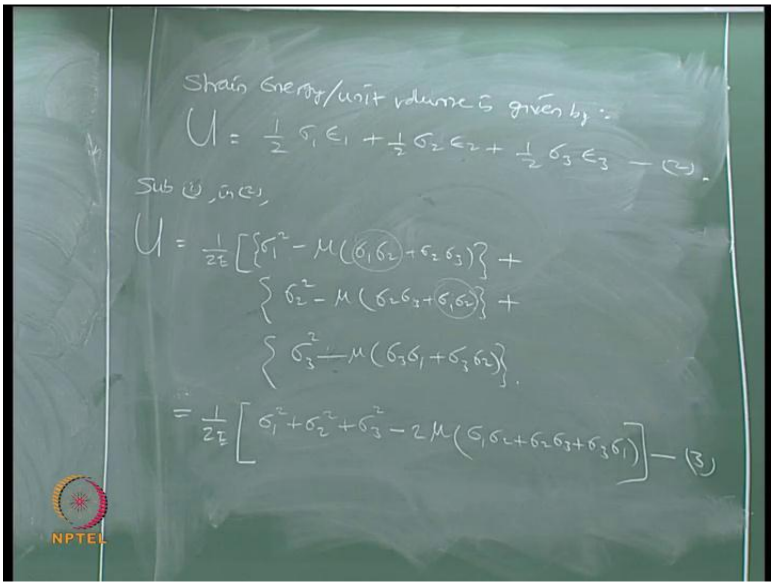
(Refer Slide Time: 17:25)



But, in the first and third viscosity is going to be only sigma Y P whereas, (( )) says it is beyond sigma Y P. It is possible, so that is the derivation of this theory with respect to the reaming 2 theories. Let us look at the fourth 1, any questions here? The fourth theory says total strain energy theory; this theory was proposed by Haigh.

This theory states that in elastic action, inelastic actions or yield, will take place at a point when energy by unit volume of the material is equal to the energy absorbed by the material under uniaxial stress state as in the case of simple tensile test. It says that the energy per unit volume of the material absorb, if this is equal to the energy absorbed by the material and the uniaxial stress state define by this particular test mechanism or the principle then yielding will start occurring in the material. It is not bothering about any stress reaching sigma Y P, shear value reaching 50 percent, sigma Y P stress exceeding sigma Y P, nothing; it simply says look at the energy absorbed in the material per unit volume. Talking about total aggregate different dimension of the failure, so in a given triaxial stress state, if sigma 1 exceed sigma 2 exceed sigma 3, then strains epsilons 1 epsilons 2 epsilons 3 are given by I, call equation number 1.

(Refer Slide Time: 21:34)

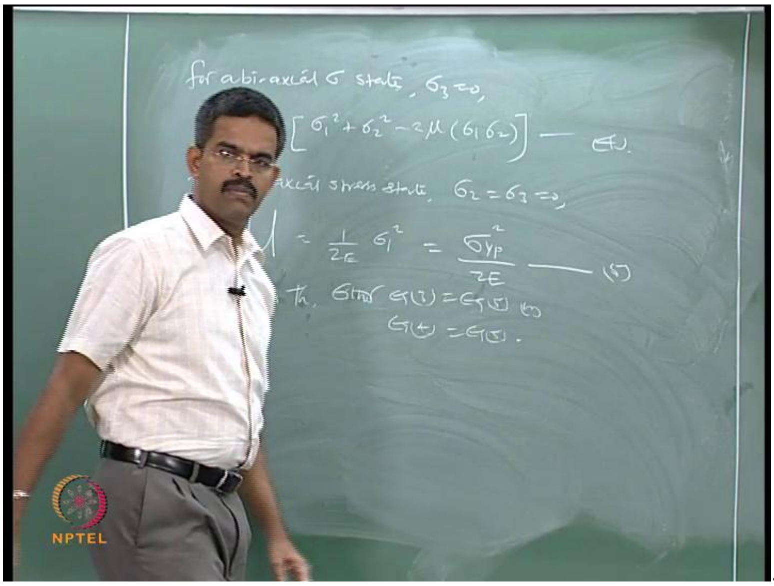


Then, strain energy per unit volume u is given by, Calls equation number 2, so the equation 2 as strain values here epsilon 1, 2 and 3 equation 1 as epsilon 1 and 2 and three separately, I want to substitute this in that equation and rewrite. Can you please rewrite? So, substituting 1in 2, so what will be the value of u; quick 1 by 2 E sigma 1 square mu of sigma 1 sigma 2 plus sigma 2 sigma 3plussigma 2 square if you get this.

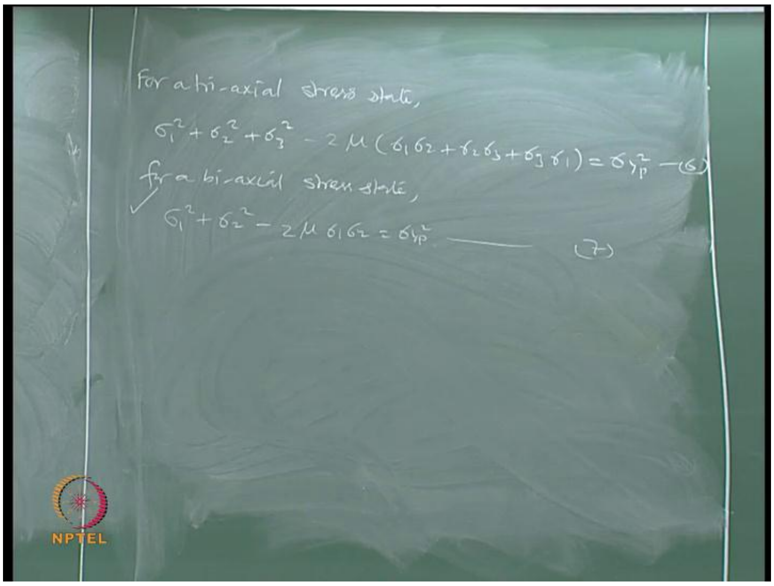
Let us simplify this I will call this equation number. I will simplify this first, so it becomes 1 by 2 E of sigma 1 square plus sigma 2 square plus sigma 3 square minus 2 mu of because each 2 products, I get each 2 products, sigma 1 sigma 2 sigma 2 sigma 3 and sigma 3 sigma 4, I call this equation number 3.

So, in equation 3 for a biaxial stress state sigma 3 will be 0, so what will happen to u 1 by 2 E of sigma 1 square sigma 2 square minus 2 mu of sigma 1 sigma 2, is that sigma 3 will be set to 0 in this case I get this. I call this equation number 4, for uniaxial stress state so I should put sigma 2 as well as sigma 3 both are 0. I simply get u as 1 by 2 E of sigma 1 square, which I say this reach yield value to get the ultimate strain energy is it not?

(Refer Slide Time: 26:10)



(Refer Slide Time: 27:29)

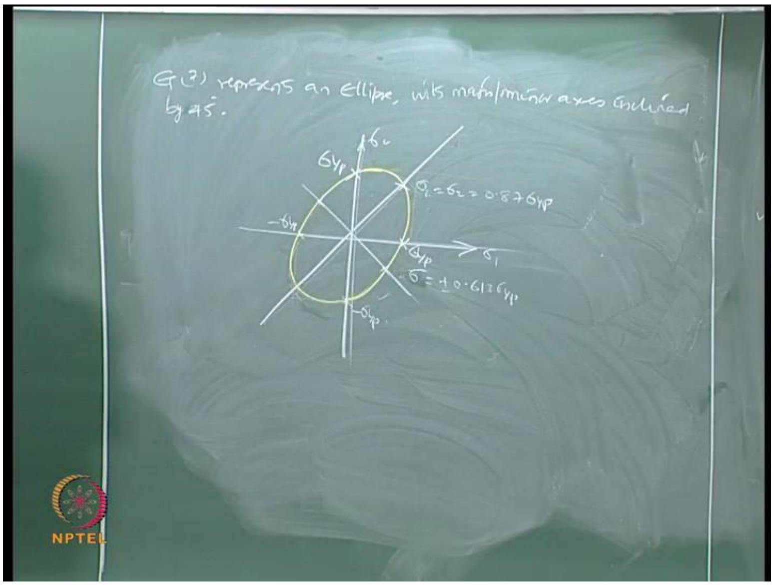


So, I can simply say this is equal to sigma Y P square by 2 E that is the yield value, is it not, so as performance this theory either equation 3 either equation 3 should be equal to 5 or equation 4 should be equal to 5, is it not. As per this theory, whether it is uniaxial biaxial or tri-axial do not bother about this stress state. Bother only about the strain energy so either 3 should be equal to 5 or 4 should be equal to 5.

So, according to this theory either equation 3 should be equal to equation 5 or equation 4 should be equal to equation 5, is it not. So, let us imply that and see what happens, so for a tri-axial system. So, I am equating 3 and 5, I will get sigma 1 square sigma 2 square sigma 3 square, I do not have equation 3 here.

But, anyway minus 2 mu of sigma 1 sigma 2 sigma 2 sigma 3 sigma 3 sigma 1 should be equal to sigma Y P, is it ok? . And you are getting this, this is equation number 6. For a biaxial stress state I am going to equate the equations 4 and 5. So I will get sigma 1 square sigma 2 square minus 2 mu sigma 1 sigma 2 is sigma Y P. I call this equation number 7; let us look at this equation because 2 dimensional plot is simpler than three-dimensional plot.

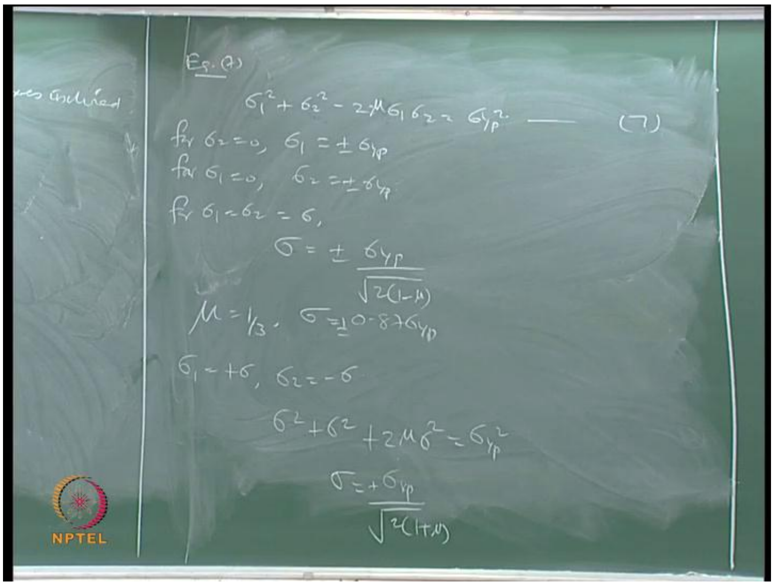
(Refer Slide Time: 29:44)



Let us look at this equation; I am going to give a very important assignment for you. You've got to plot all the tri-axial stress states of failure envelope and show it. I am plotting all the biaxial stress states, so I'll plot the biaxial stress state here for this theory. a square plus b square minus 2 a b is equal to k square, some value, this is an equation of an ellipse. Let us plot this; I'll remove this, so I should say equations 7 represents an ellipse with major and minor axis inclined by 45 degrees.

Let me draw that here. So, this value this is going to be my sigma 1 and sigma 2 and these values are all sigma Y P and sigma Y P and minus sigma Y P and minus sigma Y P. This is my major axis than ellipse, this is my minor axis than ellipse and this value this value will be equal to 0.87. I will show you how it is and this value is going to be equal to 0.613

(Refer Slide Time: 32:26)



How do we get this? Let us say I will remove this, I will write equation 7 here again, and equation7 is sigma 1 square minus 2 mu sigma 1 sigma 2 is sigma Y P square. This is equation 7, I am rewriting it again here, is it not, and now for sigma 2 equal to 0, what will happen to this equation? For sigma 2 equal to 0 you are travelling on sigma y axis what is going to happen to this equation sigma 1 will be either equal to plus or minus sigma Y P, is it not?

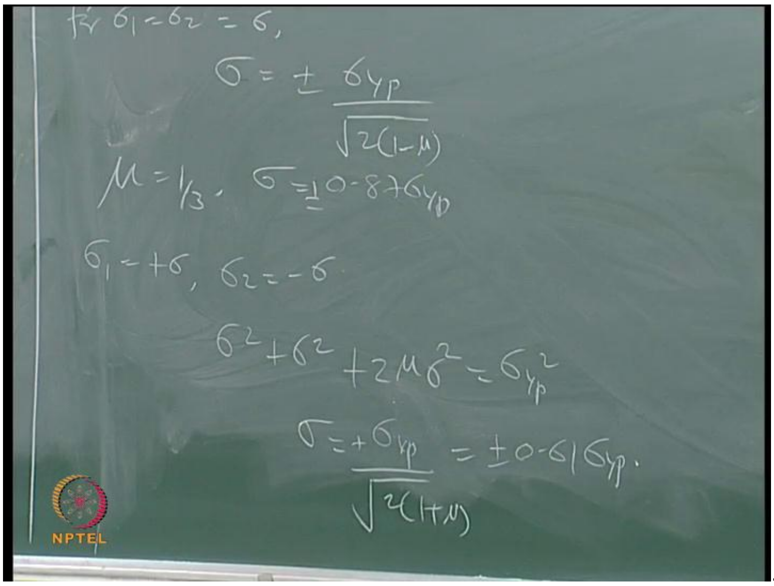
I get these points for sigma 1 equal to 0, I am travelling along the y axis sigma 2 will be equal to plus or minus sigma Y P. I get these points for sigma 1 equal to sigma 2 same value simply sigma intensities are same, what will happen to this sigma value then? Work and tell me quickly, plus or minus , ya plus or minus sigma Y P by root of 21 minus mu.

See, this becomes sigma sigma 2 sigma 2 mu, 2 is out 1 minus mu sigma square. Simplify, take a root and get this, is it not? Now for mu equal to 1 third which is the common value of steel, what is mu? Mu is the poissons ratio, transverse strain by longitudinal strain. What is sigma? Quick quick, do you have a calculator.

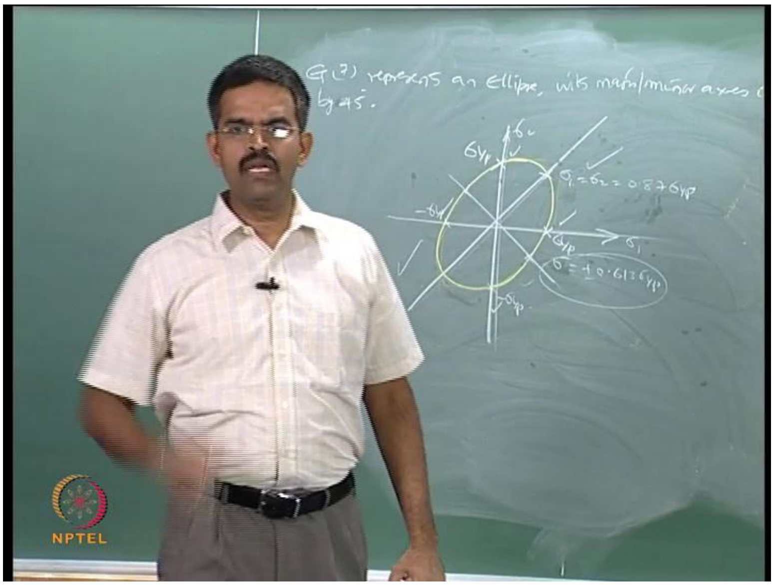
How much will I get the answer, do you have a calculator? Can you give me the value? Give me the value in fraction 0.876; that is what I am getting in the first term third quadrant, 0.87now in the second and fourth quadrant, let us say sigma 1 is equal to plus sigma and sigma 2 is minus sigma. That is nature is different then what happens then what happens here, so sigma square plus sigma square this is minus of minus, so plus 2 mu sigma square is going to be equal to sigma Y P square. So, I simplify this, I get sigma as sigma Y P plus or minus root of 2 of 1 plus mu. In this case because this will become positive now for a mu of 1third what is sigma value?

Student: 0.61.

(Refer Slide Time: 37:28)



(Refer Slide Time: 37:34)

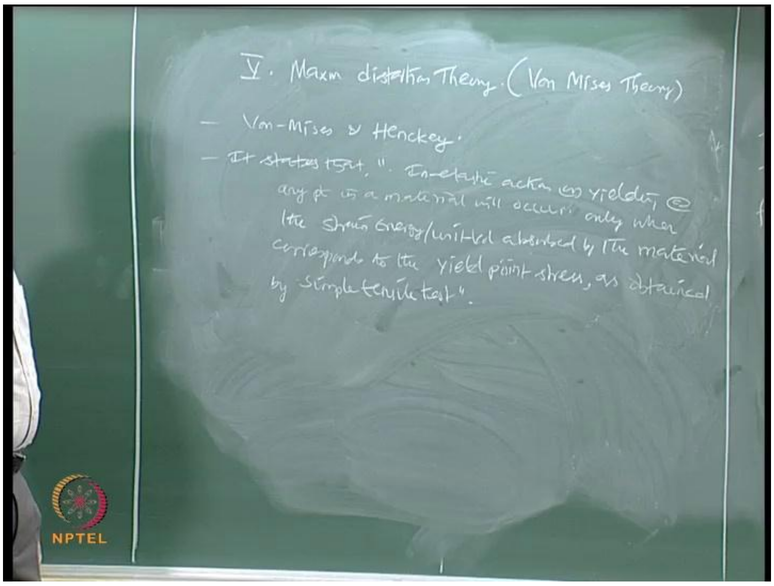


That is what I am getting here, so what is this theorem says is in the first and second and third quadrants. Sorry, first and second quadrants when the stresses are alike at failure at yielding the stress never reach a sigma Y P, it is only 0.87 whereas, in principle stress theory it has to reach sigma Y P.

In maximum shear stress theory, it has got to reach sigma Y Pby 2. In the previous theory, it exceeds sigma Y P but, in this theory it is not even touching sigma Y P, it is only 0.87. So, what does it mean is when both the stresses are of the same nature and same magnitude, the yielding will start much below sigma Y P. That is it dangerous issue the stress may not even reach sigma Y P, yielding will start even at 87 percent of itself, is it not? This theory predicted that, which is different from the previous 3 theories. Again in the second and forth quadrant the previous theory said approximately a linear line, say it may be here 50 percent 0.5 approximately but, these theory says yielding will start only when the stress reaches 60 percent.

So, there is a deviation again, are you understanding? So, different theory started giving different recommendations in all the 4 quadrants which are not matching at all but, remember all these concepts of occurrence of maximum strain energy, occurrence of maximum shear stress, occurrence of principle shear stress principle stress etcetera. All will start occurring on a material simultaneously when it is uniaxial. In a multi axial stress state, there is no guarantee which will first occur, what would be the failure, will the failure occur, when we take a sigma Y P. Will the failure occur when it reaches even 87Y P.? We do not know, is it clear, and so if the failure is occurring based on maximum strain energy as expressed in this theory which is given by Haigh.

(Refer Slide Time: 40:29)



The failure will start occurring even when the stress reaches 87 percent of it yield value. Much below the yield value the failure starts occurring failure, may be because of yielding, because of fracture as well so that is the discrepancy with this theorem. Let us go to the next theory, fifth theory maximum distortion theory which is called Von Mises theory. This theory was proposed by Von Mises and Henkey. This theory states that in elastic action or yielding at any point in a material will occur only when the strain energy per unit volume absorbed by the material corresponds to the yield point stress as obtained by simple tensile test.

So, this theory talks about distortion. What is the difference between volumetric strain energy and strain energy per unit of volume with that of distortion? So, there is a difference, distortion is change in shape of the geometry. That is also inelastic deformation, I have a body which is cube in shape, and I apply a force, the body shape change that is all inelastic deformation. So, this theory focuses on distortion of the body, it says that yielding will occur only when the total energy strain energy per unit volume reaches the yield point stress as shown. So, you will if you look at Von Mises and Hag's theory or Von Mises and the total strain energy theory which is just now there we saw the theory 4, there is a very marginal difference between these 2 theories in understanding this theory includes total strain energy the distortion also into it.

Whereas, earlier one did not talk about distortion at all, so in this theory I must clearly show the expression which term corresponds to distortion which does not corresponds to distortion. In that theory, if I say that term corresponds to distortion set to 0, this theory will be shown as a different deviation which will see in the next lecture. So, we will see the fifth theory elaborately in the next lecture, then we will pick up a problem compare all the 5 theories and as a designer what theory should I say and what is the consequence of using different theories in design. We will talk about that in this lecture as a summary I wish to state that we have discussed different failure theories, maximum principal theory, maximum shear stress theory, maximum strain energy theory etcetera.

The difficulty is here in this theory is all these values of occurrences will happen simultaneously if the material or member is subjected to uniaxial stress state. But, in reality marine structure members are subjected to biaxial or tri-axial stress state that is one complexity we have. Therefore, we do not know in a given stress state in marine structure members, we will shear stress be predominant to cause failure, maximum stress predominant cause failure and so on. The second issue is, if the stress magnitudes are same but, the nature is alike you fall in second and forth quadrants. If the stress magnitudes are same and the nature is also same if you fall in first and third quadrants, so each theory gives different recommendations of yield value in all these 4 quadrants.

So, there is big complexity confusion in understanding which will initiate failure and should I consider sigma Y P based on principle stress value, maximum stress value, total strain energy observed, maximum strain energy value or distortion we do not know, is that clear? So, far we have not been exploding this in detail explicitly when we studied plastic analysis and design. Because we understood that when stress in a material reaches yield value collapse load or collapse of the structure occurs. But, now the question is when it will raise the yield value through what mechanism, through what mode it will be through shear will it be through principle stress will it be through distortion, we do not know. Therefore, we are studying this slightly in detail to understand what would be the effect of these theories independently on estimating the failure path that is what we are discussing. So, in the next lecture we will talk about the fifth theory in elaborate we will compare them and will do problem in the designer’s interest what will happen in next class.

Thank you.