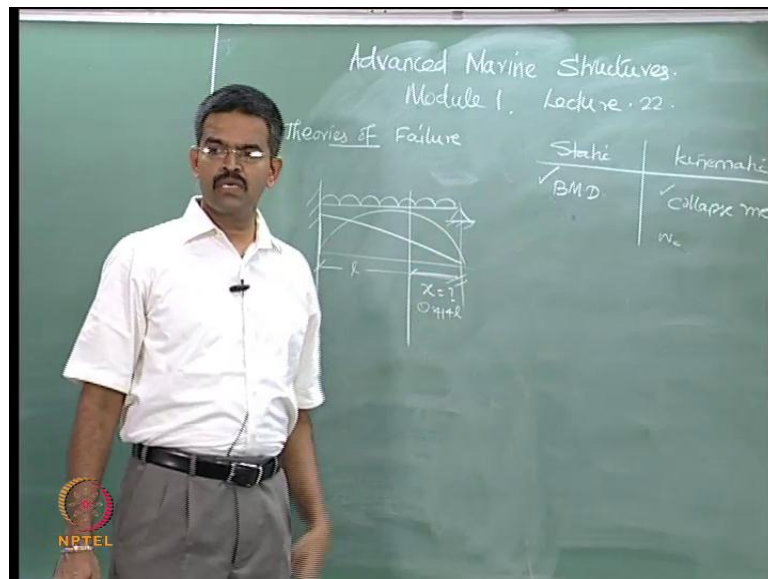


**Advanced Marine Structures**  
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**Lecture - 22**  
**Theories of failure - 1**

So, we will continue with lecture on advance marine structure. We are discussing on module one, where the focus on ultimate load design method. We are discussing about the plastic analysis and design. In the last few lectures, we discussed about the different theorems, which are useful for finding out the collapsed load or the true collapsed load using upper bound theorem, lower bound theorem or static kinematic theorem as well as uniqueness theorem or a combined theorem.

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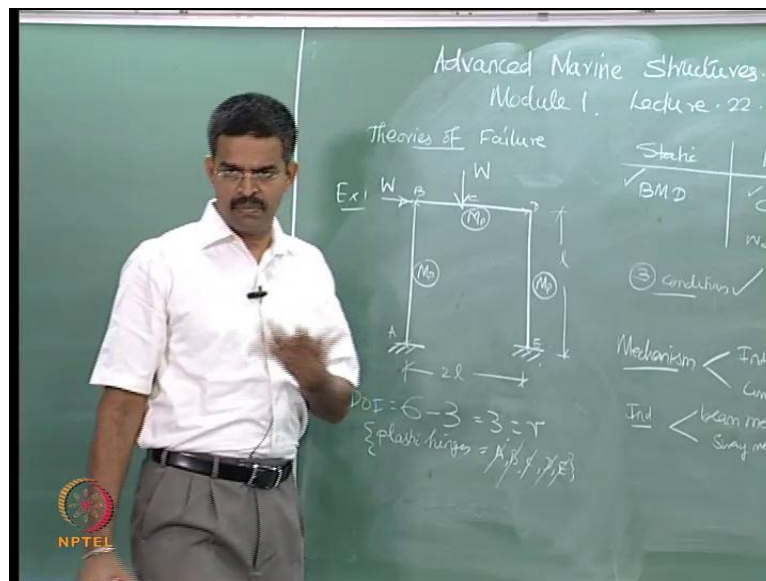


We did some examples for beams and we have understood that both the theorems for example, the static theorem and the kinematic theorem gives you the same true collapsed load provided as far as the static theorem is concerned. The bending moment distribution is correct and you have identified the maximum sections, where the plastic hinge can form, which are required to convert a given structure into a mechanism. Whereas, in kinematic theorem principle, you are supposed to assume a proper collapsed mechanism, so that appropriate number of hinges are formed and you are able to compute the collapsed load. If you are able to do either bending moment distribution correctly or

collapse mechanism assumption correctly, then it will land up in the same true collapsed load by using both the theorems. But there are certain cases, where kinematic theorem cannot be directly applied. You have got to use or take the help of the static theorem.

A classical example is a propped cantilever, where you have got a simple supported end subjected to UDL. So, obviously, you do not know where will be the maximum bending moment occurring in the cross section;  $x$  is not known. Therefore, you are supposed to compute  $x$  using the proper moment distribution methodology first; then for that  $x$ , find out  $M_p$  and then verify this using the kinematic theorem using principle of virtual work. So, we have found out this value comes to point  $0.414 l$ ; where, this is the span of the beam. And,  $M_p$  is around  $(\frac{1}{2}) w c$  is around  $11.66 M_p$  by  $l$ . So, we found out and we verified the same using the collapse mechanism or using the kinematic theorem. So, both the theorems cannot be directly used all the time, so that you can get the two collapsed loads. Sometimes you have to depend on static theorem. Certain cases, you are going to depend on kinematic theorem, which I will come to the example now.

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Quickly, then we will move on to the theories of failure; why do we study the theories of failure very briefly. We will take up another example, where I am discussing a frame subjected to a lateral load  $W$  and central concentrated load again  $W$ . And, the moment carrying capacity of the beams and the columns are all same, which is  $M_p$  marked here. And of course, the beam is for a span of  $2l$ . Whereas, the column is of a height of  $l$ . That

is the frame given to you. So, in this specific example, it will be easy for us to start with the kinematic theorem application directly and get the collapsed load. And then, verify whether all the basic conditions for a plastic design or analysis satisfied.

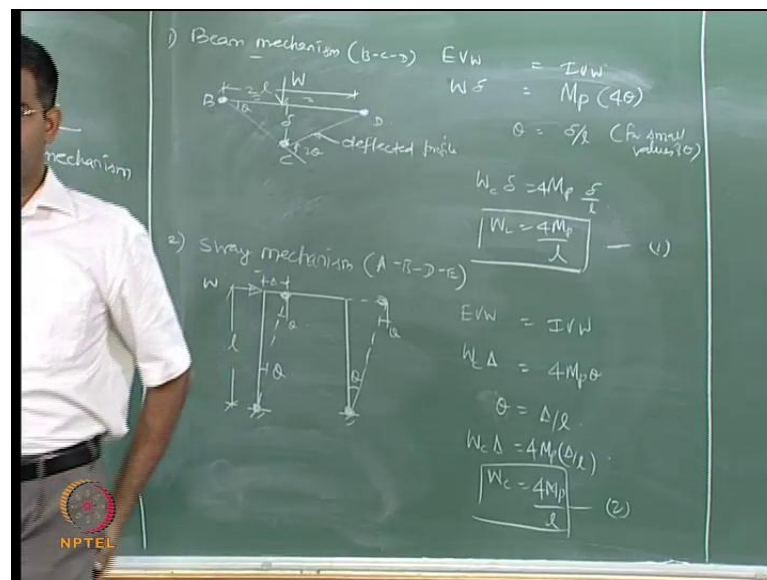
What are the three conditions which will be satisfied of plastic analysis? There are three conditions, which are to be satisfied (( )) plastic design. Can you tell me, what are those three conditions, which are to be satisfied for a plastic design? It is already given there in your literature; it means you are not looking at it back at all. Simply coming with blind head will not help. We have completed twenty two lectures earlier. So, you are supposed to read them. We already discussed this. There are three conditions. One of the important condition is that, the moment at anywhere in the cross section should not exceed  $M_p$ ; or, to be optimum, this should be can be equal to  $M_p$ . That is the case. So, we have checked whether in the cross section, whether  $M_p$  is not exceeded or not. So, what I will do is, I will assume certain mechanisms. What are the possible mechanisms I have, which I can assume for a frame? Independent mechanism and combined mechanism.

What are the possible independent mechanisms I have? What are the possible independent mechanisms I can assume for a given problem like this? The independent mechanisms are the following. One will be the beam mechanism; one will be the sway mechanism, etcetera. And of course, the combined mechanism is an assembly of both of these simultaneously acting together on a given structural system. So, I will assume. And also, you will understand that, depending upon the number of plastic hinges formed, the mechanism can be either a partial collapse or a complete collapse or an over complete collapse. It is not necessary that, you must always get  $r + 1$  hinges. Even if we get  $r$  hinges, it is called partial collapse; still the structure will collapse.

Let us quickly see what are the degrees of indeterminacy for this frame. Now, I have to look for the axial loads also coming on the frame. Therefore, there are six unknowns, because each fixed support will give me three unknowns. So, the unknowns are 6. And, the equations of equilibrium are 3. So, the degree of indeterminacy for this problem will be 3, which is nothing but the  $r$  value. Therefore, I must get at least 4 hinges to make it as a complete mechanism or even if I get 3, it can be a partial collapsed mechanism. If I get 5, then it will be an over-collapsed mechanism. Let us name these points as let us say A, B, C, D and E. So, the possible locations of plastic hinges for this problem could be all the five: A, B, C, D, and E.

Now, let us quickly revise how did we estimate what are the possible locations, where the sections where the plastic hinges can form. The sections where the plastic hinges are likely to form are the following. It can be either fixed supports. Therefore, A and E. It can be under the concentrated load. Therefore, C. Can be also the joints, which are fixed in a frame. So, it can be B and E, D. So, these are the points where the plastic hinges can form. So, there are possibly five locations, where the plastic hinges can form. But I need only 4 to make it as a complete mechanism. So, let us see how we are going to estimate using the kinematic theorem, because in the last example of the cantilever beam, we started with the static theorem and verified the example with the kinematic theorem. In this case, I will pick up the kinematic theorem, apply on this frame and try to verify it with the static theorem.

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So, I can try with the beam mechanism. The beam mechanism can be applied only on BCD. Let us say pick up BCD. So, I have a central concentrated load  $W$ . So, if I say BCD is a beam mechanism, the hinges can form here; it can form here; it can form here as well. So, this may deflect a profile of the beam. So, I call this as delta. And of course, this is theta. And, this will be 2 theta, because it is rotating twice to come back to its position. So, the external virtual work should be equated to internal virtual work. External virtual work is forced into work – work done at displacement, which is  $W$  into delta. Internal virtual work is going to be  $M_p$ . Why  $M_p$ ? Because all plastic hinges are

nothing but hinges which needs a specific moment to rotate them; and, that moment is nothing but  $M_p$  in a given section. So,  $M_p \theta$ ,  $2\theta$  and  $4\theta$ .

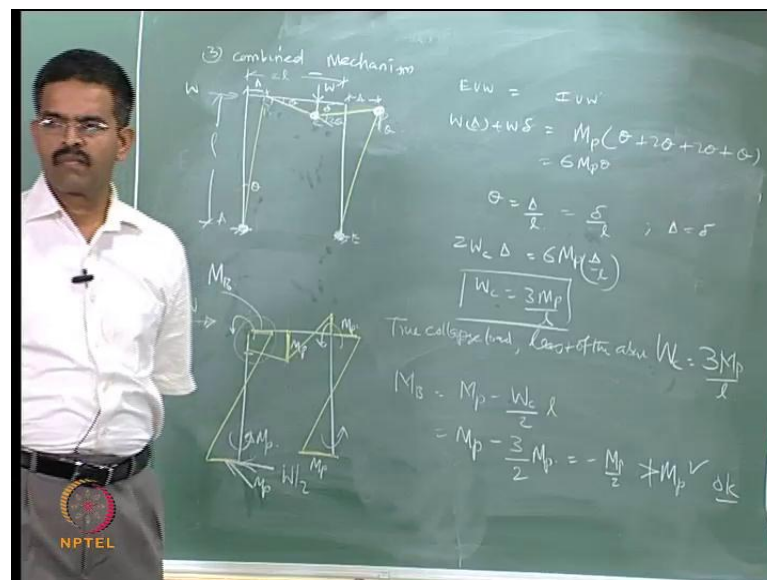
I also equate for small deflection of  $\theta$ . I can say  $\theta$  as  $\delta/l$ , because this is  $2l$ ; we can see this span here is  $2l$ . This is  $2l$ . And, this is equal. Therefore, it is going to be  $\delta/l$ . So, I can say from the figure,  $\theta$  can be simply  $\delta/l$  for small values of  $\theta$ , because you know, the principle of virtual work is applied only for small deflections; large deflection problems are not solved using this theorem. So, I can replace this  $\theta$  with  $\delta/l$ . So,  $w_c$  – I call this as a collapse mechanism. This is a partial collapse mechanism, because we have got three hinges formed; it can collapse still. It is a partial collapse.  $\delta$  is equal to  $M_p \delta/l$ . So, I can say  $w_c$  is  $4 M_p \delta/l$ . That is what one value I get for a beam mechanism, which is I should say B, C and D. I can write here beam mechanism – B, C and D. I can also have a beam mechanism if I have got the load here. If I have got a load here, I can also do beam mechanism for A, E, B, etcetera. I can do the same method here. Now, I do not have a load here; do not have to worry about this.

Now, I will think for a sway mechanism. I will not include this force; I will include only this and see that, the frame is getting pushed by one side – sway mechanism. I am going to apply sway mechanism for the full frame. So, ABDE. So, this is the original frame. I apply a load here. Now, the frame is getting swayed like this. That is swayed; it is not deflecting at the beam level of BCD unlike here. It is just swayed. Let me call this as  $\delta$ . This is a small difference – a lateral deflection; this is vertical deflection. This is  $\delta$ . So,  $(\delta)$  external virtual work, internal virtual work. So, I have to apply or locate the plastic hinges here. There are four locations: 1, 2, 3 and 4. These are the four places, where the hinges can form. And, I will call this as  $\theta$ ; this as  $\theta$ ; this as further  $\theta$ ; and, this as  $\theta$ .

Now, the external virtual work is  $w_c$  into  $\delta$ .  $w_c$  into work done is a distance  $\delta$ . And, internal virtual work is  $M_p$  of 1, 2, 3 and 4. So, again get  $4 M_p \theta$ . So, we already know, for small deflection,  $\theta$  is  $\delta/l$ , because this is  $l$  as given in the problem. Height of the column is  $l$ . I replace it here. So,  $w_c$  into  $\delta$  is  $4 M_p \delta/l$ . Therefore,  $w_c$  can be  $4 M_p \delta/l$ , which is another option of a sway mechanism. I am looking for solution of this problem using kinematic theorem. So, kinematic theorem says that, you can assume any number of possible collapse mechanisms. Each

mechanism will give you possible collapse loads. But the theorem says that it is upper bound; it means they will either be equal to  $M_p$  or are equal true collapse load or they will be higher than the true collapse load. So, I must consider the lowest, because it will keep on giving me the higher values or at least equal to the true collapse loads. So, in this case, both of them are equal. So, there is no botheration. If they are not equal, I will consider the lower value, because it is an upper bound theorem. Let us also look for a combined mechanism, because there are possible hinges, where 5 can form. But I have assumed for 3; I have taken for 4. Let me see is it possible to have a combined mechanism also. So, I will pick up this. Remove this one.

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Let us look for a combined mechanism. So, this is the original frame. So, the deflected profile goes like this. This is the deflected profile. And, this is 90 degree. I do not want to have a hinge here. It is my assumption, because I am assuming a mechanism. Let us say I have a hinge here. I will have a hinge here; I will have a hinge here; I will have a hinge here. But we do not want to have a hinge here. But I want to check, because the condition for plastic design is that, at nowhere in the section, the moment should exceed  $M_p$  – one. Secondly, we have a probability that, the hinge can form here, may not form here, because there are five locations.

We are looking only for a complete collapse mechanism. So, let us say this is A, B, C, D and E. For this assumed mechanism, let us work out the collapse load and see what

happens. So, this is if I call theta and I call this as delta and this also as delta with a different symbol; this is also delta. So, if this is theta, obviously, you will agree, this is also theta, because it is swaying with the same amount. And, this will be 2 theta, because this is theta. It is 90 degree I said. It is 90 degree. So, if this is theta, this is also theta.

Let us say external virtual work and internal virtual work; that is, principle of virtual work we are applying. All these deflections are very small in dimension compared to the sections of the frame or dimensions of the complete frame. So, external virtual work – I have got two loads: one is  $W$  and one is again  $W$ . So, let us say  $W$  into delta plus  $W$  into delta. Internal virtual load – I have got  $1 M_p \theta$  here. Let us say  $M_p$  times of theta. There is nothing happening here. There is 2 theta here. Again, there is 2 theta here – 2 theta here. And, there is again theta here. So, 2 plus 2 – 4 and 6. So, it becomes  $6 M_p \theta$ . So, for small deflections, we already know that, theta is equal to delta by  $l$ , because this is  $l$ . And, theta is also equal to delta by  $l$ , because this is  $2 l$  in the original problem. So, this theta is also equal to delta by  $l$ . So, I replace them. Wherever you have got theta, replace them. And, from this equation, we already know that, delta and delta are equal. So, can I write this as  $2 w_c \delta$ ? This is one and the same. And, let us say  $M_p - 6 M_p$ . I am replacing theta by delta by  $l$ . So, I will get collapse load as  $3 M_p$  by  $l$ .

Now, I have got a new value, which is different from what we assume from previous mechanisms. The previous beam mechanism and sway mechanism independently gave me a collapse load of  $4 M_p$  by  $l$ ; whereas, the combined mechanism of four hinges again gave me a collapsed load of only  $3 M_p$  by  $l$ . So, since I am using an upper bound theorem or kinematic theorem, the class of this theorem says that, the collapse load what we get from independent assumed mechanisms will be either equal to true collapse load or it will be higher than the true collapse load. So, I must pick up the? I must pick up the lesser – lowest possible value. So, the true collapse load in my problem is the least of the above. So, true collapse load – I should say it is  $3 M_p$  by  $l$ .

Let us quickly reiterate what we do with the design. Once I have got the true collapse load analyzed from plastic analysis, I already know what is true collapse load connected with the working load. That is called load factor –  $Q$ . We already saw that  $Q$ . Therefore, I know the working load; I know the collapse load, because I can pick up the  $Q$  factor for different combination of loads given in the codes. So,  $l$  is known to me, because that is the geometric dimension for a given frame. So,  $M_p$  is a function of  $z_y$  or  $\sigma_y$  and  $S$

into  $z y$ .  $S$  is the shape factor. So, you must know what cross section you want to adopt: circular, square, I section? You know the shape factor for the material or for the member. And, for that member cross section, you already know  $z y$ . So, everything is fine. Design is done, because  $z$  is function of  $b d Q$  by 12 or BMD. You assume one dimension. If it is rectangular, get the depth by equating this equation. Simple design. So, in this problem, we have used kinematic theorem first, but have not employed the static theorem. But, there is a catch here. How do we prove that, this point or this moment at this point or  $b$  is not exceeding  $M_p$ ? We should prove this, because none of the place exceeds  $M_p$ ; there is no doubt, because here it is  $M_p, M_p, M_p, M_p$ ; no problem.

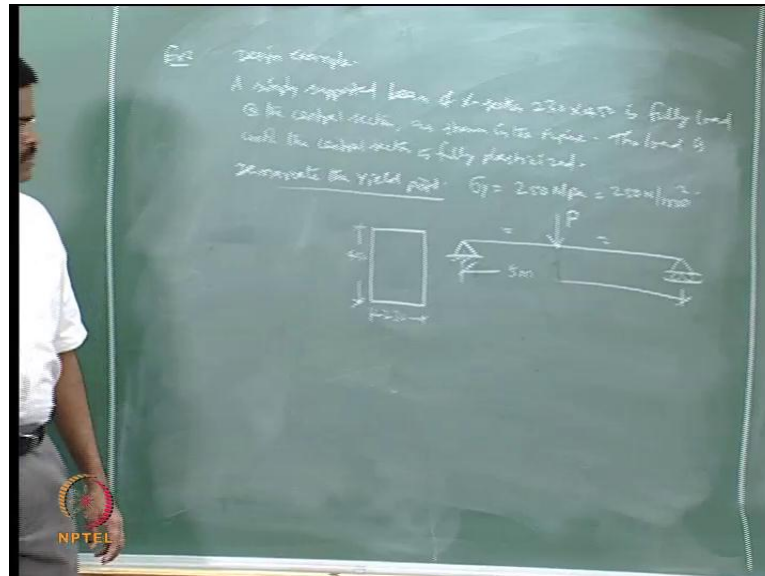
But, here is it exceeding  $M_p$ ? We want to check that. So, let us check that quickly. I am trying to plot the bending moment diagram of this problem. I am trying to plot the bending diagram for this. So, I am saying that, this is  $M_p$  and this I do not know; this I do not know. This is  $M_p$ ; this is  $M_p$ . And of course, this is  $M_p$ . This is the bending moment distribution. You know how this is drawn. So, what you try to do is, I am stating here that,  $M_p$  is creating tension on the outside. So, I am plotting the BMD and the tension side of the column. So, tension is creating outside. So, this has got to be tension creating outside. So, this is going to be... So, tension is here. Similarly, tension is here. So, tension is here. This is for the column and of course this is for the beam. So, we already know that, there is an  $M_p$  happening here. So, these values are all  $M_p$ . This is  $M_p$ ; this is  $M_p$ ; this is  $M_p$ ; this is  $M_p$ . I call this value as  $M_B$ , not  $M_p$ , because this is the point B; I want to check is it  $M_p$  or more than  $M_p$ . So, for a given load of  $W$ , we already know that, this is going to be  $W$  by 2, because there are only two supports; they have got to react to this  $W$ .

Let us take moment about this point. So,  $M_B$  will be equal to  $M_p$ , that is, anti-clockwise. So,  $M_p$ . I am taking anti-clockwise as positive; anti-clockwise as positive minus  $W$  by 2 of  $l$ , because this is  $l$ . That is the moment about this point. There are no other forces. We already know this is the... If this would have been a collapse load, then we already know that,  $M_p$  minus...  $w c$  is  $3 M_p$  by  $l$ . So,  $3$  by  $2 M_p$ . So, this gives me minus of course  $M_p$  by 2; minus indicates that, the moment is of a different nature what you have assumed here. But the value is not exceeding  $M_p$ . So, it is (( )) Since the value does not exceed  $M_p$ , plasticine does not form at point B. So, plasticine does not occur at



B; we have got hinges only at A, C, D and E as we have assumed on the section is fine. I will remove this.

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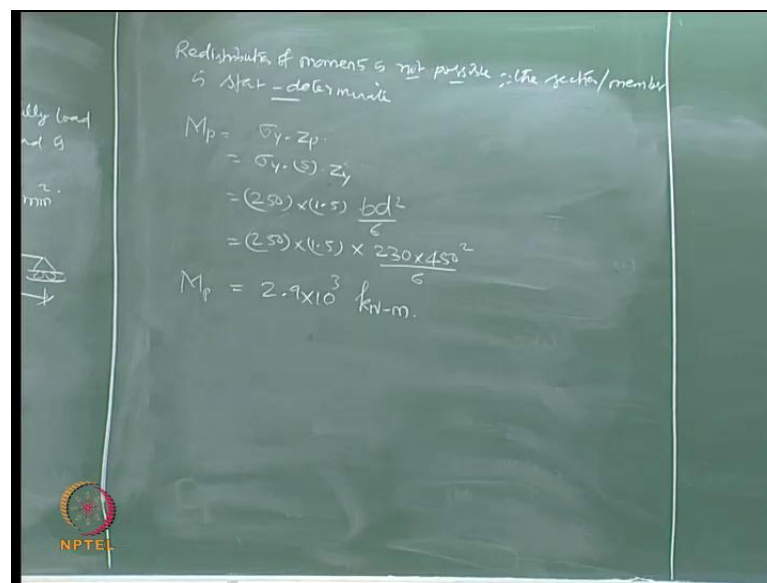


Let us quickly take another example, where we would like to plot the elastic code variation in a given rectangular section, which is design problem. So, let us take another example, where we would like to know the... This is a design example. We say a simply supported beam... I am doing a very simple problem, so that we first understand a simply supported beam of cross section 230 by 450 is fully loaded at the central section as shown in the figure. The load is until the central section is fully plasticized. What is asked is, demarcate the yield point. Take the yield value of the material as 250 mega pascal, which is 250 newton per mm square. It is a simply supported beam. The problem is the simply supported beam having a central concentrated load P. And, the span of the beam is 5 meters.

Now, interestingly, let us try to understand what is the importance of this problem. It is a simply supported beam. Therefore, the beam is statically determinate structure. It requires only one plastic hinge to make it as the mechanism. And obviously, that hinge cannot form either at left support or right support, because these are structural hinges already. So, it can form only at the maximum bending moment section. So, it has got to form only under the point load. So, one hinge is formed. So, what we hypothetically did is, we keep on loading P; we keep on increasing P until the stress at the section at the

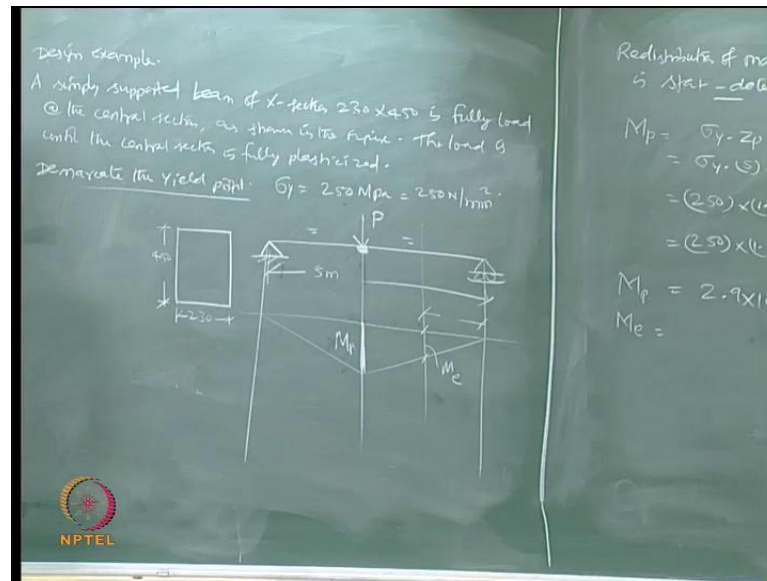
highest bending moment reached  $\sigma_y$ . And, we thought the section is plastic. In fact, we have done the design let us say. And, the section has reached... The maximum section has reached the yield value. Therefore, the section is done for a full plastic design. That is what we thought. But let us try to demarcate and see, how is this yield taking place in the entire length of the beam. We want to see that. So, the cross section is a rectangle as given here; the width is 230 and the depth is 450. The first question – is it possible to have redistribution in this specific example? Redistribution of the moments in this example – is it possible to have a redistribution of the moments?

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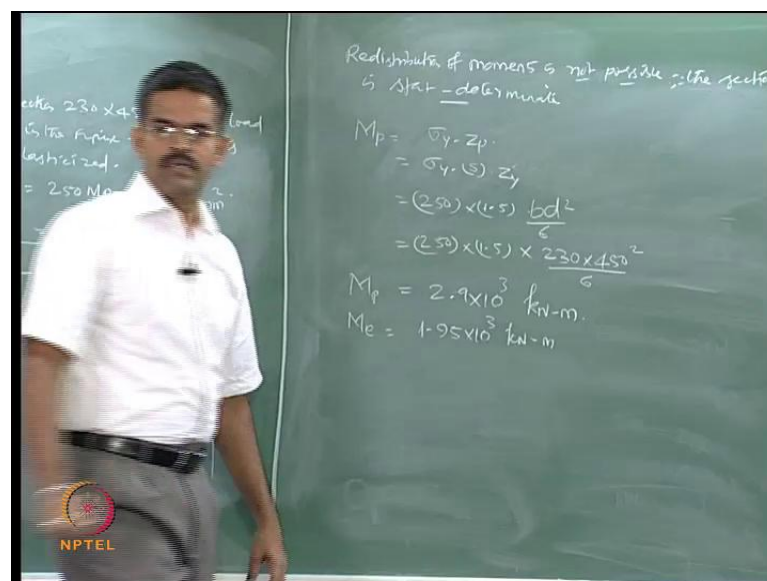
Redistribution of the moments in this example is not possible, because the section is or the member is statically determinate. Redistribution happens only when the section is statically? When the member is statically indeterminate. So, it is (( )) Therefore, no distribution occurs. Now, we already know that,  $M_p$  is given by  $\sigma_y$  into  $Z_p$  or  $\sigma_y$  shape factor into  $Z_y$ . So,  $\sigma_y$  is given to us, which is 250. Shape factor – what is shape factor for rectangular section? 1.5. And,  $Z_y$  is  $bd^2$  square by 6 for this case. So, it is  $bd^2$  square by 6, which is  $I_y$  max. So, let us substitute 250 1.5 –  $b$  is 230 – and, 450 square by 6. Can you tell me how many newton mm is this? You have to be very fast. We cannot keep on passing time on calculations. How much is this? Give me in kilo-newton meter; give me the value in kilo-newton meter. Here you divide the value by 10 power 6; you get kilo-newton meter. This is 2.9 10 power 3; that is kilo-newton. Meter. That is  $M_p$ .

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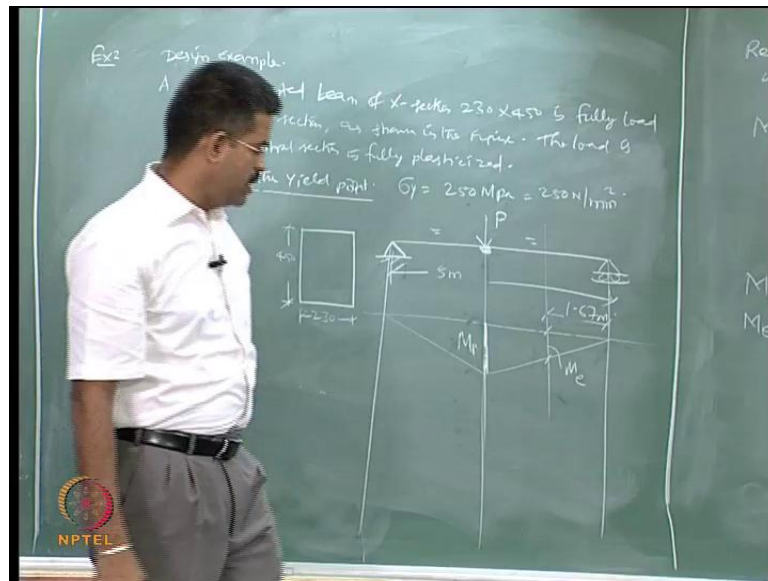
Let us try to see the bending moment variation here. Let us say, if I try to plot the bending moment variation here, a 0 here, a  $M_p$  here and 0 here; this is  $M_p$ ; this is  $M_p$ , because the hinge is formed here. And obviously, you know, here the bending moment is 0. So, there will be some point in the cross section, where the bending moment remain elastic – purely elastic – fully elastic, because it is plastic here; it is 0 here. Somewhere it will be purely elastic. I can even call this  $M_y$  or  $M_{elastic}$  – simply  $M_e$  (( )) very good. How much? 1.95.

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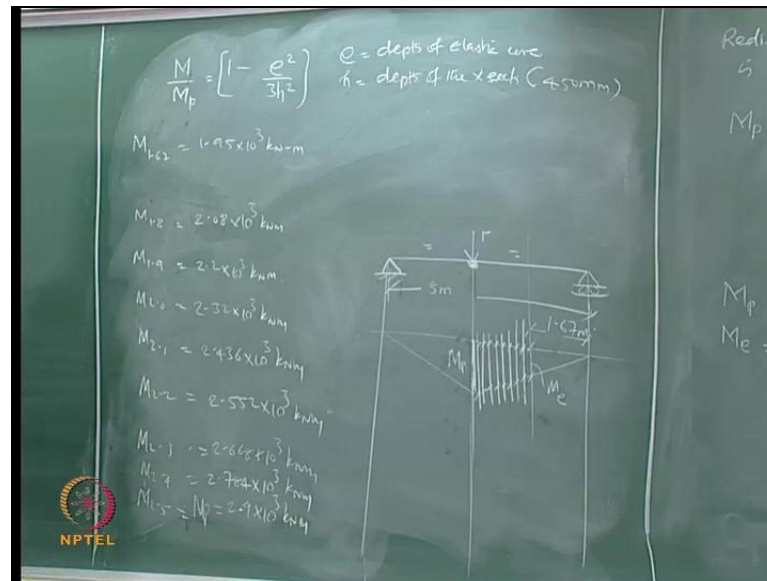
So,  $1.95 \times 10^3$  kilo-newton meter. That is  $M_e$  – elastic.

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So, if you look at this, will become 1.67 meters. So, at 1.67 meters from the support, you will see that this value will be equal to  $M_e$ ; and, at the centre it is exactly going to be  $M_p$ . Now, I want to trace how the yield has formed from this point to this point. Can you tell me, at this section, is the yield maximum or at this section, the yield is maximum? At the mid-section, the yield is maximum. And, at this section, the yield is minimum. So, I want to demarcate this. That is what they ask. That is demarcate. So, to know that, let us see the expression what I am going to use. I will write the figure, but I will remove this.

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We already know the equation  $M$  by  $M_p$  is  $1 - \frac{e^2}{3h^2}$ ; where,  $e$  is the depth of elastic core and  $h$  is depth of the cross section. In my case, this is 450 millimeters.  $M_p$  I have. So, every section, I would like to know the bending moment value. So, let us say  $M$  at 1.67 is  $1.95 \times 10^3$  kilo-newton meter. I would like to know,  $M$  at 1.7,  $M$  at 1.8,  $M$  at 1.9,  $M$  at 2.0,  $M$  at 2.1,  $M$  at 2.2,  $M$  at 2.3; similarly,  $M$  at 2.5, is nothing but  $M_p$ , which is  $2.9 \times 10^3$  kilo-newton meter. So, it is very simple. Again, linear variation – 2.5 – it is  $M_p$ . At 1.7, somewhere here, what would be this value?  $2.08 \times 10^3$  kilo-newton meter. Similarly, 1.9, 2.2, 2.0 – 2.32, 2.436, 2.2 – 2.552, and 2.3 – 2.668 and 2.4 – 2.784, and 2.5 is  $M_p$ . Now, we have these values with us. I would like to trace the elastic core.

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$$\frac{M}{M_p} = \left[ 1 - \frac{e^2}{3h^2} \right]$$

$$\left( 1 - \frac{M}{M_p} \right) 3h^2 = e^2$$

$$\left( 1 - \frac{M_{1.67}}{M_p} \right) 3 \times 450^2 = 0$$

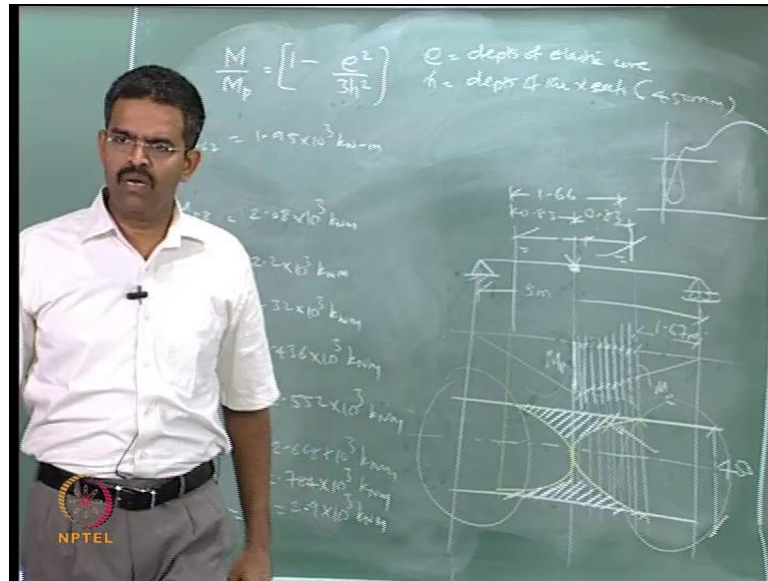
$$\left( 1 - \frac{M_{2.5}}{M_p} \right) 3 \times 450^2 = 0$$

$$3h^2 \left( 1 - \frac{M_{1.8}}{M_p} \right) = 0$$

Use the same equation. Let us say  $M$  by  $M_p$  is equal to  $1 - e^2$  by  $3h^2$ . So, this says  $1 - M$  by  $M_p$  of  $3h^2$  will give him an  $e^2$ . So, I want to know the depth of elastic core at every section; very simple. If you want to find  $e$  at 1.67, what I should do? I must say  $1 - M$  at 1.67 by  $M_p$  multiplied by 3 into? What is the depth of the section?  $450 - 450$  square. So, what will be the depth of  $e$  1.67? So, you can substitute here simply.  $1 - M$  2.5 by  $M_p$  of 3 into  $450$  square what is  $M$  2.5? So, it is going to be  $M_p$ .

So, this becomes 0 actually. So, elastic core, which is 2.5 will be actually 0, is fully plastic. Therefore, what is elastic core here? Section is fully elastic. Now, this is 450 here and this is 0 here, because it is fully elastic; it is fully plastic. Now, I want to unfortunately find out at different levels. Let us say I want to know at  $e$  1.8, what is the value? How did you do that?  $1 - M$  at 1.8 by  $M_p$  multiplied by  $3h^2$ ; I keep on finding it out. Similarly,  $e$  1.9,  $e$  2.0, 2.1, so on. You will trace the elastic core from this point to this point. So, let us plot it here.

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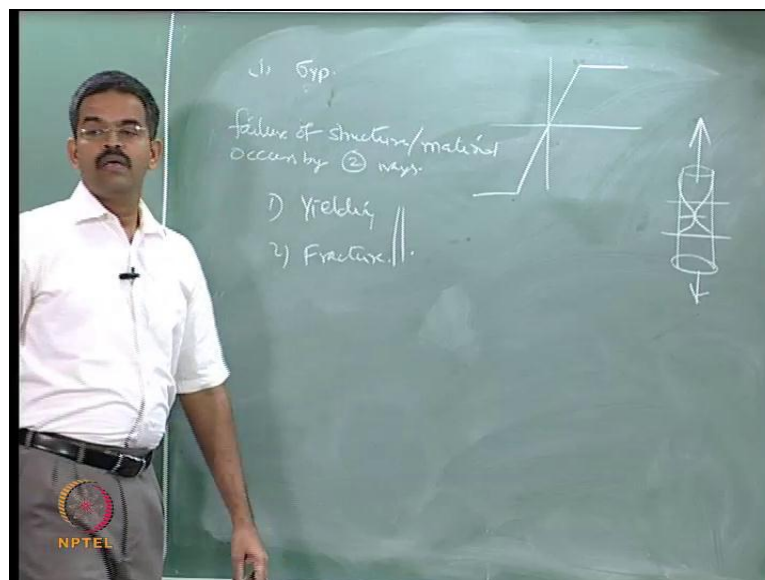
Assume that we have all the values. Let us plot it here. This is the depth of the beam. What is the depth of the beam? 450. This is the central axis. Now, let me draw the elastic core at different sections. For example, at all these sections, I am trying to find out the values; I am trying to plot them here. So, what is the elastic core here at this section? At this section, 450. So, this is the full depth. So, these are the points. What is the elastic core here? 0. So, I want to plot this. I am plotting it like this. Why? This variation is of second order. So, symmetrically, do from this side also; you will get the elastic core like this. So, this is the trace of the elastic core.

And, most important inference, what to draw from this problem is, the distribution of the plasticization has taken place only for a distance of... This is 1.67. So, this will be 2.5 minus 1.67. How much is that? Will it be 0.83? I am also getting 0.83 here; which is 1.66. Only middle-third of the beam is drawing the benefit of getting plasticized; and, that too not fully. Fully is happening only at the centre; only middle-third is getting plasticized. It means we have not utilized the reserve energy of the material in the remaining two-thirds of the beam. Is it an economical design? It is not economical design. That is why, when the section is not statically indeterminate, plastic design will not lead to optimum design at all. We are only plasticizing the... The benefit of plasticization has occurred only on middle one-third of the beam. Why? Because only one hinge is formed. You can imagine now. If the successive hinge would have been formed here as well as here being a fixed support, you would see that, the full section or

the full member for the full entire length will get plasticized. It means the load carrying capacity can be further increased. That is called optimization of the design. So, that is the design example.

We will tell you, how we can try to find out. So, just marking the plastic section alone is not sufficient; you must tell and you must ensure as a designer that, whether you have done an optimum design for a given loading. So, only one section is formed. The benefit is only for in this example, middle one-third of the beam is only benefited; remaining two-third is still remaining elastic only.  $\sigma_y$  is not reached. The stress level is somewhere here only in this plane. It has not reached  $\sigma_y$  at all. So, it is not an optimum design.

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Now, we have studied plastic analysis and design techniques. In the entire process, we have said that, we are worried about one important aspect that, the stress in my plastic design will be equal to  $\sigma_y$ . That is the yield value. I am saying  $\sigma_y$ , because it is plastic design. We also said in the plastic design that, once the stress reaches  $\sigma_y$ , it remains constant thereafter. It is constant. So, the section should have or the member should have or the structure should have enough reserved strength, which is in terms of ductility ratio to keep on redistributing the moments as the load is kept on increasing, because you can see the stress-strain curve of steel as a material. Once  $\sigma_y$  is reached, after that, before it goes to  $\sigma_u$ , there is lot of inelastic deformation



takes place. So, we assume that... Once the yield value is reached, we assume the stress remains constant at the level and it goes on deformation. That is what the stress-strain curve is. It is a standard curve, what we have seen in the last lectures. We have also said that, in tension and compression,  $\sigma_y$  value remain same.

Now, what is the method by which  $\sigma_y$  can be obtained for a given material? If you want to find the yield strength of any material, maybe steel, you would have studied in under graduate. How do you get an yield value of a given material? What is the simple test you have conducted? Simple tension test – you place a specimen in a new t m. Try to pull this. You mark the demarcation length. And, you will see that, the neck formation will be done and the metal will break. So, yield value. It is nothing but a simple tension test. So, in a given material especially in steel, which has got a pronouncing yield point, generally, the yield point value is obtained using a simple tension test. Once you keep on increasing the load after the yield is reached, the elongation forms, neck forms; then it breaks. It means ultimately, the specimen is failing. So, failure or the path to failure is through the yield point of the material. But in reality, the member or the structure is not subjected to uniaxial stress state. It is in a uniaxial stress state only.

In reality, the structure is subjected to multi-axial stress state –  $\sigma_1$ ,  $\sigma_2$  and  $\sigma_3$ . Once you have all the three stress states in position, there is no guarantee that the structure will fail in a same manner as you have been studying in simple tension test. Therefore, there are two ways by which structures are said to be failed. One is what we call by yielding. So, the failure of the structure are to be very specific; I should say material. Material occurs predominantly by two ways: one is what we call by yielding; other is what we call by fracture. There are different theories of failure, which talks about whether the failing has happened because of yielding or because of fracture. And, different theories of failure say that, even when the structure is yielded, the metal is yielded, in a given multi-point stress state,  $\sigma_y$  will not be reached. So, what does it mean?

It gives me an alarming situation that, so far on plastic design, I was thinking that, once  $\sigma_y$  is reached, collapse is initiated. What kind of collapse it is? Is it a collapse of yielding or is it collapse of fracture? What is the difference between these two kinds of collapse or failure? What failure we are looking at? And, if collapse can occur even before yielding is reached or collapse is not occurred even after yielding is reached? If

the situation arises from a theory, then our whole understanding of plastic design is in complex state. So, we have to study the theories of failure at a certain raiser. What are the different theories? How do they deal with? And, what do we understand by a failure or a collapse mechanism from these theories? These theories are not similar to lower bound and upper bound theorems of plastic analysis and design; they are different. Both of them talks about the structure having reached a collapsed state. I am now fundamentally questioning, what we understand by a collapsed state? And, what are the theories?

There are five theories available in the literature. We will talk about all of them in the next lecture. And, we will do a problem and understand them as a designer, which theory should I followed and why I should not follow the other theories; where are the (( )) of different theories in understanding a complex stress state on a given material. In an uniaxial stress state, there is no problem at all. But in reality, structures have gravity loading and lateral loading; that is, wave loading and self weight. So, at least  $\sigma_1$  and  $\sigma_2$  are there. And, wave loading and wind loading is not unidirectional; it is multidirectional, because wave and wind can act from any direction;  $\sigma_3$  can also be there. Therefore, in reality, structural system is not subjected to uniaxial stress state. It is subjected to multiaxial; fundamentally, triaxial. How the behavior differs? We have to understand this. We will discuss in the next lecture.

Thanks.