Advanced Marine Structures
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Lecture - 21
Plastic analysis - Example problems - III

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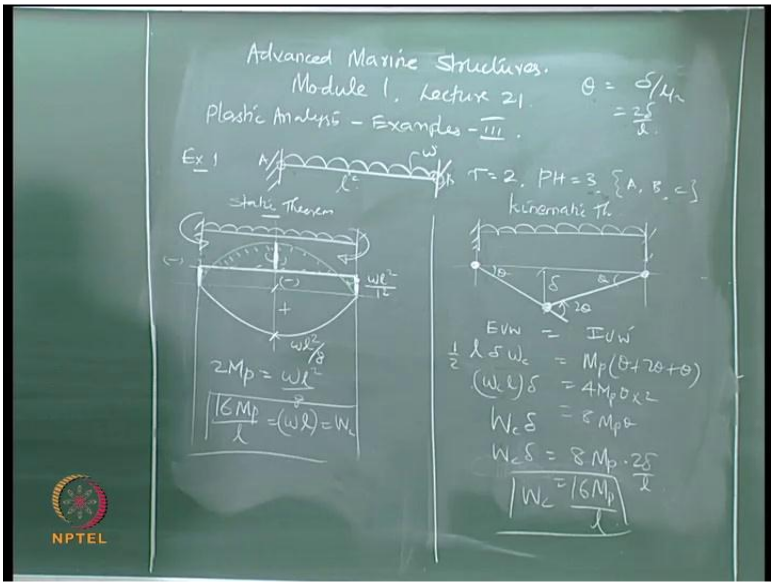

In the last lecture, we discussed about some of the examples, where we could pick up easy problems to compare the static and kinematic theorem results. And, we have understood that by selecting a proper mechanism, by selecting a proper appropriate admissible bending moment distribution, we could land up in the same collapse load whether you follow the static theorem or you follow the kinematic theorem.

In this lecture, we will pick up again examples where, we will slightly go further beyond to understand, how one theorem depends on the other in certain examples. Example 1 – I will pick up a fixed beam with UD – w. So, the span of the beam is l. And we already know the degree of indeterminacy of this beam is degree of indeterminacy is 2. So, you need the number of plastic hinges as 3. Possible locations are A comma B, that is A and B; and somewhere in the beam AB, which will be obviously the point C, where the bending will be maximum. Can you guess where the bending will be maximum in this given beam? Centre. So, I can mark a point C here, and say that is going to be the possible point, where the plastic hinge could form.

Let us solve this using the both the methods. Let us say using static theorem; static theorem wants the structure to be in position; it wants the loading to be applied. And now, it wants a bending moment distribution for both external forces and internal reactions. So, let us draw the beam. Let me draw the bending moment diagram for both cases. So, we all know that, if the beam would have been simply supported for a given loading, the bending movement diagram is going to be a parabolic, whose value at the center will be equal to w l square by 8. That is this value. And, because of this, there will be fixed-end moments generated at these fixed supports. And, that value is going to be equal to w l square by 12. So, as usual, this is going to be negative and this is going to positive. So, I want to flop this. So, if I do that, I get a figure like this. So, obviously, this portion will be negative; this portion will be positive; and, this portion will be negative. So, this is the net bending moment diagram I have.

The possibilities of plastic hinges in these sections are the following. One I can have here; one I can have here; and, one I will have at the center. So, I should say 2 M p is equal to w l square by 8, because this is w l square by 8. This is one M p; this is another M p. Now, you must understand that, I must select sections in such a manner of maximum bending moments, where the structure will become a mechanism. So, I have selected the sections have maximum bending moments, the structure has been made to a mechanism. To make it that mechanism, I need three hinges; I have made three sections, where the bending moment is maximum. So, 2 M p is w l square by 8. So, 16 M p by l is w l. w l can be called as w c. This is kilo Newton per meter into meter, becomes a load – w c. So, 16 M p by l.

Let us solve this problem using kinematic theorem. This may deflect a profile. This is not the bending moment diagram; it is a deflected profile. The maximum deflection will be at the center. So, hinges will form here; it will form here; it will form here. Three hinges. This is an assumed mechanism. So, as usual, I will call this value as delta; this is theta and theta; and, this is going to be 2 theta. So, we should say, external virtual work should be equal to internal virtual work. The external virtual work in case of UDL is nothing but the force multiplied with this area. So, half l delta w c; which is going to be M p times of theta plus 2 theta plus theta. So, 4 M p theta. I can also say w c into l into delta is 8 M p theta. I can even call this as W c into delta is 8 M p theta, because w c into l is capital W c. And, delta and theta can be connected from the figure. Theta for small angles will be delta by l by 2, which is 2 delta by l. I substitute it here. So, W c into delta is 8 M p into 2 delta by l, which is 16 M p by l. We got the same value here.

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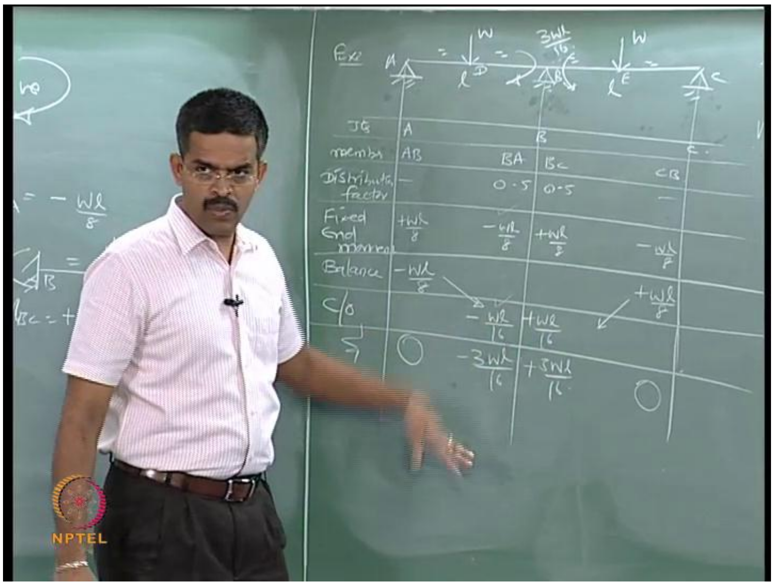

We will do another example, where we have got to straightly do some static indeterminacy calculations for finding out the elastic bending moment distributions. We will pick up this is example of a continuous beam with central concentrative loads. So, these are the points – A, B, C, D and E. What is the degree of indeterminacy for this problem? These are all hinged supports. For the modified value neglecting axial deformation, they will have only one vertical reaction. There are three reactions unknown; there are two equations available. What is r in this case? r will be 3 minus 2, which is 1. So, the number of plastic hinges required will be only 2, where they will form.

To solve this problem by static theorem, I must first know the bending moment distribution. The bending moment distribution for the span AB and span BC is very simple, is wl by 4. Simply, we can draw. But to draw for this entire span ABC, what are the moments at A and C being simply supported? They are 0. There are no moments. But there will be a moment at B here; I want to find this moment now. Since, it is statically determinate by one order, I cannot use it. By simple equations of equilibrium, I must do a method by which I can find out M B. Let me do moment distribution method. We are not discussing this method here. But I will briefly explain this here. We will now do moment distribution method to find out the moment at B. So, it is a very simple method. These are the joints, which is A, B and C. These are the member designations. I call this as AB; this is BA; this is BC; and, this is CB. I would like to write down the distribution factors – distributions factor.

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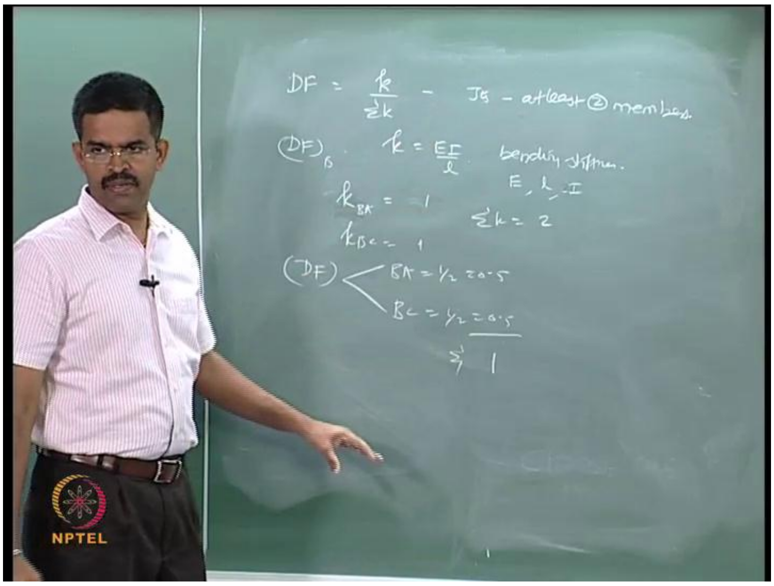

Distribution factor is given by k by sigma k. This is generally done for joints, which has got at least two members. So, in this beam, which is the joint, which at least have two members connected? B. So, I should do distribution factor for the joint B. Generally, k is given by simply E I by l. I am looking for the bending stiffness. The bending stiffness is proportional to E I by l. So, I am saying E and l of AB and BC are same. Even I – that is, the cross-sectional dimensions of AB and BC beams are same. So, I can simply say k of BA will be let us say 1; k of BC will also be 1; that is, E I by l.

So, sum of k will be 2. So, distribution factor for BA will 1 by 2, which is 0.5 and for BC is again 1 by 2, which is 0.5. Remember, the sum of distribution factor at any joint – be it any number of members connecting the joint should be always equal to 1. So, I am writing those values here. And, these are not available for A and C, because you should have at least have two members to connect this joint; I can remove this. Is this clear? Very briefly and quickly I have explained this. Now, I want to write the fixed-end moments. Let me draw the lines; it will be easy – fixed-end moments.

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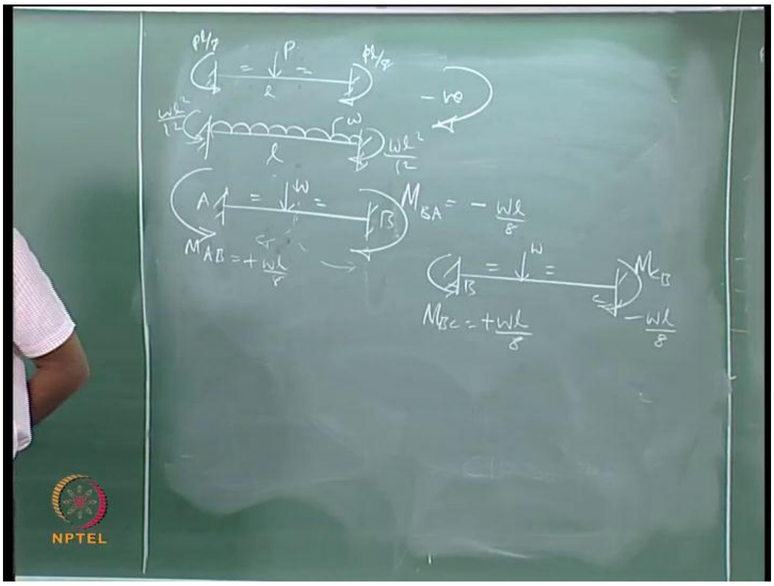

If I have a member subjected to a central concerted load let say p – equal l. This value and this value or standard available, which is p l by 8 and p l by 8. If I have a member of l subjected to UDL of w, then these are equal to wl square by 12 and wl square by 12. Now, I consider clockwise moments as negative. Some sign convention I have to follow. So, I pick up this problem; I say, initially, AB is simply supported. But I am trying to find out the fixed-end moments; I must convert these supports to fixed; I have to assume them to fixed; then release it later. I will show you.

So, this is A; this is B; central load w – equal. So, now, this will be M BA; and, this will be M AB. You may wonder, how I am marking these arrow directions; how am marking these. Hold the finger here; take the moment of this force about this point. So, this will have a direction like this. This will oppose it. Hold the finger here; take the moment of this force about this point. This will rotate like this. This will oppose. So, I said clock wise is negative. So, I can say M BA is minus wl by 8 and M AB is plus wl by 8. Similarly, let us do it for BC also. Do you agree? Same.

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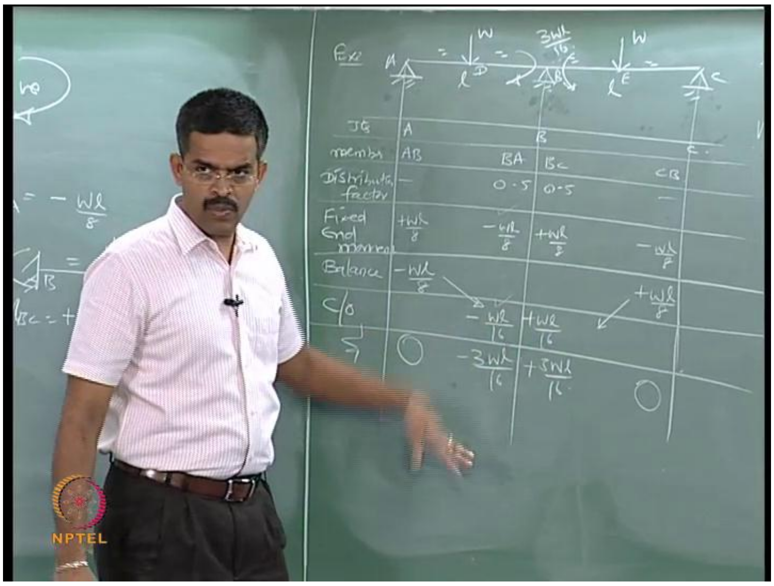

Let me write down these values here. So, M AB is wl by 8 minus wl by 8 plus wl by 8, minus wl by 8. In original beam, joint A and C were simply supported. What are the moments at the simply supported joint or a support? They are 0; whereas, the fixed-end moment caused by this load at this A is plus l by 8. So, I have to make it to 0. So, I must say balance. So, if it is plus wl by 8, I must say minus wl by 8 to make it 0. Similarly, this will be plus wl by 8. Do you agree this? Once I balance this, I should also do what we call as a carryover. Carryover is 50 percent of this. So, I must carryover this with minus wl by 16; 50 percent of that. And, this is plus wl by 16. Now, if I say this is balance 0, this is balance 0; and, this is also balanced, but the value is not 0; they are balanced, but the value is not 0. Now, this is nothing but M B. What is that value? l by 16. So, I am marking it here.

Now, I have to add this; these two values. Let us add this. It is not wl by 16; I have to add these two. Can you give me what is this value? This and this together, because there is value here; there is value here. I have to add. This is nothing but sum. 3 wl by 16. wl by 8 and wl by 16. So, I write it here. Just to find this value, we have explained this thing. This method what we call as moment distribution method. Suppose if they are not balanced, then you have got to again do the balance and carryover; and, keep on doing it for n number of cycles until the moments get balanced. So, I will remove this. I do not require this anymore. What I wanted is only this value. Now, we got this value.

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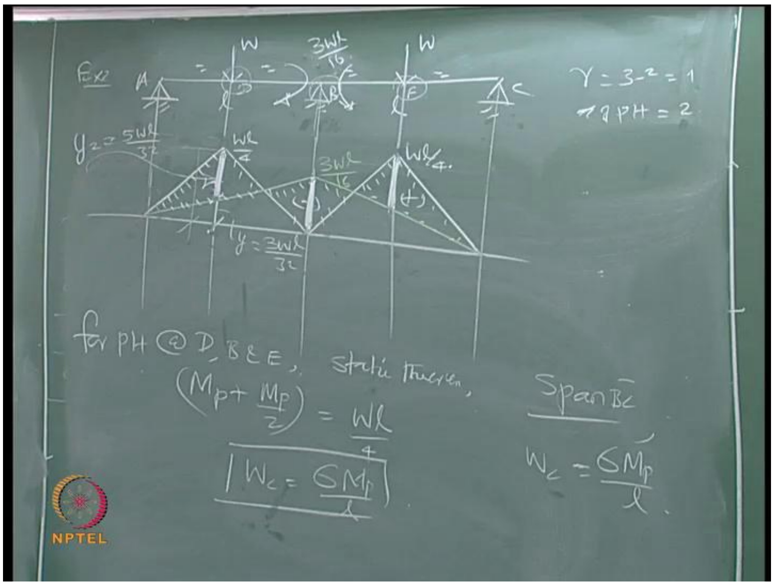

Now, let us do the static method. I am drawing both the positive and negative bending moment in the same side; instead of flipping it later, I am drawing on the same side. So, if I do this first for here, what will be this value? wl by 4. And similarly, this value will also be wl by 4. Then this value will be 3 by 16. Is 3 by 16 is greater than wl by 4 or less than wl by 4? Check 3 by 16, what is the fraction. It is 0.1875? See.

Student: 0.18, sir.

So, wl by 4 is 0.25. So, this ordinary beam will blow here. So, this is 3 wl by 16. Now, the net bending moment diagram is this. This may net bending moment diagram. Now, this value is wl by 4. I want to know, what is this value. Can you find out that? It is very simple, because for l, it is 3 by 16 wl. For l by 2, this can be easily found out. Can you tell me, what is this value? I call this as y. Tell me, what is the value? This is l by 2. So, this is 3 wl by… Half of this, no? 32. How many plastic hinges the section can have? 2; one is for sure at B, let us say for example. One will be here or here. Since the structure is symmetric, it can have more than r plus 1 also, which is called as over-complete collapse.

Now, can you tell me, what is this value? I call this as y 2. This is wl by 4; this is 3 wl by 32. I want this value. 5 wl by? 5 wl by 32. Now, if you agree that, the plastic hinges will form at D, B and E; for plastic hinges formed at D, B and E; using static theorem, I can say that, M p plus M p by 2. This is M p by 2, why? This is M p. M p by 2 plus M p is equal to wl by 4. Can you say that? Because this is wl by 4. So, can you tell me, what is w c? Good. This is for the span AB. They can also try to do this span BC; you will get the same story here also – span BC. This value will be wl by 4. This is going to be M p; this is going to be M p by 2, because the same equation; you get w c. So, if you have got the combined mechanisms like this, analyze them independently and look at the maximum value. Why? This is by the static theorem. Static theorem will give you w c's. This is w c, is also 6 M p by l. So, static theorem will give you w c's, which are either lower than true collapse load or to the maximum, equal. So, look at the maximum. I compare these two; both of them are equal. So, w c is equal to actually 6 M p by l. This by the static theorem.

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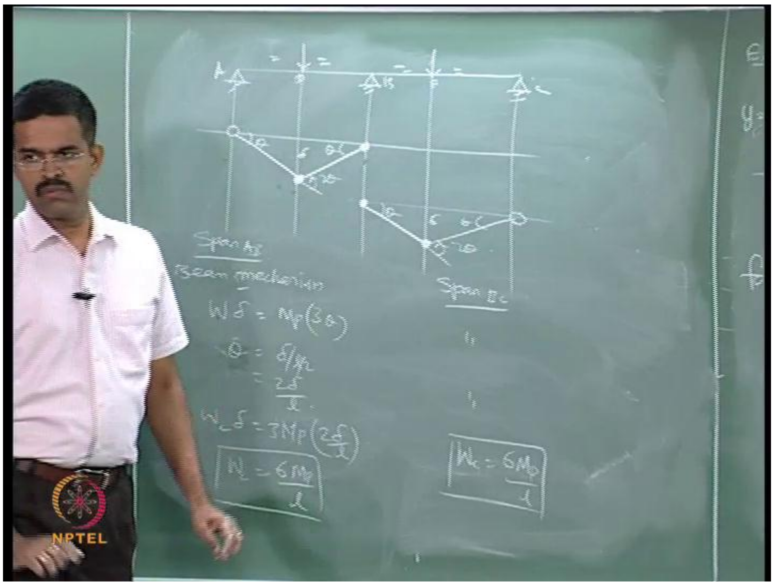

Let me do this problem by the kinematic theorem. Where are the hinges forming? For the span AB, it will form here and here. For the span BC, it will form here and here. Why it is not forming at A and C? They are simply supporting the structural hinges already. If I call this as delta and theta, this is 2 theta again; this is 2 theta again. I can do it for span AB separately. What kind of mechanism is this? This is the beam mechanism. Span AB – it forms a beam mechanism. How many hinges are required? You require two hinges. So, you get two hinges here. So, complete collapse; it is not a partial collapse.

So, W into delta is going to be equal to M p into 3 theta. 2 theta here and 1 theta here. So, we all know, theta is also given by delta by l by 2, which is 2 delta by l. So, substitute here. w c into delta is M p 2 delta by l. So, w c is 6 M p by l. This is for the span AB. Let us do it for the span BC. Virtually, it is same. It is going to be repeatedly same. So, you get both the answers same. So, w c is 6 M p by l. Now, from this quickly, one can think about, what is the design, what we are worried about. Let us pick up this example quickly to demonstrate, how do we do the design from this. Let us rub this.

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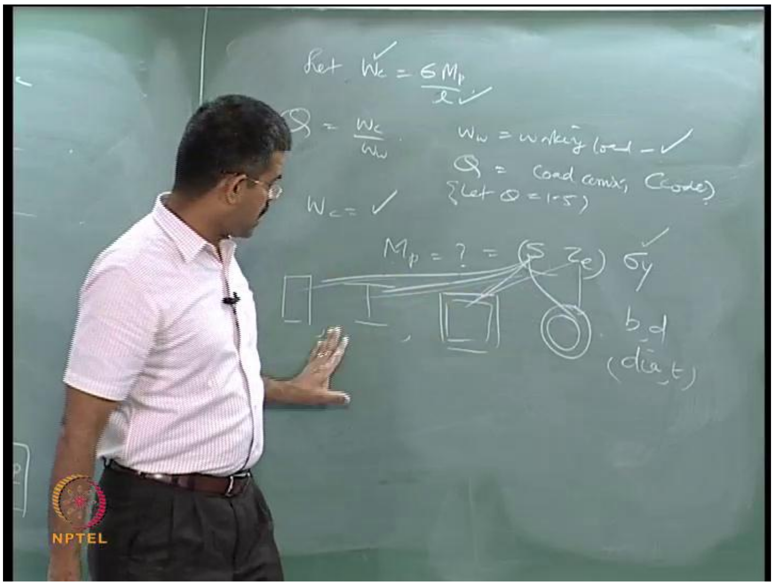

See for this example, let us say, we know that, w c is 6 M p by l. We also know the load factor Q is w c is by w w. w w is nothing but the working load, which we already know. Q can be taken for different load combinations from the code. So, Q is known to me. For example, let Q be 1.5. So, I know w c. So, left-hand side is known to me; l is known to me; I will find M p. And, M p is nothing but S into z e into sigma y. Sigma y is known to me. Now, I will select a section of rectangle I box Q.

So, I know S value. I already know the z e value for all of them. I design it, because S will be a function of… Or z e will be a function of b and d or diameter and thickness, etcetera, depending upon the section what you want. So, you equate this. You will equate this, because M p is known. Equate it here. M p is equal to this. Pick up the section; choose S – 1.5 1.15. S is a number for a given section. z e is a parameter of b and d or etcetera; you will find b and d. That is design. That is plastic design. It is as simple as this. We will take up another example, where the problem is slightly tricky.

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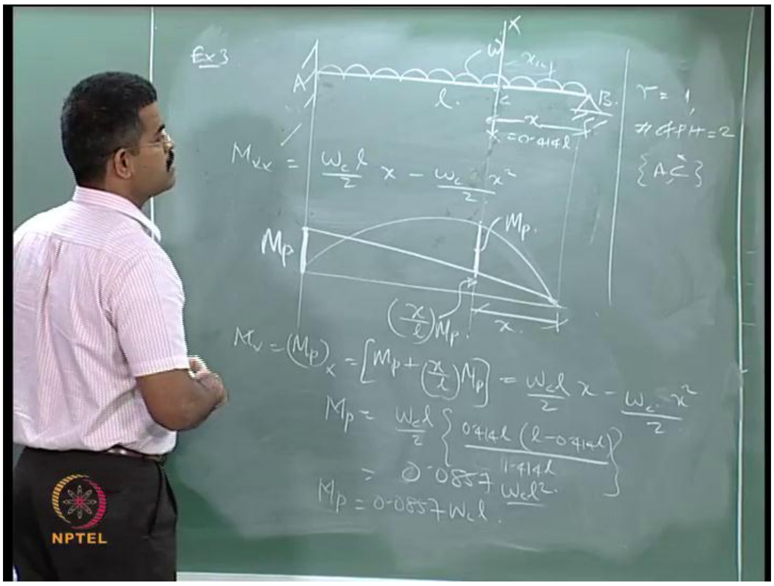

Now, I will take a propped cantilever, is w, l, A, B. Now, what is the degree of indeterminacy of this problem? There are two unknowns here: vertical and moment; one unknown here – vertical 3 minus 2. So, it is 1. So, how many plastic hinges you need to make it as complete collapse? 2 to make it as a mechanism. Where they will be from? For sure, one will happen at A, no doubt. Can the other one happen at B? Next hinge. Why? Because simply support. So, there will be a section C, which we do not know where this will occur, because it is the point where the bending moment is going to be maximum.

So, we should find that point. Are you getting the question here or not? So, it will be at C, which we do not know. We want to find the point C as well in this problem. So, if I do… For example, if I directly do kinematic theorem to this, kinematic theorem says, draw a hinged location and form a hinge here and form a hinge here; the external virtual works internal. So, I should know delta; I know theta. Now, if I do not know this, I cannot find delta and theta. So, I cannot apply kinematic theorem to this.

Can we apply static theorem to this? I do not know what is the bending moment at this point. So, let me do bending moment x x, which is w c l by 2 into x. That is the reaction here – minus w c l x square by 2. That is the bending moment at x any section. Section; look at the right – the reaction multiplied with the distance; and, this value multiplied with this distance. If I try to draw the bending moment distribution for this, one will be a parabolic – the maximum is occurring here; one will be a linear – 0. So, this is going to be M p and this is going to be M p, because two hinges. And, this distance is x of course. Can you tell me, what is this, if this is M p. For l, it is M p; for x, what will be this value? What is this value? x by l in M p. So, the total bending moment at this point should be equal to this to make it as a mechanism. There is no l here, no? There is no l here. There is no l here, please. It is w c x square by 2.

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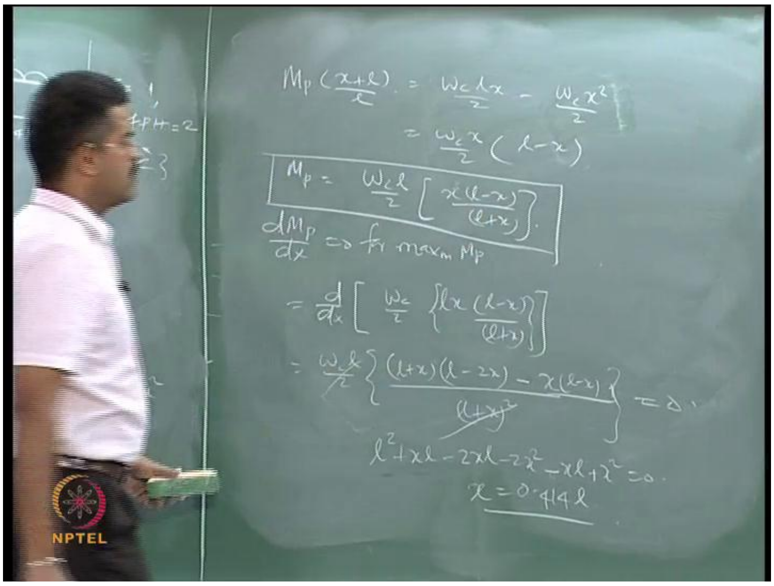

Let us say now, w c x by 2 l minus x. So, M p will be now equal to w c l by 2 x into l minus x by l plus x. Now, I want this (( )) M p to be maximum. M p will be maximum, because plastic hinge is forming here. So, I should say, the differential of this with respect to the variable should be set to 0 to make it maximum for maximum M p. Differentiate this expression. See what happens. So, it is basically d by d x of w c by 2 l x of l minus x by l plus x. So, already we have an expression here. We can use this equation. Substitute x as 0.414; get the value. We have x as 0.414; get the value. How much is that? So, w c l by 2 0.414 l minus 0.414 l by 1.414 l. Substitute this. No. No. How much? Come on, this can be easily sorted out, no? This is going to be 0.0857 w c l square. Check. Are you getting it or not? So, if I say w c l is capital W c, M p is equal to 0.0857 W c l; I can say. This small w c l square is capital W c l. So, can we say what is W c? I am interested in W c. What is W c?

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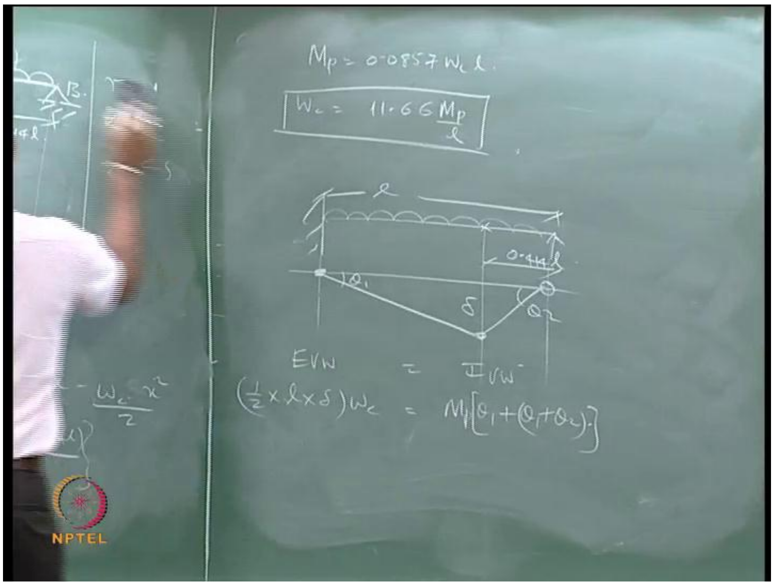

What is W c? M p is equal to 0.0857 W c l. 11.66 M P by l. This may collapse. And, the hinge will occur at a distance of 0.414 from the proper support; not from here, from the proper support. So, it is not at 0.5. It is away from 0.5. That is why we have drawn the figure like this. The center is not at 0.5; it is away. This value is more than this. Now, I can use kinematic theorem to find out M p. How? See here. This is delta; this is theta 1; this is theta 2. And, this distance already I know, which is 0.414 l if this is l. So, what is the external virtual work? Half into base into height into w c. Internal virtual work will be M p of theta 1 plus theta 1 plus theta 2.

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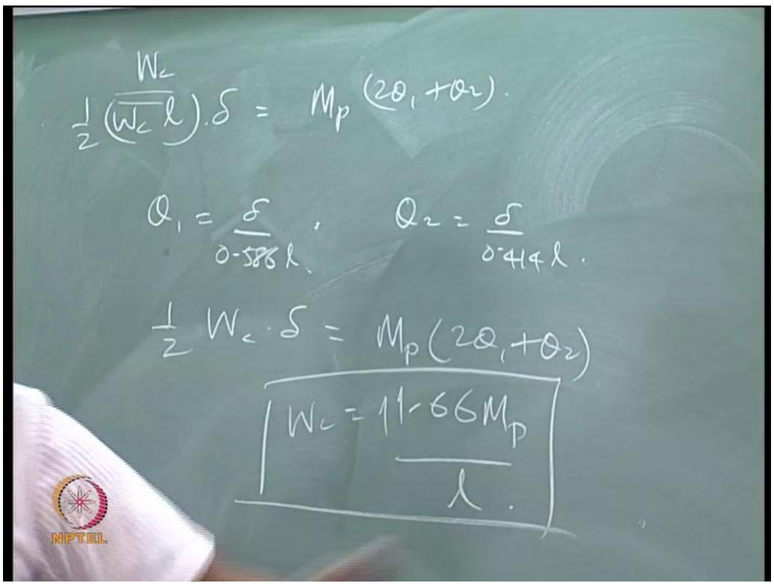

Half w c l into delta is M p of 2 theta 1 plus theta 2. Theta 1 is equal to delta by? This is how much? l minus 0.414. How much is that? 0.586. And, theta 2 is delta by 0.414 l. So, substitute here. Cancel for delta. Get me, what is M p. Whereas, w c into l can be taken as capital W c. So, half capital W c into delta is M P 2 theta 1 plus theta 2. So, theta 1 is known; theta 2 is known. We can compare them; substitute them. Find out what is (( )) tell me, what is M p? How much? You are using calculators, no? How much is that? You will see that, this will be also 11.66 M p by l; exactly same.

So, in this example, we have understood that, kinematic theorem in certain cases, cannot be applied directly, because you do not know the location of the formation of the hinge. So, both the theorems are equally important to understand. So, in this example, classically, you have understood that, we have used static theorem first to find out the location, where the hinge can form. Then we have verified with the kinematic theorem. Is that clear? So, you cannot forego any one theorem thinking that, both theorems will give me the same answer; so I can study only one theorem and be done with that. No. In certain examples like this, static theorem plays a very important role to really find out what is the value of or where the point of occurrence of the maximum bending moment.

In the next class, we will talk about the some design example and depth of elastic core calculations; where, we do plastic design really for a simple beam and a frame and will demonstrate it. Once we understand that, we will talk about theories of failure. Then we will move on to accident loads and tubular joints. So, that will end the module one; maybe in another 4 or 5 lectures maximum. But you have got to be faster in calculation; otherwise, we will be delaying things.

Thanks.