Advanced Marine structures
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Lecture - 20
Plastic analysis-Example problems – II

So, we continue to discuss the lectures on topic advanced marine structures, lectures on module 1. In the last lecture, we discussed about basically two theorems which are used for doing plastic analysis of sections.

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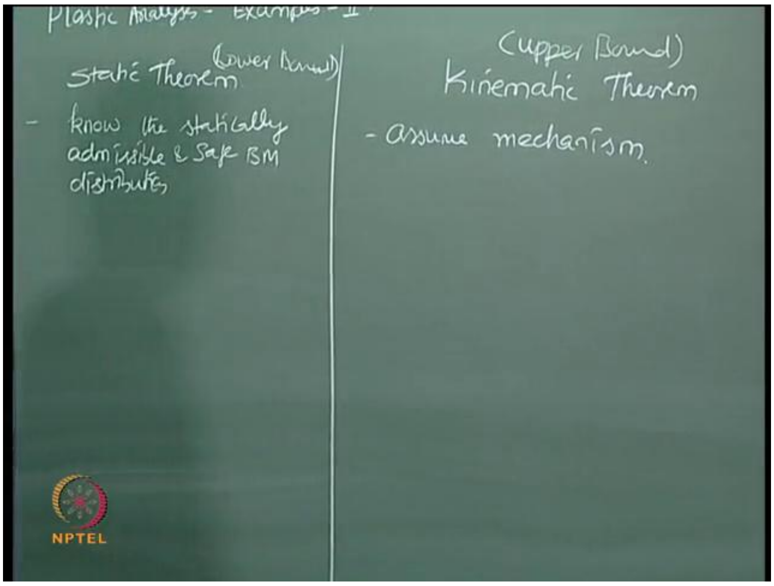

One is what we call as the static theorem, other is the kinematic theorem. Both theorem should yield me the same collapse load, though the procedure may be different. So, the main difference between the two theorems, which we saw yesterday was, in this case you must know the statically admissible and safe bending movement distribution. Whereas in this case, we have to assume a mechanism and for that mechanism you will always compute the collapse load or the true collapse load.

So, this theorem gives you values which will be lower than that of the two collapse load, whereas this theorem gives you the values which will be either greater or equal to the true collapse load. So, we call this as upper bound and we call this as lower bound. So, the moment we understand the kinematic theorem needs me to assume a mechanism or static theorem needs me to do a safe bending moment distribution. So, for the structure to remain statically indeterminate of a very high order I must know what are the possible mechanisms I will get in a given structural system or I must know how to do the bending moment distribution for a structure, which is statically indeterminate of a very high order.

So, both ways it requires a preload or precondition that one must know the classical structural mechanics to do your plastic analysis. So, plastic analysis can be seen as an advanced method of analysis with respect to the elastic convection analysis. If you do not know plastic analysis, you cannot proceed with the plastic analysis, because at least one method demands you to find out the bending moment distribution for this.

So, let us focus on this term mechanism. We already said mechanism is the structural system, which cannot offer resistance to the external load applied to it. So, obviously a given structural system, assembly of members, will have a certain distance to the forces, so we cannot say it is a system where does not offer resistance. So we say, we correct our statement saying that it is a structural system, which cannot offer resistance or does not prefer to offer resistance to a load beyond certain value. That value is what we are trying to capture. That is what we call as a collapse load. The term adjective collapse may cause a confusion to people saying that, is this method really leading towards the collapse mechanism or collapse of the structure. It is not so we had already seen in the last lecture that my load factor accounts for enough margin of safety, which is comparatively or reasonably better or similar to what we have in a conventional working such design in elastic analysis. So, talk about mechanism there are different kinds of mechanism available, which we must understand.

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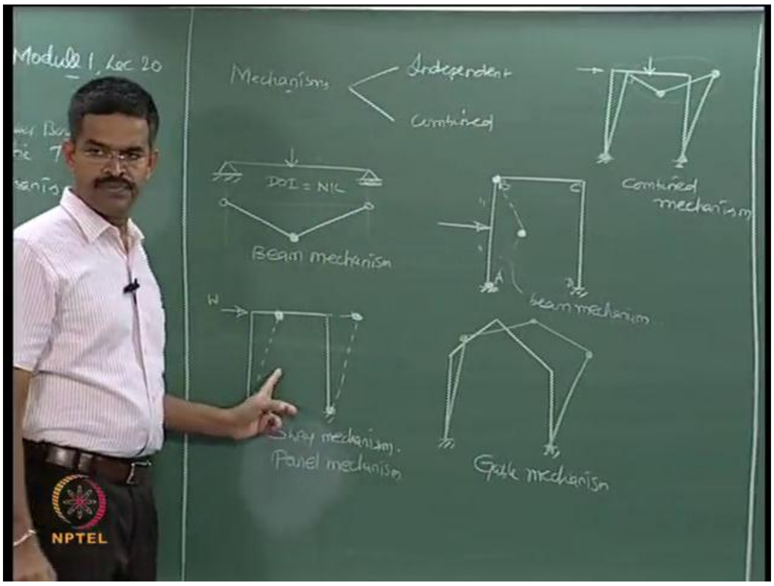

So, I can divide this globally into two. One is what we call as independent mechanism; other is what is called as combined mechanism. Suppose I have a beam which is simply supported subjected to a point load, central concentrated load, where the span on either side are equal then there will be only one hinge formed here. These two will be structural hinges. I think you now understand how I am drawing the plastic hinges and structural hinges. Why I am drawing one hinge because this is statically, static degree of indeterminacy for this beam is 0. Is it not? Is it determinate structure? So, I need only one plastic hinge to make it as a mechanism. This mechanism is what we call as beam mechanism.

Suppose we have a structure, which is a frame. Let us say a, b, c, d; subjected to a central concentrated load on the column. Let us say the span being equal. I can also draw a similar mechanism to the column like this. There can be a hinge; there cannot be a hinge here, that we are not bothered. So, this is also still called as a beam mechanism only. Some literature address this as column mechanism, it is one and the same and so on.

I have a frame, similar frame subjected to a lateral load, let us say w. The frame will have a tendency to sway. This frame has static degree of indeterminacy of, how much, two. It got two degree of indeterminacy. So, I need three hinges, or if we consider axial defamation also, I require, consider axial defamation also, then it will be three. Therefore, require four hinges. So, hinges can formed here, here, here, and here. This mechanism is what we call as a sway mechanism or panel mechanism. If you look at a gable frame, then there can be a mechanism like this. That is what we call as gable mechanism. All these are examples of independent mechanism.

If you want to look at an example of a combine mechanism, let us say, I have frame subjected to a lateral load and the gravity load. So, hinge may be found here, here, here, and here. And this is a 90 degree; there is no hinge formation here. This is what we call as a combined mechanism. You may wonder why it is called as a combine mechanism, I will explain you. This mechanism has two things inbuilt into it. One is a beam mechanism similar to what we have the here; other is a sway mechanism similar to what we have here. So, you can combine this.

Now, what do we do by identifying independent mechanisms? For every independent mechanism available in a structural system, you must try to find out the possible collapse load. Since I am assuming the mechanisms, I am trying to find out the collapse loads, each one format of the problem will give me one set of collapse loads. And since I am following the kinematic theorem, because I am assuming a mechanism, I will get collapse load for each one of the problem separately, independently; may be independent mechanism, may be combined mechanism.

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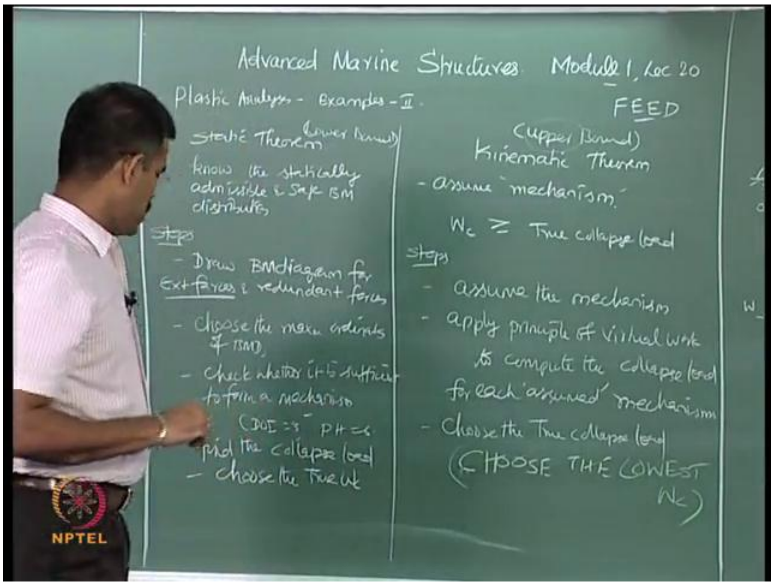

All will be align and I must pick up the lowest value. Why? Because as far as the kinematic theorem is concerned for an assumed mechanism, you will always find the collapse load either greater or equal to the true collapse. Is it not? Is it not? So, let us say I get Wc1, Wc2, Wc3, three collapse loads from three independent mechanisms. For example, from a beam mechanism we get Wc1, from a column mechanism I get Wc2, from combined mechanism I get Wc3. I must compare all these things and pickup the lowest value, because Wc1, 2, 3 at the best will be either equal to collapse load or greater than collapse load. Is it not? Therefore, I make it the lowest.

So, do not get confused that upper bound theorem means pickup the highest; lower bound theorem means pickup the lowest. It is not like that. While picking up the value it is controversy here, right? Because these values will give you in true sense more than the actual value or worst case equal. So, Wc1, Wc2, Wc3, either all will be equal or all can be greater than the true collapse loads. So, I must pick up the lowest. Do not get confused with upper bound means highest value; lower bound means lowest value. It is not like that. Be very carefully with the statement.

Now, let us see what are the steps involved before we do couple of problems today. What are the steps involved in both the methods, independently? Very simple. In this method the structure is given to you, loading system is given to you. In this also structure is given to you, loading system is given to you. Plastic analysis is not a technique, which will help you to choose a structural form. Structural form is chosen for marine structures, depending upon what is the functional requirements, what it is to be provided, what is the other kinds of loads coming on to the structure, what is the deck area required.

All these are covered in what we call as feed that is called Front End Engineering Design. So, we do this and check the layout of the platform, select a platform or select a form, geometric form, structural form of the platform. So, for all practical purposes to do a plastic analysis the structural form, and the load coming on to the structure or available to me. All these loads are variabilities, uncertainties; these are also called working loads. I multiply this with the factor make them as factored loads. Then apply them to the structure.

So, the steps involved in this case is having given both the theorems required; structural system and the set of forces or loads coming on to the system. In this case, draw the bending moment diagram for external forces and redundant forces. What do you mean by external forces, redundant forces? External forces are applied forces; redundant forces are support reactions of the structure. We must draw bending on diagram for both. Then choose the maximum ordinates of BMD; check whether it is sufficient to form a mechanism. What do you mean by a mechanism here? Say, I am having a bending moment diagram; I pick up the points in the bending moment diagram, which is having the maximum ordinates. Let us say the degree of indeterminacy for my problem is five, how much plastic hinges should I get to make it as a mechanism? Six, so I must get number of plastic hinges required is six. It means plastic hinges will generally form at sections where the bending moment is maximum. I must at least select six sections, I must at least select six sections where the bending moment is maximum in the given system. So, it forms a mechanism.

Then, find the collapse load and choose the true collapse load. How will you choose the true collapse load? If you have got different mechanisms different possibility combinations, you must select them and each one of them will give you different load sets. They will be lower than or worst case equal to the true collapse load. You must pick up the maximum. Let us say in this case, how do we do? For a given system, assume a mechanism and for an assume mechanism, apply principle of virtual work to compute the collapse loads for each assumed mechanisms. So, for each assumed mechanisms you will get one set of collapse loads all of them either will be greater or at least equal to the true collapse load. So, choose the true collapse load. Let us put a very clear statement here if you got many mechanisms, choose the lowest Wc, choose the lowest Wc. Any doubt here?

So, there is one more theorem, which we call as uniqueness theorem. We will discuss that quickly and then take a few examples on beams and frames and try to do plastic analysis and find the collapse loads. Then it will be clear for you how we are estimating.

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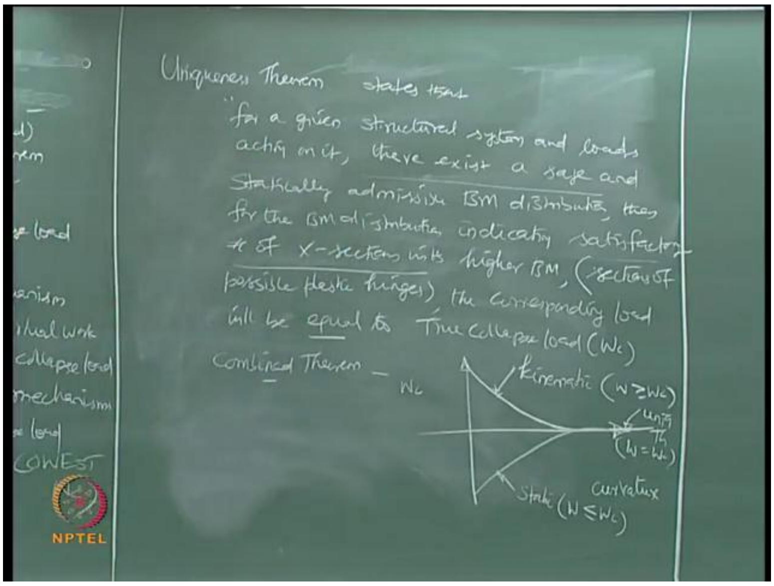

Uniqueness theorem states that for a given structural system and loads acting on it there exists a safe and statically admissible bending moment distribution, then for the bending moment distribution indicating satisfactory number of cross sections with higher bending moments which are nothing but sections of possible plastic hinges, the corresponding load will be equal to true collapse load. Will be neither higher nor lower it will be exactly equal to the collapse load. This theorem is also called as combined theorem. The name combined theorem is given to this theorem because of one reason that it is combing both the principles of static and kinematic theorems.

It combines static theorems because it requests admissive bending moment distributions. It combines kinematic theorem because it is checking whether the sections develop enough number of plastic hinges to make it as a mechanism. So, it is combining both. That is why it is called combined theorem and this theorem can be plotted like this. We talk about curvature and Wc. Wc will keep on dipping and the curvature will keep on increasing and in the another case Wc will be keep on increasing. So, one is upper bound and other one is lower bound or one is kinematic and other is static.

Can you tell me which is kinematic, white or green and why? So, this will be kinematic, because in this case W will be either greater than or equal to Wc. And of course this will be static. In this case W will be either less than or equal to Wc and uniqueness theorem is this horizontal line, this is uniqueness theorem where W will be equal to Wc. Is that clear?

Now, we have been talking about mechanisms, we studied independent mechanism, we understood combined mechanism. All these mechanisms will enable n number of plastic hinges to form, so the structure does not reflect or react back to the applied loads.

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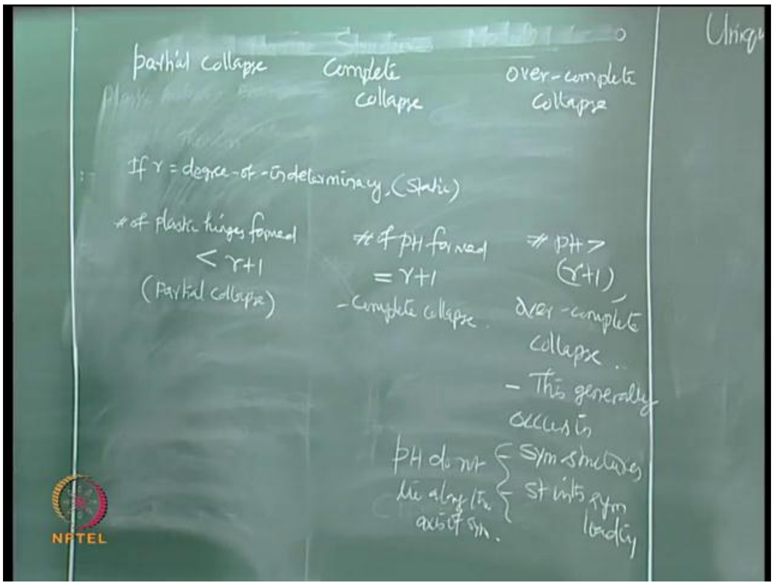

There are something called partial collapse, complete collapse, over complete collapse. If r is the degree of indeterminacy, I am talking about static degree of indeterminacy. If the number of plastic hinges formed are less than r plus 1, then it is called partial collapse. If the number of plastic hinges formed are equal to r plus 1, it is called complete collapse. If the number of plastic hinges formed are more than r plus 1, this is called over complete collapse. This generally occurs, this generally occurs in symmetric structures or structures with symmetric loading, where plastic hinges do not lie along the axis of symmetry.

So, not necessarily always you required r plus 1 to make it as a mechanism. Can have less than r plus 1, still it is called as a partial collapse. Can have r plus 1 more than that, still it is called as an over complete collapse. So, r plus 1 is a strategy to understand the structure will become a mechanism, but even if your plastic hinges less than r plus 1, still it will collapse partially. Let us take some examples now and try to understand both the theorems by solving these examples. I will take simple problems first then we will straightly take complicated problems later in this lecture. So, we can demonstrate this.

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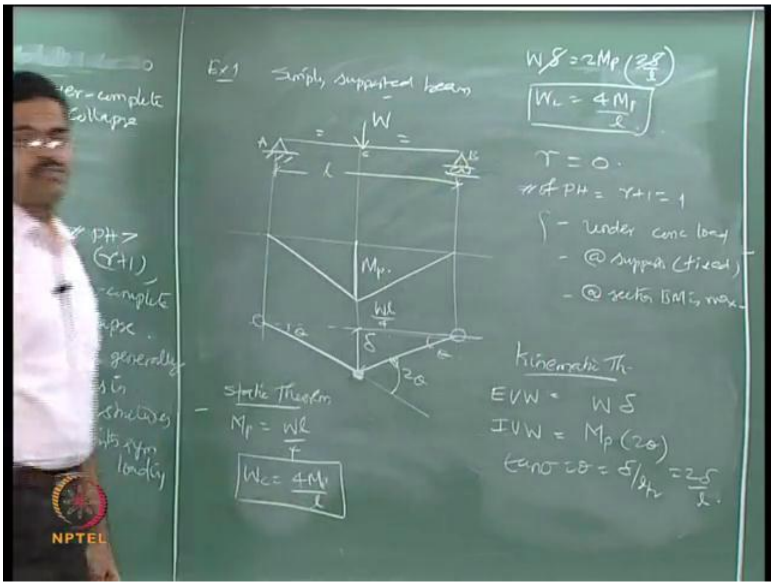

So, I will take example one we will take a simply supported beam. A simply supported beam is subjected to central concentrated load w, span of the beam is l. We all know that degree of indeterminacy of this beam is 0, it is determinate structure. So, how many plastic hinges are required to make it as a mechanism-one. So, number of plastic hinges required to make it as a mechanism is r plus 1, where plastic hinges can form? It can form at the following locations; one under the concentrated loads, two at supports which are fixed, at sections where the bending moment is maximum and so on. Is it not? This is what we have seen yesterday also.

These are two supports where there are structural hinges are already available. So, plastic hinges cannot form here because bending moment is 0 at this point. So, the only point available on this beam where plastic hinge can form to make it as mechanism is the point c. You can call this is a, b, c. So, let us first solve this problem by static method. Static method says I must draw the bending moment diagram. Is it not? Let me draw the bending moment diagram for this 0, 0 and supports and this value. What is this value going to be? W l by 4. That is a bending moment diagram ordinate here. I understand that there will be a hinge formed here and this value will become Mp directly. Is it not?

So, let me write down the value here. Static theorem: so Mp will be equal to W l by 4 or Wc will be 4 Mp by l. I am writing directly collapse load because there is any one possible bending moment distribution I can do. If I got few more, then I have to pick up the values and compare them and find out the highest of this and wrote Wc. But, there is only one possible distribution I have. Therefore, I wrote Wc directly. Is that okay? Then, let me draw the plastic hinge formation so this is the original line where the beam is just now this is going to be my deformation or deformed shape of the structure. There will be structural hinges here and there is going to be a plastic hinge here. Is it not, right?

So, I called this deflection as delta and this rotation as theta because of symmetry this will also be theta. You can agree that this tangent will have two theta rotations Is it not? Agreed? So, principle of virtual work as to be applied for kinematic theorem, what is principle of virtual work? What is external virtual work? External virtual work is the load multiplied by deformation, which is W into delta. What is the internal virtual work? Internal virtual work is the work done by the hinge. We already told you yesterday, plastic hinge is a hinge, which requires some moment to rotate. It is not rotate freely, is it not? What that moment require to rotate plastic hinge, Mp. So, it is going to be Mp multiplied by 2 theta, that is the work done.

Now, theta and delta can be compared from the geometry. I can simply say tan theta or theta is directly delta by l by 2. If this is l and this is l by 2, so I say simply it is 2 delta by l. Is that clear? Let me compare this. So, I should say, let me write down here because it is will very difficult to be focus I will write down here, so I should say W delta is Mp theta I am replacing as 2 delta by l delta goes away, I can straight away say Wc is 4 Mp by l, is that okay? I get the same answer as I have here so both the theorems will give me the same value. Okay any confusion in this example? I start with a simple example just to understand. Because, bending moment diagram for a simply supported beam is what we all know. Any doubt?

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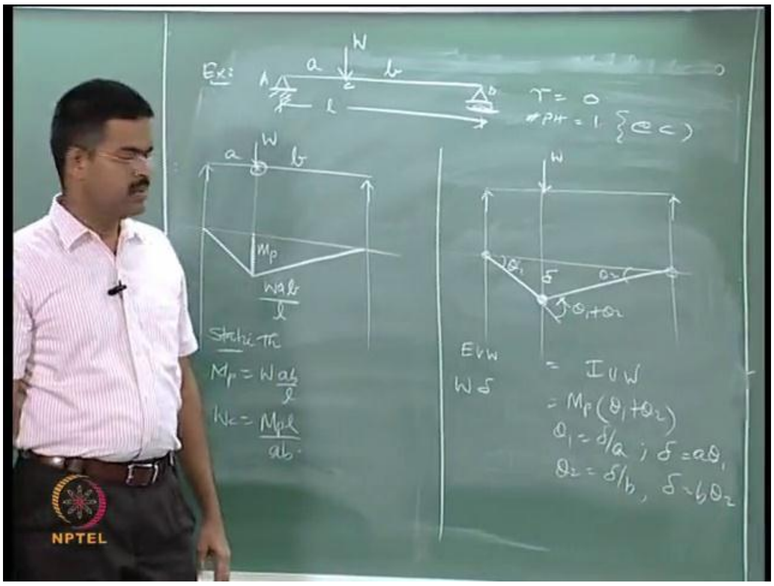

Let us do one more example. Let us take up an example where the loading a section take I have a simply supported beam again but the loading is not equal. This is a and this is b and of course this is l and this is w. So, the degree of indeterminacy for this beam is 0, number of plastic hinges required for this section to make mechanism is one, and where it will form, it will form under the concentrated load. I call this is a, b and c. The possible locations of the plastic hinges at c, that is possible location. So, let me draw the bending moment diagram for this. This is going to be bending moment diagram what is the value here W a b by l. And I know that there is going to be section where the bending moment is maximum.

So, this will be a location where plastic hinge can form which makes it as a mechanism. So, let us do the static solution for this from static theorem I know Mp is simply W a b by l. Wc is Mp l by a b. Let us solve this problem using kinematic theorem. I have a structural hinge here, I have structural hinge here, there will be a plastic hinge here. I call this as delta, this is theta 1 and this as theta2 now. And the rotation total will be actually theta 1 plus theta 2. External virtual work is equal to internal virtual work external virtual work is W into delta. Internal virtual work is nothing but Mp of theta 1 plus theta 2. Is that okay? Now, from the figure theta 1 is delta by a or delta is equal to a theta 1. Also, from the figure theta 2 is delta by b that is delta is equal to b theta . So, theta 1 can be expressed in terms of theta2 because they are equal…

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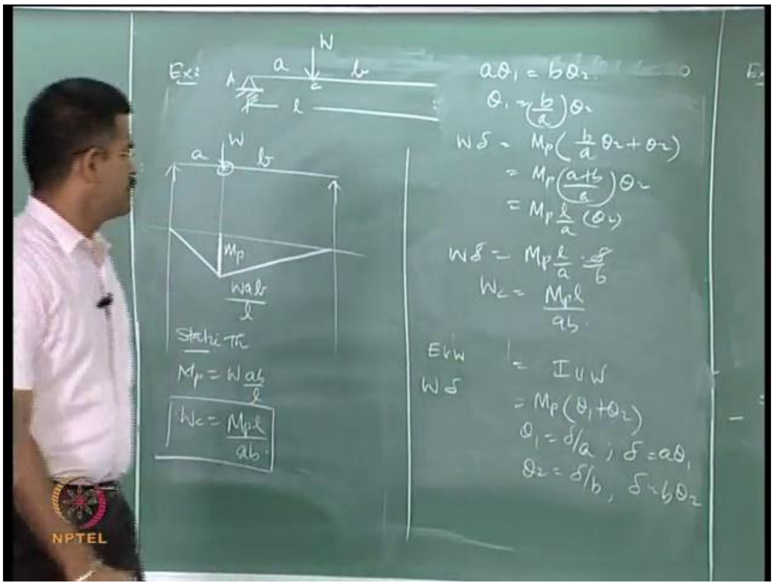

So, if I say, I will remove it here, a theta1 can be b theta2. Therefore, theta1 can be b by a of theta2. So, let me get back here. W delta is going to be equal to Mp of theta1 which is b by a of theta2 plus theta2, which is Mp a plus b by a of theta2, which is Mp l by a of theta 2. I will remove this off. Theta 2 can be said as delta by b. so W delta is Mp l by a theta2 is replaced as delta by b so delta goes away. So, Wc is Mp l by a b which is as same as I have here.

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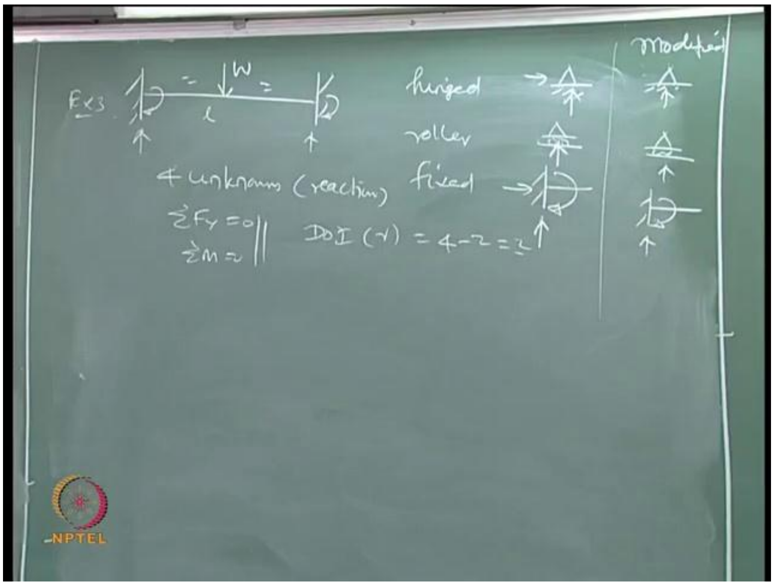

Let us do one more example. This time we will take a fixed beam. I will take a fixed beam with central concentrated load, span of the beam is here. Now, what is the degree of indeterminacy of this beam? How will you calculate the degree of indeterminacy? There are two three kinds of supports we have in general. One is what we call as an hinged support, roller and fixed. The three supports. How are they marked? Fixed is marked like this, roller is marked like this, and hinged is marked like this. Fixed will offer three reactions one vertical, one horizontal and one moment.

Roller will offer only one reaction, which is vertical. Hinge will offer two reactions one is vertical and horizontal. Now, let us ignore the axial defamation present in the system therefore, all horizontal reactions will ignore for time being. If we ignore that obviously the modified reactions-- for this could be only vertical, for this could be only vertical, for this could be vertical and moment, right? Let us use these values here to understand. This is got one, two, three, four, four unknowns. How many equal equilibrium you have? There are four unknowns, which are reactions. I am neglecting the axial defamation. So, two equations I have: sigma f y0, sigma m0. These are two set of static equations of equilibrium which I can apply here. Therefore, degree of indeterminacy, which I call as 4 minus 2, which is 2. So, the r value for this problem is 2, agreed.

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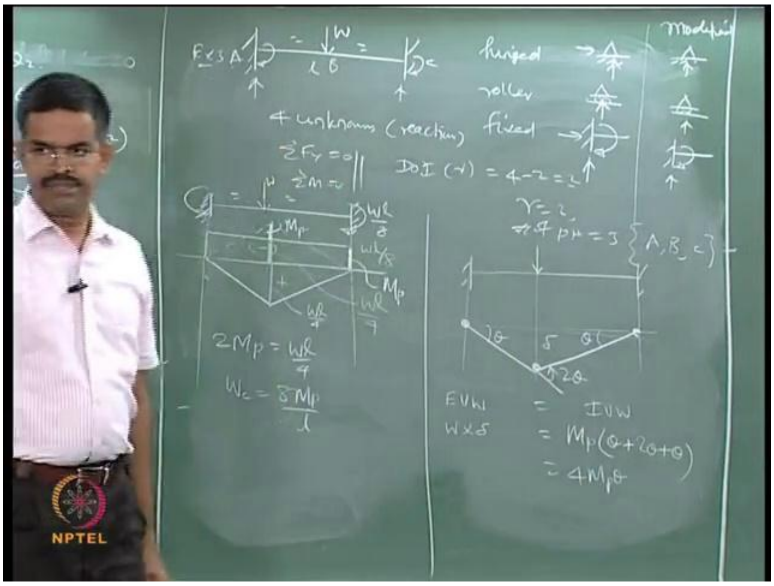

So, r is 2 therefore, the number of plastic hinges required will be three to make it as a mechanism. So, where they can form? What are the possible locations where these can be form? They can form now in all three points, a, b and c; a and b because they are supports, fixed supports c because it is a point of concentrated load, to make it as a mechanism. Let us solve this problem first using static. So, I would like to draw the bending moment diagram for this first. There are two bending moment diagrams for this: one is because of the load, other is because of the support moments. Because of the load the value is going to be, how much is value? This is central concentrated load is going to be W l by 4.

Now, I have the support moments also, from your knowledge of understanding can you tell me what is the support moments a fixed beam will develop because of central concentrated load here? It is going to be W l by 8. So, I plot this here. This is W l by 8 and this is negative and this is positive. Because this is because of the external load; this is because of the internal reaction. They will have bending moment at the bottom maximum but, these two will have bending moment at the top maximum. So, the negative moments and positive moments. Now, I want to super impose. What I do is with respect to this line I flop this, I just rotate this if you rotate it will become like this. Agreed? Why this is gone up because W l will be 4, this half of that. Now, this value what you see here is W l by 4. Agreed? I just flopped this, that is all. So, the plastic hinge which were interested is, one will form at the support, one will form at the section. Is it not? So, this will also be Mp and this will also be Mp, the net moment. So, I should say now 2 Mp is equal to W l by 4, two Mp is equal to W l by 4.

Therefore, I can say Wc is 8 Mp by l, is it not? Let us do this problem by kinematic theorem. Are you following or not? If you do not follow, please stop me and ask. As I told you, plastic analysis requires certain basic understanding of static indeterminate structure solution. So, unfortunately in this course we will not discuss about the how to solve this static indeterminate structures for analysis, elastic analysis. How to bending moment diagram and all? This is something which you must know. But still I am trying to explain to the maximum possible, within the given span of time. Now, I am trying to do a mechanism here. The mechanism is well understood that there will be a hinge here, there will be a hinge here and there will be hinge here.

This is going to be my deflected profile of the beam. So, this is delta, this is theta, this is also theta, this is 2 theta. Agreed? Now, let us see what is external virtual work? Nothing but W into delta, W into delta. Internal virtual work has happened in four places , two, three and four; all are Mps. Is it not so? Mp times of theta, 2 theta and theta is that okay? It is 4 Mp theta, agreed? I can now connect delta and theta by a simple relation. I will do it here I will remove this.

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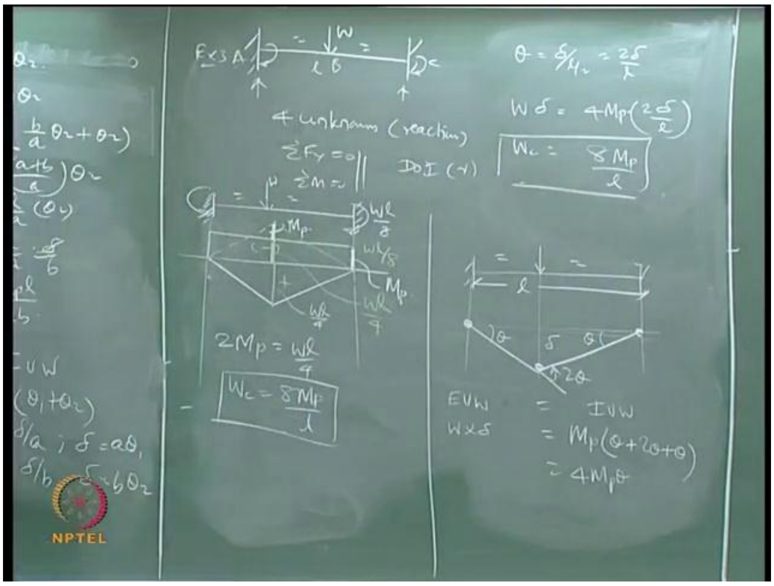

So, from the figure this is l by 2. So, W into delta is 4 Mp times of 2 delta by l. So, I should say Wc is 8 Mp by l, which is as same as what you get here. I can solve this problem by both the methods, using both the theorems; I will get the same answer. So, for in all these problems of examples there was only one possible mechanism which can directly give me the Wc value. Is it not? Suppose if we have more mechanisms, I must select the higher or the lower depending upon the method I am using. We will do those examples also possibly in the next lecture.

Thank you.