Advanced Marine structures

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Lecture - 19
Plastic design-IV - Example problems - I

Now, we continue discussing on the lectures on module 1 of advanced marine structures course. So, let us quickly see what we have so far discussed in a brief summary in this course. We started to understand, what are the different kinds of marine structures for its application for specific purpose in oil exploration? How do they react for the environmental loads on which they are acted upon? How these structures are designed or evolved based on form special modifications? The form modification, the geometric modifications make the structure suitable for different levels of oil exploration from shallow water to ultra deep waters. So, it is form based design and it is not a function based design.

Therefore, we were critically understanding or trying to understand, what are the different varieties of environmental loads, which act on these kinds of structures. So, we classified them in five permanent P, P category, live load L category, deformation loads D category, environmental loads E category and accidental loads A category. We understood that these loads, they are various series available for computing let us say wave load, wind load, current load, earthquake load, different methodologies etcetera. But in nut shell, all of them do have lot of variability's and uncertainties in their estimates.

Therefore, ultimately if you work out a load encounting a marine structure, it should be deterministic value, but does not remain deterministic, it becomes probabilistic. Because the probability of exceedance of any specific value, let us say you say you want to design a marine structure for wave height of 15 meters and a wave period of 15 seconds for a given sea state. What is the guarantee that the wave height and wave period at that sea state, where your structure is located within the service life of the structure will not be exceeded by these values?

So, we are looking for a question of probability of exceedance. The moment the probability term comes in, we associate this with the value called characteristic load. Then, we said even though the load and the strength of the load and the resistance can be normally distributed, but the normal distribution will not help us. Because I would like to see what would be those cut off values of the distribution of the load and the resistance in terms of S and P value, which is connected by a factor called gamma, which we call as a safety factor.

So, we said that the characteristic value of the load means 95 percent, this value will not be exceeded. The probability of exceedance will leave only 5 percent. If I say the characteristic value of the strength of the material, why characteristic value of the strength of the material, because strength of the material has got lot of imperfections, residual stresses set in the material or in the member, because of welding process, because of flame cutting, because of geometric un equality, because of manufacturing defects, because of construction processes errors etcetera.

So, we have to also account for these variations, these uncertainties in the strength of the material also. So, we say characteristic strength of the material, which we say as gamma M and we say 5 percent is the variation. So, we account for these and ultimately we connect these two and say there is a factor of safety or a margin of safety, which should account for these uncertainties and variability's present in the given system. Once, we said this we discussed about ultimate limit states.

Different varieties of limit states were limit state of serviceability and limit state of ultimate that is ULS and SLS are very important. They have got to be met as a mandate in a given design. If you follow ULS even then, limit state of serviceability for control or deflection should be checked for its safety of serviceability. So, we said that ultimate limit state is one process where we allow the structure to the load level beyond which the structure tend to collapse or we say the structure will get into a mechanism. Now, the question comes what is a mechanism?

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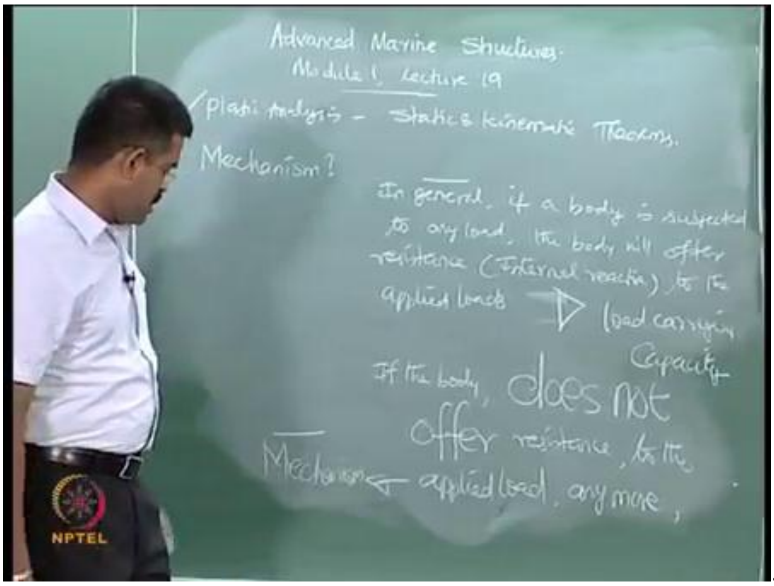

In general, if a body is subjected to any load, the body will offer resistance which is called as internal reaction to the applied loads. This process of offering resistance to the applied load is what we call as load carrying capacity. This is general behavior. Now, if the body, if the body does not offer resistance to the load anymore, then this is called a mechanism. So, mechanism is a system of structural members which does not offer resistance to any external load applied on to it.

Now, obviously you cannot have a structural system, which does not offer resistance at all. So, to be very implicit we must say that beyond a specific load level. It will not offer resistance beyond a specific applied load level. That load level is what we call as collapse load. Remember, very importantly mechanism is not the collapsed state of a structure. The structure has not collapsed. Structure has stopped offering resistance to the load at that instant. Once, the load exceeds even that then only it will collapse.

There is always a difference between a mechanism and a collapsed state of the structure. Mechanism is that state of the structural system or the body which does not offer any resistance or resistance any more to the applied load. And this applied load will have a level beyond which this will be implemented. That load level is what we call as collapsed load or ultimate load. Mechanism is not the collapsed state of the structure. If the load level exceeds even beyond that then it will collapse.

So, the structure becomes a mechanism in ultimate limit state design methodology. There are two theorems available with help of which we can easily find out what is the plastic moment of resistance of a given section or what is the collapse load the structure can sustain? First let us understand the difference between analysis and design.

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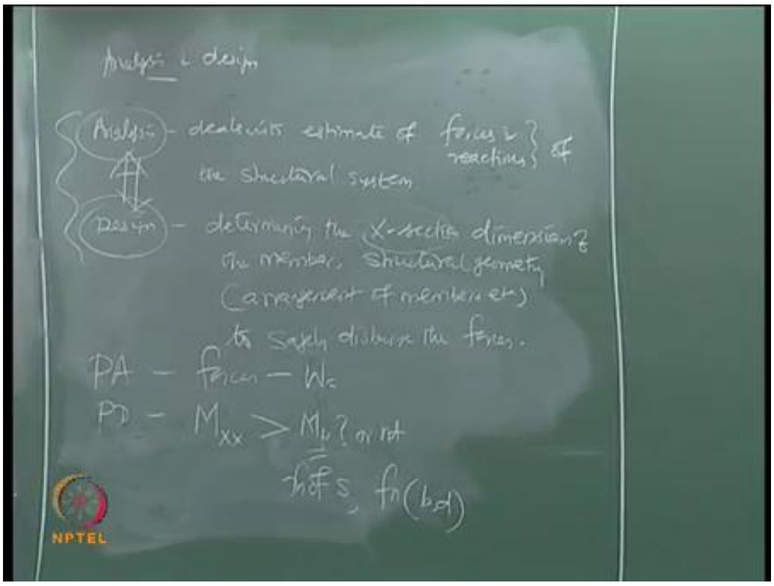

Let us understand this first because I am talking about plastic analysis here whereas; ultimate limit state is a design mechanism. So, what is the difference between these two? Interestingly, we have undergone both these terminology commonly people mix this. Analysis deals with estimate of forces and reactions of the structural system. The reactions may not necessarily be only the forces; the reactions can be rotations, displacements also.

You can react in any manner. Not necessarily only the force will be pumped back it can be any reactions. Design is to determine the cross sectional dimensions of the member, the structural geometry which talks about arrangement of members etcetera to safely disburse the forces. So, in any process the first step is to find the analysis, do the analysis. Then, the second step is to do the design. But interestingly, for doing the analysis, since we are interested to know what is the reactions offered by the member you need to know the member dimensions for example, you need to know I E, you need to know A, area of cross section, moment of inertia which are all geometric characters of a given structure or a member, is it not?

So, if you do not know the cross sectional dimension you cannot actually find out the reaction to the forces on the member. You can find the forces, but the reaction cannot be found out. Now, see these two are in a close loop, they are interconnected, they cannot be separated, they cannot be separated. So, what people generally do is assume certain dimension, find the reactions, then find the forces. Check those dimensions, whether it is safe. If they are not safe repeat the process again.

That is what the general theory about analysis and design is. In this context, let us see what are you talking about plastic analysis and plastic design? In plastic analysis I must find out the forces, what are the forces? The forces which will cause or tend to cause collapse to my system. In plastic design, we can check whether the moment coming in the cross section anywhere, anywhere in the given system because of these collapse forces does it exceed the capacity or not?

We are checking, but M p is a function of shape factor and shape factor is a function of b, d etcetera cross section dimension. Therefore, what you generally do is equate the maximum moment to M p and for that M p find the optimum dimensions of the section. So, that is what we say as plastic design. So, interestingly if you do not have the collapse forces to which the structure will subjected to, I cannot do the design, is it clear? So, far we were talking about the design principles where given that the collapse load is known to me, I can find the optimum dimensional section to safely distribute that load coming on to the section.

Now, I am going to talk about how to get that collapse load itself for a given structural system, are we clear? This is what we are trying to do. One may wonder sir, why we discussed about the design first then the analysis later. Why we did not talk about the analysis first and then the design? Because that is the sequential step we do. As long as you do not appreciate what is M p as long as you do not know how M p depends on the sectional characteristics. There is no point and there is no sense in making and finding W c. It is very simple. As long as they do not have a section with you, you cannot do an analysis. So, first we found out the section, we understood the section first. Now, we will talk about analysis. To do the analysis we have got two theorems available which is called static theorem and kinematic theorem.

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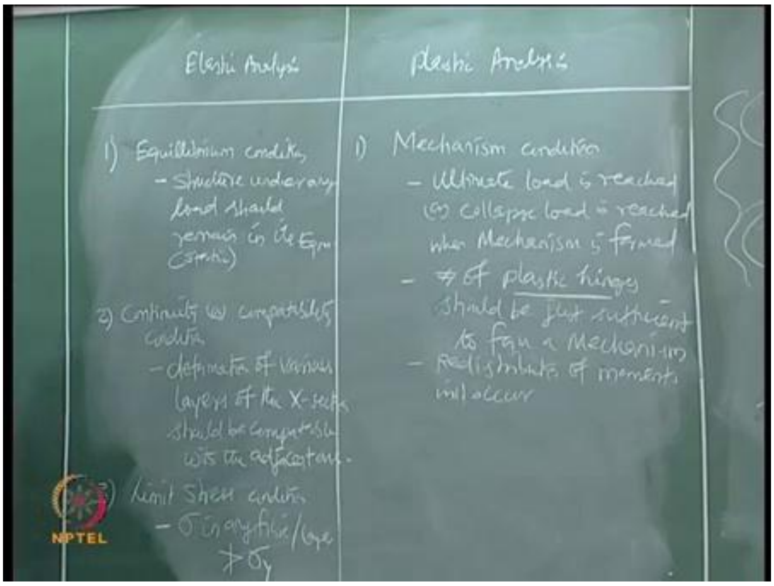

Before going to these theorems, we will quickly compare the elastic analysis and the plastic analysis let us quickly compare this. In elastic analysis, you must fulfill three conditions. One, equilibrium condition; this condition says the structure under any load should remain in its equilibrium. To be very clear, it should remain in static equilibrium. There is another you call dynamic equilibrium. We are talking about static equilibrium here.

The second condition, which is to be met for a successful elastic analysis is that continuity or compatibility condition. This says the deformation of various fibers of the cross section, the deformation of various layers of the cross section should be compatible with the adjacent ones. If there are two layers, which are touching each other, then the deformation on the upper layer A should be compatible with the lower layer B. We have got to check that; that is called continuity or compatibility.

The third condition is very important which is called limit stress condition. This condition says that the stress in any fiber or layer should not exceed the yield value. These are essentially three conditions, which are to be satisfied for a successful deployment of elastic analysis. Taking it granted that you all know how to do an elastic analysis because elastic analysis is also not that easy. It is complicated when the structure becomes statically indeterminate of a very high order you have got to use different tools. There are various methods.

We will not focus those methods in this course, because it is purely dedicated to something else on the area of topic, but you can refer to standard text books which you have studied already in your undergraduate program. We will touch upon some important aspects in the example problems later just to re brush whether you have understood them or not? But my focus is not explaining the elastic analysis in this course at all. We all think that we know this.

Since, we know this we can quickly compare the equivalency of this with the new method, which you are studying now. The first condition, which must satisfy in plastic analysis, what we call mechanism condition. It says that ultimate load is reached or collapse load is reached when mechanism is formed. The number of plastic hinges I am introducing a new term, the number of plastic hinges should be just sufficient to form a mechanism.

We already know, what is a mechanism? We already defined what is mechanism. Mechanism is a structural system or assembly of members, which does not offer resistance to external loads at all. Obviously, the load level cannot be 0 because structural will have an internal resistance offered to any load. Therefore, inherently we say that the load level is beyond a specific value, that value is what we call as a collapse load. I have introduced a new term plastic hinges. We will talk about this slightly later. So, the number of plastic hinges introduce should be just sufficient to convert the given structure to a mechanism.

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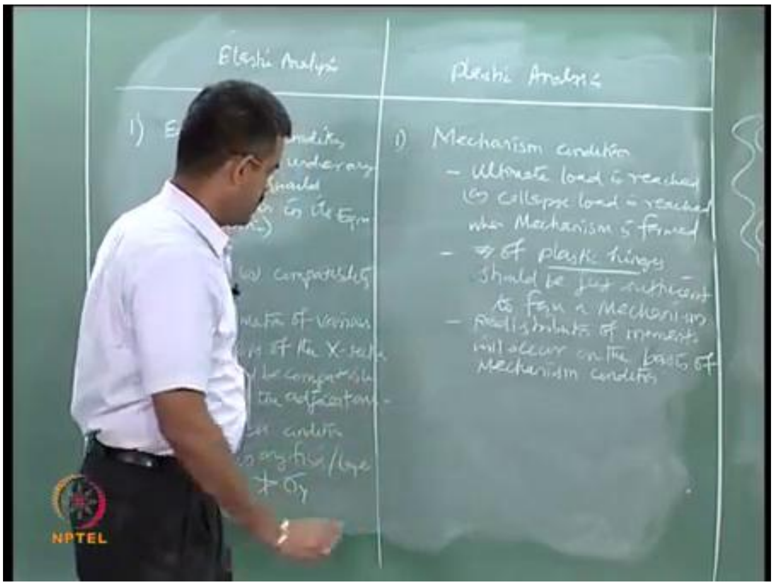

It is assumed that redistribution of moments will occur on the basis of mechanism condition. What does it mean? It is a very interesting statement. I will explain this statement. Once, a mechanism is formed it means plastic hinges will be formed at critical sections in a given structure. What is a plastic hinge? We will come to that. Let us understand; we know what is a plastic hinge? Plastic hinge form at a given section or sections which converts the structure into a mechanism, it does not mean that the structure will collapse.

Once, it will become as mechanism redistribution will start taking place. So, the highly stress sections will no more take any load, whatever load comes on to section will be redistributed to the successive highly stress section which enables sequence of formation of plastic hinges, one by one. When all possible hinges are formed completely, in totality then the structure will collapse, is that clear? That is the mechanism condition. The second condition I think I will, can I rub this and write here? I will rub this.

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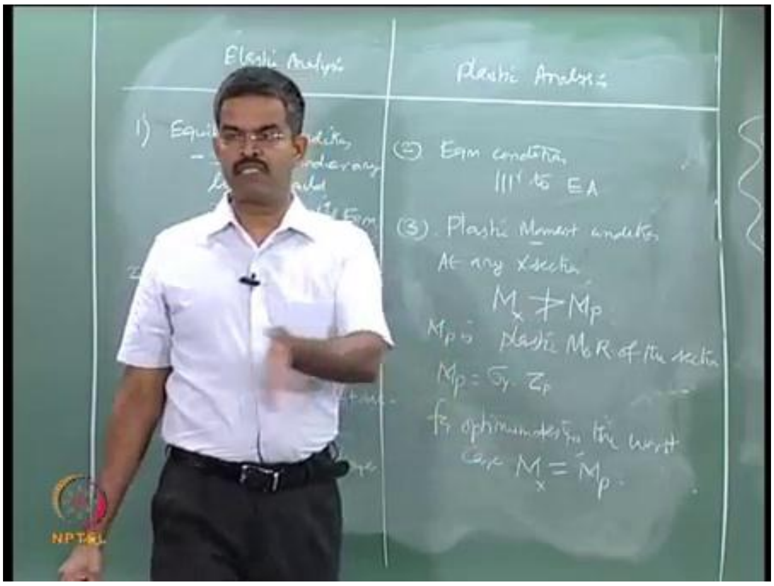

The second condition is equilibrium condition, which is similar to elastic analysis. Same, the structure should remain in static equilibrium under the given set of loads. Third condition which is called plastic moment condition. At any cross section, moment should not exceed the plastic moment carrying capacity of the section where M p is plastic moment of resistance of the section which is easily given by, is it not? We already know this.

So, for optimum design the worst scenario is M can be equal to M p. Not necessarily, M should be less then M p. M is moment at any section, let us say M x, not necessarily M should be less then M p. M should be not greater then M p, but M can be equal to M p if you want to really optimize the design. These are the three conditions which one must satisfy for successful understanding of or deployment of plastic analysis. We can see here obviously condition one and two they match exactly.

Condition three and three they match similarly, because here it should not exceed sigma y. It is talking about the stress limitation, is talking about stress limitation. It talks about moment condition. The condition of compatibly continuity is been replaced by a new condition called mechanism condition where people say that, if the structure gets enough number of plastic hinges just sufficient to convert into the mechanism. Then redistribution will be start taking place and it is believed that the ultimate load has reached.

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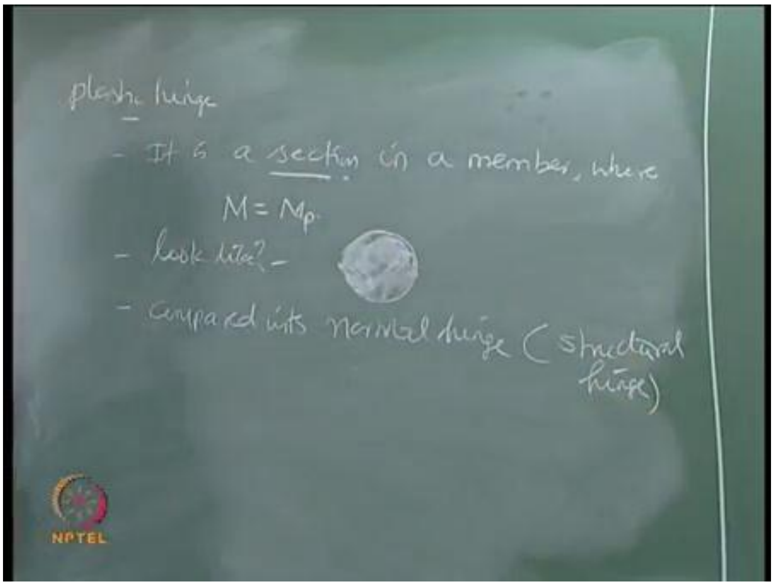

Now, the question is what is the plastic hinge and where are they formed? What is a plastic hinge? Plastic hinge is a section, it is a section actually in a member where the moment is equal to the plastic moment of resistance of the member. It is not less then or not greater than. It is exactly equal to M p. How does it look like? It looks like a filled dot. How can it be compared with normal hinge, which we call structural hinge? We can compare this.

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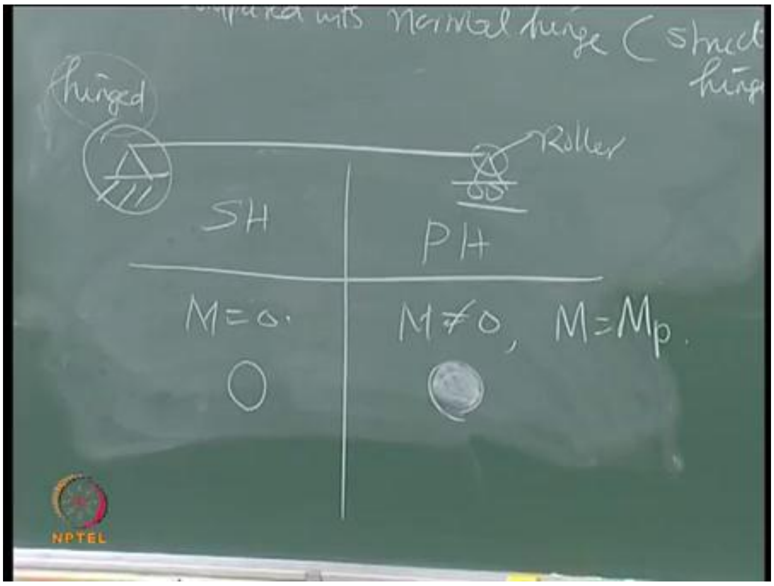

Let us say, I have a simply supported beam where one end is hinged, other end is roller. This end is on roller, this end is what we call as a hinged joint. Now, here also we say there is a hinge, here also we say there is a hinge. This is called structural hinge, this is called plastic hinge. Now, let us see, what is the difference between these two? Structural hinge and plastic hinge has got explicit differences. At structural hinge the moment is actually 0, it does not transfer any moment at all, whereas in plastic hinge the moment is not 0 it is equal to some capacity of M p. Structural hinge is indicated by a circle without filling. Plastic hinge is indicated with a circle with filling.

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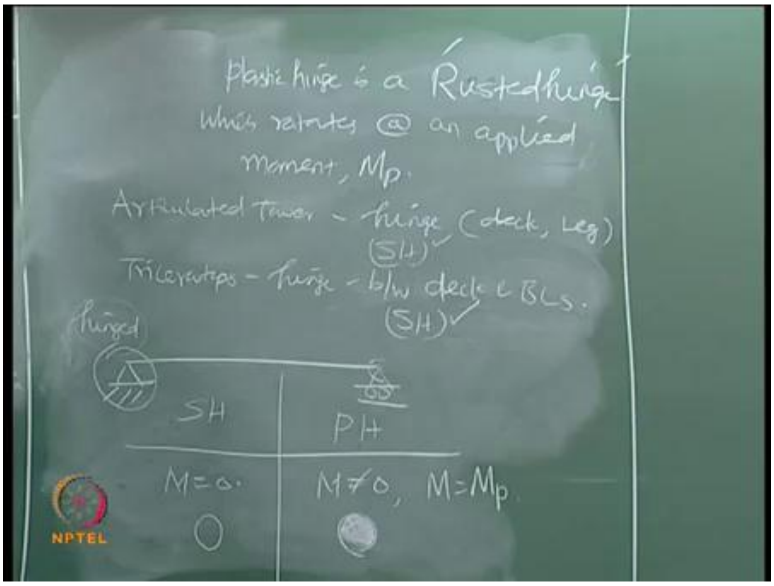

So, I can say plastic hinge is a rusted, which rotates at an applied moment M p. It is a rusted hinge. It requires some energy, some moment to rotate this, whereas structural hinge does not require any moment, it is 0. Now, can you tell me in articulated tower you have a hinge between the deck and the tower legs. What kind of hinge is this? Structural hinge. In triceratops, you have a hinge between the deck and the buoyant leg structure. What kind of hinge is this? Structural hinge. These are all structural hinges which does not transfer any moment at all. Moment is 0 at these joints, whereas plastic hinge is a section whose moment capacity is not 0, but M p. Now, the question comes where will they form?

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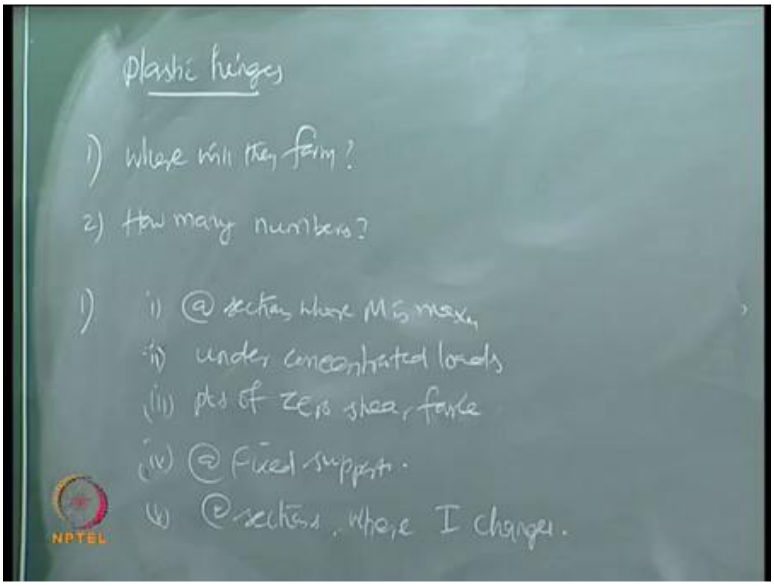

Plastic hinges, questions. One, where will they form? How many numbers? Both questions; where will they form? They will form at the following sections. One, at sections where moment is maximum; under concentrated loads, if you have got any concentrated load applied on the platform deck, at those points plastic hinges will form. Three, points of 0 shear force. Four, at fixed supports, five at sections where moment of inertia changes. How many of them will form?

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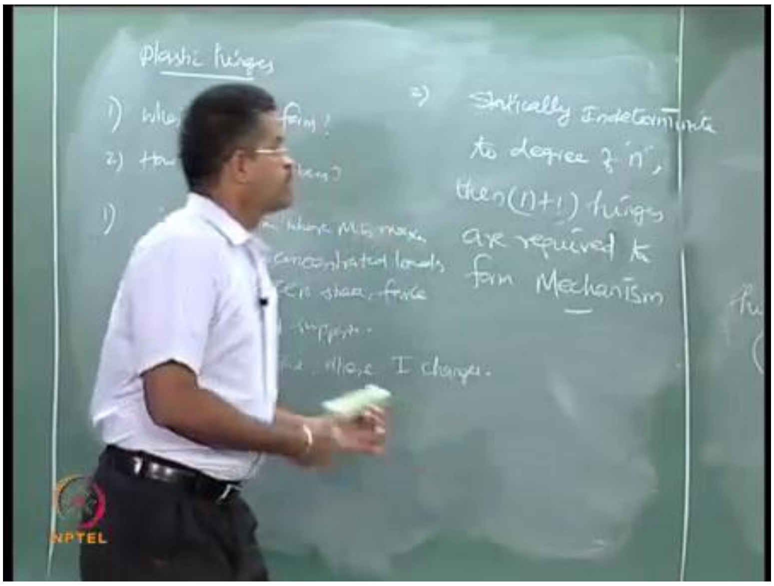

If the structure is statically indeterminate to a degree of n, then n plus 1 hinges are required to form mechanism. I believe, you all know how to find the degree of indeterminacy for a given structure. There are two kinds of indeterminacy, static indeterminacy, kinematic indeterminacy, total degree of indeterminacy. For a given structural system, you must know; how to compute the degree of indeterminacy? If you know the number n, then n plus 1 hinges are required to convert the structure into a mechanism.

What is a mechanism? Mechanism is a structure, which does not offer resistance to load any more. Earlier it was offering, earlier it was offering when the load was applied. When the load reaches specific state or specific value, now the structure stopped its reaction of offering resistance. Structure is then called by a new name called mechanism. So, ideally a structure is called as a mechanism when sufficient number of plastic hinges are formed in a structure I am redefining the mechanism now.

A structure is called a mechanism when sufficient number of plastic hinges are formed in a structure. How many n plus 1? n is degree of indeterminacy. Obviously, now you will thoroughly understand that for plastic analyst and design the structure need to essentially be a static indeterminacy part. But, if you have determinate system like a simply supported beam whose static degree of indeterminacy is 0 still then one hinge will be formed to make it as a mechanism, remember that.

That is very, very interesting. Even determinate structure can also be analyzed using plastic design because one hinge will form at a specific section of critical location and it becomes a mechanism. So, plastic design can also be applied to statically determinates systems. The disadvantage is redistribution of moments will not occur because there is only one hinge, only one section. Once M p reaches structure is going to collapse that is the advantage.

Now, there are two theorems which will help us to do plastic analysis. What does it mean, to determine the collapse load in a given structural system. Now, we all agree that what is the difference between the load encountering the structure and the collapse load. What is the difference between these two? What is the difference between the collapse load and the load, which we have calculated to come on the structure? There are differing by what we call as a load factor Q, W c by W w; that is what we have seen. Is it not?

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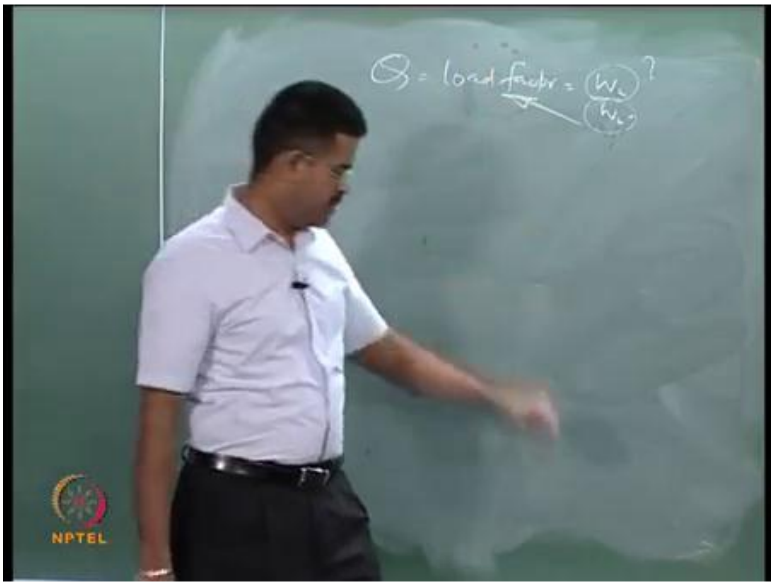

What you have so far computing will be all working loads. It is not elastic load, it is working load. The load coming on the structure, you enhance this load, you multiply this load by a number whose values always more than 1. We obtained collapse load. Now, you may wonder say then why do we do plastic analysis because we can directly get the collapse load from this mechanism. The load will call as a collapse load only when the structure becomes the mechanism.

All loads multiplied by load factor are not collapse loads. The loads which are responsible to cause collapse are collapse loads. So, how to get this? You must have a procedure, you must have a methodology to obtain the collapse loads. Is it clear? Then, the question comes, there are two confusions here. One, the collapse load what again the directly from this equation. The other is the collapse load, which will do by doing a plastic analysis from two theorems.

How are they different? They are exactly same, but the catch is here. I do a plastic analysis using the theorem, get collapse load for a given mechanism condition. From the collapse load I get the working load. That will be my design load for the structure. Is it clear? Therefore, I am designing the structure for that design load which has got a factor of safety or a margin of safety and this margin of safety will never allow the structure to collapse.

It means even though the structure is designed by a traditional term called collapse mechanism, the design principle will not lead to actually a collapse. Is that clear? It will only make the structure to become a mechanism. That is all, is that clear? So, we are not designing or we are not using a method by which we are purposefully collapsing a structure. It is not like that. This margin of safety will take care of that, is that clear?

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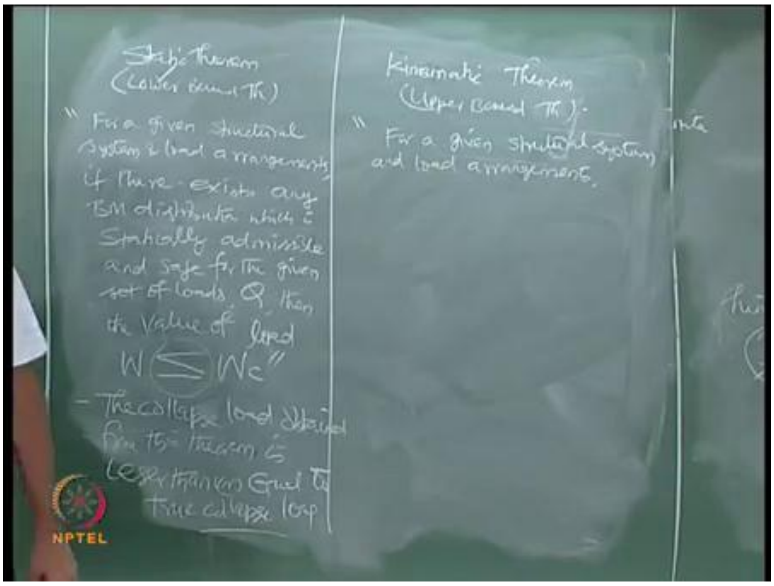

So, there are two theorems available. One is called static theorem, other is called kinematic theorem. This otherwise is called lower bond theorem. This otherwise called upper bond theorem. The statement of this theorem are the following. For a given structural system and load arrangements for a given, this statement is very important. You please try to understand these two theorems thoroughly. Both will lead a same answer, but which is easy is depending upon how you want to practice or how you want to do the analysis.

The statements once I complete they be will very, very confusing. I am writing slowly to explain you. These are available almost all text books on structural engineering. Everybody will be teach this in plastic design courses, but understanding them will be a great problem because once I complete the statement it will, as if you have got to memorize this. You will be not able to understand this. So, I am writing in part and parcel, I am writing it parallelly, so that you can compare in your mind what is happening?

The difference between these two for a given structural system, so to apply static theorem you must have a structural system with you. It means plastic analysis does not tell you what a system you have to have in place. The system is always chosen by the designer. For example, you want design a TLP, you want to employ a nautical tower, you want to employee a triceratops, these are all form based systems which is not arrived from any design or any analysis procedures.

So, you must fix, choose the system readily available to you. So, for a given structural system and the load arrangements where the load will act for example, on the deck, what are all the tops side details? Which are all the loads coming on that? That arrangement you must already have with you. For a given structural system and load arrangements if there exist any bending moment distribution which is statically admissible. So, for a given structural system be it indeterminate or be it determinate.

Obviously, one will not look for a determinate system, because of obvious reasons. Can you give me one reason why I am not interested in looking for a determinate system? Why I cannot go for a determinate system? Why I should look for indeterminate system? In a indeterminate system you require n number, large number of plastic hinges to form, to convert the system to a mechanism, whereas in determinate system you require only one hinge to form to convert into a mechanism beyond which it will collapse.

So, there is no reserve strength available in the geometry. There are two reserve strengths. One is from the material; one is from the geometry or arrangement of the members. Material strength is what we are looking at from the elastic to post elastic or plastic state. That is called ductility. There is the reserve strength, the material. The reserve strength in the geometry is arrangement of members, which we call as indeterminate structure, determinate structure etcetera.

So, even though the given structural system is statically indeterminate for a given load arrangement, you must know how to get the bending moment distribution. Now, how many of are very good in finding out the bending moment distribution or drawing a bending moment diagram for a statically indeterminate structure of a very high order. So, it is a pre request to apply this procedure of analysis. The pre request is this. You must have the values of bending moment distribution in advance with you.

So, if that bending moment distribution which is admissible exists, then I have to add one more adjective here. Statically admissible and safe for the given set of loads Q, then the value of Q or the value of load let us say the value of load W will be less than or equal to collapse load. Since, the value of load W is less than or equal to collapse load this theorem is called lower bound theorem. Therefore, I can say the collapse load the statement ends here, the collapse load obtained from this theorem will be, is lesser than or equal to true collapse load.

That is why it is called lower bound theorem? The upper bound theorem has a different statement, for a given frame and loading. So, in this case also you should have the structural system in position, you must know the load arrangements.

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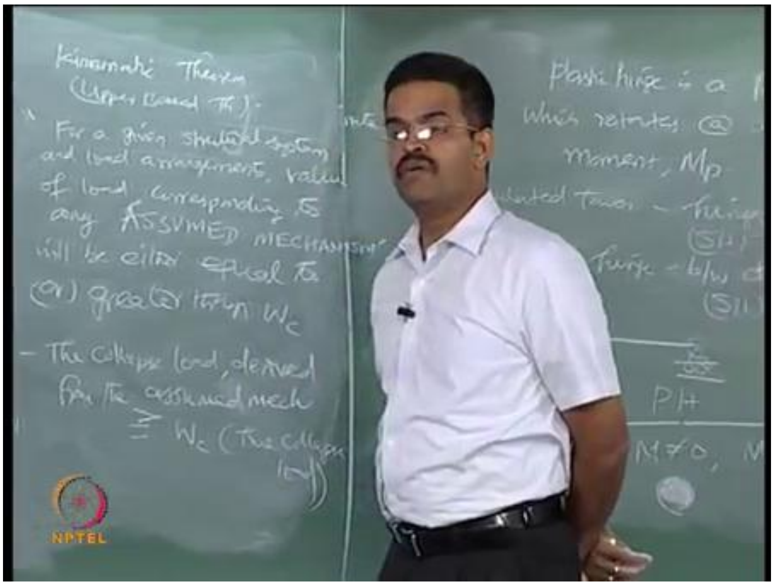

Value of load corresponding to any assumed mechanism; any assumed mechanism will be either equal to or greater than W c. So, what is the catch in both these methods? So, I should say the collapse load derived from the assumed mechanism will be either equal to or greater than true collapse load. Now, what is the catch between these two theorems, which may stick distinctly different? There is a very important statement or very important catchword here, which makes both this theorem distinctly different.

What is that? Both the theorem bonds structural system and load arrangements, therefore this cannot be a difference. One distinct difference can be this theorem gives me the lower bound values; this theorem gives the upper bound values. Of course, that is in the name itself. So, that cannot be a distinct difference. The distinct difference is this methods wants you to assume a mechanism. These methods want you to have bending moment distribution.

Now, what would be or which method would be difficult? For a statically indeterminate structure from a very high order it is generally difficult to find the bending moment distribution. Comparatively, it is easy to assume a mechanism, what is a mechanism? Mechanism is that structure which has got n number of plastic hinges which can convert the given structure to a mechanism. That number n will be nothing but one value more than the degree of indeterminacy.

So, plastic hinges; if the degree of indeterminacy is 5 you require 6 hinges, where they can form? We have got a list. So, keep on selecting these junctions, put the plastic hinges, assume a mechanism, find the collapse load. So, this method appears to be simple because this method does not require you to solve the problem to find the bending moment distribution. If that is so simple then why this method is not popular compare to this.

The reason is if we miss out any specific mechanism, if we do not know how to assume a correct mechanism you can skip certain basic critical mechanisms, your collapse load estimate will be wrong, you land up in a wrong value. That fear is there, in this case the fear is not there because you are only assuming a bending moment distribution which is safe and admissible, but here you are not assuming the bending moment distribution. You are assuming a mechanism directly.

This method though appears as if it is very simple it can lead to wrong estimates of collapse loads. Then one can ask me a question, sir, will this method always lead to correct value of class mechanism? The difficulty here is, if you do not know correctly how to do a bending moment distribution then also you can have an error here. Both ways there are difficulties. So, let us quickly look at the steps of first theorem, second theorem.

Then combining these two, there is a theorem available in the literature called uniqueness theorem. That is also we will see. Then we will do couple of examples by both the methods and see which is easy and why, is that clear? Possibly, I think this we should do in the next lecture that will take some time for me. To solve, I cannot rush through in 5 minutes, I have to complete this. We will do couple of problems to understand. So, in this lecture we understood very interestingly that why a plastic analysis and design procedure cannot be employed for a static determinate system.

You can do, there is no, nothing stops you from doing that, but the advantage is it will not give you enough reserve strength for redistribution of moments which is one of the important advantage of a plastic design. Second point, what we learnt is plastic analysis and design is not a theorem or not technique, which leads to collapse of a structure. Though, you call the load as a collapse load, though you call the load as an ultimate load, the ultimate term adjective may give a feeling to the people that that is the ultimate load beyond which the structure will collapse and the loads can exceed etcetera.

All these probabilities are there in back mind, but still this methodology is not leading towards collapse of a structure because Q is what we call margin of safety which is generally and comparatively good with respect to working (( )) methodology. Now, a question comes very simply when this method is economical, when this method is utilized in the maximum reserve strength of the material, when this method is utilizing the reverse strength of the geometry. In terms of static indeterminacy, redistribution of moments, why this method is not popular for a longer time in engineering practice?

Even now it is not very popular. Why this method is not popular? This method has higher risk if you do not do the design properly. So, as a beginner one is not encouraged to do a plastic design because he does not do that kind of error checking in the initial stages of design. It does not mean that plastic design done by a engineer is always unsafe, but it needs rigorous checking. You may skip certain mechanisms, you may skip or you may do a wrong distribution of moments, you may land up in a wrong collapse world value which you think that is an ultimate value; in reality that may not be what is called as true collapse load, is that clear?

So, there is always a sense of risk available in this. That is where we talking about the difference between elastic analysis and plastic analysis. We have seen the three conditions to be satisfied for both the analysis separately. We have also tried to understand what is the difference between analysis and design in terms of plastic design. Why do we talk about this? In the next class, we will talk about the example problems on both the theorems, solve them and try to understand which is comfortable to us and why?

Thanks.