Advanced Marine Structures
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Lecture - 18
Plastic Design - III

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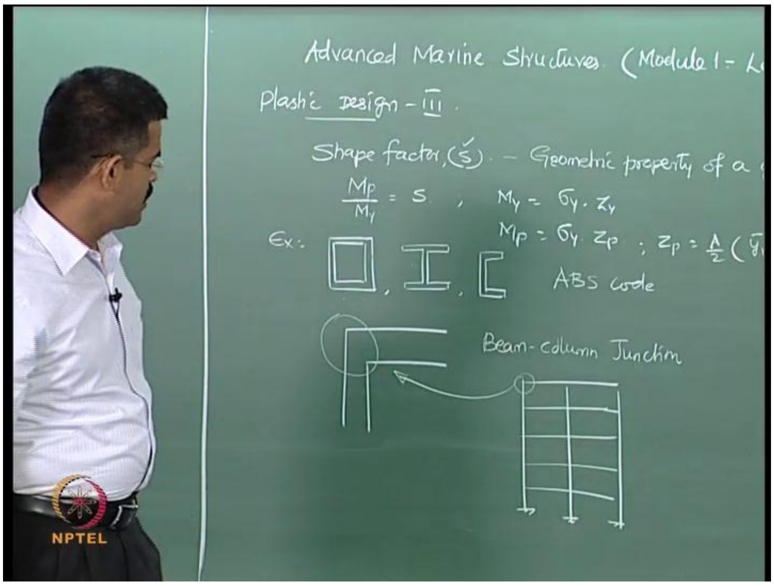

In the last lecture, we discussed about the importance of what we call as the shape factor, which is actually a geometric property of any given section. For any given section, the shape factor is available in the standard table. So, if we know the shape factor, we can easily find, what would be the additional load carrying capacity of any system. When you are talking about the plastic moment of resistance of the section, because you can multiply the elastic moment of resistance, which is nothing but sigma y into z y or z e or simply z, which is a section model, which we have already know for a given section. Whereas, M p is again given by sigma y of z p, where, z p we already know, is given by conventionally, area by 2 of y bar 1 plus y bar 2, where, y bar 1 and y bar 2 are the centroids of the respective tensile and compressive area with respect to the equal area axis, where, A is the total area of cross section, which is getting plasticized.

We have worked out couple of examples to estimate the shape factor. So, I think it is a home work for you to find out, what all the shape factors for different sections. Like for example, a box section, which is square in shape, what would be the shape factor for this? Then an I section; then of course, a channel section, etcetera. You should find out the shape factors for these. Of course, this x is not very necessary, because for different standard conventional geometric forms of steel sections, which are likely to be used in marine structures, we already have the equations available in the literature for finding out the shape factor. I will give you these equations later. It is available in ABS – American Bureau of Shipping code. It is available in the ABS core for different shapes. So, it is not necessary that you have to memorize this, but you must know how to estimate that.

Now, we spoke about a very important transformation from an elastic analysis to a plastic analysis, where the designers felt that, the reserve strength of the material can be utilized provided the structure should remain statically indeterminate, where (( )) high order. Whereas, redistribution can effectively take place and the material should have enough ductility. The moment we say ductility, there are two aspects of ductility here: one is the displacement ductility, whether the deformation can sustain, other is what we call curvature ductility, because in certain cases… For example, let us say I have a beam column junction. This is a typical beam column junction. It may be a stiffened seated connection. It may be an ISMB, it can also be an ISMB. It can be a stiffened seated connection. It can be a moment restrained connection between the beam and the column if it is a steel structure. If it is concrete, of course, this is also reinforced, this is also reinforced.

Now, the structure becomes (( )) indeterminate. For example, let us say I have system like this, whereas, all the vertical lines indicate columns and all the horizontal lines indicate beams. A typical junction, which is a beam column intersection, looks like this. If this structural system is subjected to lateral loading and gravity loading, the system becoming very highly indeterminate, if you want to enable plastic analysis to find out the M p value for this system. There are two things, which are inherently required. One is the system should be statically indeterminate to a very high degree, it is true in this case. The second requirement is though the material is steel, we must check whether at the junction, there is enough curvature ductility, otherwise, if the curvature ductility is not sufficient, then effective redistribution of moments from the section to the next critical section will not happen, because when the structure is subjected to a collapse mechanism, the joints where the plastic hinges will be found, should be sufficiently rotating to transfer the moments from the section to adjacent section. So, it requires enough curvature ductility. So, one must check, what is a relationship to derive the curvature ductility or essentially moment curvature relationship, what we call M-phi relationship.

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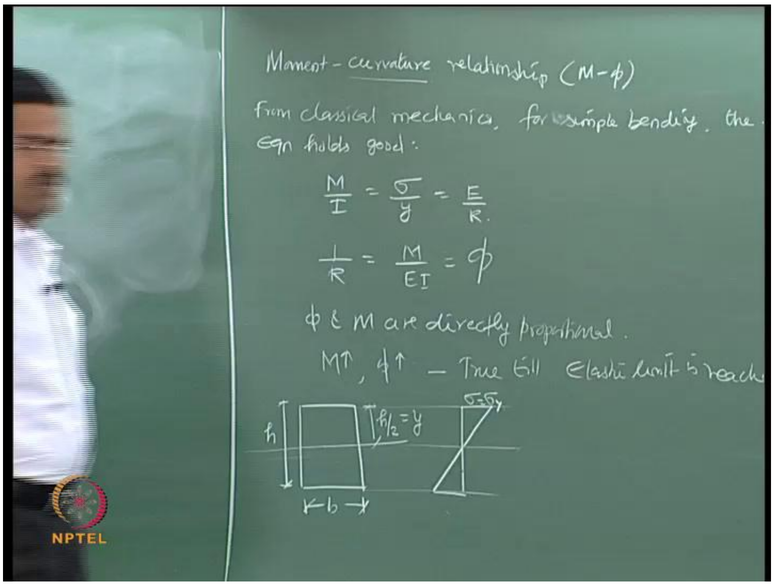

We must check this, which we will do today. We already know from classical mechanics, for simple bending problem, the following equation holds good. M by I, stress by y, E by R, where, we all know the conventional terms of this M is the moment at this section, I is the moment of inertia of the cross section about the bending axis, stress – sigma is the stress value – bending stress, and, y is the distance of the extreme fiber from the neutral axis, e is the Young's modulus of the section or modulus of elasticity of the material and r is the radius of curvature of the bending profile of the beam. It is a classical equation, which is valid when the bending remains elastic.

So, let us pick up this equation and say 1 by R is M by E I, which I call as phi. 1 by R is called curvature. So, from this equation, it appears that, phi and M are directly proportional, that is, if M increases, phi also increases. This is true only till elastic limit is reached. Let us have a triangular cross section just for illustrating an example. Has a breath of b, has a depth of h. This is my equal area axis as well as neutral axis for this cube. And, the distance of extreme (( )) is h by 2 from here, which I call as y.

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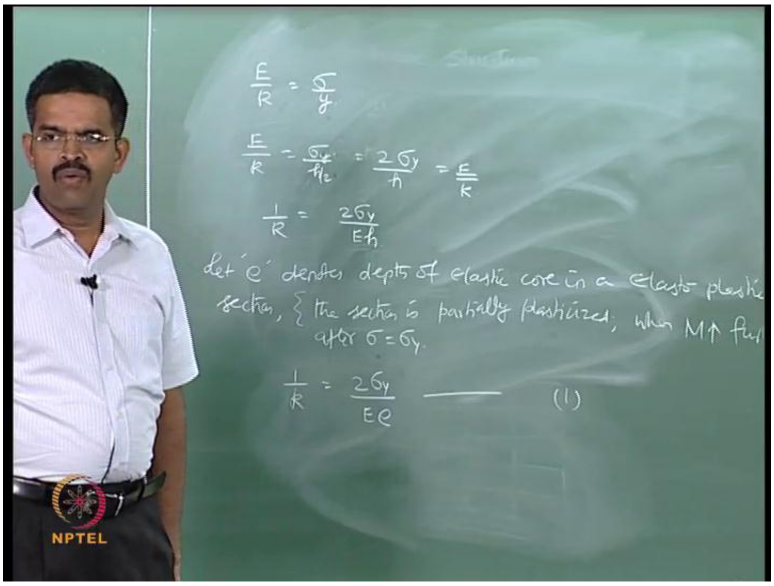

Let us say E by R is stress by y. So, the distance of the extreme fibre in the present case is h by 2. So, E by R is stress by h by 2, where, stress remains the yeild value, because extreme fibre is yeilding only the extreme fibres is yielding, only the extreme fibre is yielding. We have given a special name from an elasto-plastic section. We call this as depth of elastic core. Let e denotes depth of elastic core in an elasto-plastic section, where, I am talking about partially plastisized section. The section is partially plastisized. And, when this will happen? This will happen when M increases further after stress reaches the first yield. After the stress value reaches the first yield, plastization will start.

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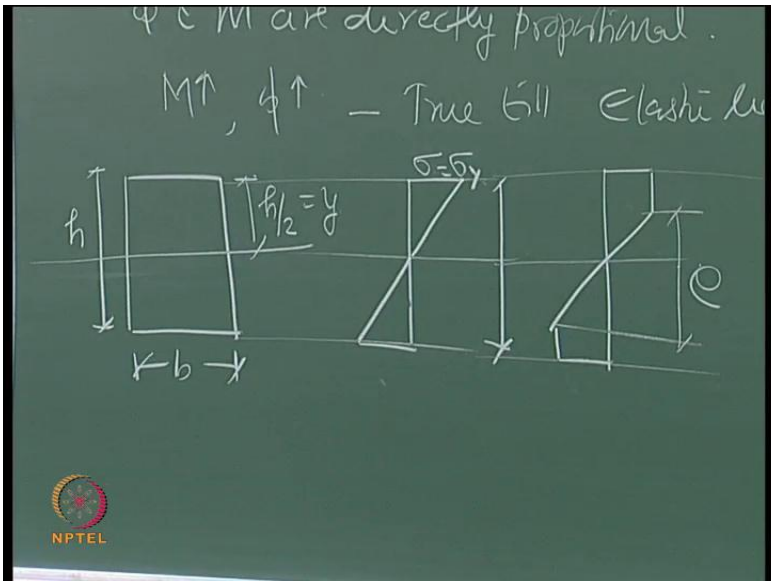

So, the stress-strain diagram will start getting modified like this as we saw. And, this becomes mine, what I call depth of elastic core. This is what we have seen yesterday. So, at that condition, 1 by R value can now become 2 sigma y by E e. I call this equation number 1. I am replacing h with depth of elastic core, because I am looking at the elasto-plastic section now. Why I am looking at this? Because I am studying the moment curvature ductility effect when the section is getting plastisized. In elastic, there is no problem, curvature is there. In plastic, we must check this.

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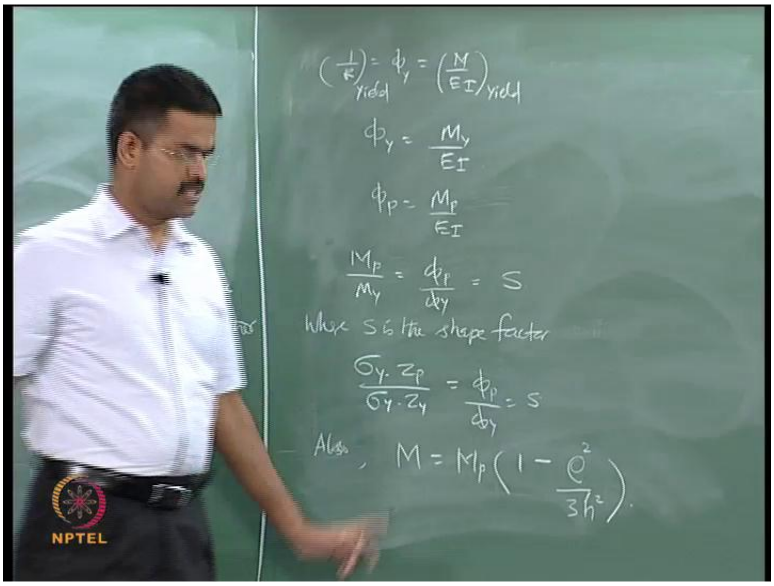

We can now say, 1 by R, which is phi, which is M by EI. I can use a suffix here, I say 1 by R at yield will be at yield. I can replace this phi as phi y. Therefore, phi y can be simply M by E I. It means phi and M are proportional. Similarly, I can say phi p is also equal to M p by E I. And, M p by M y is phi p by phi y. And, we already know, this M p by M y is nothing but shape factor, where, S – this is shape factor. Let us expand the left-hand side of this equation. We know this is going to be equal to sigma y of z p by sigma y of z y, which is phi p by phi y, which is S. Also, we already saw, moment at any section can be traced with the depth of the elastic core in a given elasto-platic section by a classical expression, which we saw yesterday. This expression we have already derived yesterday.

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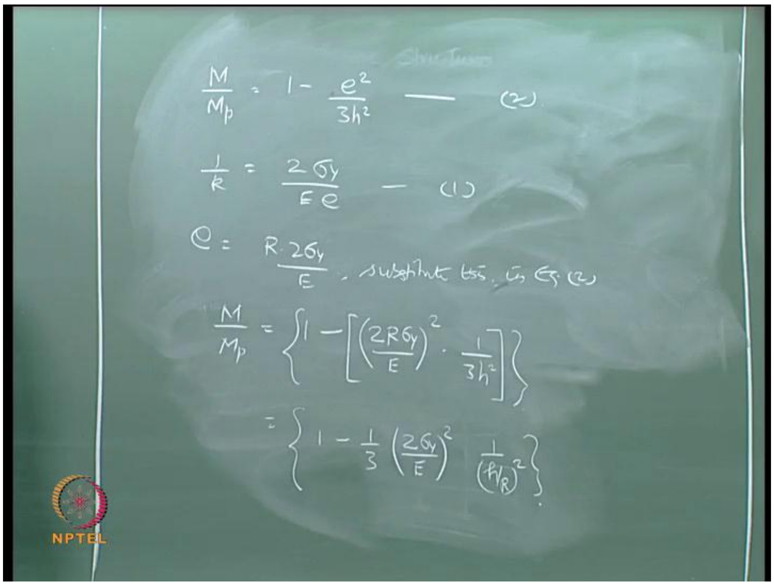

Let us say M by M p – 1 minus e square by 3 h square. I call this equation number 2. Look back equation number 1. 1 by R – 2 sigma y by E e. This is equation number 1. Already, we have this. So, from this, e will be actualy equal to R 2 sigma y by E. Substitute this in equation 2, because e is here, e is here. So, M by M p – 1 minus 2 R sigma y by E the whole square 1 by 3 h square. We can simplify this further – 1 minus 1 by 3 of 2 sigma y by E of 1 by h by R square.

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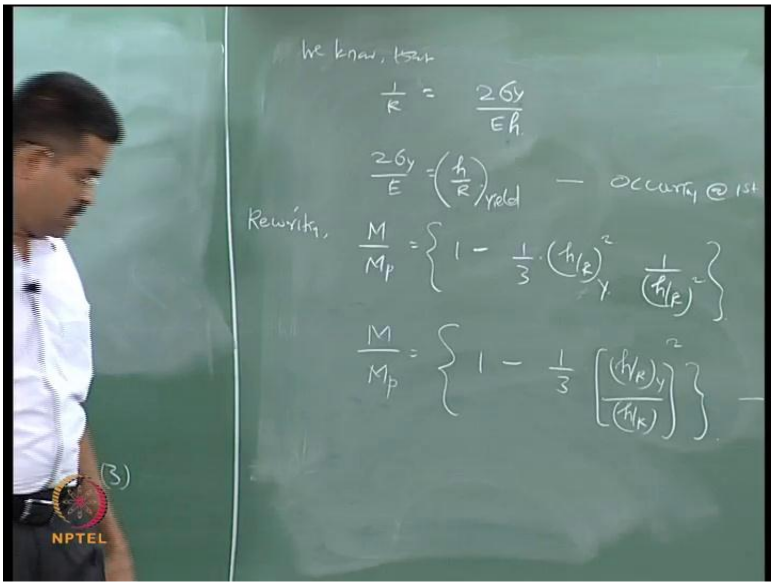

We already know… We know that, 1 by R from here – 2 sigma y by E h. So, 2 sigma y by E can be replaced as h by R. So, rewriting – M by M p – 1 minus 1 by 3 of h by R square 1 by h by R square. Now, there is a difference between this value of h by R and this value of h by R. What is the difference? What is the difference between this value of h by R and this? This is at yield. So, I can say here yield. Is it yield? The distance of extreme (( )) of h by 2 and I got sigma y here. So, it is yield, first yield point. So, this value is occuring – first yield. Therefore, I can say this is h by R yield. Substitute it here. I can rewrite this equation as M by M p is equal to 1 minus 1 by 3 h by R y by h by R the whole square – equation number 4. This was 3, this was 4. Now, equations 2 and 4 are very important for us. Equation 2 gives you the relationship between the moments – the plastic moment and the moment at any section with respect to the elastic core. Equation 4 gives me the relationship of the moments, that is, the moment at any section with the plastic moment capacity with respect to the curvature ductility. This can be plotted.

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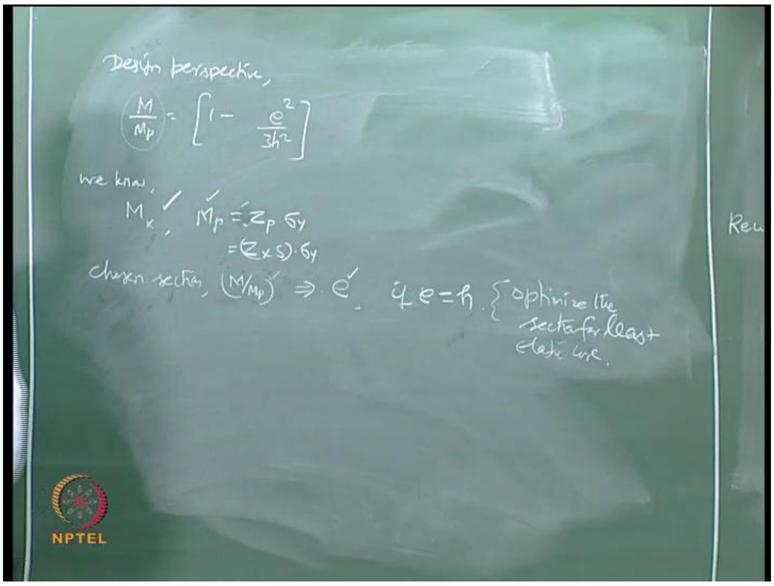

Now, just to understand, physically, what does it mean for the designer is that… Let us take the equation 2 in design perspective. In design perspective, let us take equation 2, that is, M by M p is 1 minus e square by 3 h square. Now, what you know in this equation are the following. Moment at any cross section is known to me, be it determinate, be it indeterminate. If the structure is indeterminate, there are methods to find out moment at any section using classical structural mechanics principles. For example, moment distribution method.

For example, stiffness method. There are many methods available. One can easily find out moment at any section you want for a given loading. M p is known to me, M p is not a problem, because M p is nothing but z p into sigma y and z p is nothing but section modulus into shape factor into sigma y. And, for a given section, I know the section modulus and I know the shape factor. So, M p is also known to me. It means the left-hand side variation is known to me for a given selected section. So, for the choosen section, M by M p is known to me for the choosen section. Design is nothing but finding the section. You choose a section, know this relationship, and, from this relationship, try to find out e. If e matches h – depth of the section, it means section is fully plastisized. So, the design what you have done is optimal. So, one can optimize a section for least elastic core. Elastic core should be minimized. Plastic should be the maximum. So, that is the design perspective of this.

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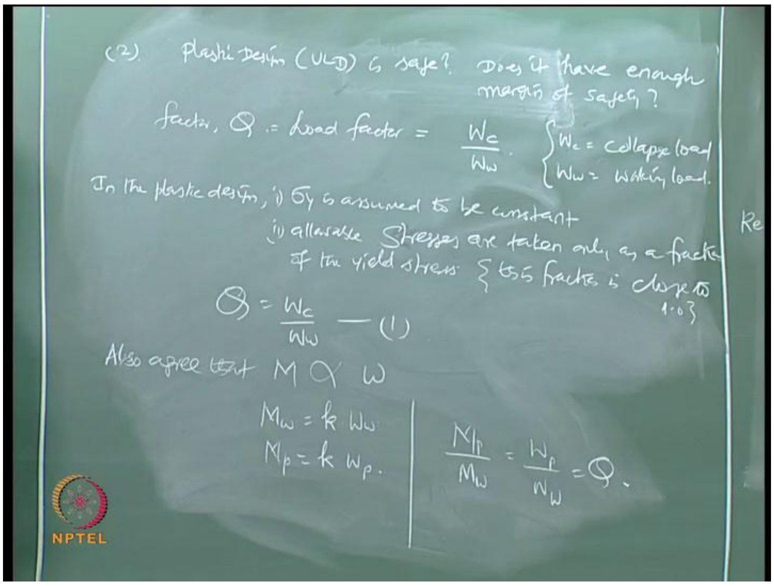

Having said this, a very interesting question asked, whether the plastic design or ultimate load design is safe? It means, whether does it have enough margin of safety? I asked this question in the beginning. Why this question is a concern? This question is a concern, because in ultimate load design or in ULS – ultimate limit state design principles, we are utilizing the reserved strength of the material till the ultimate strength of the material. So, any increase in the load will have a tendency to cause failure to the structure, because the material will fail beyond the point. In working such design or a classical working such design principles, this problem does not occur, because you do not stretch the strength of the value of the material beyond the elastic limit, whereas here I am trying to stretch the load carrying capacity beyond the elastic limit till the ultimate limit. So, there is a question asked, does it have enough margin of safety.

Now, as I said, let us introduce a factor Q, which is called as a load factor; which is nothing but the ratio of collapse load to working load. So, in the plastic design, the yield strength is assumed to remain constant, the allowable stresses are taken only as a fraction of the yield strength. But more interestingly, this fraction is close to 1.0 – 0.95, 0.97, 0.90; it is very close. But it is never 1, it is close to 1. Of course, it does not exceed 1. So, we know now that, Q is W c by W w. And, we also agree that, moment carrying capacity is always proportional to W, that is the load. Therefore, M w can be some constant of W w, M p can be some constant of W p – some constant. So, M p by M w, which is W p by W w, which is nothing but Q. Why I am saying W p as same W c, because beyond that value, the structure will collapse, p stands for plastic design. So, it is… I can always say this is same as collapse (( )).

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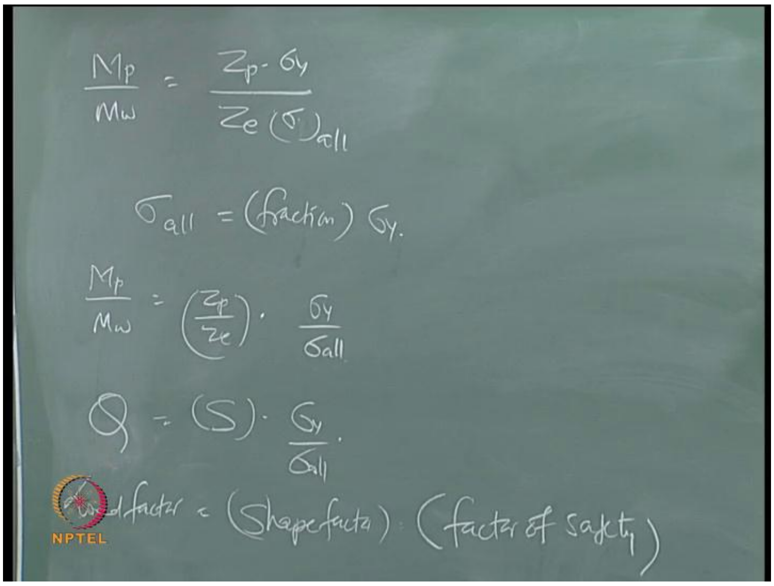

Once we agree this… Let us expand left-hand side of this equation. Let us say M p by M w. M w stands for moment based on working loads, which is z p into sigma y and z e into sigma y allowable, it is not sigma y, it is sigma allowable. And, sigma allowable is always a fraction of sigma y. We already said that here. This fraction is close to 1, but it is a fraction. Now, let us say M p by M w is z p by z e and sigma y by sigma allowable.

And, I can rewrite this as shape factor in sigma y by sigma allowable. And, we already know that the relationship of M p by M w is load factor. We have already said that in equation 1. So, load factor is a product of shape factor multiplied by… I put this as factor of safety. Why I called this factor of safety? Because any fraction, which is applied on the yield strengh to arrive at the allowable stress is always factor of safety. Now, let us take a quick example and see what is happening actually in the plastic design.

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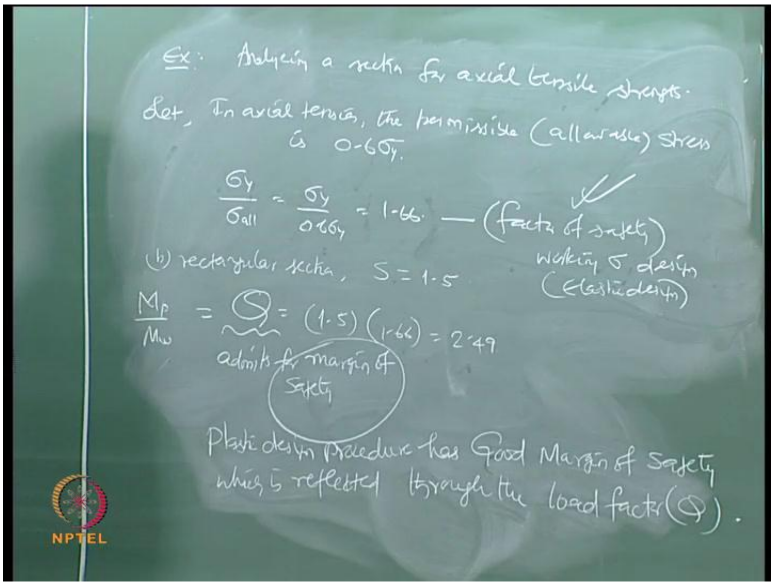

Let us say, I am analysing a section for axial tensile stregnth. I am having a bar, I am trying to pull the bar, I am analysing the section for axial tensile strength. In axial tension, the permissible or I can say allowable – allowable stress is 0.6 sigma y. Let I am saying let, because it depends upon the code, depends upon the method of design, let us say 0.6 sigma or sigma y. Therefore, sigma y by sigma allowable will be sigma y by 0.6 sigma y, which will be 1.66. This is what we say as factor of safety in working stress design or elastic design. Why elastic design? Because here the stress levels are limited till the proportional limit of the material.

So, working stress design principle has an explicit factor of safety, which is seen in the calculation, which is nothing but the ratio of the yield value to the allowable stress value for a specific application. This changes. If it is talking about bending strength, it will be 0.66 f y, talks about shears – about 0.4 f y. Keeps on changing. This fraction keeps on changing. But I have picked up an example of axial tension, which is 0.6 f y or sigma y.

So, people are happy in working this design method, because explicitly, the factor of safety is seen in the design calculation. I am trying to show, what is the effect of this on plastic design.

Now, let us apply this plastic design. Let us take a rectangular section for which the shape factor is 1.5. We have derived yesterday. So, you will see that, the load factor, which is 1.5 times of 1.66, which is about 1.85 I think. You please check this. How much is that? 2.49. And, Q is nothing but… What is the value of Q? This M p by M w. So, Q admits for margin of safety, which accounts for a value phenomenonly higher than the conventional factor of safety working in this design. So, plastic design procedure has good margin of safety, which is reflected through the load factor Q. So, the question, which was bothering us as a designer, whether plastic design principle has enough factor of safety compares into working this design is eliminated, because it is having enough factor of safety as margin of safety, which is seen in the load factor; which is nothing but the ratio of plastic moment to working moment or elastic moment, where, elastic moment is the moment what we get using elastic design, plastic moment is the moment carrying capacity, what you arrive using the plastic design. The difference between these two is Q, and, that Q accounts for margin of safety multiplying shape factor.

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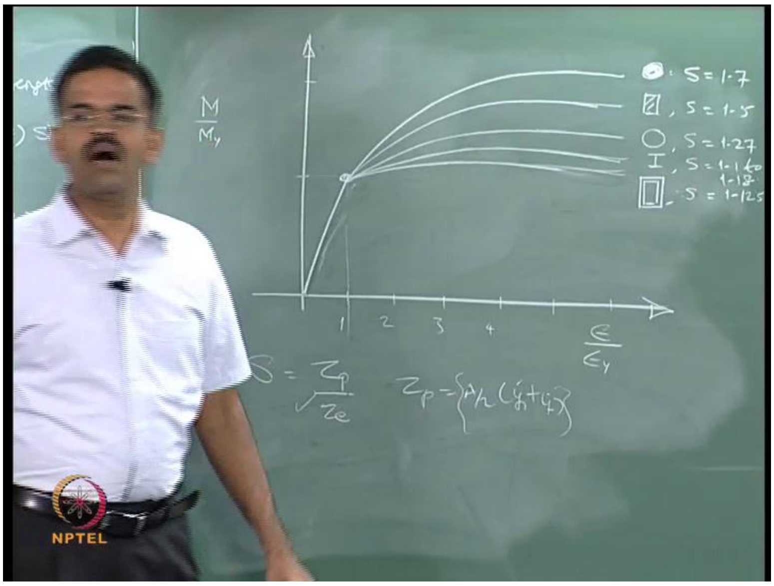

Now, let us quickly see, what is the effect of shape factor in terms of this ratio. Let us try to look at a plot. I am plotting this as M by M y and epsilon by epsilon y – 1, 2, 3, 4, 5, 6.

And of course, this is 1. M and M y are equal and then this is 2 and so on. So, the grid starts from here is linear till here about this point, then keeps on changing. For different shape factors, this is solid circular – 1.7, rectangular – 1.5, cubes – 1.27, and, box sections – 1.125, and of course, I sections – it varies from 1.1 to 1.18. They are horizontal lines. There is not dipping. These are horizontal. So, you can see from here, there is no section, which is choosen for marine structure design, whose shape factor is less than 1. So, always, the load factor will be higher than the conventional factor of safety what people have in mind for working stress design. So, plastic design is a safe procedure. There is no doubt. You can select any section like this.

Conventionally, I will just finish in few minutes. Then we will take up the next lecture. Conventionaly, if you really wanted to find the shape factor, which is nothing but z p by z e. z e is not a problem, you can easily define for a given section. But z p is a problem, because z p conventionally is A by 2 of y bar 1 plus y bar 2. So, ultimately if you want to really find z p, do not try to look for this equation, it is nothing but the first moment of the area about equal area axis. So, for a given section, locate the equal area axis, locate upper and lower parts, locate the CG's of that – y bar 1, y bar 2. Take the moment of the upper part and the lower part seperately, independently with respect to the equal area axis. Mechanically, this is how you will find the z p. Once you know z p, for a given section, you know z, you can find shape factor. Try to understand this very clearly. Otherwise, shape factor is directly available in equation forms in standard literature and in codal provisions for different sections, which are conventionally used for marine structures. We need not have to derive them at all. But we should know how they are obtained.

In this lecture, we discussed about two aspects important. One is what we call curvature ductility, how to arrive at curvature ductility. And, we have seen the relationship between M p and M in terms of ductility, in terms of elastic core depth. Then we went down to explain that, how the margin of safety is inherently available in the plastic design procedure, because S – shape factor is nowhere less than 1 for any section. It means that, the margin of safety is always higher than the conventional so-called factor of safety, what people follow in working stress design. So, in the next lecture, we will talk about how we will arrive the collapse loads for different systems using two interesting theroms, what we call static therom and kinematic therom, respectively. We will see in the next lecture.