Dynamics of Ocean Structures
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Module - 1
Lecture - 6
Missing Mass Correction, Example Problems

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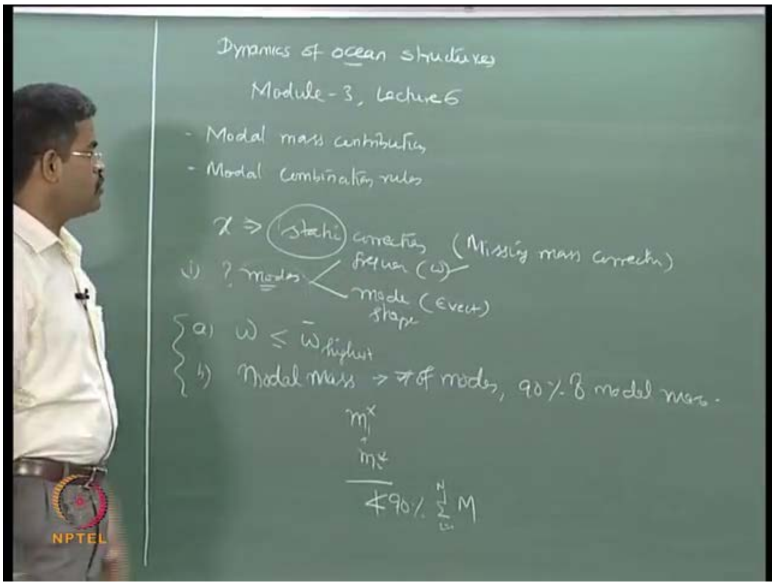

So, in the last lecturer we discussed about the static correction method so if you want to find the response I can always say take the response only from a specific mode and do a static correction for not including the higher modes. We say static because there is no dynamic component present in this correction; we also call this as missing mass correction because mass from the higher modes may not be participated.

So, now the question fundamentally is to ask how many modes should I consider in my analysis when I say modes, I have got two components in this one is the mode shape other is what I call is an Eigen vector other is the frequency which is the eigenvalue. If I say the number of modes automatically I am actually asking question indirectly how many modes correspond to what frequency, I must consider the answer to these questions will be of 2 kind one do not include any frequency in your argument which is lesser than or equal to omega bar higher. For example, if a forcing function frequency has a highest value of omega bar do not avoid any frequency content in your analysis, which is lying within the highest frequency of the forcing function.

Secondly, you should look for the modal mass participation and include only those number of modes up to which you can say 90 percent of the modal mass is included that is if I say m 1 star m 2 star are the respective modal mass in respective modes. Then if I sum them up I must say if I am including 2 modes or the modal mass participations then the sum should not be less than 90 percent of the total m so that is what we are. So, the 2 arguments are holding good for truncating the number of modes even if I truncate the higher number of modes you can do a static correction to find out the correct value of the response. Now, we will talk about modal mass contribution back with an example.

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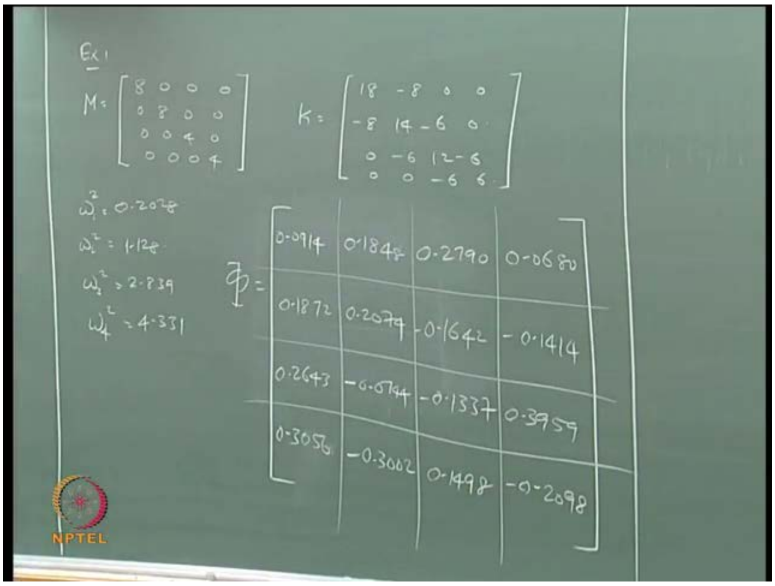

So, let us say I have an example here the mass matrix is given as a 4 by 4 matrix which is diagonal, the stiffness matrix is given as so as I solve this using a classical Eigen solver. Let us say I get the frequencies in the mode shapes and the following omega 1 square is 0.2028 omega 2 square is 1.128 omega 3 square 2.839 and omega 4 square 4. this problem we already solved, 4.331. And my phi matrix each column corresponds to a specific mode shape of that corresponding frequency which gives I should write it bigger no. So, we just estimate the modal participation factor for this problem in each of the modes we will also work out the modal mass contribution.

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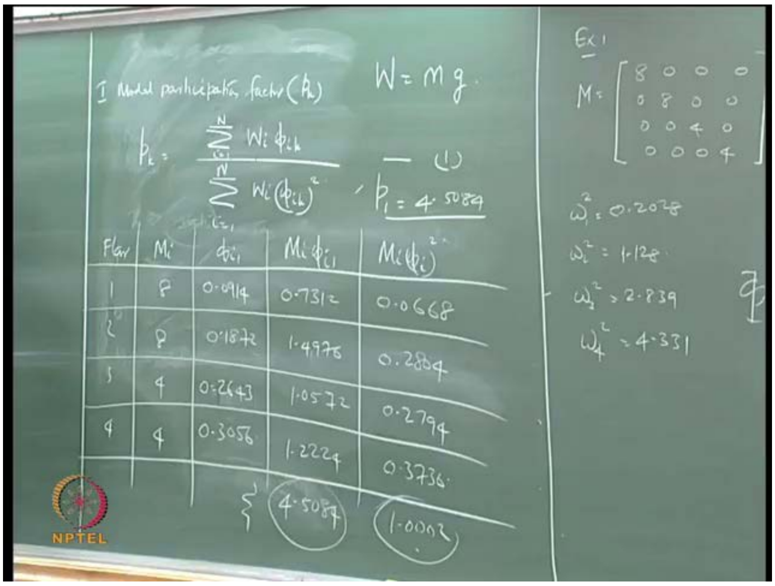

So, first let us do the modal participation factor which I call as p k the k th mode, p k is given by summation of I is equal to 1 to n in my argument I am picking this as n I am not talking about truncation of modes if it is truncated then it is in hat whatever may be the case which ever considering. Call equation number 1. I can use it to this using a table of column let us say is this flow m i phi i 1 I am talking about p 1 so the summation is from 1 to n, that is only for I this k will always associated with this p 1 so it is 1 the first column of the vector.

So, I should say m i phi i 1 and m i phi i 1 square so you may wonder that here I have got w's here m i's, w is m g so strictly speaking I must multiply the numerator as well as denominator with g so they get cancelled so that I can use m as well. There is no implication in the calculation as well as this concern, so 1 2 3 and 4. 8 8 4 and 4 these are the values I have in mass matrix they are not w's they are mass so if we expand it should be phi 1 1 phi 2 1 phi 3 1 so the first column I got to copy it here. So, 0.0914 1872 2643 and 3056 can you fill up these 2 columns quickly and let me sum of this so I get p 1 as 4.5084, that is this value divided by this value can you get me quickly for p 2 p 3 and p 4. So, you want find out really p 2 I have got to make changes is anybody who is not understood this column or this table is clear though here it is weight I am using mass. Here, I must strictly multiply this and this as well as g's here g as well as here g they get cancelled so I did not have to do it implied here so I am not doing that. So, I have to replace the second calculation. So, what I should do here is I should go for p 2 so the summation varies from 1 to n only for i, k stands for two so its phi 1 2 2 2 3 2 and 4 2 and look for the second column.

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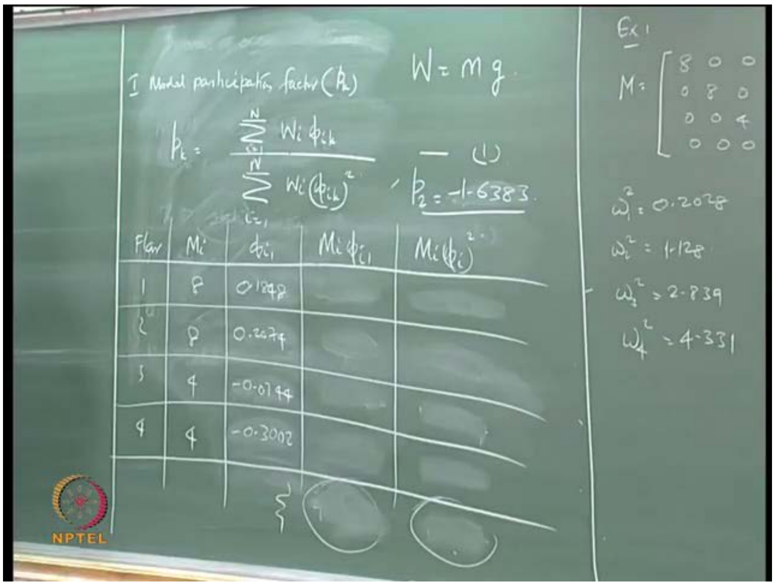

So, 0.1848 0.2074 minus 0.0744 and 3002 so calculate this and give me what is p 2 and rub it here and write it here so p 2 quick p 2 how much are you getting, anybody, is it minus 1 minus 1.6383 can anybody confirm this answer is it ok. Let us quickly get p 3, so p 3 I should replace second column here by third column of my mode shape 0.2790 I am looking for now p 3 so p 3 value will now change so I am looking for p 3.

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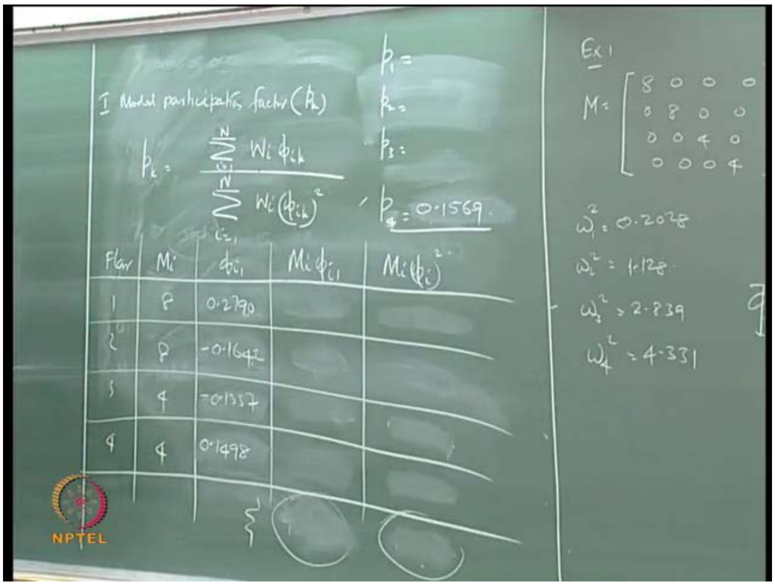

My summation start from 1 to 4 I am looking for all the 4 frequency mode shapes we are not truncating anything so the summation of phi will run from 1 to 4, but k will be at 3 so I should read this as 1 3 2 3 3 3 and 4 3 look at this modal matrix 1 3 is here 2 3 is here 3 3 is here 4 3 is here I am looking for the third column of this. So, I am replacing it here can I have p 3 is it minus .9831 minus or plus.

Student: Sir, p 2 also plus sir. (( ))

That is the bad thing; no it is a bad news, because you are asking me to do all the problems; you can come without the pen, pencil I think you can relax.

Sorry.

Student: P plus (( ))

What happened to p 2. Also, plus. The number is 1.6383.

Student: Yes, sir.

So, 0.9831 and what is p 4, I will replace this is the fourth column give me the value of p 4 and p 4 all values are becoming positive is it, is it 0.1569.

Student: 1572

1 5

Student: 7 2

1 5

Student: 7 2

That is fine is it positive?

Student: Positive.

I will take away this now looking at the modal participation factors p 1 p 2 p 3 p 4 I can write the following physical inferences from this.

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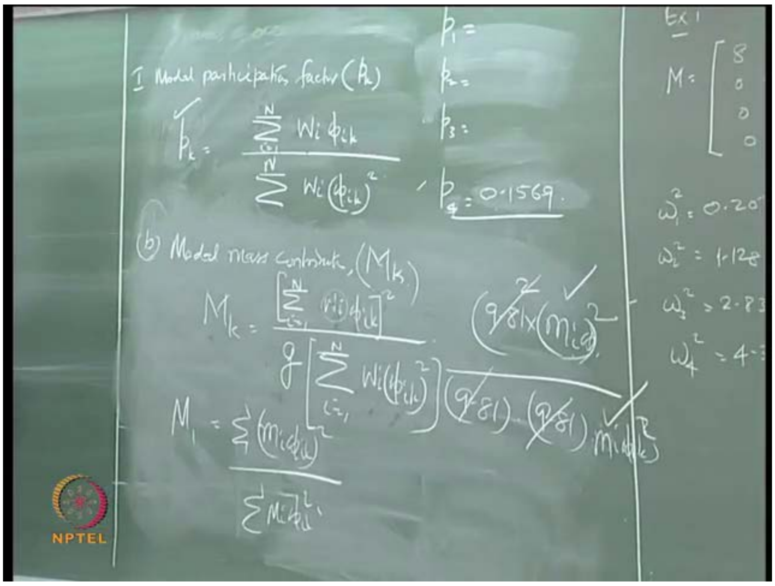

In this problem p 1 is much higher than p 2 higher than p 3 let us say or more than p 4 what is it mean is the first mode is dominating the participation so this participation or this domination of participation will can be seen in 3 phases, one it can be seen in mass two it can be seen in p k 's three you can also see this in x that is a final response. So, this will give an index that which mode is participating to the maximum in your phases of mass modal mass contribution on your participation factors or on your final peak responses I will show you that.

Now, let us try to compute what is the modal mass contribution and how much mode should I consider in my analysis for this problem. So, the modal mass m k the modal mass contribution in k th mode can be computed by a simple equation as same as the participation factor, which will be m k is given by summation of I is equal to 1 to n. I am not truncating anything, I am just trying out fine the modal participation in all the modes and see whether first mode or first and 2 will sum up to 90 then I will truncate. This is given as w i phi i k the whole square divided by g of talking about the mass now I am using weight inside summation of 1 to n w i phi i k square, the square is only for the phi i k

This equation is similar to what we have here except that the numerator is squared and I am talking about strictly the modal mass dividing by g. Now, let us see if I substitute this if I want w, I am having mass here so I should say let us say 9.81 into m i it gets squared because there is a square term here. Of course, all this is already available to me except that m i phi i k square this already available I will show you where it is available to me and here again 9.81 and again here 9.81 of this value phi i k square so this value and this value are already available to me in p k calculations. Now, this square and these two actually gets cancelled so I can easily find my m 1 as sum of m i phi i k the whole square divided by sum of m i phi i k square which I already have in my third and fourth column of a earlier table. So, use that find the ratio and get me m 1 m 2 m 3 m 4 is this clear?

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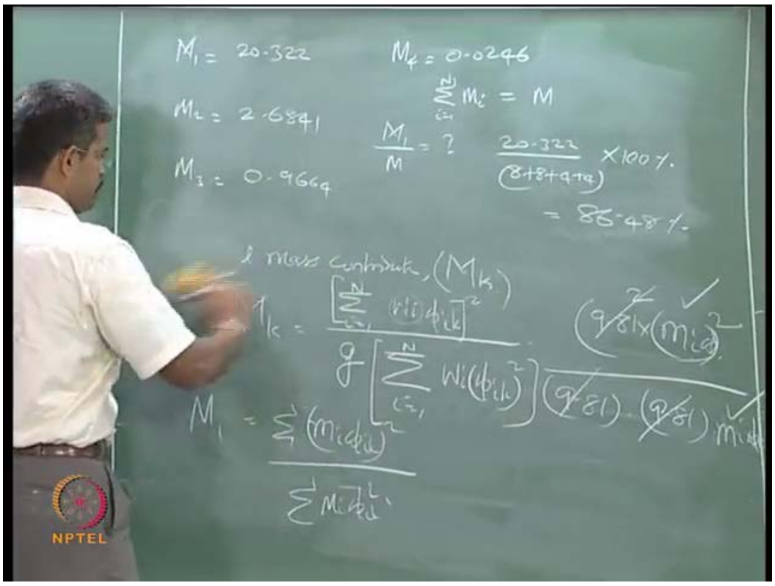

So, I can find m 1 m 2 m 3 and m 4 yes is it 20.322? m 2 if you have completed those earlier 4 tables you can easily work out m 2 m 3 and m 4 in minutes if you have skipped those things then it is difficult what is m 2 2.6841 what is m 3 0.9664 and what is m 4 0.0246 is that. Let us check whether m 1 my m is what percentage I want to see what is the modal mass contribution so m 1 is around 20.32 divided by 8 plus 8 plus 4 plus 4 that is my total m, in terms of percentage this comes to about 87 percent you can check that.

What it means is that my first mode alone or the modal mass in first mode alone contributes about 90 percent close to my analysis this is indicated even in p k's also because p 1 is predominantly high compared to p 2 p 3 and p 4 so that is a cross indication between these two so this can be checked like this. Of course, all these 4 that sum of m I, i 1 to n should become the total m you can check that, then it should become 24 if you do not have any calculation error. So, this problem demonstrates this example actually demonstrates how to compute the modal participation factor and the modal mass that can contribute to my analysis. We will take up one more example then will talk about modal combinational rules in finding out the responses. I will remove this so will do one more example quickly.

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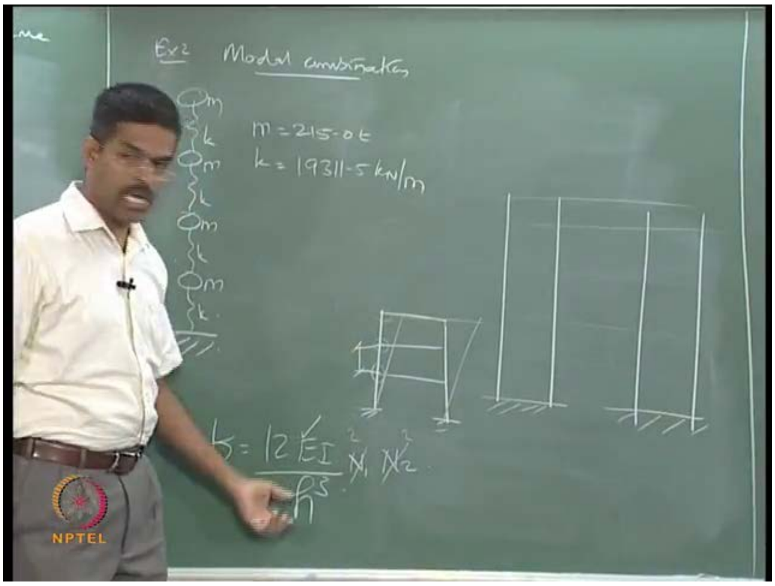

I want see how can I combine the modes first of all one should know why should I combine the modes I will explain that how to combine the modes we will see. I am just checking an example of 4 degree freedom system model of all m's and k's where m is 215 tones and k 19311.5 per meter. Now, there is a small catcher which I want to explain how to estimate k before we solve this problem, this problem will take about another 15 minutes we can solve this problem.

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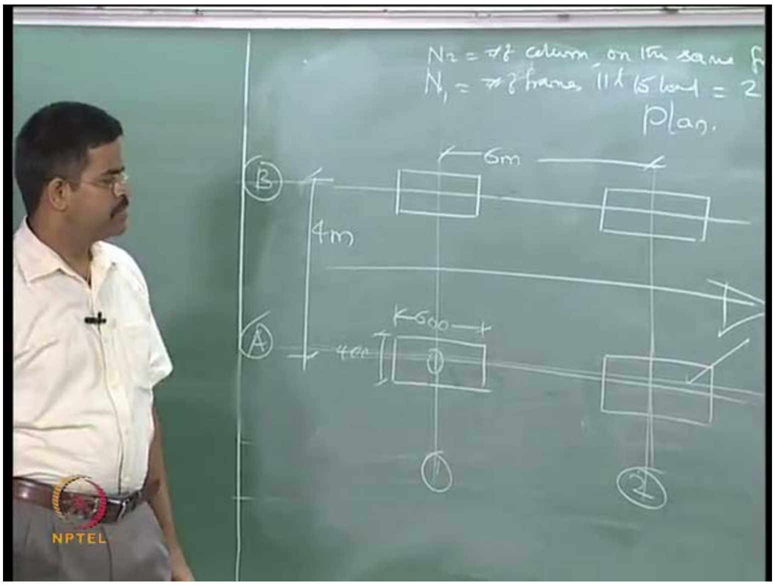

Now, I have in plan I have a system which has got let us say I am deliberately taking, I am deliberately taking a rectangular system or a structure usually in astro systems you will see they are circular, but I am taking deliberately rectangular just to explain how I can compute the k value. Let us say this is plan, this is plan it is having some dimension let us say about 600 Emma by 400 Emma spacing let us say 4 meters 6 meters and column 1. Let us say this is my a this is my b this is my 1 this my 2 I can nominclate this column as a 2 b 2 a 1 and b 1 I can read like this this is my plan.

If I draw a subsequent elevation of this it will look like this, let us say I assume that these members are founded in soil at a fixed support there can be a n number of flows we are not bother about that this is my elevation. I can also draw this as a line diagram like this may be 3 floors 4 floors whatever may be. Now, the question is how to compute k, k is actually the bending stiffness of this member so look at one frame this one frame I call this as a frame this a frame this b frame this my predominantly loading direction may be x axis, but this way unidirectional wave load so predominantly load direction for me. So, I have got 2 frames which are parallel to my load direction I have got 2 frames a frame and b frame.

So, n 1 is the number of frames parallel to loading which is in my case 2, I pick up any one typical frame either be a frame or a frame which I am drawing it here so line diagram. When I try to impose a lateral load to this frame this frame will swing and this members will all invoke bending stiffness is it not they all get bent I can always find the stiffness of this, this and this I can derive the stiffness matrix which you did in the first module form the fundamental principle of structural mechanics.

Now, there all will be a combination of k a bending stiffness nothing, but 12 e i by h q of N 1 of N 2. So, N 1 is the number of parallel frames I have so in my problem this is 2, N 2 the number columns I have on the same frame which is again 2. So, N 2 is the number of columns on the same frame which I am considering. So, e is not a problem because this is experimentally or analytically determine for any given material e can be found out Young's modulus, h is unsupported link between the floors without any imparting any boundary condition is not that both ends are fixed you say 0.65 l one end fixed, one end free 2 l nothing like that simply center the center of 0 moment connections of h nothing, but the floor to floor height in simple terms ok or between the brazing in jacket structures.

Now, the question of i we all know that i is b d cube by 12 or d b cube by 12 or d b. Now, which is to be used here is the important whether specifically picked up rectangular section. Now, look at the frame, the frame is bending this way you must always look at the axis where the bending axis looks like a line, now this is the bending plane in the top view looks like an axis it is a line. So, it is bending about this line is it not the frame is bending about this line which is vertical so plane in the top view looks as a line so b d.

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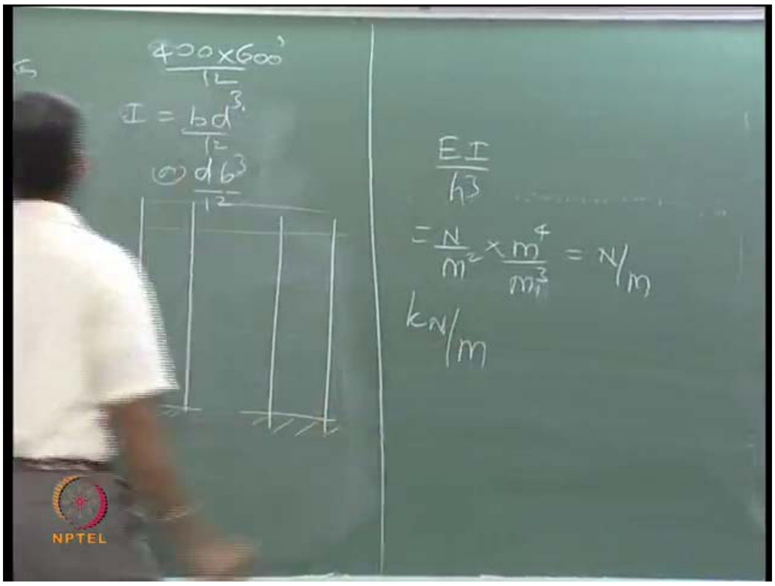

So, it becomes 300 sorry 400 of 600 cube by 12 you must look at which plane you are looking at which is a lack some bending accordingly you must use b and d. So, I had a problem where I picked up an worked out m and k which I have given, you can look at the units here e i by h cube Newton per meter square meter 4 meter cube this becomes Newton per meter I can convert this in the kilo Newton per meter which I have here. To work out the mass I must find out the weight of all the material at this floor considering half of the length of the column here, half of the length of the column here it means at every floor the height of the column will be equal to one storey height where as in the top it will be only half.

So, I can easily find out m 1 m 2 m 3 m 4 I got m in this case they are equal because I have a top side weight on a jacket structure which is also compensating to this which is m 1 m 2 m 1 m 2 m 3 m 4 all m which is 250 tones. So, I have a problem which is now designated which is an input data so far whatever we discussed in input data which is of course, given to you now.

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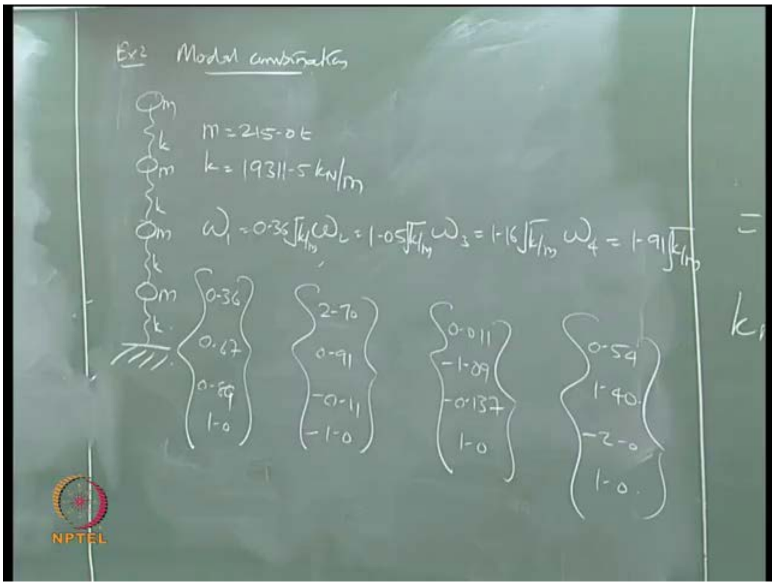

So, after doing k and m matrix I have got omega's and phi's which are calculated .36 k by m 1.05, 1.16 and 1.91. Let us say my phi 1 is 0.36, 0.67, 0.89 and 1.

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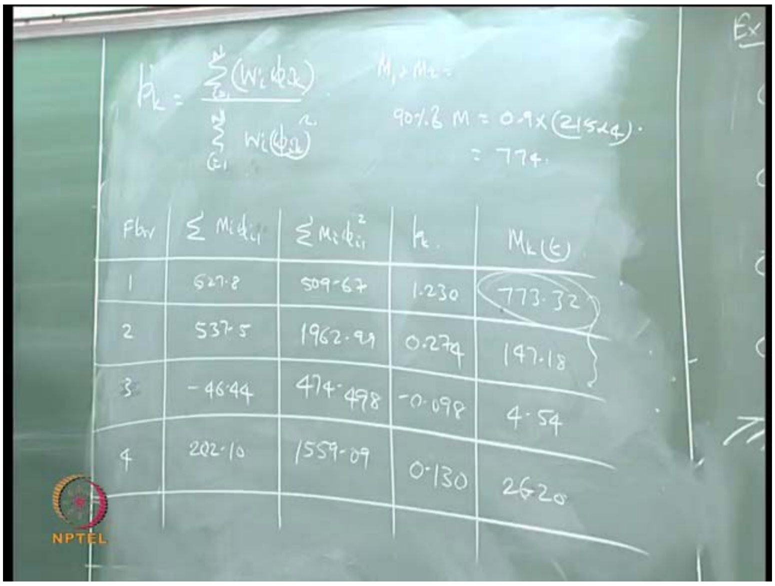

So, can you quickly find out p k's which is sum of make a table and quickly find out this I will give this is the slightly different form which will be easy for you to compare. I am giving the results I am not working on the problem you have got to work out and give me the values of p 1 p 2 p 3 p 4 m 1 m 2 m 3 m 4, but I will give the results in slightly different form which will help you to compare quickly. I am giving the results in a different form do not look at this table first work out a values then compare this table 0.4047.

Student: Sir, instead of p is it 0 percent.

There you check up that actually it is starting from 0 to positive it is taken as one level of correction here the mode shape starts from 0 to positive so that is why it is again a shift. Any way you can check up these omega's and phi's later using a classical by the time your program should be ready you can check up that. So, these are the summaries I have so far p k's and m i's check at least one or two rows of this and see are you getting it same or I am a wrong.

You have to be fast is the first row all right then let us believe all of them all right. Let us also quickly check for the sum of 1 and 2 let us say m 1 plus m 2 or let us find out what is 90 percent of m which is 0.9 of 215 of 4. There are 4 floors or 4 mass points each is 215. This becomes 774 you can see here approximately my first mode itself is contributing to 90 percent of the whole system I need not have to look into the higher modes at all if this value is right.

So, this will give me an indication that how at what mode should I truncate it I can truncate to 1. Let us quickly see if I consider 1 and 2 what happens in my modal combination. Now, will talk about the combination rules I will remove this I may need this values I will remove this you may give me these values I hope you have taken this table have you copied this table, any way you can create this table.

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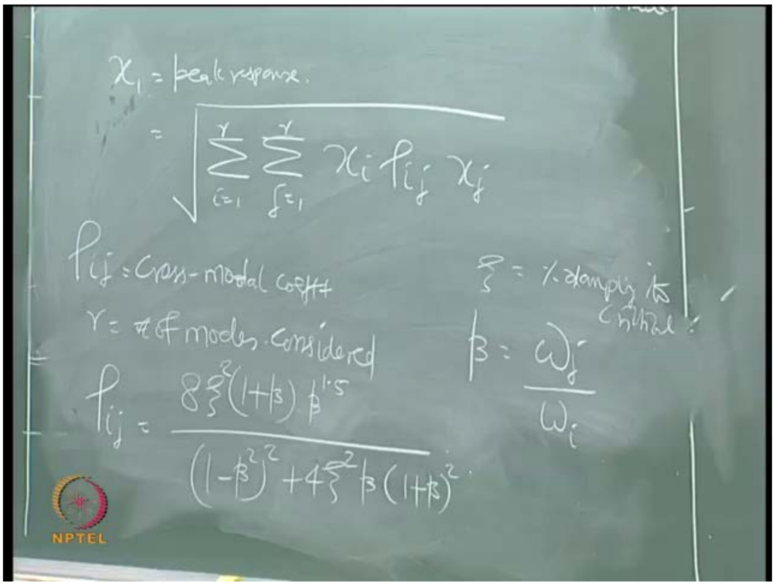

Talk about modal combinations I want to know combine the modes and see what be the effect if we ignore the higher modes. There are 3 rules available here, the first rules is it is c q c rule first let us see what we are trying to combine it is complete quadratic, complete quadratic combination rule that is the c q c rule these are all mathematical applications. What it says is we want to find the response in any level not in mode any level that is first level, second level, third level, fourth level, fifth level that is response either in the top or on the bottom whatever where ever you want to find the response. This is nothing, but the peak response we are interested in the maximum response.

Peak response I want to compute this can be simply given by this rule as summation of i is equal to 1 to r j is equal to 1 to r x i rho i j x j. Where, rho i j is called cross modal coefficient r of course, the number of modes which you want to consider. And rho i j which is cross modal coefficient this is given by an expression 8 zeta square 1 plus beta of beta to the power 1.5 1 minus beta square the whole square plus 4 zeta square beta 1 plus beta the whole square.

Now of course, in this expression zeta is a percentage damping to the tough critical 2 percent to 5 percent that is what we are varying and beta is the ratio of you are talking about the cross modal coefficient between i and j it is called rho i j. Then I should say beta is ratio of frequencies of j over i, if I am talking about j i then it is i over j.

So, I have these frequencies with me beta can be computed where as omega i and omega j respectively frequency in i th mode and j th mode we are talking about the modal combinations right each frequency will have a specific mode attached to it first mode, second mode, first frequency, second frequency we have these values with me. Now, in this example we already know that modal mass one is contributing to closely 90 percent of the total mass we do not have to look at the higher responses, but just for completing this to understand remove this how to operate this rule.

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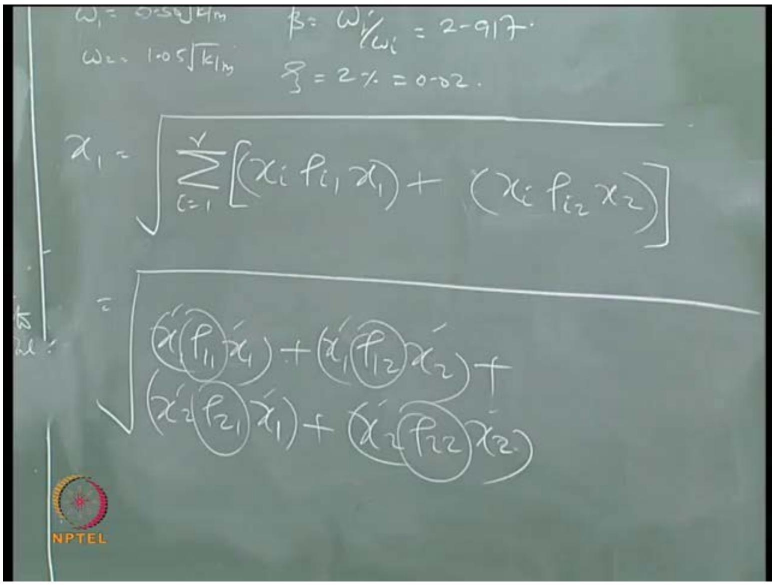

Let us pick up for example, let us pick up let us consider first and second modes so then I should say omega 1 and omega 2 are .36 k by m and 1.05 k by m and beta which is omega j by omega i for rho i j is nothing, but 1.0358 .36 which will give me the value as 2.917. And of course, I take zeta in my problem 2 percent is 0.02 in my calculation.

Now, to find the response at any level this is indicating the level where you want to find the response not the mode first level, second level, third level, fourth level of a mass point. Where ever you want to find the response usually one may be interest in finding out the response at the top near the deck one can also find the response at the foundation if it is a free floating system where ever you want to that is what we are looking at we are not looking at that response peak response which has contributions from all possible modes.

Why I am saying all possible you may say sir there are only 2 modes here i and j, but it is summing up know so I am picking up here only 2 modes I can do for 4 also. Now, let us expand this equation see how do I write this let us say x 1 will now become square root of I retain one summation now for the time being i is equal 1 to r expand the inner summation there. So, I should say x i x i rho i 1 x 1 is it not plus summation of x i rho i 2 x 2 is it not summation, the summation is common for all i is common for both.

Now, when I expand this also I will have 4 terms I got 2 terms, I got 2 more terms square root of x 1 rho 1 1 x 1 plus x 1 rho 1 2 x 2 plus x 2 rho 2 1 x 1 plus x 2 rho 2 2 x 2. What I am interested in finding out is the peak response provided know the response or every floor or every mass level independently I am looking at combination rule. Now, if I do not know x 1 and x 2 I will never get x 1 total right that is fine if I do not know any one of the responses this this combinational rule will not give me the response it will give me how to combine the response on different modes so I must have the responses in individual modes separately.

So, these are all not problems for me, provided I know them I must evaluate these let us first evaluate the first co-efficient 1 2 and 2 1 see what happens remove this we will not be able to complete this problem because this take time we can continue in the next lecture. So, we will evaluate this we do not want to hurry up we will evaluate this in the next class or if you find time complete it so it becomes easy for me to just write down the values. So, one more class I will have tomorrow there I will talk about the Duhamel integral which we have not discussed and there is a question asked some where during the lecture of module 1 and 2 that if you do iterations for example, stored law for example, iterative scheme why do my iteration will converge in the first mode first why not the higher modes.

Why I am getting always store law and iterative scheme, why I am always getting the omega n natural frequency lowest mode and why not the highest mode. Mathematically this can be proved; I will prove that tomorrow in the lecture how iterative scheme land up in lowest frequencies, I will prove that when we will discuss about Duhamel integral we will talk about evaluating step functions. If I got step loading then impulse function actives only for a specific time duration very short duration how do you do the analysis for this using special kind of integral which is called as Duhamel integrals, we will talk about that I think there I will close tomorrow's lecture.