Dynamics of Ocean Structures
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Module - 3
Lecture - 5
Modal Response Method Modal Mass Contribution

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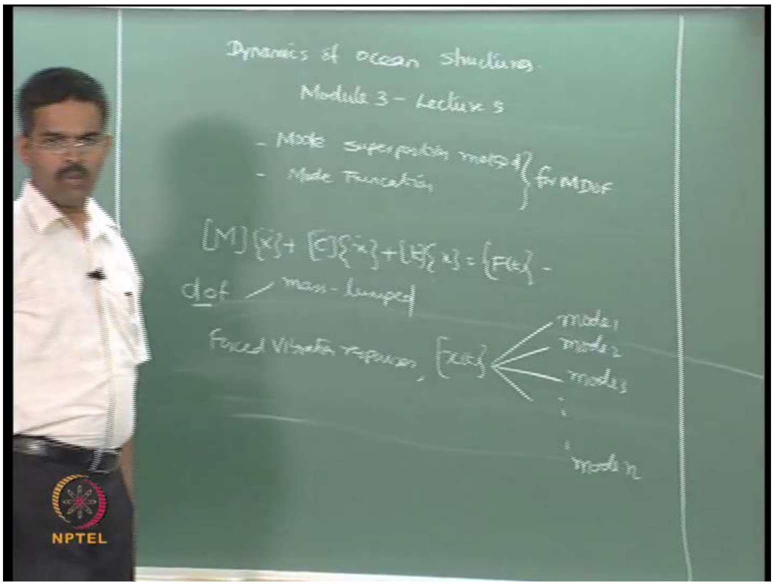

So, in this lecture, we speak about mode supervision method. So, when you talk about multi degree freedom system model, I can say the equation of motion can be written as you see here and the size of these matrices depends on the degree of freedom. They depend on the degree of freedom, usually the mass or lumped mass. The moment I say lump the mass at the same point where I am measuring my displacements, then I am expected to get a diagonal matrix of mass. Of course in offshore structures as you have seen in the second module, there can be half diagonal elements present in mass matrix despite, you measure the responses in the same place, where degrees of freedom are marked.

And of course look at the k matrix, because of the dependency of coefficients of stiffness matrix on the response, which is an integral built up, because now this equation of motion cannot be solved explicitly. There is a dependency on left hand side to the right hand side of this equation, because f of t, if I take look at the Morrison loading, it demands the water particle kinematics, which has got relative motion of the vessel or the body of the offshore structure.

Therefore, x is required to compute or x dot is required to compute f of t even and x dot is not known, because that is the one of the unknown here. Now, instead of looking at the forced vibration responses, we always ultimately interested in finding out the response vector, which essentially comes from different modes. So, one may wonder that why one is interested in knowing responses in different modes.

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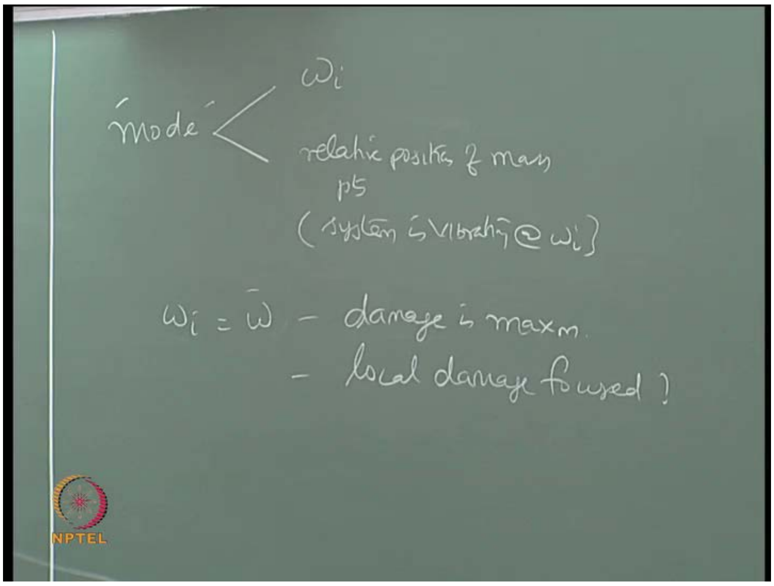

For example, the moment I say mode, I have got two inherent characteristics, which is assigned to this. One is, I will know at what frequency this response is activating or contributing, and of course I will also know the relative position of the mass points, when the system is vibrating at this specific frequency. So, I will know what would be the relative position of the mass at this frequency. So, I can easily find out, if this frequency matches with my excitation frequency, you can always expect that the damage is going to be maximum and by knowing the mode shape, I can always know where will be the local damage focused, because I know the relative position of the mass moment.

So, the response always is contributed from different modes. Now, the question comes, if in a multi degree freedom system model, if the degree of freedom is very large, practically let us say infinitely high, do you have to consider all the modes for working of the response. Now, the question comes, how many modes do I consider for finding out my response summation?

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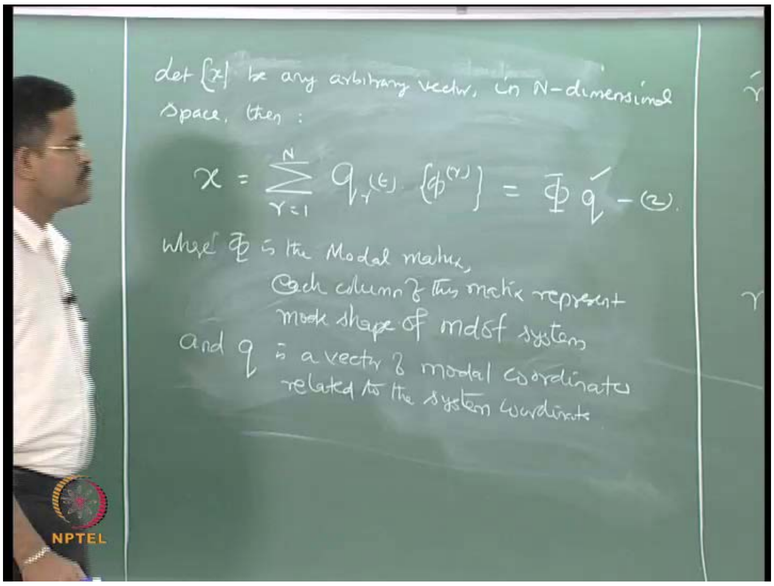

Let us say, x be any arbitrary vector. Let x be any arbitrary vector in n dimensional space, where n talks of the degree of freedom. Then I can write this x as simply summation of r equals 1 to n q r of t phi r, which I call as phi q. Let us say equation number 2. Equation number 1 was the equation of motion, which we had in the previous slide. Now, in this case, obviously phi is the modal matrix. Each column of this matrix represent mode shape of the m d o f system and q, the q is a vector of modal coordinates which is relating to the system coordinates. We will come to this.

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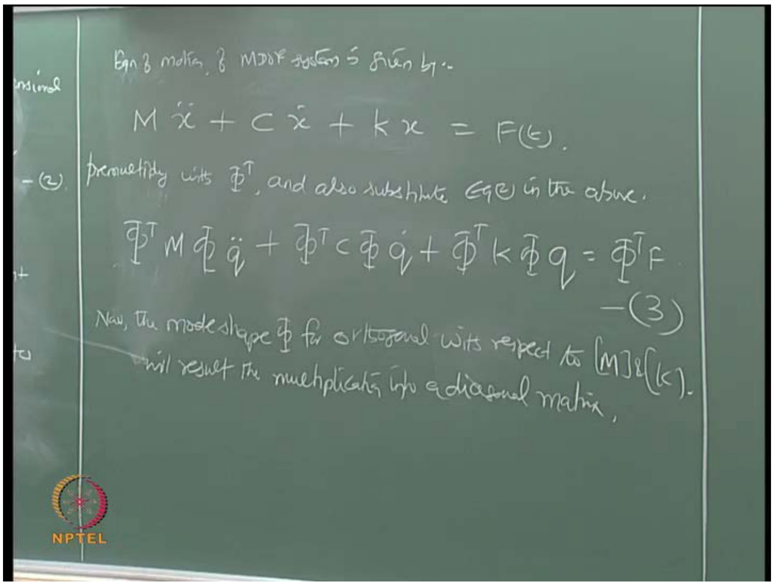

Now, my equation of motion of m d o f system is given by, let us say m x double dot c x dot k x is f of t. I pre multiply this with phi transpose and also substitute equation 2 in the above. That is, x is a vector multiplier of the modal mass matrix or the mode shape matrix. Therefore, I can say phi transpose m x is phi q. So, I should say this is going to be phi q double dot plus phi transpose phi q dot plus phi transpose k phi q as phi transpose of f. Let me call this equation number 3. Now, the mode shape phi for orthogonal matrices, I mean mode phi for orthogonal with respect to m and k will result in a diagonal matrix. It will result the multiplication into a diagonal matrix; this multiplication for example. They will become diagonal. Let us write each one of them for an r th mode separately and see what happens.

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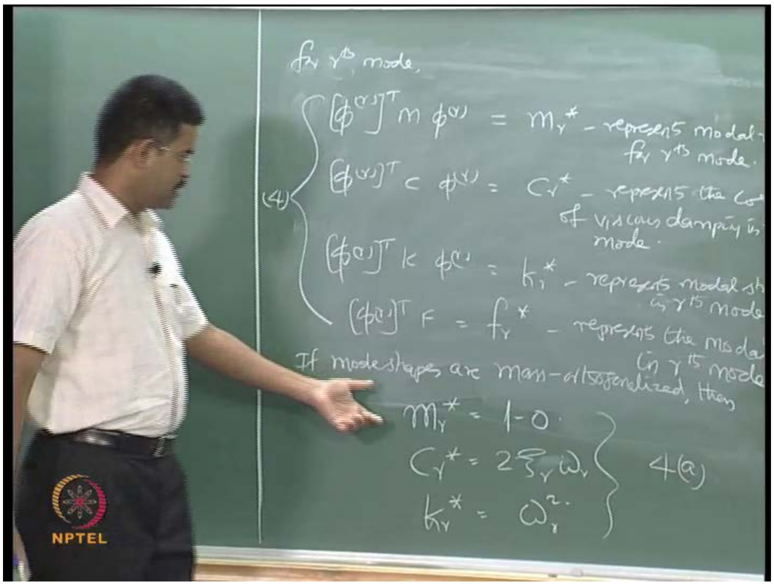

For example, for r th mode phi, let us say r transpose m phi r. I am going to call this as m r star, which represents the modal mass for the r th mode. Similarly, phi r transpose c phi r can be called as c r phi, which represents the coefficient of viscous damping in r th mode. Similarly, phi r transpose k phi r, I call this as k r star, represents modal stiffness in r th mode. Phi r transpose f is called as f r star, which represents the modal force in r th mode. All these equations, I am calling them as 4. What I have done is, from this equation 3, I just wrote a typical equation at an r th mode and see what is happening to each one of the terms. I will now apply a special condition to these set of equations 4 and see what happens. That condition, what I am going to apply is, if the mode shapes are mass orthogonalized, then you can orthogonalize it with the stiffness also. Then m r star, what will happen to m r star, if it is orthogonal to the mass matrix. Sorry, 1.

C r star, I am expanding c as ratio of the critical damping. So, 2 r zeta omega, so this becomes 2 zeta r omega r and k r star will give you the natural frequency. Is it not? Now, I will write this as equation 4 a. These are all actually special conditions of these values when the mode shapes of phi remains mass orthogonalized. Now, based on 4 a, I can now write an expression for modal participation factor.

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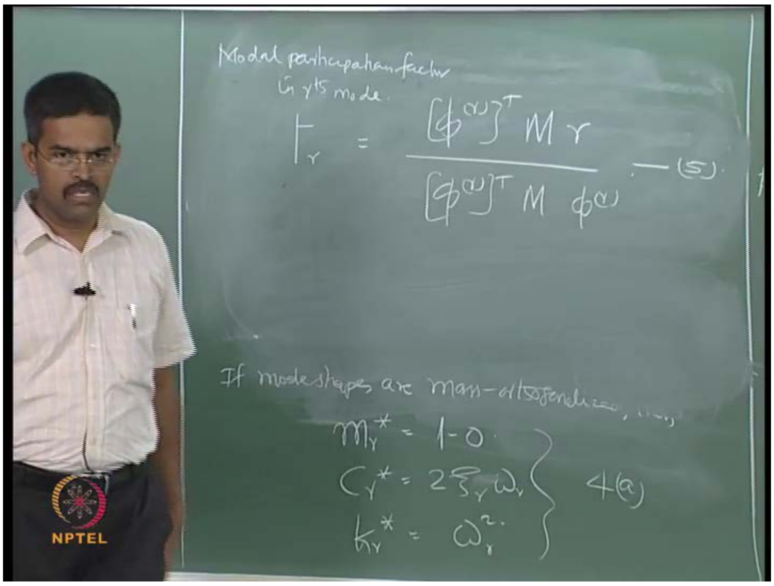

We will see what is the importance of this factor. Now, I can find what is the modal participation factor in r th mode indicated by gamma r, which is given by phi r transpose m r; r is a multiplier here and not a suffix, phi r transpose m phi r. Equation number 5. Before I want to find the modal participation factor, I want to know what all the modal mass present in every mode are. Because, I have given equation, set of equations for 4, saying what would be the modal mass in the r th mode. I would like to know what the modal mass in every mode is because depending upon the conditions implied on the modal mass, I can truncate the modes. How many mode I must look at.

If there n degrees of freedom, I will get n mode shapes and n frequencies. Should I use all of them to find out? So, there are two ways of looking at it. One, estimate the modal participation factor and use this factor on every mode shape and then get your x value. The other way is, truncate the modes depending upon what is your modal mass participation in the whole system. Why we are talking about mass because mass is the major source of contribution because this gives me the inertia force to the system. I am not talking about the restoring force. Of course, this is a dependent on these two. So, we do not talk about this because there can be systems, which can be un damped also. But, this gives me the inertia force, which is predominant for dynamics. Already we discussed, even though the response be time variant, even though the excitation force be a time variant but if the system does not have a reproductive mass, indicative mass, which can give you or produce good substantial significant inertial force, we do not do dynamic analysis.

For example, I have a paper, which is floating. The motion of the paper is in a time domain variant. The force which I am blowing like a wind is also a variant. I do not do dynamic analysis to trace the response of the paper because paper has no mass actually. The inertia component will significantly remain absent. So, I focus on the modal mass participation. So, we will quickly work out an example to find how I will get m 1 m 2 m 3 for this example. Then I will take it forward to talk about truncation of modes after we understand this example. Any question here? I will use this equation later. I will take away this now.

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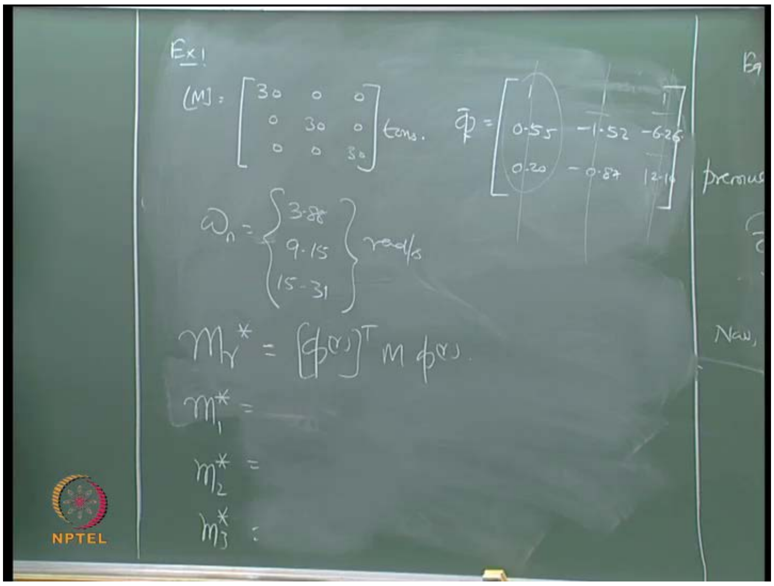

So, let us say, I have an example of a three degree freedom system, whose mass is given as in tons. Of course, we have the modal matrix, which we know how to get. But, anyway I am giving the values, if I know my k and m. This is 6.26 and 12.10. You can easily make out there are no 0 crossings. There is one 0 crossing here. There are two 0 crossings here. So, first mode, second mode and third mode. The corresponding frequencies, 3.88 and they are in radiance per second, 9.15 and 15.31. So, we already know m r star is given by phi transpose m phi. So, can you find out m 1 star, m 2 star and m 3 star.

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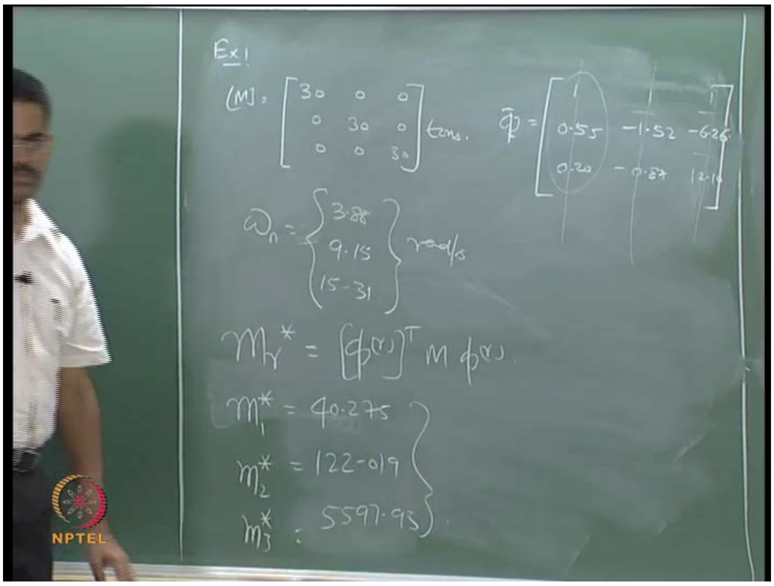

So, if you want to find m 1 star, I must use phi 1. Phi 1 is the first column. I have to use a transpose of this. So, 1.55 and 0.2 multiplied by the mass matrix, which is 30 0 0 30 0 0 0 30 and multiplied by the vector, which is 1.5 and 0.2. You can see the compatibility. This is 1 of 3, this is 3 of 3, and this is 3 of 1. You will get only one value, which can be quickly worked out. Is it 40.275? So, this is 40.275. Similarly, workout for m 2 and m 3. Yeah, fine, I will give you the answers. It becomes 122.019 and m 3 becomes 5597.93.

Having said this, now let us see how we truncate the modes. What will be the condition for truncating the higher modes, where do we stop and what is it frequency governed or is it mode shape governed and what would be the effect in natural, in dynamics, if you truncate them. Can this error be corrected because you very well know you are creating an error because you are truncating the upper modes. Can you correct this error? This is called static correction. We will see that also. Remove this and then I will take up this same example and then apply it on a different perspective.

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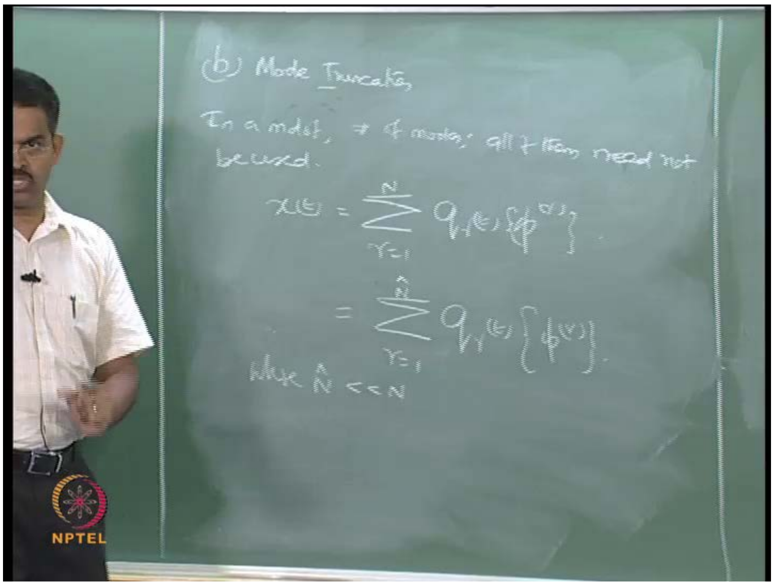

Now, what we are talking about is the mode truncation. So, what you have learnt in the A part of the lecture is that, we know how to find out the modal mass contribution in every mode. In an r th mode, I know what is the contribution of mass from the equation. We also studied an equation, which gives me the modal participation factor. Based on these two understandings, I will take forward and see what will be the logic to truncate the higher modes. So, we already saw in a multi degree freedom system, there can be many number of modes. All of them need not be used. Now, the question is where to use them. Where do we need them actually? So, if you look at the response, we wrote in equation 1 earlier; I will rewrite it again. We said, the equation is varying from r to n, which is having q r of t and phi of r. I am retaining this equation because n becomes the degree of freedom and I am rewriting it saying, do not take it till n, but take it only till n hat, where hat is far lower than n. I am truncating it.

Now, the question comes, what are the factors which will govern how many modes I should include. What should be the n value? n value for a 6 degree freedom system is 6. For 30 degree freedom system is 30. n hat can be any number of your choice. Lower than 30 or 6. Can be very much lower, if n is very high. For example, 1000 degree freedom system modal you have, you can pick up only 10 20. can be very much lower. Now, the question is, how many modes, how many numbers you desire in hat and what will be the factors. The factors which will decide the number of modes to be considered in your response computation remember. We are talking about the response computation, right.

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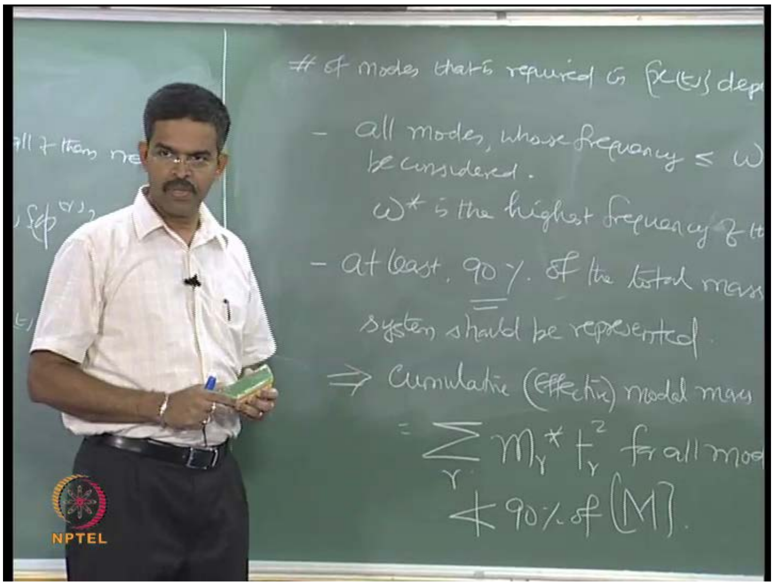

The number of modes that is required in response computation depends on many factors. One, all modes whose frequency is lower than omega n, sorry, what is that omega star is to be considered. Now, the question comes, what is omega star? Omega star is the highest frequency of the forcing function. That is, the highest frequency content of the forcing functions. You must pick up the frequency till that. Two, the second condition is also important. At least 90 percent of the total mass of the structural system should be represented. Where do you check this? This can be physically seen, because for a multi degree freedom system modal, the classical Eigen solver will give you different frequencies. You can always physically check that a frequency is not omitted, so that, omega star is not deleted from your system. You can always pick up those numbers of modes and you can do.

How will you check the second condition? A second condition similarly says, that the cumulative effective modal mass, it leads to the cumulative effective modal mass is simply sum of r; r is the number of degrees you are considering, m r star gamma r star for all modes and this summation should not be less than 90 percent of the total mass of the system. Now, in this equation, m r star is already known to me. I got equation, set of equations 4 a, which can give me m r star. Equation 5 gave me the modal participation factor at the r th mode. So, if I know this and sum them up, and if that sum should be lower than 90 percent the total mass of the system, I can truncate the remaining modes.

Now, let us see how do you correct this error because this will result in error. We can correct this error. This is called static correction.

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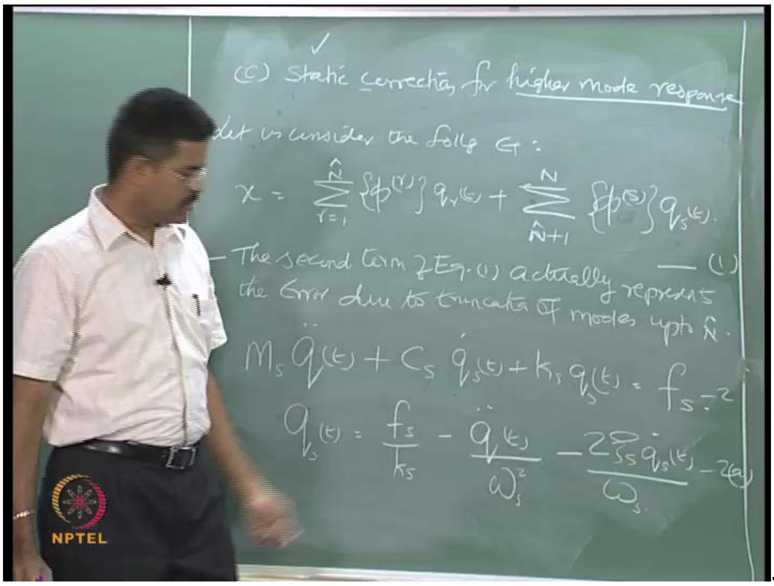

Any question here? The moment I truncate the modes, I result in error. I can correct this error. This is called static correction. Why thisis called static? I will come to that. We are talking about dynamic analysis. why is it called static? I will come to that. What are you correcting? We are correcting for higher modal response. I mean, you are not including the response from the higher modes. We are applying correction to them. Let see how.

Now, let us consider the following equation, x is given by, x is may response, which is given by r equals 1 to n hat, n hat is the truncated mode I am having, which I say is phi r vector of q r of t plus n hat plus 1, the next mode till n. I am just separating this phi s q s of t, for equation number 1. Look at the second term of this equation. The second term of this equation actually represents the error due to truncation of modes up to n hat. Is it not? Originally I have n modes, but I am truncating them into n hat, where n hat is much lower than n.

Having said this, let us come back to this equation of motion. I am talking about this part. So, I am using s suffix here. m s q double dot t, because my response is no more in x. I am ten got into the q because there is a multiplier value of phi. It is a general expression, plus c s of q dot of t plus k. So, of q of t, which I have of course called as f s. The suffix s says that I am not looking at the original equation of motion and I am looking only at the error created by this part alone. From this equation, I can write q s of t, which can be simply f s by k s minus q double dot of t. I have to divide by m as well as k. Is it not? I get omega a square minus c is of course propositional damping respect to critical. I can say 2 zeta s q dot s of t by omega s. Is that ok? First, this was 2 zeta omega n, k will be omega square and I get omega n at the bottom. I call this is the equation number 2. and 2 a.

When we look at the equation number 2 a here, there are two interesting observations they can make from equation 2 a. One observation is, the response q, which is caused from the error term here depends only on the static response. That is why it is called static correction. For any value increased value of omega, you will see there will be a depletion of these terms. Is it not? They will keep on be decreasing because it is inversely proportional to square of that. The dynamic term, which comes from inertia is inversely proportional to square of higher omegas and the damping term proportional to inverse of omega for any higher values of omega, these terms will get deleted. They will become insignificant. That is why I call, if I correct this to this, I call this correction term as static correction because these are having only static term here. There is no dynamic term involved here. Is it not? Any question here? I will take ways this.

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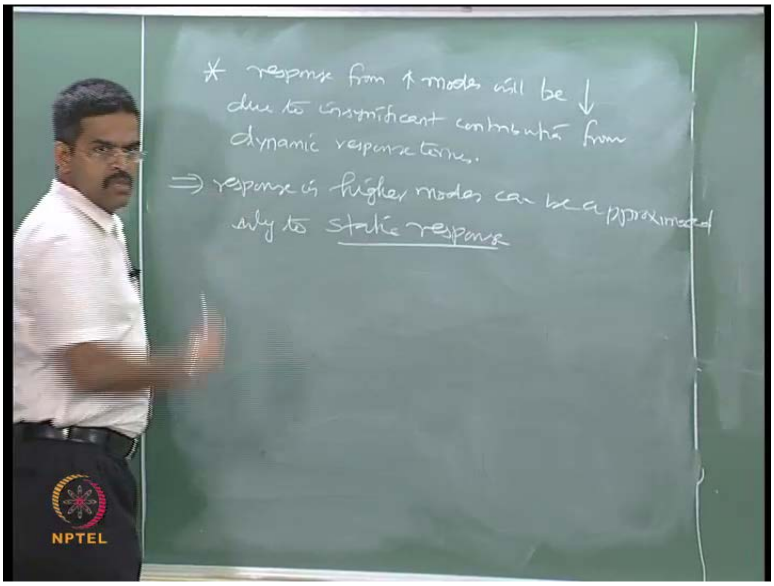

Having said this, they can put a word here saying, the response from higher modes will be decreasing due to insignificant contributions from dynamic response terms. Therefore, the response in higher modes can be approximated only to static response. What does it mean is, in the higher modes, I do not look in the dynamic content of response. Having agreed upon to look only at the static correction, the numerator has f of s in the static correction. Let us work on f of s now.

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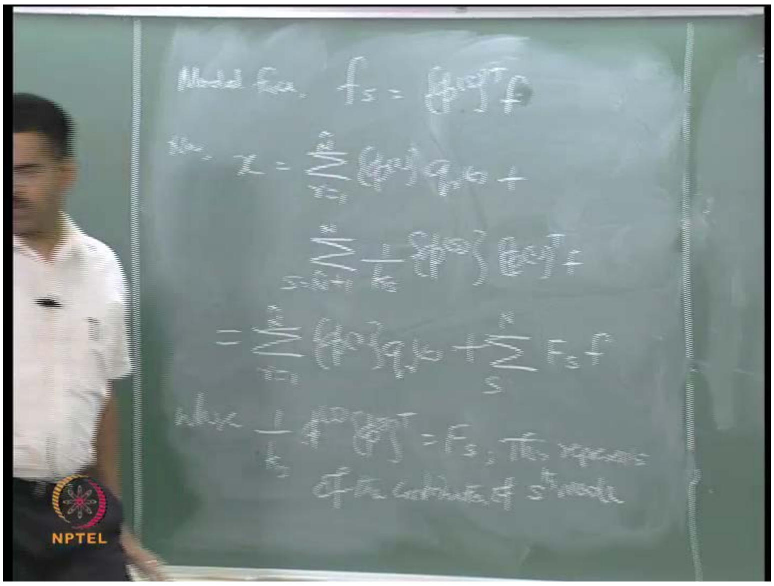

We already know the modal force equation 4 a will give me f of s as phi of t. Sorry, it is not a vector, phi of s of t, small f. That is my f of s. Now, I am looking for my response. So, x already I know summation of part one of n hat, which is phi r q of t plus summation of n hat plus 1 to n, which is, I call as s 1 by k s of, I had numerator f of s and I am replacing it with phi s of phi s transpose f. Is that ok? It can be now rewritten as summation of r to 1 n hat phi r q r t plus summation of s to n. Can I say this as f s f, where 1 by k s of phi s of phi s transpose is what I call as f s. What does is it represent?

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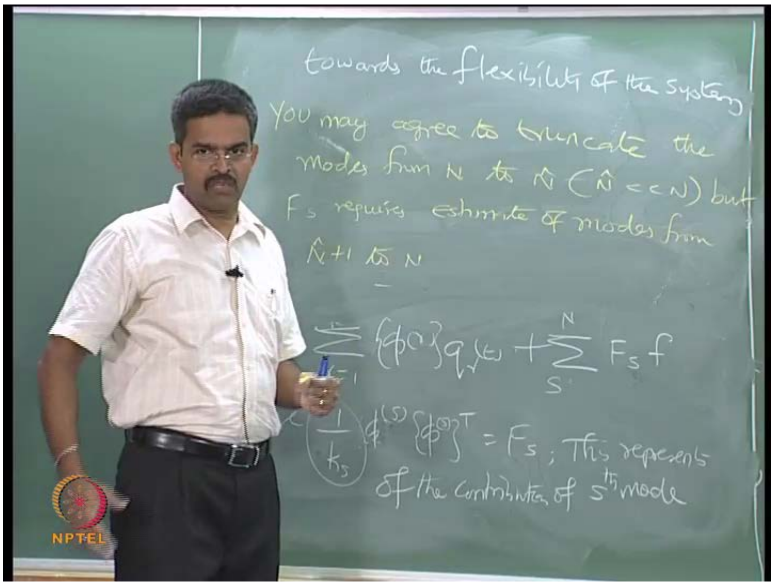

This represents, this term represents the contribution of s th mode towards; I can write it here, I can write it and I will remove this, towards the flexibility of the system. Why I am saying flexibility? It is 1 by k. Now, there is small problem here. The issue is, you may agree to truncate the modes from to n hat, where n hat is much lower than n. But, to evaluate f of s, you require the full mode shape.because, f of s wants the mode shape from s, which is nothing but n hat plus 1 to n. Is it not? But, f of s require estimate of modes from n hat plus 1 to n. Is it not? Otherwise, I cannot compute this. because, the s is summation from n hat plus 1 to n. So, there is no benefit actually I am getting because anyway I am working all the modes.

How to get rid of this problem? See, my original problem is, I do not want to compute all the modes. I agree I can truncate. I can do a static correction. So, when I say static correction can be applied, because the dynamic effects in higher modes are not responding or represented when it is agreed up on. When I start elevating f of s, I require this s value, which is contributing from n hat plus 1 till n. It means, practically I need to have all modes in front of me.

This problem will intelligently get rid of in the dynamic. See how we are doing it. Do you understand this issue? This does not give me actually a benefit because though I am truncating till n hat, but I require phi for work load f of s, which is a static correction, if you want to account for the errors of response in the higher modes. These errors do not arise from dynamic effects. But, there is a static error coming from this, which I have got to correct. To correct that, I need all the modes. Practically, if I have all the modes, there is no question of the doing a truncation at all. Is it not? Right? How will I get rid of this? Having understood there is a complexity here; we will move forward and see how will I get rid of this problem.

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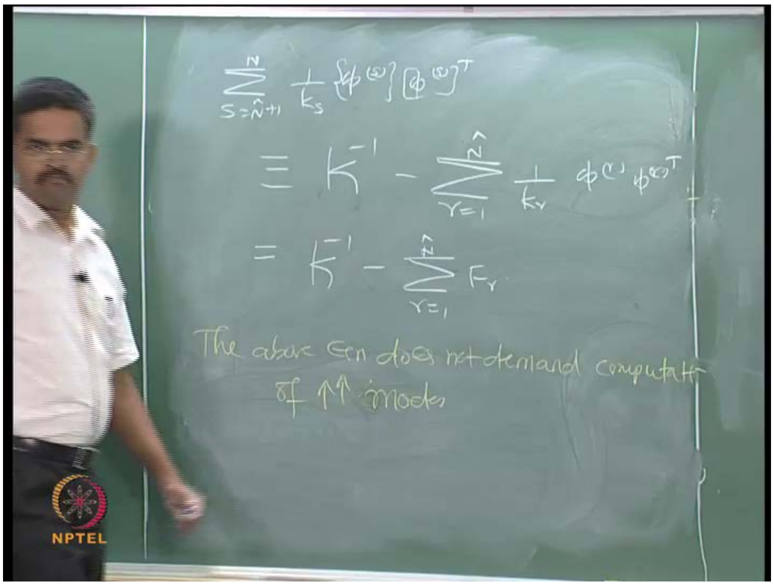

Let me now pick up that term only, which is s of n hat plus 1 to n of 1 by k s phi s phi s transpose. I can rewrite this equation as k inverse minus summation of terms from r to n hat 1 by k r phi r phi r transpose. What I am trying to do here is, instead of looking for a correction in the higher modes, I take the stiffness of the flexibility for the whole system and subtract the correction from the lower modes. You see I am doing only from n hat and till n hat, I know this value. So, I do not have to compute the mode shapes beyond n hat till n. I can rewrite this equation as k inverse minus r equals 1 to n hat, which I call this as f r. Is it not? That is an f term here. We said f s and this is f r. So, it is interestingly, the above equation does not demand computation of higher modes. Is it not?

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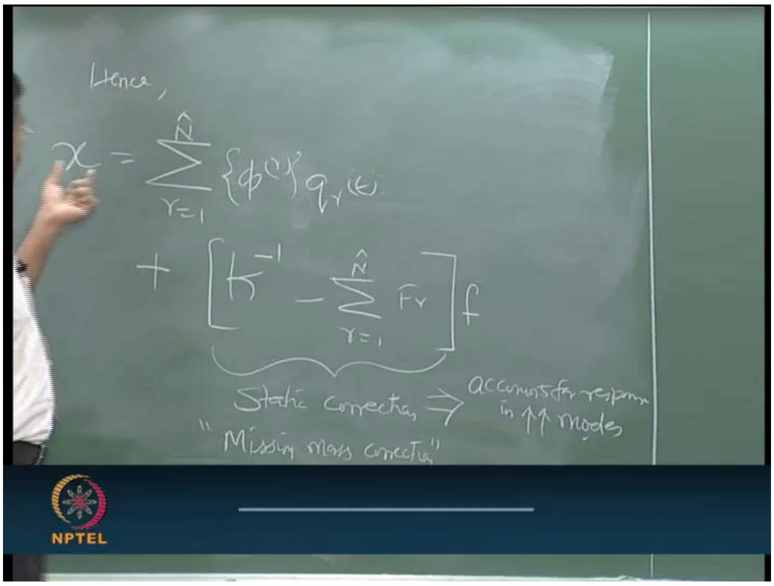

Therefore, x can be said as, into of course f, there is a multiplier here, right. This is what we call as static correction. Some literature address this as missing mass correction. What does it do? This accounts for response in higher modes. Is it not? That is what it is doing. Interestingly, this only takes care of from the lower modes. That is the beauty of this whole experiment. I am not touching on the higher mode at all. I am truncating at n hat, but still, I am making a correction, which accounts for higher modes. That is beauty of the whole exercise. I can stop at n hat.

Now, the question again, reiterates is where will I stop my n hat? Two cases, I must check for frequency which will be either equal to or lower than the highest frequency excitation force. I must look till that part and I must look at the modal participation from the mass should be at least 90 percent of the total mass. If these two conditions are satisfied, then I truncate the modes at n hat and apply this correction. This will automatically account for the error what you have obtained from the higher modes. x of t can be interestingly done. So, the next class we will take up an example and see how the truncation of modes can be applied for, let say a 4 degree freedom system problem. There I will have to all the 4 or only 2 or only 1, we will see by demonstrating this in two examples. That is the focus in the next class. Any question here?