Dynamics of Ocean Structures
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Module - 3
Lecture - 3
Narrow Band Process

In the last lecture, we derived the transfer function equation; has a response ratio; beta becomes the ratio of omega by omega n; and, zeta is the ratio of critical damping in percentage.

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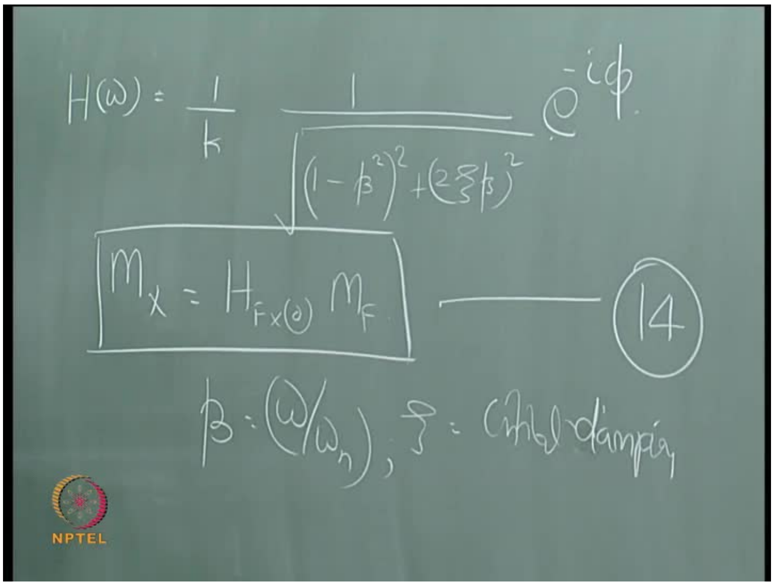

Based on which, we derived a simple relationship between the variance of the response to that of the variance of the force. But, here we are showing the corresponding connectivity between the mean value of the response and mean value of the force. And, we already know, in stochastic process, we are talking about realization of the force and the x value.

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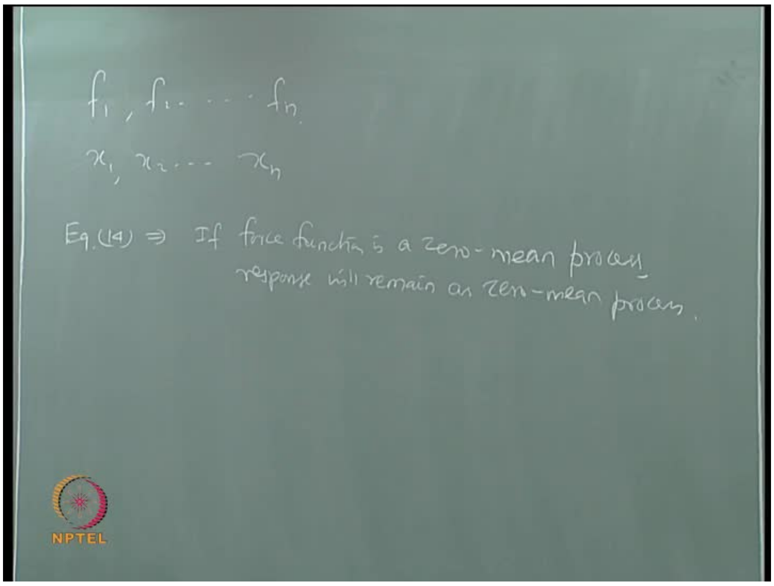

So, f y, f 1, f 2, f 3; f 1, f 2, etcetera or small n are realization of f of x. They are the set of team of f of x. Whereas, x 1, x 2, x n are realization of the response. So, connecting these two is the transfer function. So, we also see from equation 14, that equation 14 implies that, if the force function is a zero mean process, then response will also remain as zero mean process. So, let us quickly get the details of the response spectrum and see how does it look like.

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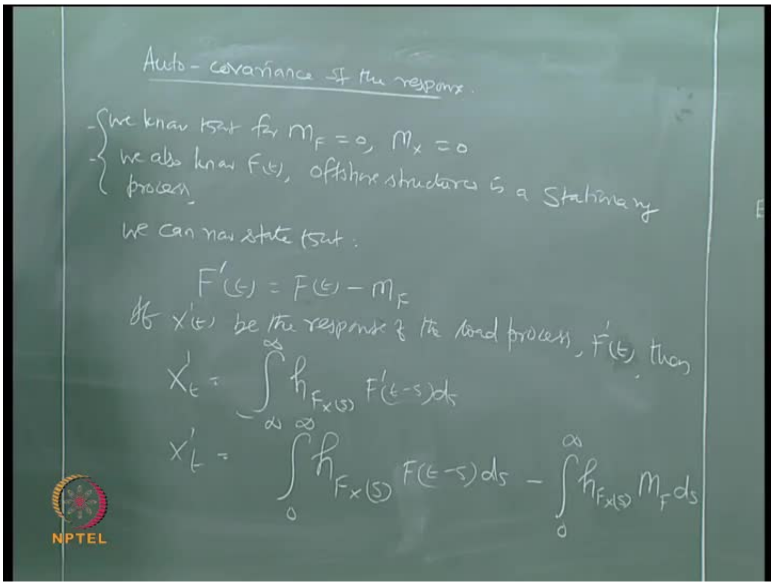

Now, let us look at the auto-covariance of the response. We already know that for m F is 0; m x also remain 0. And, we agree that, F of t especially in offshore structures is a stationary process. So, based on this, we can now write a new function, which is F dash of t – can simply be said as F of t minus m of F. I can always deduct the mean value. Now, fortunately, in my problem, being the problem remains stationary, this may tend to 0 for a zero mean process. So, it is as same as this. So, in that situation, if x dash of t be the response of the load process, F dash of t; then, I can write X dash of t as integral minus infinity to plus infinity h of F x of s F dash of t minus s ds. This is the standard form we are writing the expression for the response. Now, substituting F dash of t as F of t minus m of F, same way, I can write now this equation as 0 to infinity, because the realization of the negative value becomes invalid for us. So, F of h of s F of t minus s ds; I am using this relationship here – minus 0 to infinity h of F x s of m F ds.

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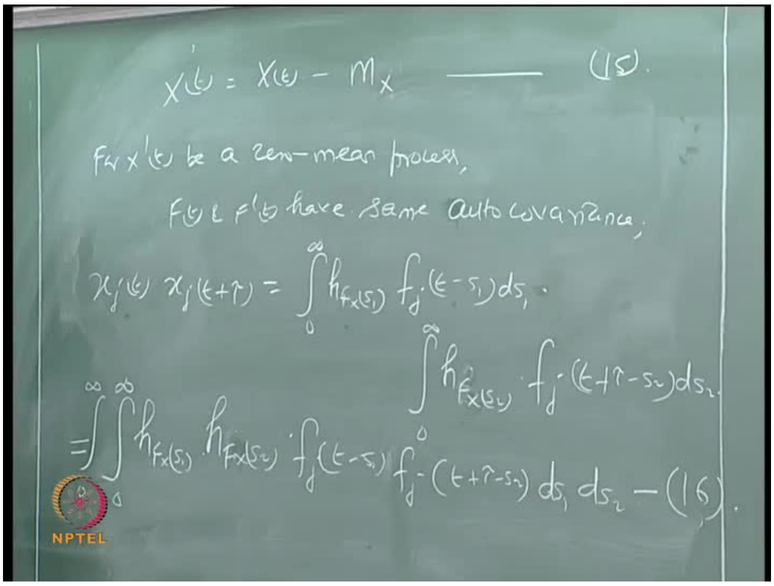

Which we can say; this is nothing but X of t. This is X of t minus… This can be said as mean value of x, which I call as equation number 15. Now, for X dash of t be a zero-mean process, F of t and F dash of t will have the same auto-covariance. Now, using this statement now, the auto-covariance can be simply given as x j of t x j of t plus tau is expressed as 0 to infinity h F x of s 1 of f j t minus s 1 ds 1. This is one part of corresponding to x j multiplied by 0 to infinity h F x of s 2 s f j of… I think I will write it here – multiplied by 0 to infinity h F x s 2 f j t plus tau minus s 2, because I am looking for a time shift minus s 2 of ds 2. So, I can rewrite this equation slightly by arranging the terms accordingly. So, now, this becomes integral of infinity h F x s 1 h F x s 2 f j of t minus s 1 f j of t plus tau minus s 2 of ds 1 ds 2; which I call as equation number 16. So, F x s 1, s 2 f j (( )) Now, we already know; I will remove this.

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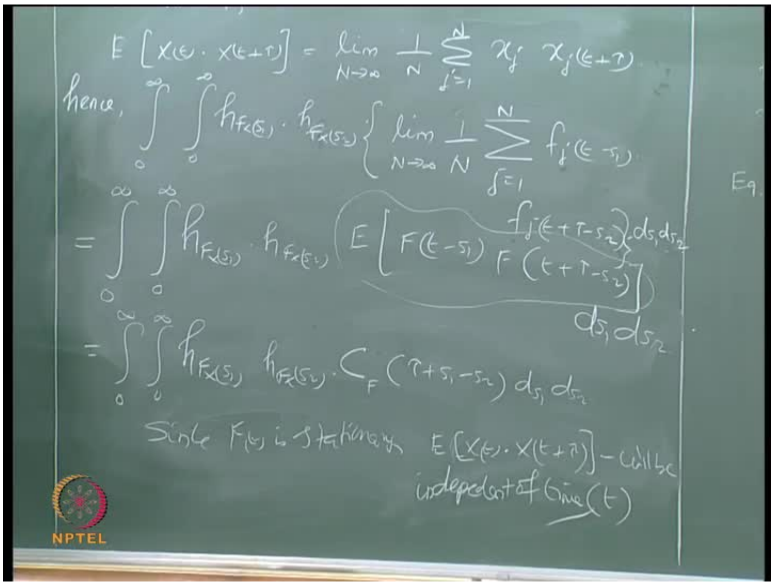

The expected value of X t X t plus tau can be expressed simply as limit N tends to infinity 1 by N of summation of j is equal to 1 to N x j and x j t plus tau. This is a standard expression for finding the expected value of this at a difference of tau. Therefore, I am just using the same methodology for expressing equation 16 now. I should say double integral 0 to infinity 0 to infinity h F x s 1 h F x s 2 limit N tends to infinity 1 by N of summation of j is equal to 1 to N f j t minus s 1. I think I will write it here – f j t plus tau minus s 2 ds 1 ds 2.

So, rewriting this equation slightly in a different manner, double integral 0-infinity, 0-infinity h of F x s 1 h of F x s 2. Now, this is nothing but expected value of F of t minus s 1. And, this is nothing but expected value of F of t plus tau minus s 2 of ds 1 ds 2. These are the two domains of integration – s 1 and s 2. Now, slightly re-alter this equation saying 0 to infinity, 0 to infinity h of F x s 1 h of F x s 2. This I am replacing by saying covariance of the force with tau plus s 1 minus s 2 ds 1 ds 2. I can write this, because since F of t is a stationary process, expected value of X of t X of t plus tau will be independent of time; to be very specific, independent of t, not tau. Tau will be there.

Therefore, now, I can write the auto-correlation function. Any questions here? So, I will rub this part, take away this.

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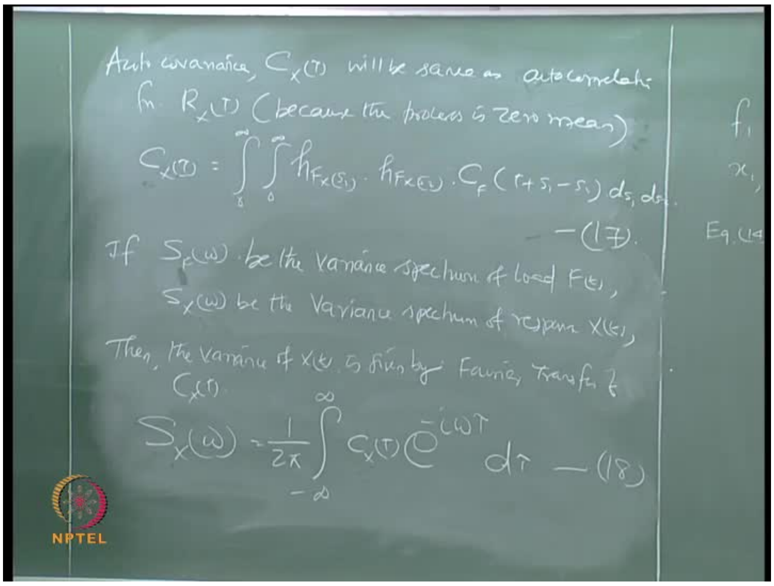

Now, I can write the auto-covariance function for this condition. The auto-covariance of the response, which I call as C x of tau; which will be same as the auto-correlation function R of… We call this as R x of tau, because the process is zero mean. So, I can now write C x of tau is 0 to infinity, 0 to infinity h F x s 1 s 2 – h F x s 1 h F x s 2 C f tau plus s 1 minus s 2 ds 1 ds 2. I call this equation number 17.

Now, we already know that the response spectrum is connecting between the force variance and the… Let us say if S F omega, that is, the forcing function be the variance spectrum of load, that is, F here – F of t; and, S X omega be the variance spectrum of response X of t; then, the variance of X of t is given by the Fourier transform of C x of tau. You can do a Fourier transform of equation 17; I can get the time domain to frequency domain. So, if I try to do that, I get simply S x omega; that is nothing but the variance of the response in the frequency domain. It is simply given by 1 by 2 pi of minus infinity to plus infinity of C x of tau E to the power of minus i omega tau d tau. That is the standard expression for FFT from the time domain to frequency tracing. I call this equation number 18. Now, I have the equation for C x tau from 17; substitute this in 18 and expand this and see what happens. So, I will should say substituting C x tau from

17 and expanding. So, I can remove 17 anyway, because 17 we already have. I will remove this; I can retain this equation as it is; remaining I will remove.

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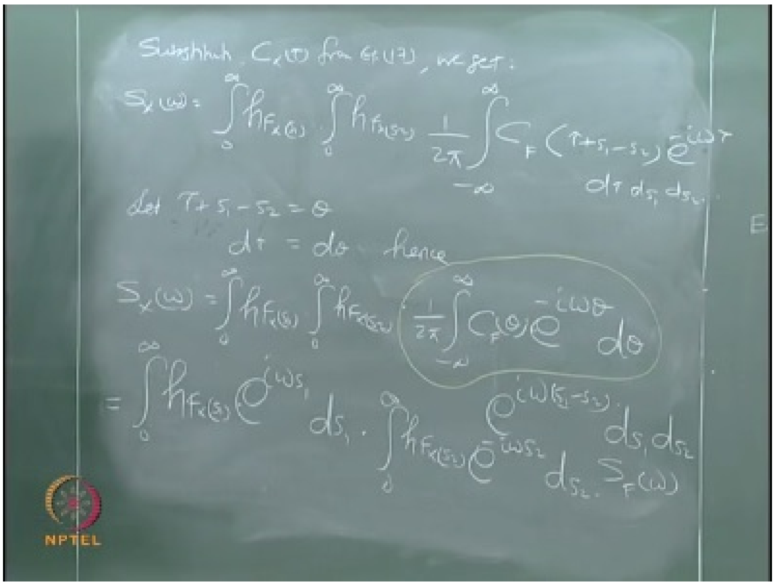

So, substituting C x tau from equation 17, we get… Let us see what do we get. So, S x of omega is equal to… C x has h F x terms; I will take it out. Let us do separate integral. This is h F x s 1 h F x s 2. There is an advantage of writing it like this. Then, I say 1 by 2pi of integral of minus to plus C F of tau plus s 1 minus s 2 e to the power of minus i omega tau d tau; then, ds 1 ds 2.

Now, let tau plus s 1 minus s 2 be theta. I substitute it here to be theta; d theta will straight away be d tau – differentiate. Hence, I will remove this. S x of omega now can rewritten as two integral 0 to infinity separately – h F x s 1 0 to infinity h F x s 2 1 by 2 pi minus to plus C F of theta, because t s 1 s 2 is theta. Now, from this expression, tau will be theta minus s 1 plus s 2. I have got minus sign here. So, what I do is, I take s 1 separately, s 2 separately and t separately. Let us see how we do that. So, I need to have i omega tau; tau is nothing but theta minus s 1 plus s 2; I separate it. So, I want theta separately; I write that value here e minus i omega theta. Theta is here with me; d theta – I club it here, because that is the integral here. Then, I say e i omega s 1 minus s 2. There is a minus sign; it is a minus sign; there is a minus sign attached to s 1, which goes away. There is a plus sign at s 2; with this, minus becomes minus. Is that all right? ds 1 ds 2.

Now, I can rewrite this equation slightly, rearrange it; maybe I will write. I will write it here; which can be 0 to infinity h of F x s 1. I have separate terms here; I am picking up separately – e i omega s 1 of ds 1 with second term of integral 0 to infinity h F x s 2 e of minus i omega s 2; I think I will… No problem – d s 2. And, this term 1 by 2 pi of d theta – this can be simply, because this (( )) to the force, can be simply called as S F of omega. Look at the equation back; it is S F of omega.

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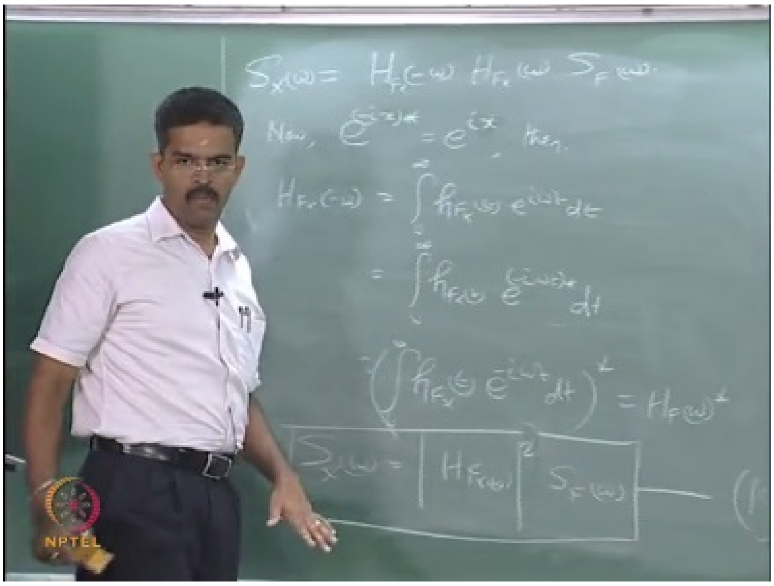

Now, I rewrite this last equation once again slightly in a different manner; which can be now said as… I am talking about S x omega, which can now be… This is F x 0 to infinity; this is nothing but impulse response function. We already remember this. Small h is nothing but impulse response function. So, I do for integration; I get this value as H of F x of minus omega; and, other value is – H of F x of plus omega – two values – multiplied by S F of omega.

Now, e to the power of minus omega star or any multiplier will have the same implication as e i omega or e i x let us say; we will not talk about omega; any value of star will have ix. Then, I will remove this. H of… H F x of minus omega can be written as this value – can be written as 0 to infinity of h of F x of e i omega t of dt; which can be also said as 0 to infinity of h F x of t e to the power of minus i omega star of dt. So, this can be written further as 0 to infinity of h F x of t of e i omega t dt and put a star here, which is as same as star. Therefore, I can write S x omega as mod value of omega square of S f omega. This will be equation number 19, which gives the relationship between the response spectrum S x omega to that of the force spectrum S f omega or the load spectrum.

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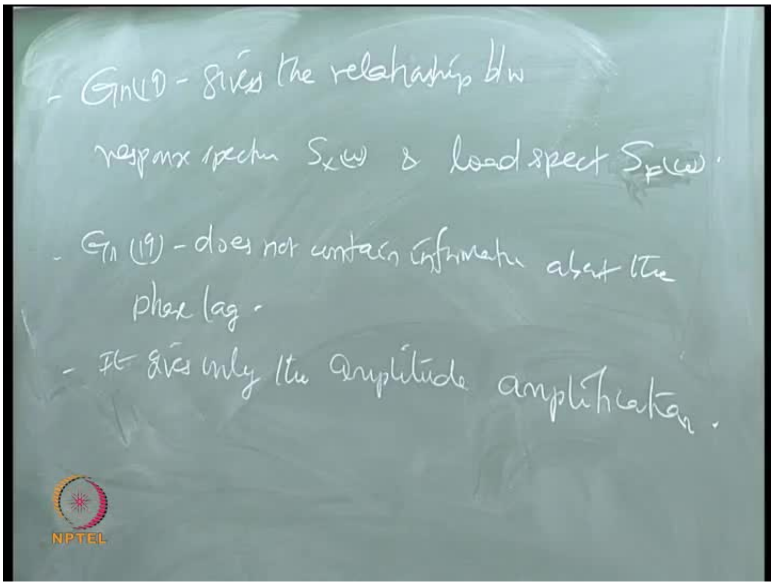

So, equation 19 gives the relationship between the response spectrum S x omega and the load spectrum S F omega. But, interestingly, equation 19 does not contain information about the phase lag. It gives only the amplitude amplification let us say.

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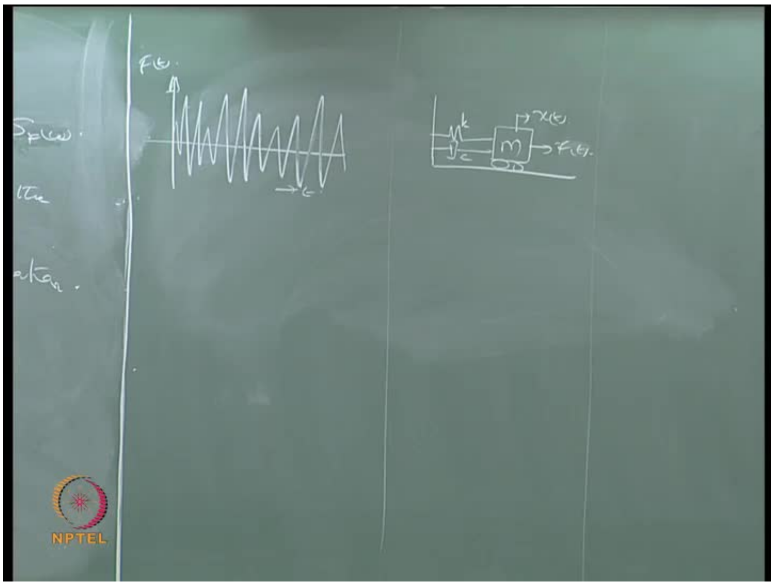

Now, let us see how does it look like for weak dampened systems; that is very important. So, let us try to plot this. Let us say I had a system subjected to some F of t. This is time and this is variation of amplitude of the force with respect to time. I impose the system – this force to a mathematical model having k, c and m having x of t with some force F of t; this is F of t; this is x of t.

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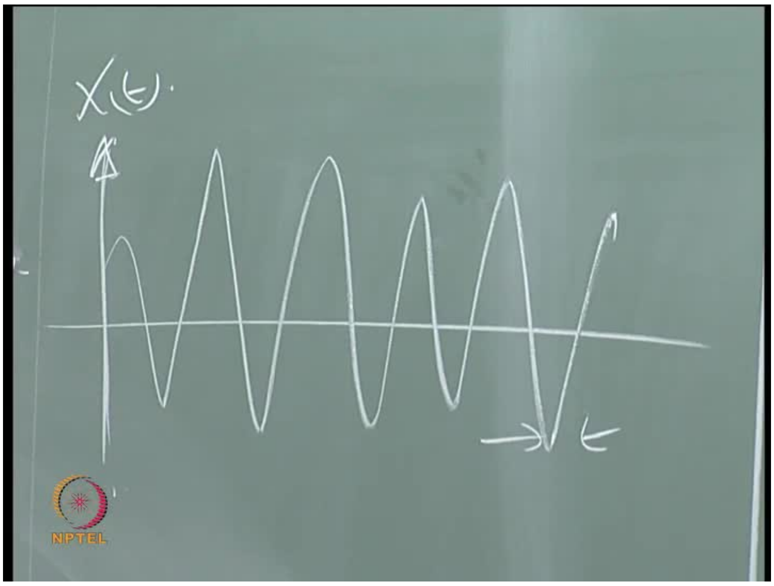

I have to measure x of t from the mass centre; which gives me a response like this.

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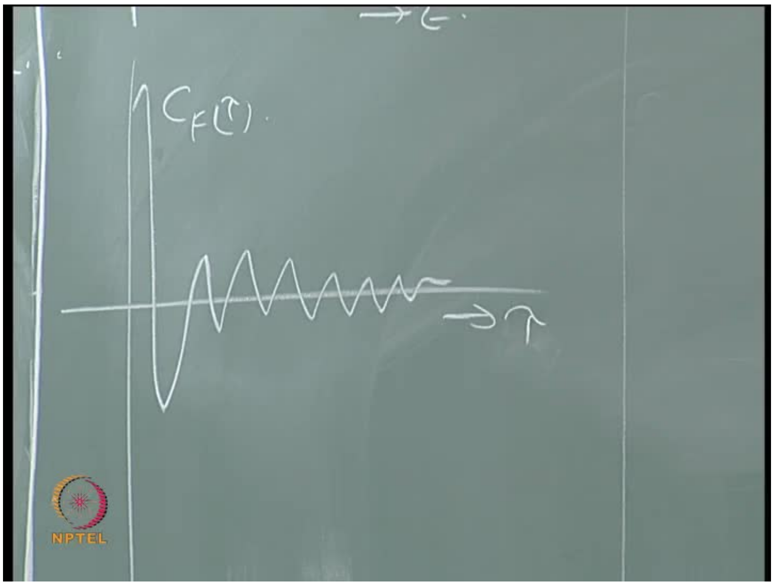

If I try to plot the covariance function of this, it appears to be like… This will be t or now, instead of t, because it is a stationary process, we put this as tau. And, this becomes C F of tau, which we already had the equation.

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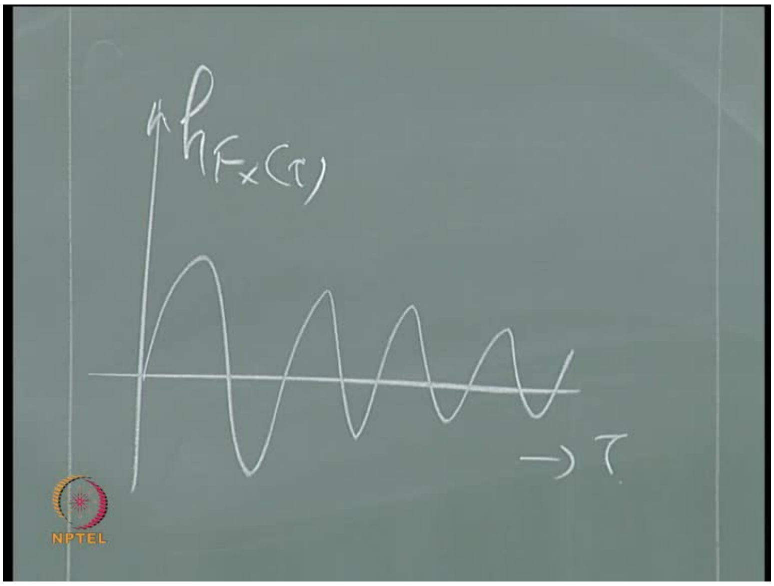

Correspondingly, I can also have the impulse response function, which can look like tau, which is h F x of tau.

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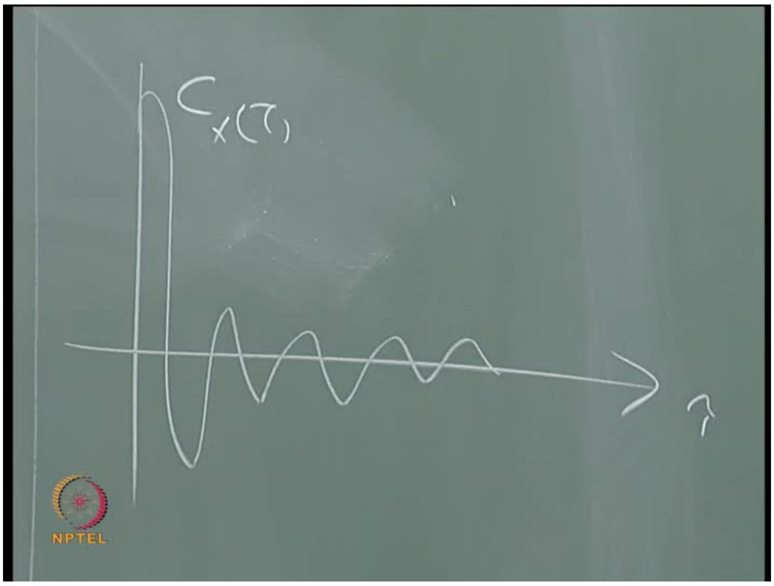

On the other hand, if you look at the response function x f t when it plots like this. This can be C x of tau.

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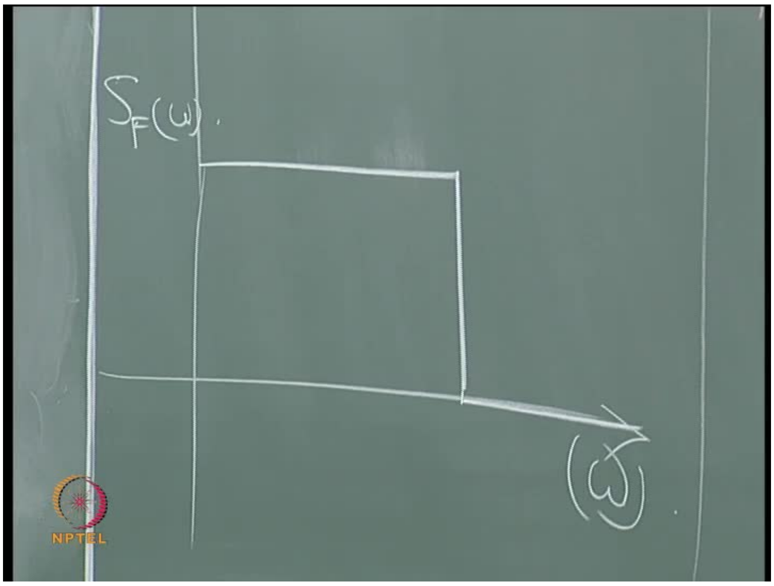

So, if you look at the load spectrum; it looks like this – some load like. Let us say the content only you are looking at. So, this becomes omega. And, this becomes S F of omega; looking only the energy content of the spectrum.

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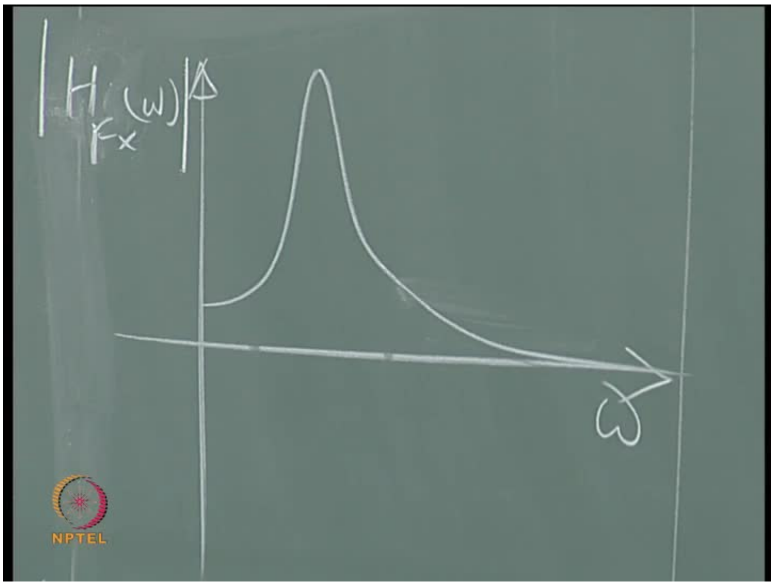

And, the transfer function, which connects this spectrum to the response will look like this; which I call H of F x omega absolute for different omegas of course. Basically, it will come to 0.

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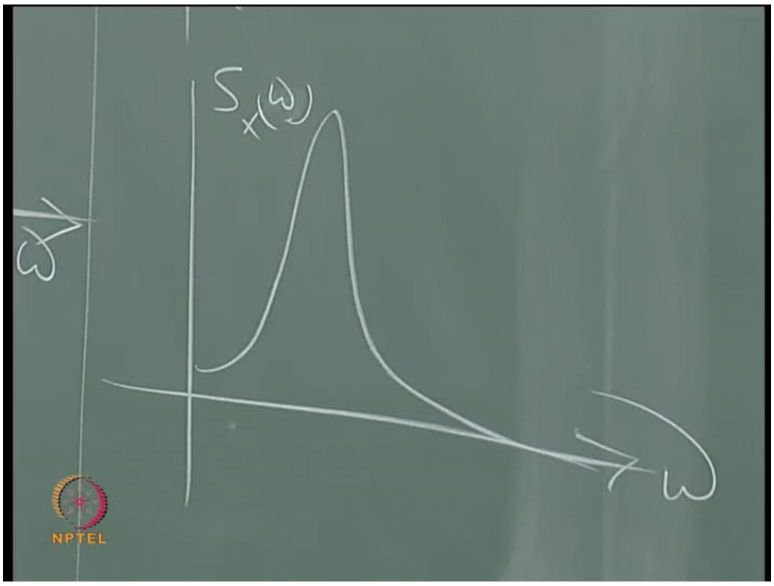

And, the response spectrum after connecting the transfer function to the load spectrum will look like this.

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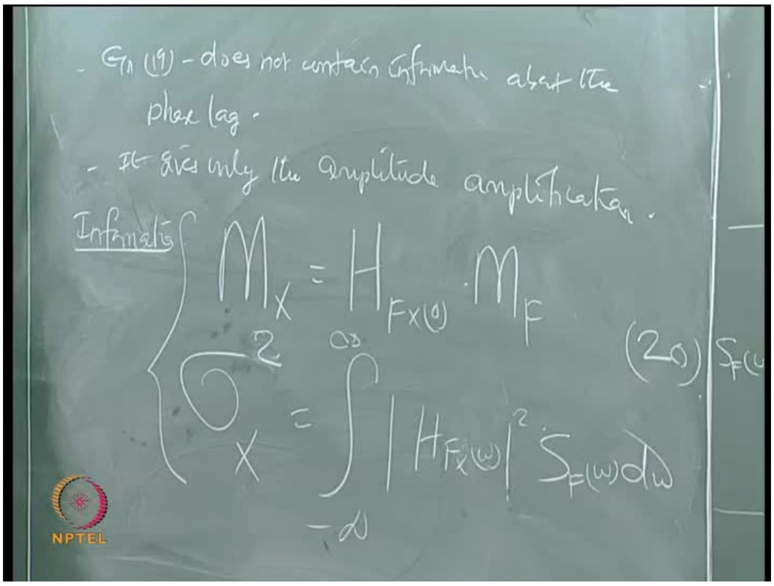

See we have very important information from this; very important information – let us see what are they. We already know that M x is given by H F x 0 of M F. This is what we have already derived, because you remember in stochastic dynamic analysis, unlike deterministic analysis in time domain, we do not look for the exact values of x of t; we look for the first order and second order moments of the statistical responses –

realization of these values. So, the first order is the mean value we have here; which can be obtained from the transfer function. And, already we just now saw the expression for the response spectrum, which is also said as the variance of the response. So, I am saying, the variance of the response – that is the standard symbol, is nothing but minus infinity to plus infinity of absolute value of H F x omega square of the transfer function of S F omega d omega. So, the square root of the variance will give me the standard deviation, which we wanted for my analysis.

The right-hand side of this equation – I can call this – both these equations as equation number 20. So, the right-hand side of the equation of 20 b – this is a and this is b – is evaluated numerically; it is evaluated numerically. Now, the question comes if zeta is very low; you remember this function; this is having the zeta component into it. If the zeta is very low, what will happen to my response factor? Let us see that. Let us try to plot and write important inference from equation 20 b and understand the quality of the response spectrum or the transfer function at very low zeta values, because we have zeta value only close to around 2 percent or maximum 5 percent. So, for zeta very very less than 1, let us see how does it look like.

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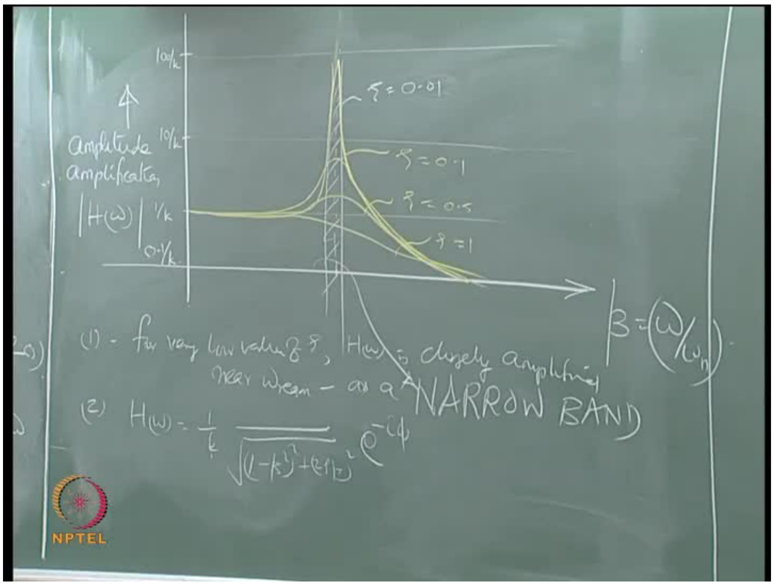

The spectrum looks like this. We will try to plot the amplitude amplification given by H omega. Let us plot it here. And, of course, here we plot beta, which is nothing but omega by omega n at beta equals 1, that is, at resonance. And, these values are all written as –

let us say start from 0.1 by k; there is a denominator k in H of omega. So, 1 by k; maybe 10 by k or 20 by k, 100 by k. So, 10 by k; let us say 100 by k. These are the values, which I am trying to plot for different zetas. All curves will start at 1 by k and it goes very steep and then comes down to 0 if you have zeta 0.1 percent – very low; I mean 1 percent; I will write zeta value to 100 percent; simply say it is 0.01. For any increased value of zeta, the curve will arc… So, this is for zeta of 0.1; this is for zeta of 1; this is for zeta of (( )) This is 0.5 and this is 1. So, we can write an interesting information from this curve saying, for very low value of zeta, H omega is closely amplified near omega resonance as a narrow band. Of course, for our clarity, H omega is given by 1 by k of root of 1 minus beta square square of 2 zeta beta square of e minus i phi in this equation or in this plot. So, the band is very close; the band is very close. This is the band, which we call as narrow band. Now, interestingly, in literature, people call this aspect as white noise approximation. Let us see what is that white noise approximation. I will remove this.

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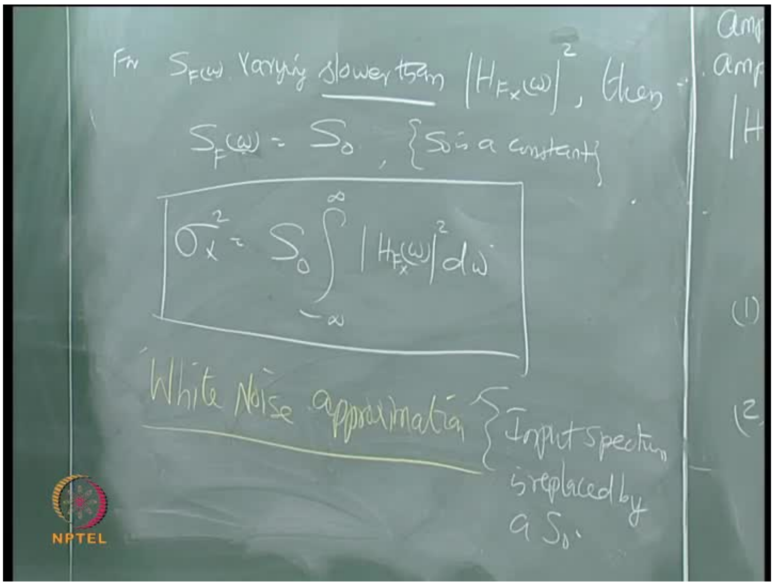

Now, we already know that, sigma x square is given by minus (( )) H F x omega square S F omega d omega. For S F omega, that is, the variance in the load – for S F omega varying slower than H of F x omega square, because you can see the variation is drastic here; then, there is an approximation made in the literature saying that, S F omega can be replaced by a constant called S naught; where, S naught is a constant. Therefore, sigma x square can be written as constant out minus integral to plus integral of the transfer function square d omega dot. And, this is what we call as white noise approximation. What does it mean? The input spectrum is replaced by a constant. That is called white noise approximation.

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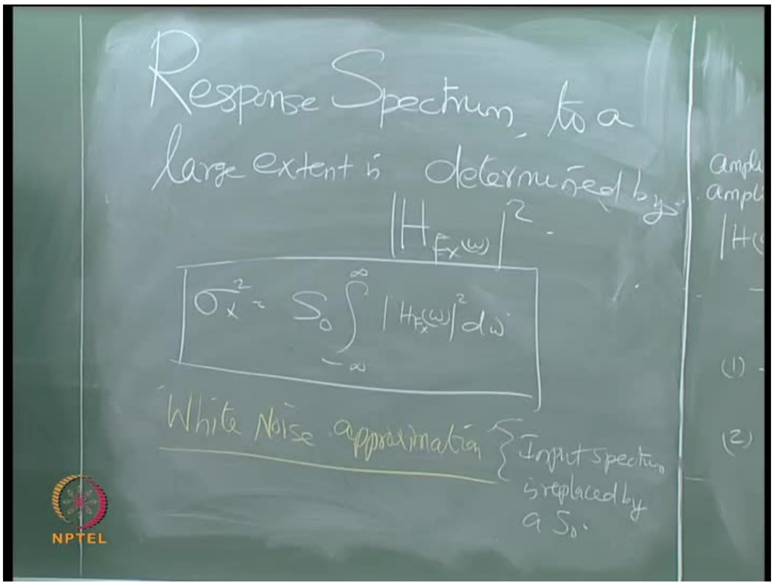

So, we have a final inference from this statement saying, the response spectrum to a large extent is determined by the square of the transfer function. So, some people write the equation as response spectrum is equal to square of transfer function of that of the load spectrum or constant of this one. So, we have seen two important inferences for the input load process remaining stationary. It amounts to a zero mean process, where the transfer function becomes time independent. For a weakly damped system, the amplitude amplification is focused only near the resonance region. What does it mean? If we really wanted to control the response of any system of this order, you have got to only play not on a broad band system, only a narrow system. That is why generally every case of response control algorithms people tune the secondary frequency with the fundamental frequency of the system. That is the reason.

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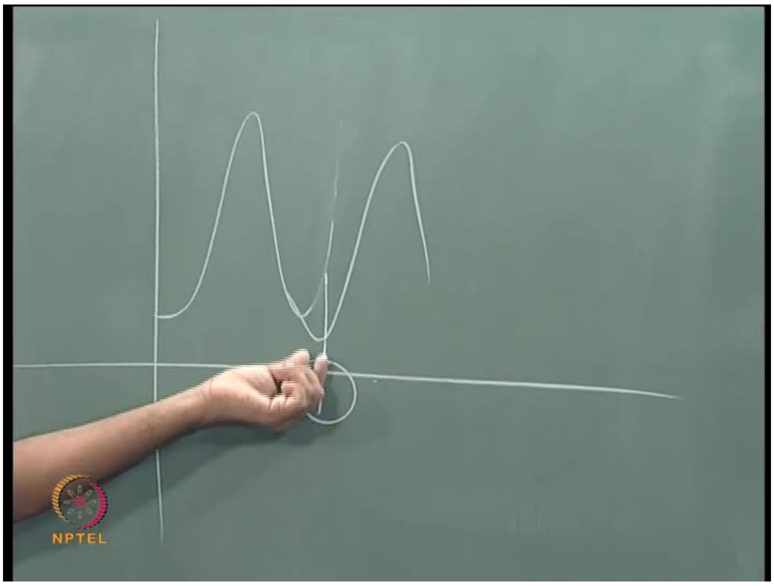

One may ask, why the response need not be controlled and tuned for the entire band of omega? It is not required actually. That is the reason why people do it. In fact, we have also seen one of the example, which we solved on a TLP as well as on the tuned mass damper system on articulate tower, where we had tuned the secondary mass system omega with that of the fundamental frequency only. Of course, the benefit was looking like this. So, at practically, omega equals omega n; it becomes… It does not becomes 0; it is marginally 0; whereas, you get two subsequent peaks. That is what we have already seen in one of the case studies what we did in the last module in multi-leg articulate towers using tuned mass dampers. So, this is the reason why for weakly damped systems, people look only the focus on narrow bands.

And, response spectrum in such cases essentially depends, highly dependent on the transfer function. So, in the next class, we will talk about some important item called return period – what do you mean by return period, because in stochastic dynamics, return period… Or, what should be the period you must take for the load process. It is very important. We will talk about that. And, we will also talk about modal response analysis. How many modes should I use? Where should I truncate my mode? Why and how? That will end the discussion on stochastic dynamic part. We will of course solve couple of problems – hand written problems – simple problems on modal analysis or modal participation factor. We will evaluate and see, because there has been questions from the students asking that, how many frequencies or how many modes should I consider in a multi-degree freedom system model.

We will address that in the coming lectures. So, any question? So, the fundamental deviation of stochastic dynamics compared to the conventional deterministic analysis is that, here we are not talking about the absolute values of the responses; we are talking in terms of its first and second order moments mean in standard deviations; and of course, variance and tuners also. So, based on this, one can design the system (( )) That is what the deviation what we had from the conventional analysis what we did in the second module. Any questions?