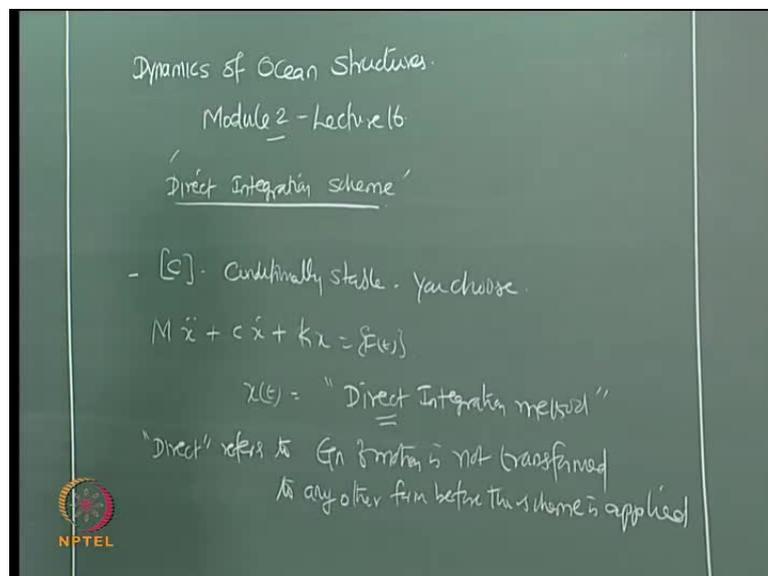


**Dynamic of Ocean Structures**  
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**Module - 2**  
**Lecture - 16**  
**Direct Integration Method**

So, in the last lectures, few lectures, we saw how to estimate the damping matrix for different methods.

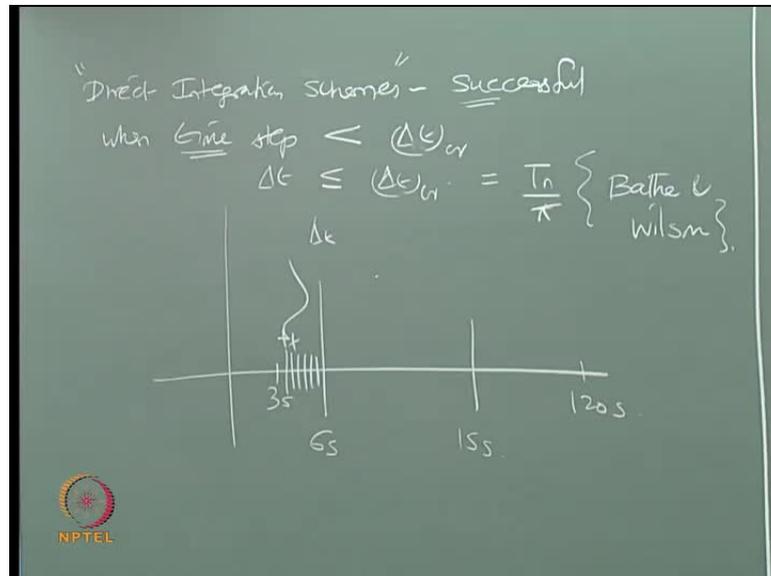
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And, we have seen one important assumption that, the damping matrix accuracy can become conditionally stable if you choose an appropriate time average method for solving the equation of motion. So, the equation of motion, which I write here, looks like this for a given system. So, hypothetically speaking, I want to find  $x$  of  $t$ , which is nothing but the integration of this equation of motion, because there is a double derivative here – second order differential equation. So, I have to integrate this equation. I have to follow what is called direct integration technique – scheme, method, technique, whatever you call. Where is the term direct coming into play here? What we do here is the term direct refers to the equation of motion is not transformed to any form before the scheme is applied. So, before you apply the scheme, do not transform this equation of

motion directly; keep it as it is. So, the term direct refers to that. So, I am keeping this equation of motion as it is; trying to solve this.

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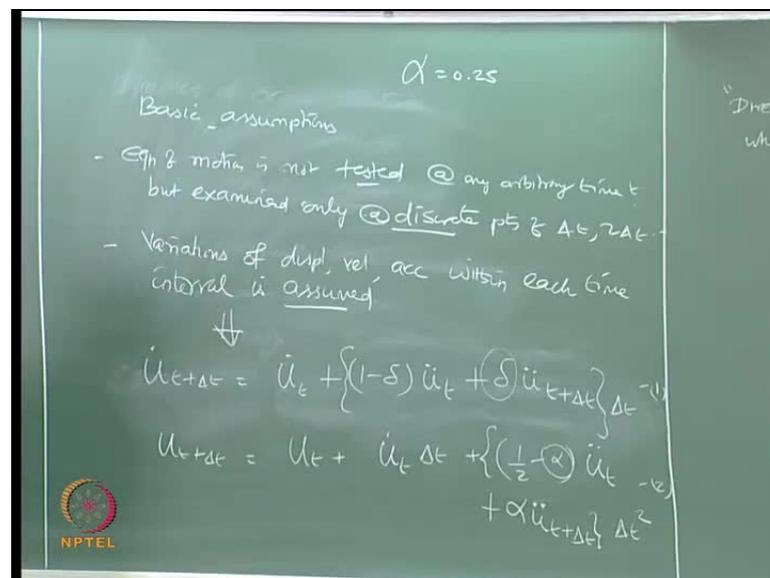
Now, literature says that, the direct integration schemes generally are successful when the time step is less than delta t critical. So, you pick up a time step. Now, first, let us understand what the time step is. So, I have got a problem; I have got a structure whose period varies from let us say 3 seconds to 120 seconds. For example, a TLP. So, this is the period of the structure. Within this period, obviously, I have got wave loading acting on the structure varies from let us say 6 seconds to 15 seconds. I have got a wind loading acting; it is a low frequency, high period acting in some other range. So, I am looking their whole encompass of the problem from a specific range of period. Do you have to run the analysis for the entire period? because it is then going to be relatively expensive computationally. So, I must select a time step; which means that, I must divide the period of interest in small segments and I call each segment as the time step. So, that is what I call as delta t.

And, this delta t should be less than or equal to delta t of critical; where, delta t of critical is given by  $T_n$  by  $\pi$ ; where,  $T_n$  is the fundamental period or the lowest period of the structure. So, picked up the lowest period of the structure; lowest period of the structure divide by  $\pi$ . That is delta t here. If the time step is lower than this, literature says that, the direct integration scheme will give you successful stable converging results. This has

been proven. So, I must judiciously select delta t, so that it is in this range. So, even to select a time integration scheme to find delta t, I must know  $T_n$  of the structure. So, where I know already, because we have got different mechanisms and methods by which we can always find the natural time periods or natural frequencies and the corresponding mode shapes.

For the given geometric form of the structure, we have already studied them in detail. So, I know. Out of which I can find the fundamental period; pick up that period. And this is only a thumb rule actually, because this has been proven by many schemes saying that, the reference for this could be... I have already given you in the last lecture; look for Bathe and Wilson. So, he has conducted many schemes and he has proved that this as one. Many researchers said this, but instantaneously they remember this. Therefore, I said this. But there are two assumptions here in any direct integration scheme. There are very serious assumptions, which we must understand, because the convergence, accuracy – all depends on this assumption. So, what are those assumptions? I will remove this.

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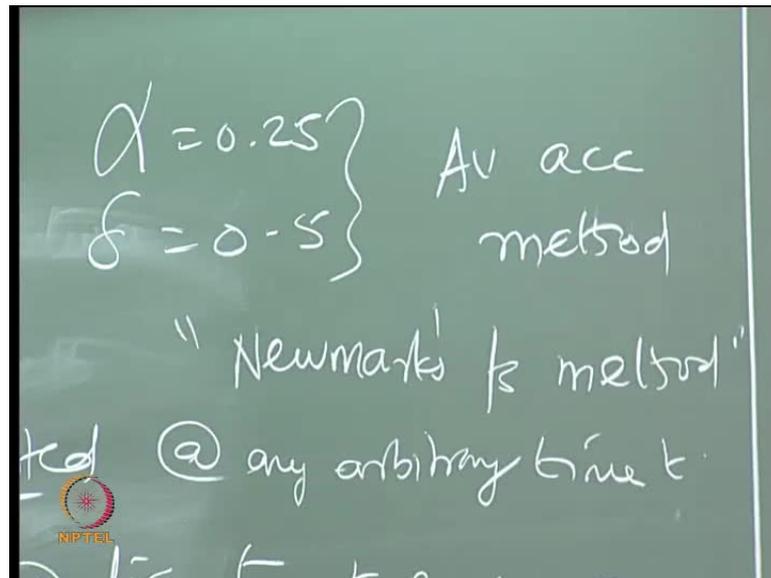
The basic assumption number 1 is that – we can even say limitations – is that, the equation of motion is not tested at any arbitrary time  $t$ . See in a given space, I do not check up the equation of motion validity at any time  $t$  in the given range. Then where will I do? but examined only at discrete points of delta t, 2 delta t and so on. I have

checked the validity only at these discrete points; of course, that is the serious assumption.

Then the second assumption is that, the variations of displacement, velocity and acceleration within each time interval, is assumed. I assume this variation. So, these two are very serious constraints in a given problem. So, these two leads to basic two forms of equation; this says that, if you want to find velocity at any time interval  $t$  plus  $\Delta t$ , then this is nothing but the value of the velocity at  $t$  added to it  $1$  minus  $\Delta t$  of acceleration of at  $t$  plus  $\Delta t$  of acceleration at  $t$  plus  $\Delta t$ . And multiply this with  $\Delta t$ . Not this; of course, this is already a velocity; this is velocity –  $\Delta t$ . These are all accelerations.

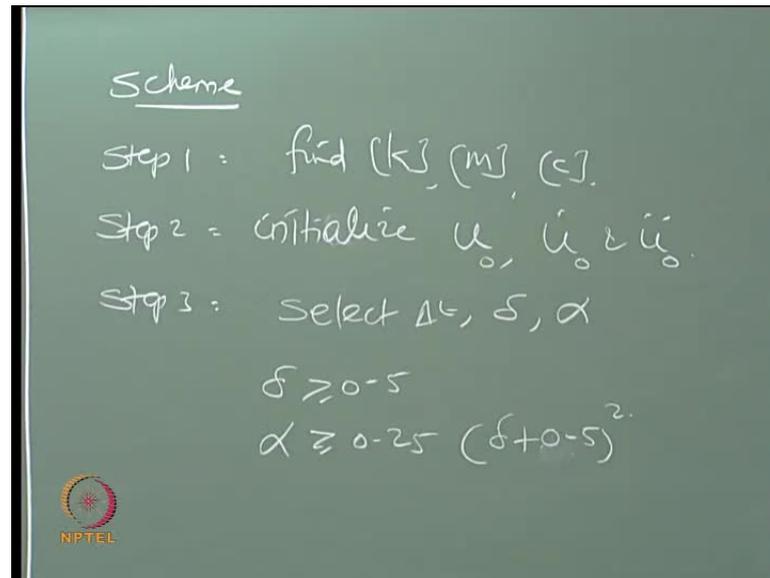
And, if you want to find the displacement at  $t$  plus  $\Delta t$ , then use the displacement at  $t$  and of course, the velocity at  $t$  and half minus  $\alpha$  of acceleration at  $t$  plus  $\alpha$  acceleration at  $t$  plus  $\Delta t$  – acceleration. Of course, multiply this by  $\Delta t$  square, because these are all acceleration terms; I want displacement. Let me check up this equation again.  $t$  plus  $\Delta t$  is  $u$   $t$   $1$  minus  $\Delta t$   $u$  double dot plus  $\Delta t$   $u$  double dot  $\Delta t$ ;  $u$   $t$   $u$  dot  $t$  plus  $\Delta t$ ; and half minus  $\alpha$   $u$  double dot plus  $\alpha$   $u$  double dot  $t$  plus  $\Delta t$  and  $\Delta t$  square. Call this equation number 1 and 2. These two are considered to be valid. So, what does it mean? This implies the same statement back again that, the variation within a time interval or each time interval, depends on the previous values and some constants of  $\Delta t$  and  $\alpha$ .

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If this alpha is considered as 0.25 and delta or del is considered as 0.5, we call this method as average acceleration method; otherwise, famously called Newmark's beta method. So, specific name is given to this. So, these are actually assumptions, which are carried forward from these two ideologies. So, let us quickly look at the scheme – how do we perform iteration. We will do a problem and try to understand how this is done. Then we will see what would be the special complexity coming out from this problem or this scheme applied to a TLP problem. Let us see that, because that is our end. In the last lecture, already you have noticed that, we have solved the TLP problem using a time domain integration technique like this and we have given you the results.

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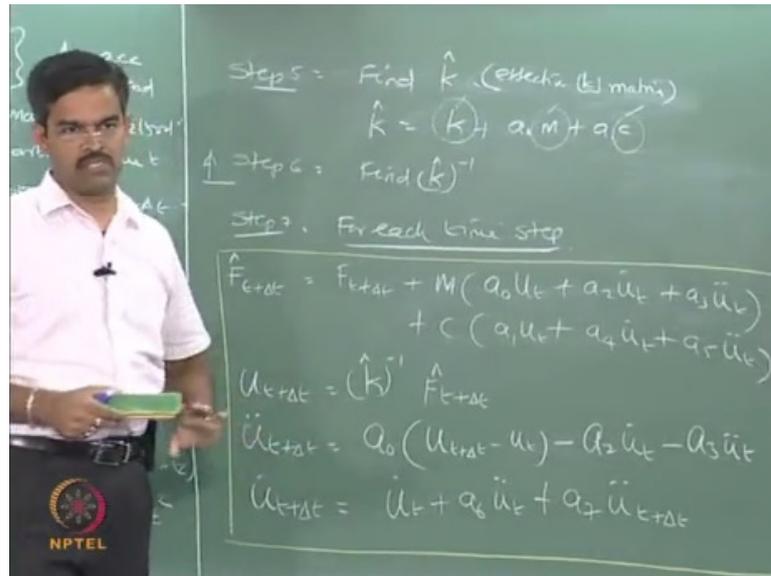
Now, we are explaining how this has been solved. So, the scheme says, for a given problem – business scheme; I am writing the scheme here, because we will retain it later for the problem. The scheme says step number 1 – find  $k$ ,  $m$ ,  $c$ ; very simple; we already know this. So, for a given problem, derive these matrices  $k$ ,  $m$  and  $c$ . The second step says initialize  $u$ ,  $\dot{u}$  and  $\ddot{u}$  at zero of course. We can say  $u$ ,  $\dot{u}$  and  $\ddot{u}$  at 0. Step number 3 – select  $\Delta t$ ,  $\delta$  and  $\alpha$ . Of course, these are all variables. You see here; a specific name given to this for using a specific value is available in the literature. You can select any other value. Then what is the range?  $\delta$  should be more than or equal to 0.5; and  $\alpha$  should be more than or equal to 0.25 of  $\delta$  plus 0.5 the whole square. So, if you look at  $\delta$  as 0.5 in the given problem, substitute here; you get  $\alpha$  as 0.25. Can have any value; more than this, we have picked up the basic values.  $\Delta t$  – of course, I will come to the problem later.  $\Delta t$  should be within the  $\Delta t$  critical; we will find that.

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Select  $\Delta t$ ,  $\delta$ ,  $\alpha$   
 $\delta \geq 0.5$   
 $\alpha \geq 0.25 (\delta + 0.5)^2$   
 Step 4: Integration constants  
 $a_0 = \frac{1}{\alpha (\Delta t)^2}$        $a_3 = \frac{1}{2\alpha} - 1$        $a_6 = \Delta t (1 - \delta)$   
 $a_1 = \frac{\delta}{\alpha \Delta t}$        $a_4 = \frac{\delta}{\alpha} - 1$        $a_7 = \delta \Delta t$   
 $a_2 = \frac{1}{\alpha \Delta t}$        $a_5 = \frac{\Delta t}{2} \left( \frac{\delta}{\alpha} - 2 \right)$

Then, I will find out some integration constants, which are varying from a 0, a 1, a 2, a 3, a 4, a 5, a 6 and a 7. So, 1 by alpha delta t square. These are all constants; you have got to work out for a given problem. Del by alpha delta t; 1 by alpha delta t; 1 by d alpha minus 1; del by alpha minus 1; delta t by 2 del by alpha minus 2; delta t 1 minus del; del delta t. So, these are all integration constants; let me check up these values once again whether it is written correctly. Remember, these a 0's, a 1's, a 2's are different from what we have from Rayleigh damping and (( )) damping. They are all different. This is integration constant. The nomenclature may be same, but they are all different meaning; they are different meaning. They depend only on the constants of alpha, delta t, and del for a given problem. They do not depend on anything else; whereas, a 0, a 1 are all different functions in damping estimates.

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Step number 5 – find  $\hat{k}$ , which is called effective stiffness matrix, which is given by a simple equation  $k$  plus a  $0$   $m$  plus a  $1$   $c$ . These expressions all will look similar to what we have in Rayleigh, etcetera; do not get confused. These are all different constants. After you do this, find  $\hat{k}$  inverse; then for each time step. These are all till now they are all constants. They are all constants for a given problem. Whereas, now onwards, for each time step, we have got to compute the equations what we have here  $\hat{F}$  of  $t$  plus  $\Delta t$ , that is, a force in the new time step, which is depending on the force at the old time step –  $M$  of a  $0$ , a  $2$ , a  $3$ ;  $u$   $t$ ,  $u$  dot  $t$ ,  $u$  double dot  $t$  plus  $c$  of a  $1$ , a  $4$ , a  $5$ ;  $u$   $t$ ,  $u$  dot  $t$ ,  $u$  double dot  $t$ .

Then, find  $u$  of  $t$  plus  $\Delta t$  – the new displacement, which is  $\hat{k}$  inverse of  $\hat{F}$  hat  $t$  plus  $\Delta t$ , which you know from the previous step. So, you have found the displacement; then you found the acceleration. This is acceleration – double dot – a  $0$  of  $u$   $t$  plus  $\Delta t$  minus  $u$   $t$  minus a  $2$   $u$  dot  $t$  minus a  $3$   $u$  double dot  $t$ . And of course, the velocity and the new time step –  $u$  dot  $t$  plus a  $6$   $u$  double dot  $t$  plus a  $7$   $u$  double dot  $t$  plus  $\Delta t$ . So, these are all important for every time step. This is my scheme. So, up to step number 6, which depends on a fixed value of  $k$ ,  $m$  and  $c$ ; step number 6 requires no iteration – till step number 6. From step number 7 onwards, I have got to find iteration, because I want to find what is the new value, what is the old value or they are converging or not. After they converge, substitute here; then get this value. What is the new value

and old value of this? Are they converging or not? Go back and find all the values of displacement, velocity and acceleration, which I actually want from this equation.

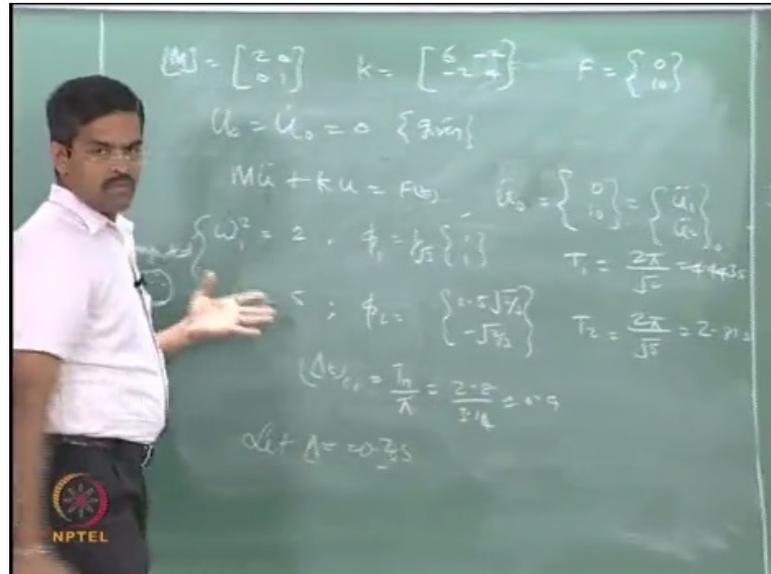
Now, let us look at the complexity of this scheme applied to a problem like a TLP. Till step number 6, the problem is simple here, because there is no iteration required; you think only iteration is here. But the unfortunate part is the iteration starts from here onwards, because  $k$  depends on response; I do not know the response; I have assumed them here. When I assume response, I have a  $k$ . For that  $k$ , I also have a set down of set; I additional mass matrices; I have  $m$ . For a given combination of  $k$  and  $m$ , I find  $\omega$  and  $\phi$ ; substitute and get Rayleigh damping and get  $c$ . I do not know whether I assumed the correct value in the beginning and so on.

Now, this value after I find out the displacement, will get replaced. So, on the whole, except here, the whole scheme becomes iterative and uses it for a compliance structure. Whereas, the scheme does not only have this iteration; it will also have iteration here, because  $k$ ,  $m$  and  $c$  are all changing. You may wonder that, certain researches said, I will not change  $k$ ,  $m$  and  $c$  updated at every time. I will pick up  $k$  initial,  $m$  initial,  $c$  initial and do the iteration and see what is the error. The error is not more than about 6 to 7 percent in the displacements. People have seen this. Numerically, we have shown; there is no error much. Even then there is a complexity. I will show you that, what is the complexity after I proceed with the problem. You may wonder that, if you do not, it would not iterate  $k$ ,  $m$  and  $c$  with respect to  $x$  or  $\dot{x}$  and  $\ddot{x}$ . Can this scheme become simple for a compliance structure like TLP, we will see there. Again there is a complication; I will come to what it is. So, we will pick up a problem quickly and try to solve this.

Any doubt here? Are we through with all the constants and coefficient required for doing this iteration scheme? This is one of the scheme; there are many schemes available; I am discussing only one per sample. We are supposed to know this kind of schemes for problems applied on FEM, etcetera and different courses; that for completion sake, I am just discussing this scheme here in this lecture, because I have used this scheme for the analytical investigation, which has showed the results in the last lecture. Therefore, it is mandatory for me to understand and explain you that, how the scheme is in operating. That is why the reason we are discussing this. We will pick up a very simple example of 2 by 2, because I want to invert it. It is difficult to do by 6 by 6; I am picking a very

simple example, so that we can quickly invert 2 by 2 and see how you are able to do faster as I am doing.

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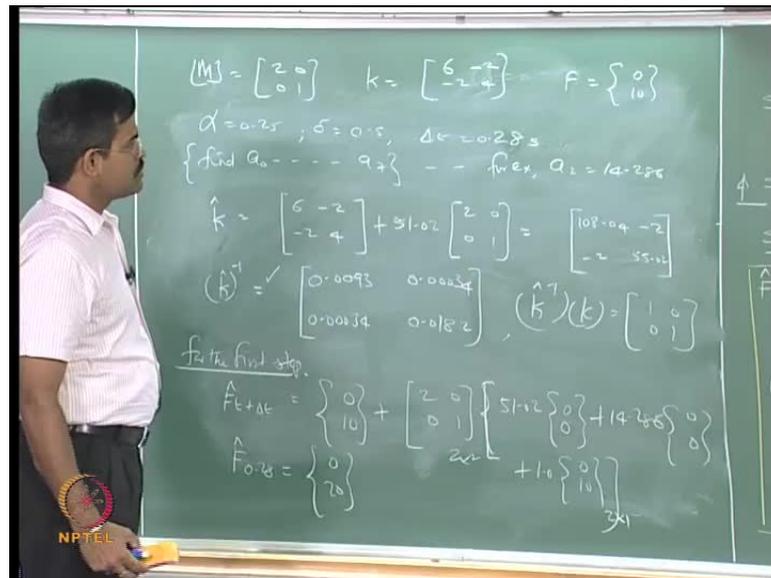
So, I have m as 2, 0, 0, 1. Whatever may be the units, do not bother about them. Maybe it is multiplied by m and m has 3000 kg; let us forget about all those constants; we already know how to handle them. We will simply take only the notified values of m k. And k is 6, minus 2, minus 2, 4 of some multiplier. And F of t is 0 and 10 applied to the system; some load – point load is applied. We already know that, for this problem, u 0 and u dot 0 are 0; they are given. Initial displacement velocities are not there in this problem; it is initially at rest. But u double dot 0 will not be 0.

Here I said it is not 0; I said initialize them. Initialization does not always mean it starts from 0. I have to find u double dot. How? m u double dot plus k u is F of t. I am taking an undamped system, because I do not want to improve complication on this problem. So, you can substitute m; you can substitute k; you can substitute F of t. u and u dots are 0, but u double dot is not 0. You will find u double dot from here. Can you quickly find out that? You have got to invert a very small matrix of 2 by 2 quickly. So, let us see how quickly you can get me u double dot of t of 0. We have only fifteen minutes left over. We have to do very fast. So, this comes to be 0 and 10; you can check that. It means this vector is u 1 double dot and u 2 double dot at 0.

Now, I want to select  $\Delta t$ ; over  $\alpha$  and  $\Delta t$ , I will select from here. But I would also like to select  $\Delta t$ , so that I can find all the integration constants  $a_0$  to  $a_7$ . So,  $\Delta t$  should be lying within the  $\Delta t$  critical for a numerical stability of this problem. To find  $\Delta t$  critical, I must know  $T_n$ . We have fundamental period of the system. I have a system here; I can find the fundamental period. We believe you know this procedure. So, I will give you these values.  $\Omega_1^2$  is 2 and  $\Omega_2^2$  is 5 for this problem. You can verify this. And  $\phi_1$  is  $1/\sqrt{3}$  and  $1$ ; and  $\phi_2$  is half of  $0.5\sqrt{2}$  by  $3 - \sqrt{2}$  by  $3$ . There is no half here; half is inside. So, these values are let us say... I will write it here computed; I believe you know how to do it. You must know this.

So, let us quickly find out  $T_1$  and  $T_2$ . You know  $\Omega$ s. Can we find  $T_1$  and  $T_2$  quickly? So, this is  $\Omega_1^2$ , that is, 2. So, for example, in this case, it is going to be  $2\pi/\sqrt{2}$ ... This is  $\Omega_1^2$  – nothing but  $\sqrt{2}$  of  $\pi$ , which is going to be 4.443 seconds. The other one is  $2\pi/\sqrt{5}$ , which comes to 2.81 seconds. So, let us find out  $\Delta t$  critical, which is the lowest by  $\pi$ , which is  $2.8$  by  $3.14$  – approximately 0.9. I can pick up any value lower than this, which can help me to have a numerically stable answer. Is that right? So, in my problem, let  $\Delta t$  be 0.28 seconds, because  $2.8/10$ ; I have got ten steps point to it. Do not wonder about this fraction; I am the counting number of steps for the given problem. I am saying that, divide it by number of steps of 10; I have taken 0.28; 0.28 is less than  $\Delta t$  critical. My solution should give me a stable numerical answer. I picked up  $\Delta t$  as this. Can we quickly find out  $a_0$  to  $a_7$ ? Because I know  $\Delta t$ , I know  $\alpha$ , I know  $\Delta t$ . Can you find out this? Any doubt here? Any question here?

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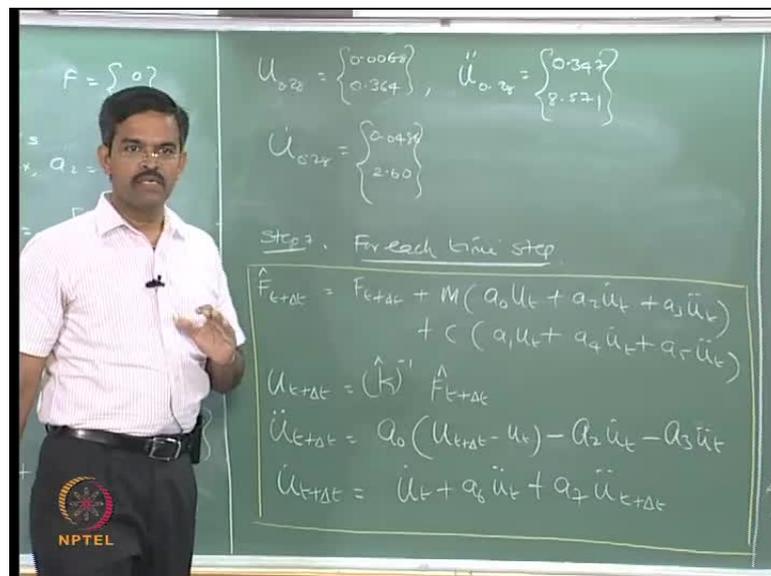
I am rubbing this; I will use this space, because I have to retain... I can rub this also. I can rub this onwards. I can retain m, k; remaining I can remove. Any questions? a 0 and a 1, a 2, a 3 – they are very simple substitution. You should be able to do them quickly. So, I am using alpha as 0.25, del as 0.5 and delta t as 0.28 for my scheme – seconds of course for my scheme. So, I am getting the values of a 0; I am writing here. You should be able to find out the values of the constants a naught through a 7 for the given... using equations as shown there. For example, we write one typical value. a 2 is 14.286; for example, a 2 is 14.286; I think you should check whether it is right. So, if it is, then believe that everything is all right.

Let us quickly find k hat. That is my next step. k hat; I know k matrix; I know m matrix; I do not have c here in my problem. So, I have a naught. Can we find k hat? k matrix – 6, minus 2, minus 2, 4 plus a naught of... a naught is equal to 51.02 – of m, which is 2, 0, 0, 1. I get k as again 2 by 2 matrix, which is coming to be 108.04, minus 2, minus 2, of course 55.02. Can I have k hat inverse. So, we will give you one minute; then I will write the answer here – 0.0093, 0.000364, 0.000364; it is not 364, it is 34. This is what I am getting as k hat inverse. Let us perform quickly k hat inverse and k and see am I getting identity matrix. Please check this quickly. I have checked it; you can see. It is becoming 1. Just verify the inverse is correct or wrong.

So, let us try to find out for the first step,  $\hat{F}$  of... that is, for the first step,  $\hat{F}$  of  $t$  plus  $\Delta t$ ; I start at  $t$  as 0. That is why I said  $\ddot{u}$  naught,  $\dot{u}$  dot naught and  $u$  double dot naught as 0. So, I start at  $t$  as 0 from the beginning. So, essentially, this value is at 0.28 seconds; that is, in my scheme, I know this value, that is,  $\hat{F}$  hat or new or old at  $t$  is equal to 0. I know this value is 0, 10 in the vector. I am trying to find out the new value here, which is 0.28 away from this in the progressive direction, which is given from the scheme here. This should be as same as 0 and 10, that is,  $\hat{F}$   $t$   $\Delta t$ . And the mass matrix of 2, 0, 0, 1 of all vectors, that is, 51.02 of a vector, which is 0, 0, because  $u_0$  is 0 plus 14.286 of again 0, 0, because  $\dot{u}$  dot 0 is also 0 plus a 3 is 1.0 of 0 and 10. So, this is a 2 by 2 matrix. This is a 2 by 1 vector. I have got 2 by 1 vector; I add this vector, I get  $\hat{F}$  hat, which will be 0, 20; that is,  $\hat{F}$  hat 0.28 is 0, 20. Yes or no?

Now, I want to find  $u$  at 0.28. I have  $\hat{k}$  hat inverse; I have  $\hat{F}$  hat at 0.28. Substitute this; multiply these two and get me  $u$  at  $\Delta t$ . I am writing only the answers; I am not doing the mathematical multiplication here. You are supposed to do it quickly and give me the values. It will multiply this matrix  $\hat{k}$  hat inverse, which we already have here with a new value just now computed; you will get a new vector, which I want. I will remove this off.

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So,  $\hat{u}$  hat 0.28 will be again a vector. Can anybody give me the value? Minus? No.

Student: 0.0068...

Fine, agreed; 0.0068 and 3.64?

Student: 0.364.

Can you give me  $u$  double dot of 0.28? How do I get it?  $U$  double dot is here. I know a naught; I know  $u$  at 0.28; just now I had.  $u$  at 0 is already 0 and a 2 I already know.  $u$  dot at 0 – already know it is 0 in my case. a 3 I know;  $u$  double dot of 0 is 0, 10 vector. So, substitute this; get me  $u$  double dot of 0.28. You have to understand the scheme. As long as you understand the scheme, I am happy. Numbers can always go this way and that way. So, you have the privilege of putting the numbers this way and that way; I have the privilege of giving you the numbers in the examination this way, that way. So, this comes to 0.347, 8.5471. Believe me I have worked out this; I may be wrong also; please correct me if I am wrong. Do not copy these values. I have worked out; I got this value. I may be wrong. Is this right or wrong? Right.

So, then can you happily find out  $u$  dot at 0.28 also? So, look at this equation. Please one minute; look at this equation here.  $u$  dot of 0.28 depends on  $u$  dot at 0. I already know this value. In my problem, it is 0, 0. a 6 – I already know the constant.  $u$  dot of 0 – 0, 10. a 7 I know.  $u$  double dot of 0.28 – I already have just now here. Now, the error in this scheme is cumulative. Whatever error  $u$  commit, it will keep on adding. That is very dangerous in this problem. They are not independent. The error you commit in displacement will go to the acceleration, then come back to the velocity. So, can I have this value? It is 0.0486 and 2.60 I think. I have just done for one step. Is it correct or wrong?

Student: 1.6

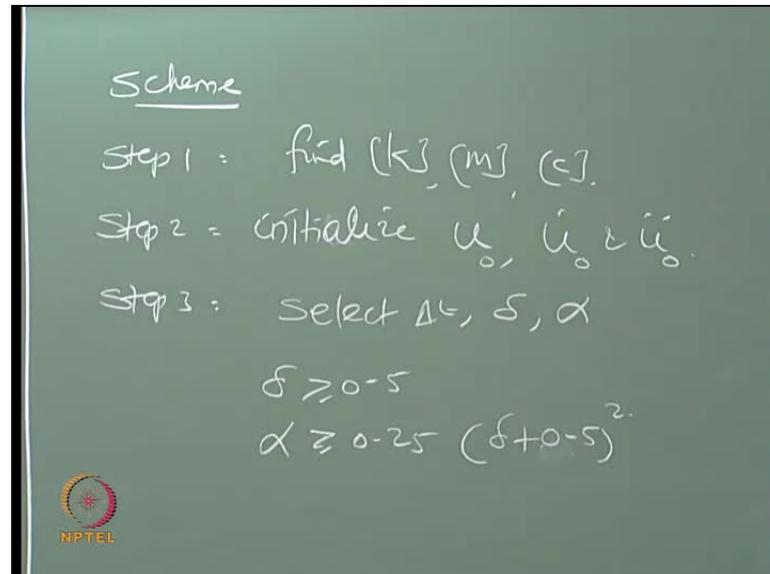
Is it wrong or right?right or wrong? Many of them do not know whether it is right or wrong at all. So, they are very happy. They say sir, it is 12:50; whether it is right or wrong, we do not care; we otherwise also do not care.

Student: Right.

But, I may be... The probability of me becoming wrong is very high. But you should never become wrong. So, I must correct it if it is wrong. So, now, where is the question of iteration coming here? We already said in this scheme that, at every discrete points of

delta t at 0.28 in this example, I am integrating the equation of motion and validating it at every point to a part displacement, velocity and acceleration.

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This must match with the system properties of displacement, velocity and acceleration of the original geometry. If it does not match; for example, if I substitute this displacement and velocity and acceleration in computing the added mass term of mass matrix and computing the stiffness coefficients at a variation of small 0.28 seconds, practically it should not vary. A very small variation of 0.28 should not vary. You have to check that. If it varies; in this case, which will vary; iteration starts, because now this becomes u dots and go to the iteration scheme. That is how the iteration scheme starts. So, there are two iterations happening here. One iteration is you are trying to check this with the previous value and keep on iterating. And when will you stop this? When the previous value on the same time step converges with the new value, you stop the iteration. But the question is – will all get converged or only any one of them?

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The chalkboard contains the following handwritten content:

$$u_{0.28} = \begin{Bmatrix} 0.0068 \\ 0.364 \end{Bmatrix}, \quad \dot{u}_{0.28} = \begin{Bmatrix} 0.347 \\ 8.571 \end{Bmatrix}$$
$$\dot{u}_{0.28} = \begin{Bmatrix} 0.0488 \\ 2.60 \end{Bmatrix}$$

Step 7. For each time step.

$$F_{t+\Delta t} = F_{t+\Delta t} + M(a_0 u_t + a_2 \dot{u}_t)$$

So, we focus on displacement. We focus on displacement, because we wanted the response of the system; so, we focus on displacement. In addition, we have a complexity in our problem, because our problem of  $k$  also depends on the displacement and the  $k$  should also get converged – the new  $k$  and the old  $k$  should not vary much. Just we converge the displacement alone;  $k$  may not converge. Remember that, because  $k$  has got other parameters also, which will influence  $k_{ij}$ . So, I have got two schemes running inside: one is of course the iteration for the displacement; other is the coefficients also getting iterated and upgradation happening in the time. You may wonder, sir, the variation is only very small; I am looking only at 0.28; let me look at 0.1; let me look at 0.01.

Why do I actually bother about  $k$  at 0 compared to  $k$  at 0.01? That is not going to vary much. But the error at 0.01 will get carried on till 100 times of 0.0, which is 10 seconds. So,  $k$  at 10 will be completely different from the  $k$  at naught when we started the problem. So, you are looking for a problem whose stiffness is let us say  $xyz$  value; you are solving a problem actually whose stiffness is not  $xyz$  value at all. So, I am looking for some other geometry; the results are all non-compliant with the problem what we started at. So, the iteration is mandatory to converge for the stiffness coefficient also how less the time interval is, because in this problem or in the scheme, the error what you commit will get added up; so, very simple illustration for a Newmark's beta technique or a time integration procedure. We say it is stable when you have got  $\Delta t$  less than  $\Delta t$

critical; which also says the damping can be used using  $(\zeta)$  or Rayleigh if you have got the time-average technique for solving integration or equation of motion. We already said that in the beginning in the last lectures. So, they are integrated. So, your time scheme or your solution scheme and your problem formation for  $c$  – all integrated; or, you will connect it in the literature. So, this will give you a response time history, because I get displacement at every time along the travel of wave. So, I get a response time history.

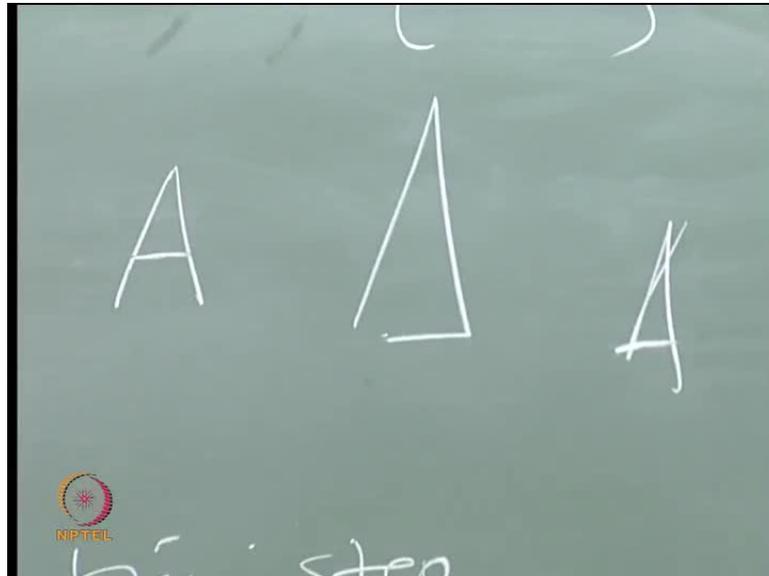
I can always find the frequency content of this response using the fast fourier transform. Many people also do the other way. They solve the whole problem in the frequency domain and bring it back to time domain. We have done this problem in time domain; wish we can take it back to frequency domain, because in the last example, I showed you that, we have got time history of the responses and the power spectral density functions also in a frequency content. So, I can do both. So, it all depends up on where you want to start the problem. Does not mean that, when you start the problem only in frequencydomain, you are more intelligent; or, you start the problem in time domain, you are more intelligent; nothing. Where you are comfortable, you have got to do. But this has a very good cause reference of using  $k$ ,  $m$  and  $c$ , because all the constants or all the coefficients are time dependent.

I would like to see at what time of the passage of waves the system is becoming super critical; system not the solution. There may be a possibility that the system may become highly flexible at that point. So, I would like to see the  $k$  value and  $m$  value; on the other hand, I would like to see the  $\omega$  content of the value. So, when the  $\omega$  content drastically changes, then it is that point where either the fatigue action of the column members have started or the structure has failed, because it has become too flexible or too stiff; too stiff is not possible; it will become only flexible.

So, system becomes flexible;  $\omega$  will drastically change. So, I would like to monitor this also. So, in my program, I cannot also write, when the  $k$  is changing drastically; when the  $\omega$  is changing drastically with the difference of let us say a 10 percent, 15 percent, 20 percent. I can give this imposition in my program. I can always stop the iteration there, debug and find out where actually is the point of degradation of stiffness; so, where the plastic deformation will start. So, I can always use this coding for my design of structures in a plastic limit or plastic theory. So, time domain analysis has got

other parallel advantage in design phenomenon also; whereas, frequency content will not give you this; I am not discouraging or encouraging any one scheme; both of them.

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See you have learnt A this way; Romans learnt A this way; this is what A; Romans write A like this. I write A like this. All are A. As long as you believe and understand, yes sir, you have written A; I am happy. So, I am using time domain for solving my problem. You can use frequency domain. I have showed you a problem, where people have used iterative frequency domain also. So, there are many methods by which you can approach the problem.

But the whole summary of this exercise is you must be able to solve the problem; which most of us start with an enthusiasm, but we stop in between, because only 0.28 I have done. I have got to go 10 times of 0.28 to do this scheme to find out the whole time history. Anyway that is not required for this problem. I wish that, you must attempt to do this problem by writing a coding on a TLP whose stiffness, mass and c are known to you or on any gravity-based structure, where you can find out the mass, stiffness and damping matrices. Solve one case of this order and see how complex it is just to know the quantum of numerical work involved in the problem. It is not simple.

And, these insides you will never get when you attempt to solve this problem using a classical numerical method available in the problem software, because none of the software will be able to help you to debug k, m and c at any instant of time during the

solution process. It will never give you; whereas, you do analytical study like this, where you write the coding; you can always interface and pick up  $k$ ,  $m$  and  $c$  and  $\omega$ , so that you will know where the stiff integration actually started in the passage of waves and passage of time. You will know that. So, if that is the case, for every 10 seconds, every third second, iteration starts and degrade at. You can find out the number of cycles of this happening for 100 period return wave, 100 year return wave; you know the fatigue strength; you can find the ascend curve and design the member for that strength. So, the extension of this study is much large in wide spectrums in design of structures. Ultimately, that is what we are interested in; my sustenance to survive for a given loading. That is nothing but design. What we are doing is an analysis.

I will find out the response; I am not going to bake a cake with this response. I want to see, with this response, my system is surviving or not. And how have all these – the system giving me a comfortable outlay on my top side. If the displacement jumps in surge degree by 3 meters plus minus, that is 6 meters; no human being can work on the system. That is an imposition. Therefore, then I planned for response control of structures using tuned mass damper, secondary mass system. All these I would like to know the response history; not the maximum, minimum value; not the bandwidth, etcetera. Bandwidths are required if you want to know in a frequency band, what is that band where the energy is concentrated. You will know at this band, the structure will fail. Whereas, in this case, I will know exactly, at what time of passage of the wave, the structure will start failing. That is very important point. So, it is how you look at it. So, does not mean that both methods are right; they have superiorities, inferiorities over one over them.

We will anyway discuss frequency domain methods also in the third module. We will pick up a problem and solve. So, this is where we end our discussion on Newmark's beta method or time integration method, which is already known as an inbuilt phenomenon in the last lecture. Any question? Any question? So, you should be able to solve a given problem. For a given geometry of any funny shape, size, etcetera, you should be able to derive the stiffness matrix, mass matrix, damping matrix using Rayleigh or (( )) or superposition, model superposition and form the whole problem integrated in direct scheme; get the solution; plot the time history; you will have to give me the value. So,

this is what the whole objective is of total dynamic analysis of any problem of any new geometry.

Now, the question comes in the next class is, if you have got a very funny fancy geometry, how do we actually handle this problem in the formulation stage itself; which we will discuss in the next class. That is about... For example, I have a (( )) problem, where we have tried a new innovative model, which can suit ultra deporters.

So, how do you start the problem? How do you mathematically model it? How do you examine this dynamically? And then how do you fit the responses for wind, wave, current, tides, etcetera? So, we will just show you the series of results, which are otherwise not available in open domain literature for any reference. So, only research papers, where we have got to contribute and try to purchase the paper, we can read this particular article. Of course, in IIT library, we have this; we can read this. So, we will talk about new geometry. So, this will give a good intuition to you that, yes, I am confident to apply my basics on the new problem; we can solve the problem. That is where we stop; that is where you will be starting. Once we do this in the second module, then in the third module, we will go only to advance methods, speak only fantasies, admire them and forgot them. So, we stop here.