Dynamics of Ocean Structures
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Lecture - 14
Estimate of Classical Damping

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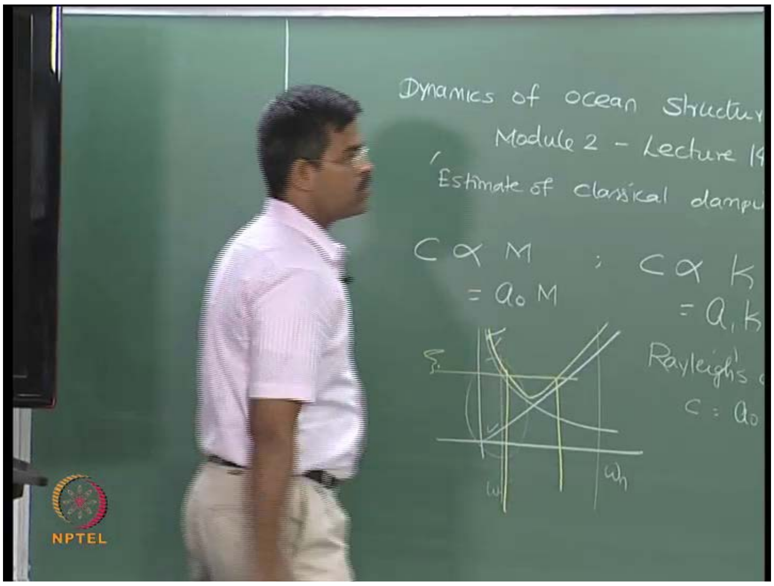

So, in the last lecture, we discussed about estimation of damping for off shore structures. We said that, the damping matrix can be either proportional to mass, where we said, this can be a 0 M or alternatively, the damping matrix can be proportional to the restoring component force K or it can be, let us say a 1 K; in both cases, we had problems; because if we look at the variation of zeta with respect to omega n, you will find that this is inversely proportional, whereas this is related to the mass, whereas when we talked about lateral stiffness, it was varying increasingly.

So, for any higher values of omega or any lower values of omega, either you consider this or, you have got a drastic variation. And, this did not match with the experimental studies conducted by people. Therefore, we said that let us result in a new model which is suggested by Rayleigh and the Rayleigh damping says that, the damping matrix can be the proportional of both stiffness and mass.

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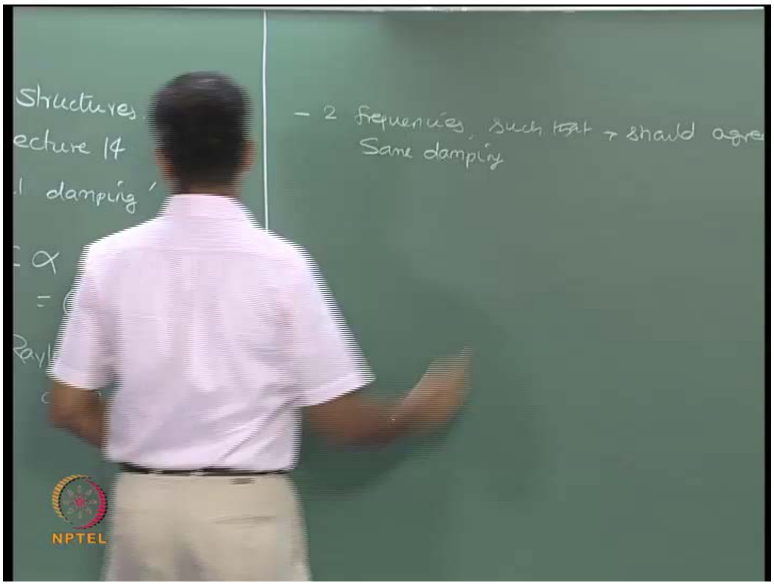

Now here, the difficulty was with Rayleigh is, you must select two frequencies, such that, these frequencies should agree on the same damping ratio. So, the plot was vary with a correction like this; so, two frequencies can be picked up which will have the same zeta value.

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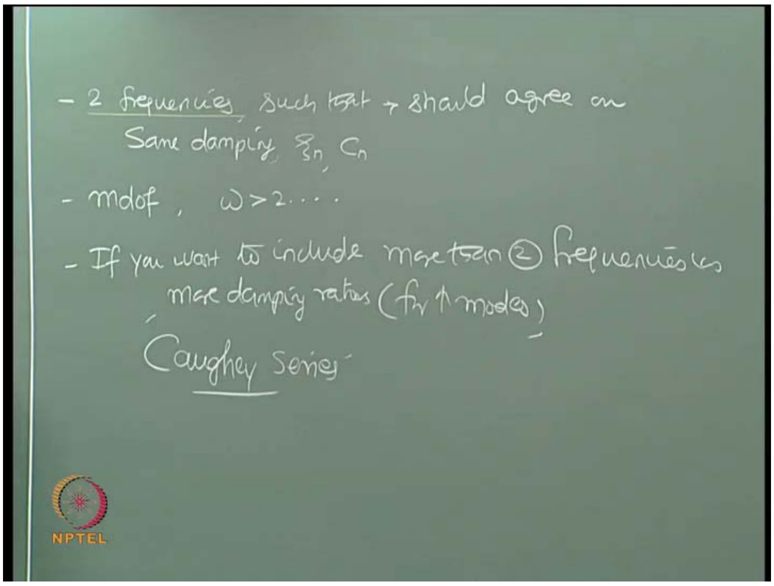

So, if you are able to pick up those two frequencies from a given set of frequencies, which will agree on the same damping ratio, then, we can easily find out the damping ratio as well as c n for a given problem. But again, here, if you got multi degrees of freedom, which has omegas much more than two, then you will not have the damping ratio participation from all the modes. Then obviously, you will see, that in Rayleigh damping we can pick up only two at a time and keep on checking, whether it is solving, or whether it is validating for the remaining frequencies. If it does not validate, then you will change the zeta number and pick up a couple of more frequencies and keep on iterating; you may arrive at a solution that, that zeta may not agree for all modal participation factors happening in the analysis.

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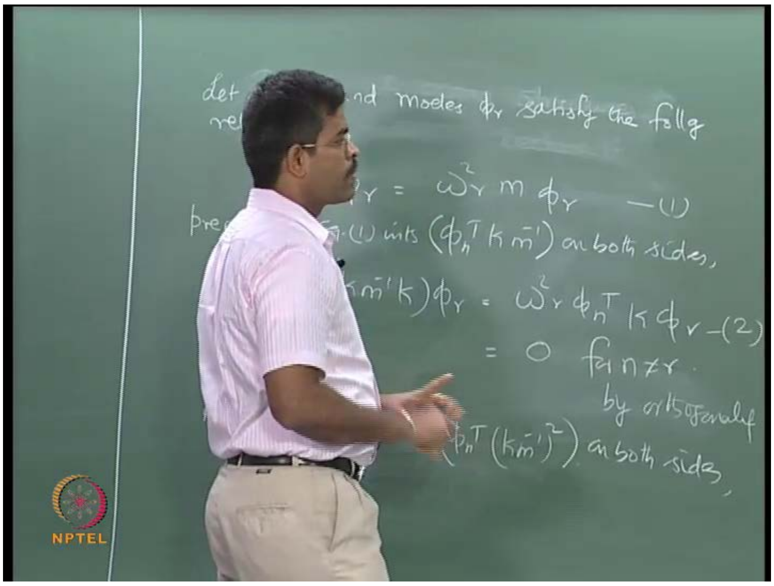

So, if you want to include, if you want to include more than that two frequencies, or more damping ratio, in fact, for higher modes, then you should look for more classical damping which is given by Caughey. Caughey supposed, proposed a method by which you can include higher frequencies also, or higher damping ratios and modes. So, let us see how this can be handled. We will solve a problem and see how this can be applied to find out zeta and c, for a given problem .Is there any doubt in the previous class, for estimating c using classical damping proposed by Rayleigh. So, this is what we will discuss today in the lecture, that how we can use Caughey’s series for estimating damping. So, let us say omega and modes, let us say, satisfy a simple relationship which is K K phi r; K again is omega square m. So, omega square r m phi r.

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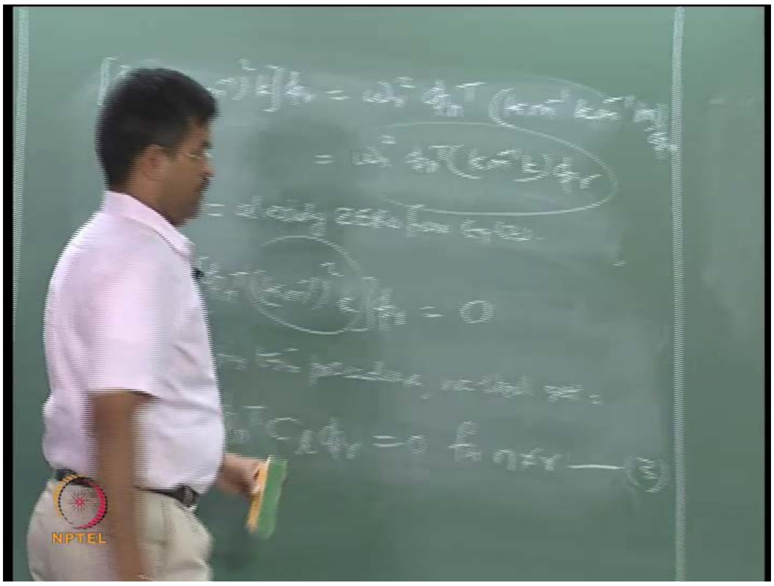

Let us say, we have a relationship which is satisfied. Now, you pre-multiply equation 1 with phi n transpose m K inverse, sorry, K m inverse, phi n transpose K m inverse, on both sides. So, let us do that; let us say, phi n transpose, K m inverse K of phi r omega square r phi n transpose K phi r; is that ok, where m inverse m will become identity. Now, I can say coolly, this as 0 for n not equal to r, by orthogonality. So, equation 2. Let us say, again pre-multiply equation 1 with square of this; that is, phi n transpose K m inverse square, on both sides. Let us do it here. So, phi n transpose K m inverse square of phi phi r, which can be omega r square; yes, this is square here; omega r square phi n transpose K m inverse K m inverse K of phi r, is it not? Of phi r. There is a m here, is it, or K?

This is omega square r m phi r; there is a m here. Equation number 1 has a m here, yes, which will simplify to omega square r phi n transpose K m inverse K of phi r. So, phi n transpose K m inverse K of phi r is already 0 from equation 2. So, this is already 0 from equation 2. Therefore, phi n transpose K m inverse square of phi, of phi r is 0. So, we are repeating this procedure by n number of powers of phi and K m inverse to the power l. By repeating this procedure, we shall get the following equation, where I can say, phi n transpose C l of phi r. So, I am replacing this with C l, is 0 for n not equal to r, where I call this as equation number 3, where C l is given by, C l which we use here, is given by K m inverse of K to the power of l of K.

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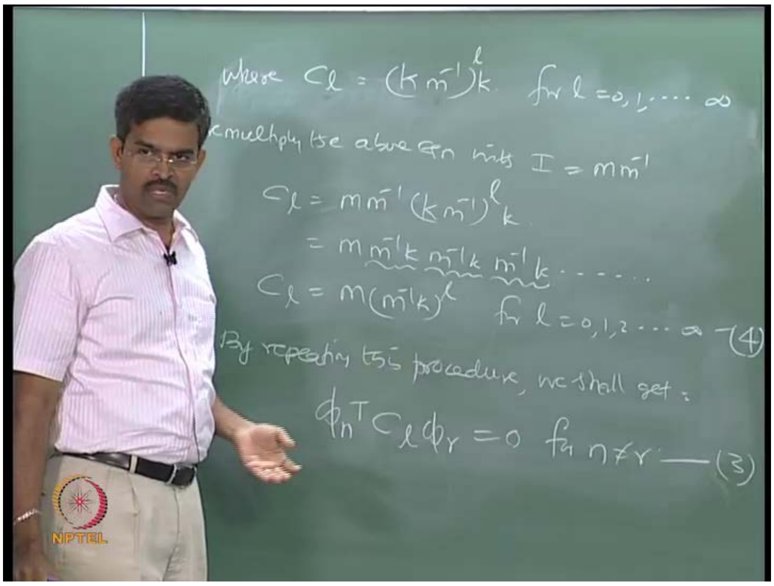

I will put it like this; where C l, this is what II am actually multiplying 1, 2, 3; this is what II am actually multiplying in my whole process; for l equals 0, 1 till infinity. Now, I pre-multiply this equation, the above equation, with an identity matrix which is m-m inverse. So, C l now becomes m m inverse K m inverse power l of K. I keep on expanding this for different values of l. I can expand it like this; m m inverse K m inverse K m inverse K and so on, is it not?

So, I get a group now, which will be m inverse K m inverse K m inverse K like this. I can now write this as m m inverse K of l, for l varying from 0, 1, 2 to infinity. Let me call this equation number 4. This is nothing but, C l; any confusion here? Having said this, now instead of m inverse K, I will process the same equation back again from the first step with K inverse m; see what happens. We have reached equation 4, where I process the original equation of knowing orthogonality with m inverse K to the power l; that is what I did, is it not? I will do the same process again, but the multiplier is now going to be a different multiplier; I will remove this.

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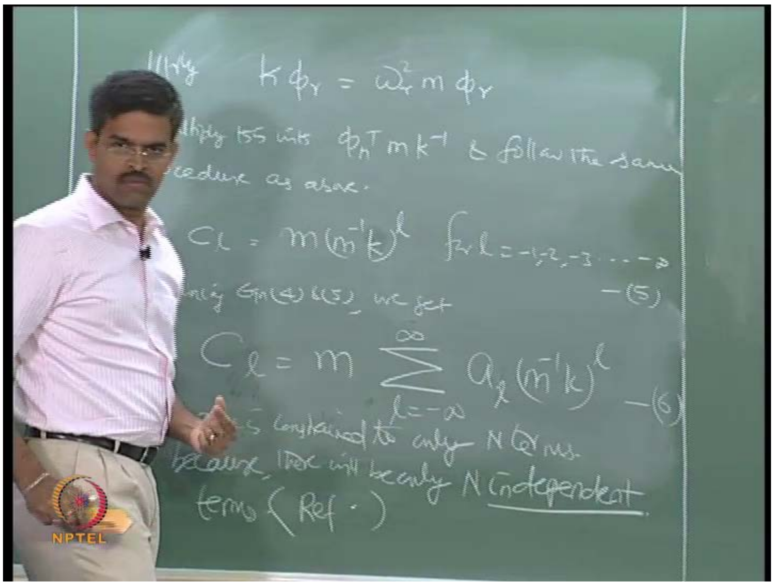

So, the original equation what I had is, similarly, K phi r, which is omega square m phi r; I use r here. I pre-multiply this with phi n transpose m K inverse on both sides. I am just reversing it and follow the same procedure as above. There is nothing above here, but, you have something in your paper, right. So, can you, can you do it and see what happens to my C l? So, do it for once, do it for twice and then, you will know what is happening to my C l. Summarize C l in terms of l. You will see that, you get the same equation back. I get the same equation back. Now, the values of l will be minus 3, like this; it will go to minus infinity; I call this is equation number 5. I mean, this is a very simple mathematical process; you can do that; we will get this equation. Now, I combine 4 and 5; let us see what we get.

If I combine four and five, I can write equation for C l as m of summation of l equals minus infinity to plus infinity, a l of m inverse of K to the power l. I call this equation number 6. If you look at equation 6, basically, the C l value is varying for a range of minus infinity to plus infinity of l value, where l is nothing, but omegas, or omega squares, is it not? It is powers of omegas actually; this K by m is nothing, but omega square. So, this hypothetically includes all omegas in a given model, is it not? But in reality, people have seen in the literature, I will give you the references back, later, that after n number of terms, the independency of this term will not be existing; it keeps on repeating. So, the above equation is constrained to only n terms, because there will be only n independent terms. I will give you the reference. You can please see the reference later. So, I rewrite this equation again; instead of minus infinity plus infinity, I will rewrite it now.

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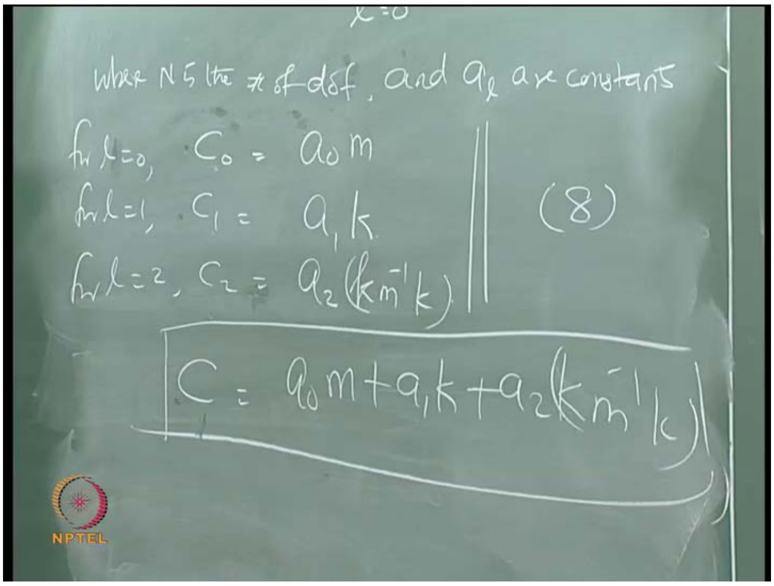

Have you understood? Caughey model is including all the frequency ranges in my whole system, where Rayleigh was not doing it. That is the difference now; that is the deviation of this model from the Rayleigh's damping. So, I rewrite this equation 6 with finite terms, like this. Hence, C l now can become, I am using this left hand side board; it is easy for me to refer the literature; is there any difficulty for anybody from other side to see? You are able to see here, no? So, let us say, m, l equals 0 to n minus 1; if there are n terms, there will be n minus 1 summation; a l of m inverse of K of l; call this as equation number 7; then, where n is the number of degrees of freedom. It is because of this problem, when you try to model numerically any structural system and find out the Eigen values, instead of 6 degrees of freedom, you will get n number of Eigen values. You have to see that, how many of them are independent.

So, I say that word independency related to my degrees of freedom, actually. So, n is the degree of freedom here and of course, a ls are constants. Now, let us quickly see what happens to my a l value, for different values of l; let us say, for l equals 0, C naught will be equal to, which will be a 0 m; can I write it like this? And, for l equals to 1, C 1 will be equal to a 1; m m inverse goes away; K. So, I am getting back the Rayleigh damping again here. For l equals to C 2, becomes a 2 K m inverse K. So, let us stop at 3 degrees of freedom and let us try to apply Caughey damping and see what happens; because we have an example, where, when degrees of freedom are 3. So, I will use it. I can continue for n number of degrees. So, we can very well see here, in this equation, or set of equations that refers 2 terms represent actually the Rayleigh damping.

Student: m inverse e to the power n, then C 1 should be, you know 10 plus a 1 K.

No, no, no; I am looking, how we can do that? So, C l, you are, I am trying to find out separately m. When I will give a complete stiffness matrix, I will add all of them. So, I should say, I will come to his argument. Let C will be a 0 m plus a 1 K plus a 2 of K m inverse of K, is that right? I wanted this coefficient separately; I am going to use them in the next derivation. So, I am having this value. Ultimately, this is my damping matrix; yes, this will be 2. So, this is going to be square. As long as I am writing this correctly here, this has got to be correct. After this, now, my aim is not only to get C, but I also want to get the zeta value, is it not? From there only, I will get my a 0, a 1 and a 2, because what I know in my system is not C. What I know in my system is, I am fixing up zeta; that is what we did in the last example also; we fixed the zeta as same for omega i and omega j.

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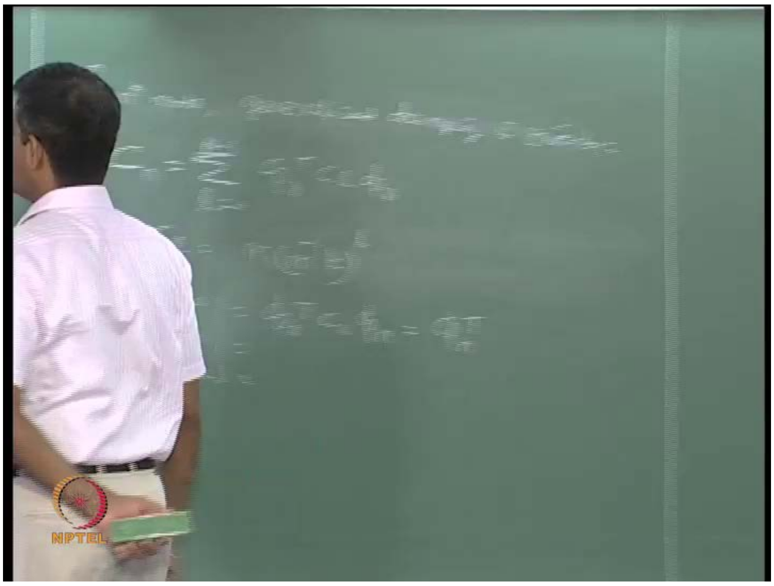

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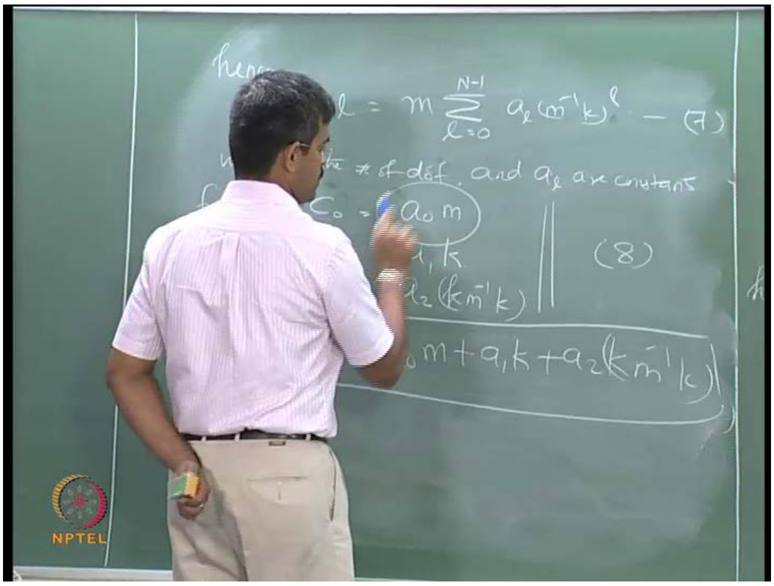

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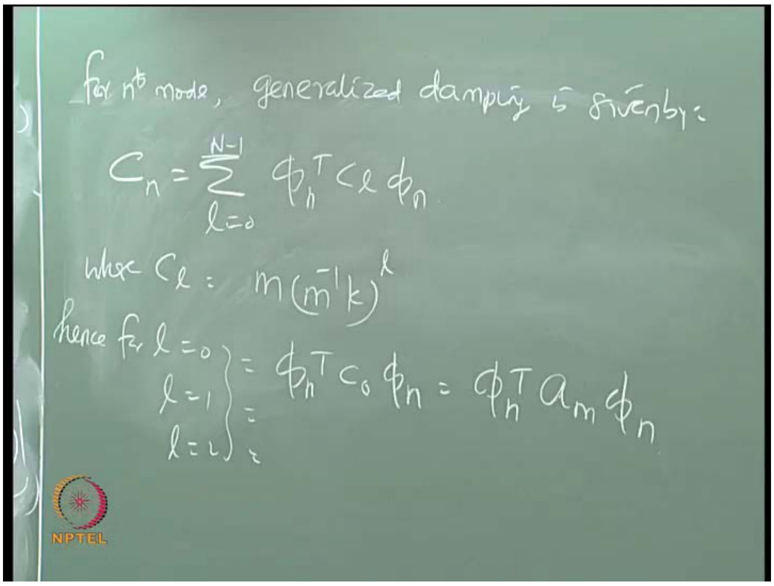

We inverted the matrix and we got a naught and a 1, is it not? So, we know zeta; we fixed zeta. So, having said this, I call this set of equations as 8. So, for n th mode, the generalized damping can be given by the following equation, which is C, here I am using n, because nth mode; the summation of l 0 to n minus 1; I am using the same equation back again here, I am rewriting this slightly in a different form; phi n transpose C l phi n, where C l is m m inverse K of l. Hence, for l equals 0, l equals 1, l equals 2 and so on, I can get phi n transpose C naught phi n, which will be phi n transpose, C n naught already I have here, a 0 m phi n.

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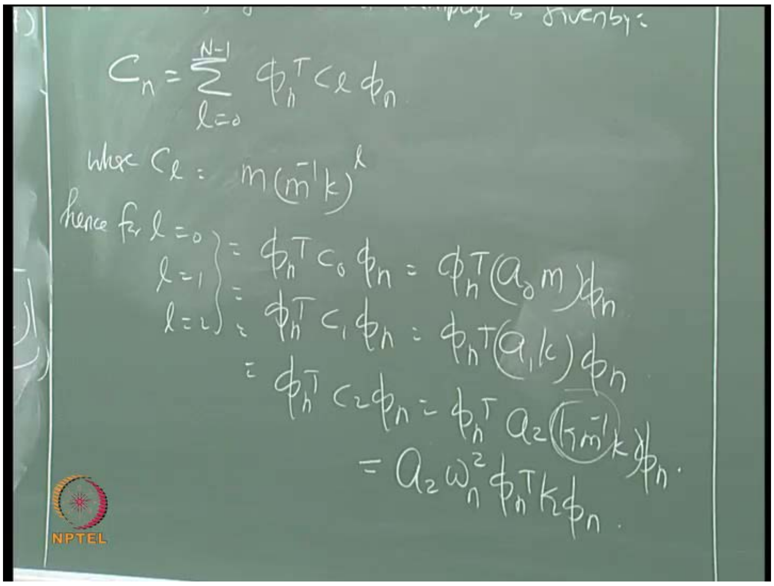

Similarly, phi n transpose C 1 phi. I can rewrite slightly the last term, as a 2 omega square, K by m omega square.

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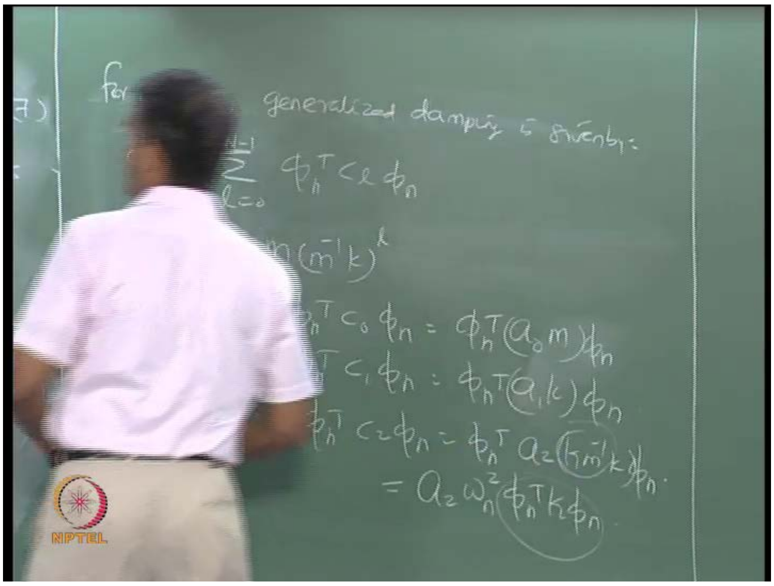

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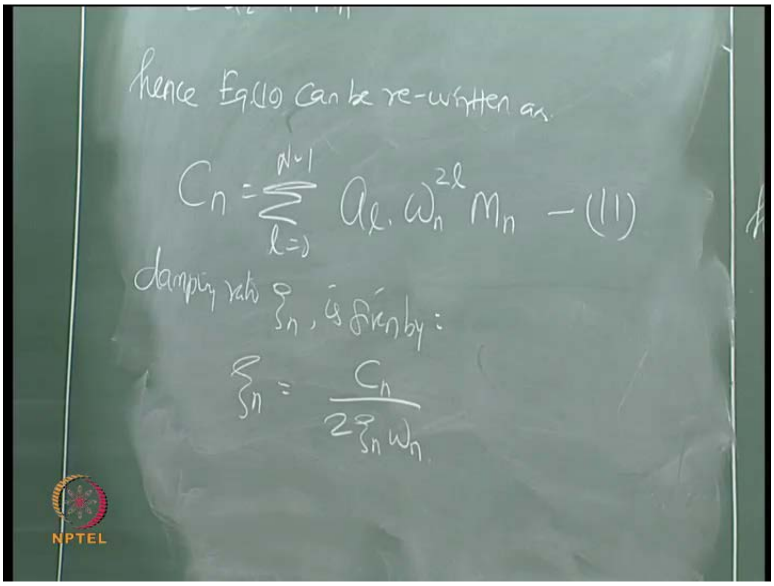

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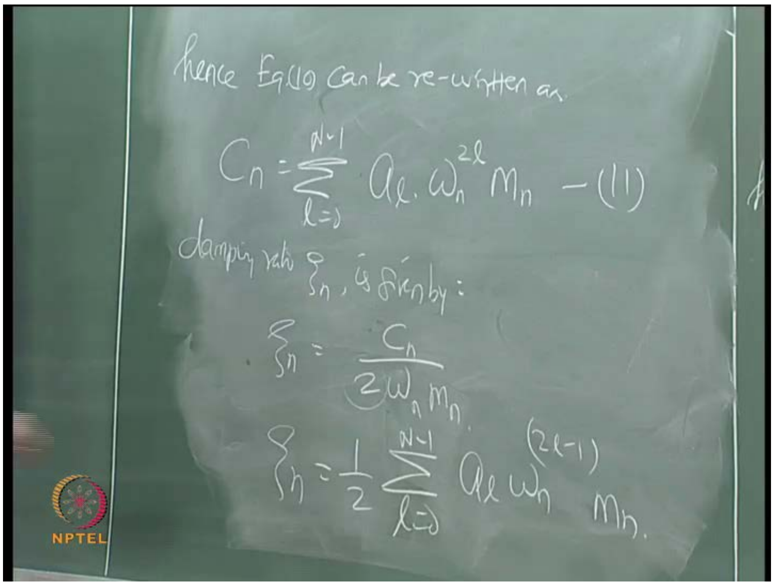

And, what is phi transpose K phi m? From the principle of orthogonality? Omega square m; it is also equal to omega square m. So, I can rewrite now this as, further, a 2 omega square n. Let us say, I rewrite it here phi n transpose K phi n, which can be said as a 2 omega square, or omega 4 m n; because this will be omega square m. Hence, this was equation 10. Equation 10 can be rewritten as C n, l equals 0 to n minus 1 a l; I am using all these values here, back; a l omega n to the power 2 l, m; call this equation number 11. And, damping ratio zeta is given by, zeta n of course, is equal to C n by 2 zeta omega n; C n by 2 zeta omega n; oh, sorry; omega n m n, because t is equal to 2 zeta omega n.

Substitute back for C n here. Omega gets cancelled. Ultimately, gets zeta n as 1 by 2; half. This half is here. Summation of l equal to 0 to n minus 1 of a l of omega n 2 l minus 1of m l; no, m n goes away; sorry.

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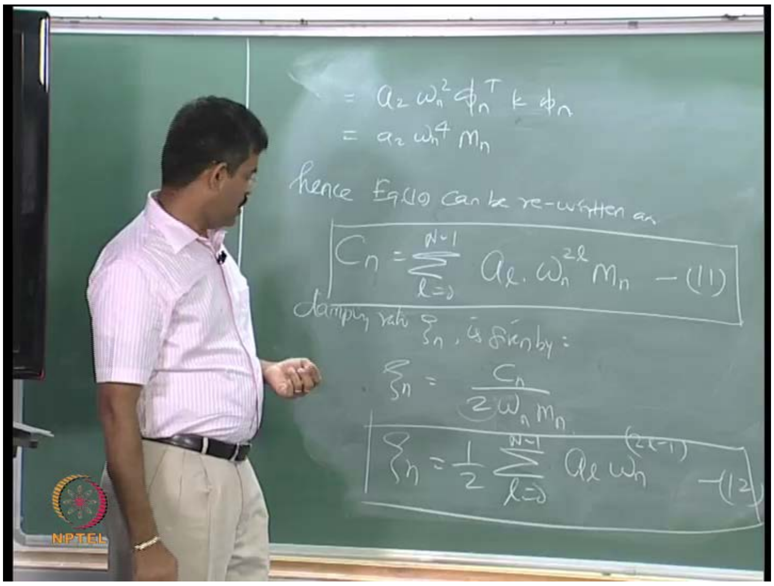

That is my equation number 12. So, I get the damping matrix from here. I get the damping ratios for different modes from here. Once you know all the a ls for different modes, we can easily find C n. Now, there is a very specific problem associated with Caughey series; what is that problem? Look at this equation 12 or 11, and tell me what is the problem associated with the specific series of damping? There is a critical problem here. Look at the series and tell me what is that problem. So, this is a l omega n 2 l minus 1, half of l 0 to n minus 1; equation is fine. And 11 is 0 n minus 1 a l of omega n 2 l m n, fine. There is a very critical problem here. Let us check what happens when I put l is equal to 0 in this equation; l is equal to 0 in this equation.

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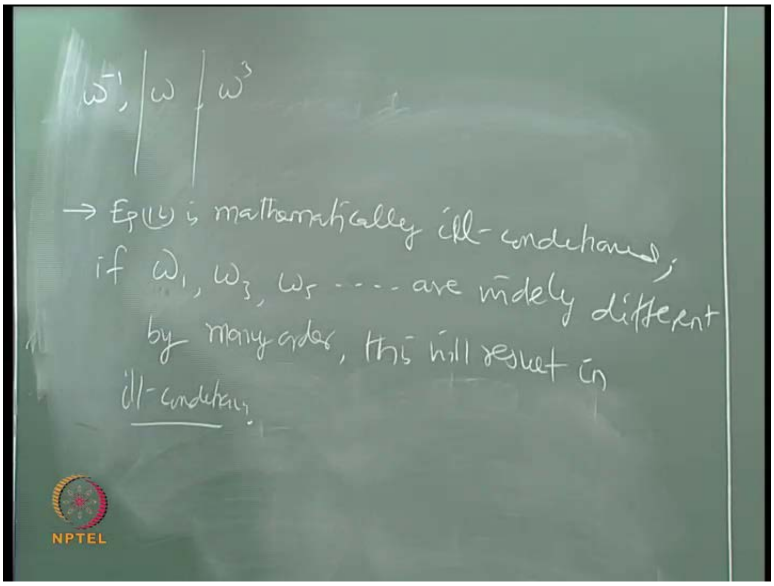

I will get omega to the power minus 1, is it not? So, I get 1 by omega. Let us put l is equal to 1 here. I get omega 1, omega 1; 2, cube. I am skipping in between frequencies, no? It is not covering actually all the frequencies. So, I call this as equation 12 is mathematically ill-conditioned. So, mathematically ill–conditioned; does not cover all the frequencies; skips. And, how this will affect my calculation? If omega, omega 3, omega 5 are widely different by many orders, what do you mean by many orders? For example, this is 1; this is 15; this is 200, etcetera; very widely different by many orders. This will not give you the correct representation of damping; and that will be the problem. This will result in ill-conditioned; that is the difficulty with this damping. But this damping is capable of covering all the frequencies as we saw, is it not?

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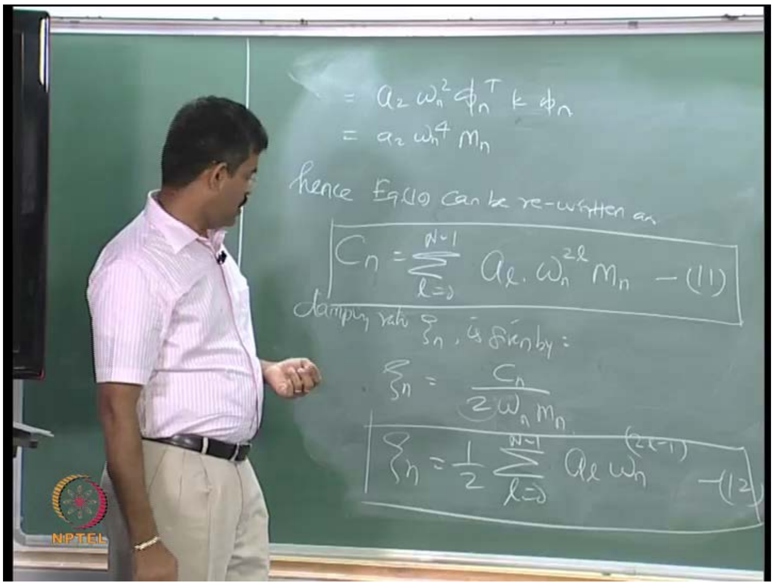

The damping ratios are skipped. See, I am not including the damping ratios of all modes. I am including only selective modes by order; damping ratios are not included, but here, I can keep on adding from 0 to n minus 1. I will have all frequencies included, but the damping ratio contribution from all frequencies is not coming in in the zeta, is it not? That is the problem here. So, let us pick up the same example quickly, and try to solve for a 0, a 1 and a 2 and get C matrix and see how this C is different from the last C matrix, what we got in the earlier lecture; they are different, is it not. Because there, we got only a 0 and a 1. I picked up only 2 frequencies, omega 1 and omega 2. I showed at omega 3, zeta 3 is approximately equal to 5; it was around 6 point something, is it not?

But here, I will fix zeta for all frequencies same and get C matrix, and compare how this C is different from the earlier C. So, we are looking at the estimating of damping matrix in this problem, is it not? It can be either mass proportional alone, stiffness proportional alone, but it is not successfully operating in the practical considerations, in experiments. So, we are looking for an amalgamated model, from Rayleigh, from Caughey, and one more method which we will discuss on Monday's class. So, these are some interesting, applicable, practically possible methods for multi-degree freedom system. There are many more methods. We are not discussing all of them here. By sample, I have discussed three methods. I have discussed now two. I will discuss one more method on Monday class. So, (( )) three methods discussed. There are many more methods available. So, let us do the same problem back. I have to retain this equation. I will write it, or you will give me this equation when we do the problem; I will rub this. Do you have any questions, any observations, in both this methods of estimating damping? Now, please understand, in all the problems in dynamic analysis, where fluid structure interaction, or material non-linearity, or geometric nonlinearity, or dependency of restoring coefficients on responses, like we have to see in many of the offshore structures, estimate of damping is most critical. This is further critical, if I am talking about response control of offshore structures. For example, you want to control the response; you have to play only with the damping coordinate. So, mass, you cannot reduce; because if you reduce the mass, like complaint structures for deep waters, like t l ps, pars or triceratops, etcetera, they will become highly flexible; you will not be able to install it. We restore, I mean, if we change the restoration matrix, K matrix, the system will become very flexible; you will not be able to put a desired pay load on the structure. So, K and m you cannot touch, because K and m are from the geometry; and geometry, you are not altering, is it not? We are only altering the form. Remember very importantly, in offshore design, or in offshore evolution of platforms, we are not touching the geometry; we are only altering the form of the Geometry, by introducing hinges, by introducing teethers, by introducing external connections, by putting more (( )), etcetera, is it not?

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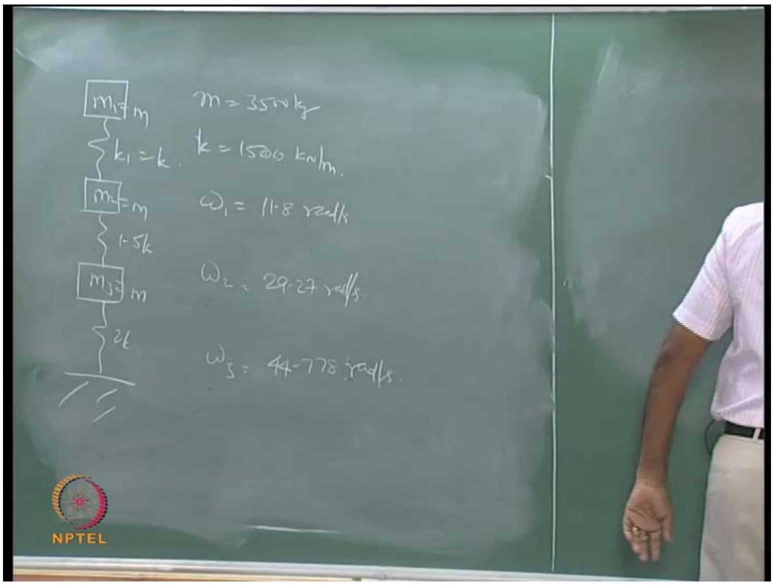

We are only changing the form; structural form. Geometry may be square; you have the same diameter, the pipe, a cylinder as 12 meter earlier; now also, 12 meters, etcetera. To take the same structure to deeper and deeper waters, we have to have an innovative structural form; we are working only on that. So, K and m will actually not help you much to response control, right. So, if you are talking about, let us say t m d, which we saw in the previous lectures of the same module, we have used unit mass dampers. There are many other ways by which one can control the responses. We have to play only with the damping methods. So, estimation of damping, because very important in the analytical as well as practical significance of offshore structures. So, let us have the same example as we saw in the last lecture. Just for comparison, I am picking up the same problem. This was m 1, m 2 and m 3, which were all equal to m; this was K 1 equal to K; this was 1.5 K; this was 2 K, where m was 3500 Kg and K was 1500 kilo Newton per meter. So, I had omega 1 and omega 2 and omega 3 as 11.8 radians per second, 29.27 radians per second and 44.778 radians per second. By the way, have anybody verified whether these two answers are correct, by the classical methods what we already know? This, I have shown you yesterday; we derived this value from this total loss method; but I wanted you to verify this two, by either influence coefficient, or matrix inverse method, to find out whether they are actually having the same value. Have you verified anybody? Has anybody attempted to, thought of verifying it, at least? See, this is where actually you are diverting from the original focus of the course. The moment you do not show interest in carrying the dynamics as your class work, at least, take it granted, the research will seriously suffer, if you are working on dynamics; because the one which you do not do now, you will never ever do it; because of two reasons; you will not have motivation to do; two, you will have no technical, let us say, lecturer assistance to do it. The problem what you had, or what you will have in future on estimating these, will remain unsolved until you come out with some other parallel course like this, or break your head, try to solve it out; you will not able to do it. I am repeatedly telling you, unless and otherwise you do all this exercises very seriously, for one hour teaching of dynamics, you must work at least 23 hours in your class, before you come for the next class. If you do not do that, you will not be able to follow this course completely. So, my objective will go waste; but still, I am insisting, at least today, before you come for Monday, verify omega 2 and omega 3. Are they correct, or wrong? Verify it. Now, let us try to work out the values of a 0, a 1 and a 2, to find out the C matrix from the Caughey's model. Really, we already have.

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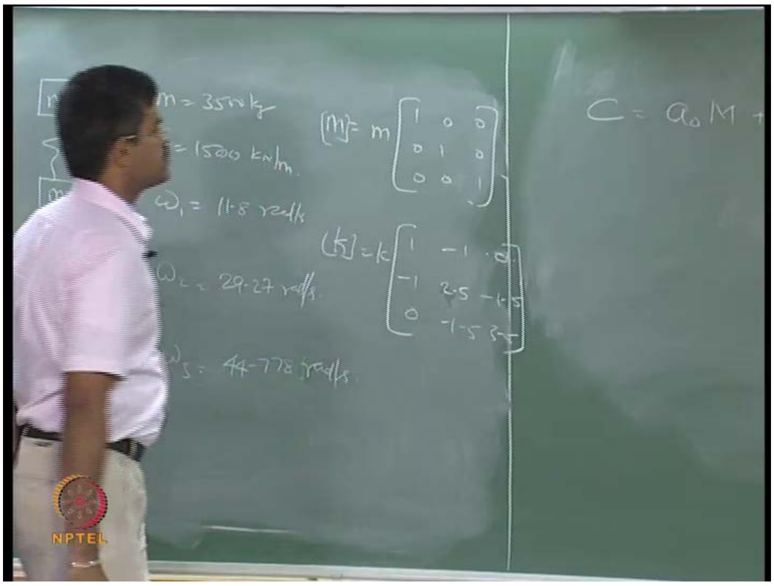

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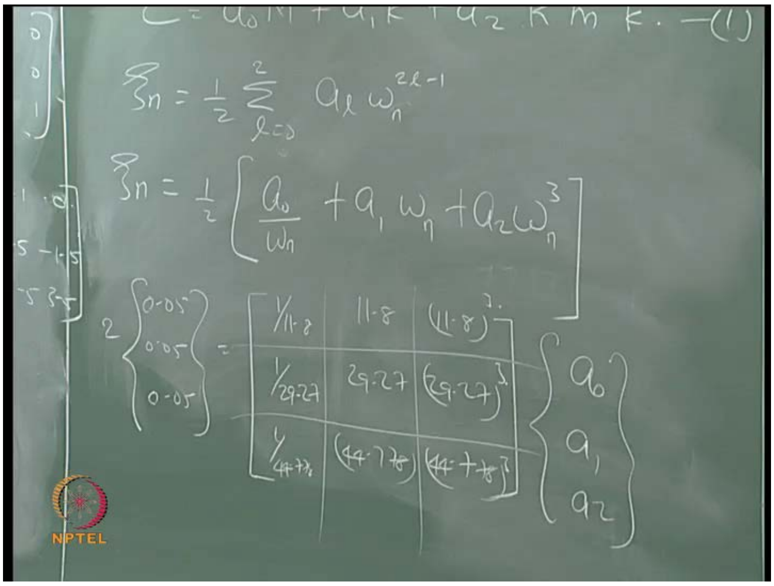

So, we already know C matrix is given by a 0 of m, plus a 1 of K, plus a 2 of K m inverse K; call this as equation number 1 from a problem. We already have the mass matrix. We already have the stiffness matrix. I do not have it; what is the value of stiffness matrix? And, zeta n is given by, the equation is half of summation of l 0 by 2, because it is 3 degree freedom system; n minus 1, a l omega n 2 l minus 1. So, I can say, I can expand this and say, zeta n is half of a 0, a 0 by omega n plus a 1 omega n. So, exactly what we are getting in the Rayleigh damping also; plus a 2 omega n cube. So, let us write this in a matrix form. So, 2 zeta n is going to be, let us do it like this; twice of, I am taking 5 percent damping. So, I say, 0.05, 0.05, 0.05; that is what I am interested in now; I am fixing damping for all the frequencies same, which we did in the Rayleigh's case also; is equal to, I should say, this is 1 by 11.8, that is my omega 1, and omega 2…Let us complete this; 11.8, 11.8 cube of a 0 a 1, is it not? That is the matrix form. And similarly, 1 by 29.27, 29.27, 29.27 cube and 1 by 44.778, 44.778, 44.778 cube; 5 minutes time, invert this matrix and get me a naught, a 1, a 2, quick.

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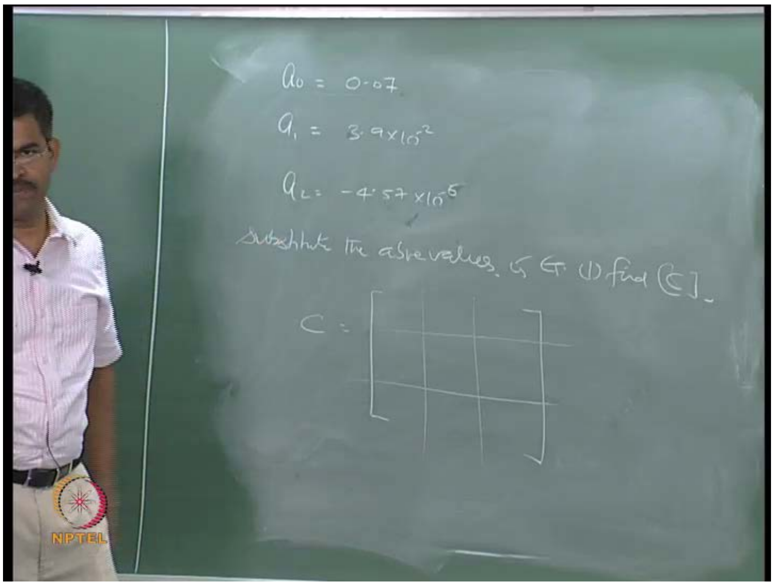

So, what are the values of a naught, a 1 and a 2? Yes, a 0, a 1 and a 2? For a 1, very high; this one, or this one? a 1, 8.9, 10 power minus, minus 4.57. 6. You will see that, this values are, this two values are closely matching with what we had for (( )), a 0 and a 1, and the contribution of higher modes is limited; s very low; but still, substitute the above values in equation 1 and find C. So, it has got many multipliers; you have to multiply with the m, K; then again, do this processing and multiply with a 2 and get C matrix. And of course, C matrix will be now a 3 by 3 matrix, which you can find; that is how I estimate the damping matrix using Caughey series, which is including the contributions from all frequencies, omega 1, omega 2 and omega 3, keeping conditional that zeta is same, or the damping ratio is same, or it is uniform in all the modes of vibration; damping ratio, not the frequency. Damping ratio is assumed to be same in all modes of vibration. In the Rayleigh's case, we could not do this more than 2. Now, in Caughey's case, it is general; we can do for n minus 1, if n is a degrees of freedom; that is the deviation between this particular method, with that of Rayleigh. Of course, we cannot compare the damping matrix obtained from a 0 m, damping matrix obtained from a 1 K, damping matrix obtained from a 0 m plus a 1 K plus damping matrix obtained from a 0 1, a 1 K, a 2 K m and K inverse, etcetera; we cannot compare them. They are all different formats, because they are including different contributions of modal frequencies, or modal damping ratios in your calculation. So, it cannot be compared. But which one we will use depends on the justification, or the decision of the analyst, right. So, that is why we stop here. Now, the problem with this particular series is that, it skips on a different frequencies contribution; forms an ill-conditioned system. If we want to make a numerically stable, numerically stable, then I should include straightaway, the superposition of modal responses directly; a modal damping ratio is directly on my C matrix; that is the third method will see on Monday. We will pick up on same example; will solve this problem slightly in a different manner and see how I get C. So, I get different methods to estimated damping matrix. That is where we stop. So, these are the three methods we have for estimating damping matrix, which is commonly practiced in literature, for structures which has widespread of frequency ranges. We can go ahead with Rayleigh damping, because we have got, in t l p, we have got 2 distinct set of frequencies. Though we have 6 frequencies, there are 2 distinct set of frequencies; one is stiff and one flexible. So, I can easily handle t l p problem of finding C using Rayleigh damping; but if you do not have that facility in geometric form of an offshore structural system, you can use Caughey series, or you can use modal damping ratio series, which we will discuss on the next lecture; you can find C. Now remember in this particular problem, that C will get updated every time, because K and m are changing and omega is changed.

Even otherwise also, C will get updated, because m or K may change in the system, when the system is under damage. So, C will get updated, right. So, once we discuss that on Monday's, the next lecture, then, we can speak about coming back to the original t l p problem. We have got m; we have got K; we have got f of t, understood. Now, we will show how we have got C. Then, we will run the analysis using a specific iteration scheme, because it is an iterative process, get the results, discuss them, quickly from a paper; that is all, we will complete the discussions on t l p, and how f of t handles different kinds of forces, like wave force, current force, earth quake force, stability forces, how it is handled and how it is modeled and how it is taken care of in a dynamic analysis. We will address that f of t part separately. So, that ends the discussion on t l ps.

Then, we will go to the next generation platforms, where we will talk about Triceratops; we will talk about perforated tension leg platforms, for response reduction control mechanisms. So, how the problem formulation can be done; just an idea. Then, we will move this to the next module, where we will talk about application of dynamics on simple manufacturing instruments; like, I want to manufacture an accelerometer; I want to manufacture a displacement transducer; I want to, because accelerometers are costing around half a lakh when you import it from US; can I manufacture an accelerometer? Can I have an instrument which will work as displacement transformer, as well as accelerometer, in different ranges? Can I manufacture it, from fundamental principle of dynamics? It is very much possible; we can do. So, I will explain that concept in the third module. Then, we will move on to application of this, for modal response estimations, or participation factors, doing some problems. Then, we will move on to the stochastic dynamics part, which is again an intensive research area in dynamics. That is how we will conclude this course, around 47, 48 lectures in total.