Dynamics of Ocean Structures
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Module - 2
Lecture - 13
Development of Mass, Stiffness and Damping Matrices of TLP from First Principles

In the last lecture we discussed about the derivation of Stiffness and mass matrix for TLP.

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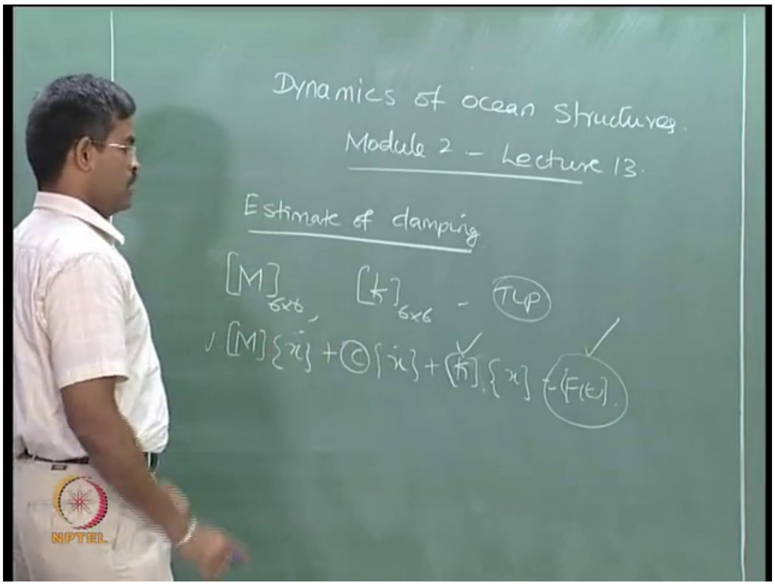

So, we already know how to derive the mass matrix and the stiffness matrix from first principles for a tension leg platform. So, for a given equation of motion as we see here, we already know these elements, and of course we believe that we know how to compute the f of t for a given geometric form for example, for a TLP, and we will be focusing on how to estimate c in couple of lectures from now onwards.

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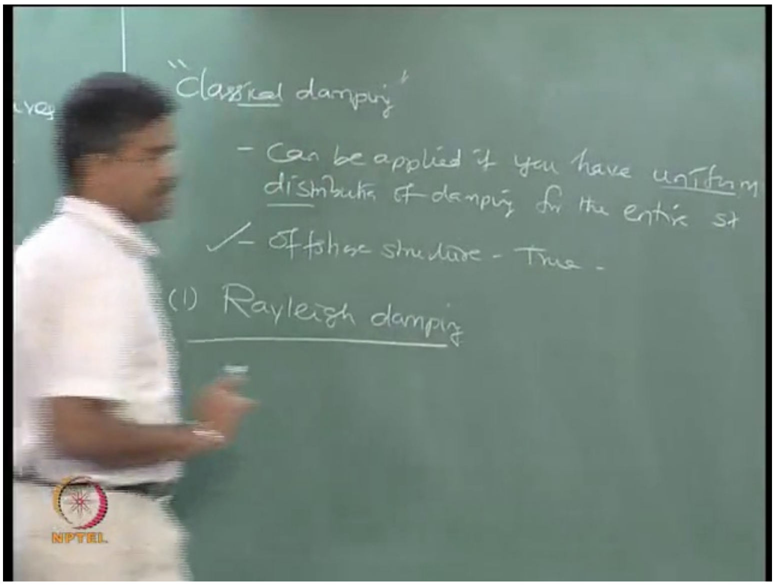

So, people generally use what we call, classical damping for estimating damping characteristics in structures, classical damping can be applied if you have uniform distribution of damping for the entire structure, if you have uniform distribution for the entire structure then we can go for a classical damping. Now, this is true for offshore structures; this is true, because we have damping which dissipates, the energy produced vibration of the structure in the water body.

So, it is more or less uniform, so we can always try to work out the classical damping for offshore structures. Now, there are two ways of looking at the classical damping, the literature suggests one can go for Rayleigh damping, the two cases we will discuss in the successive lectures. In this lecture we will talk about Rayleigh damping. We already know the damping can be either proportional to mass or proportion to stiffness.

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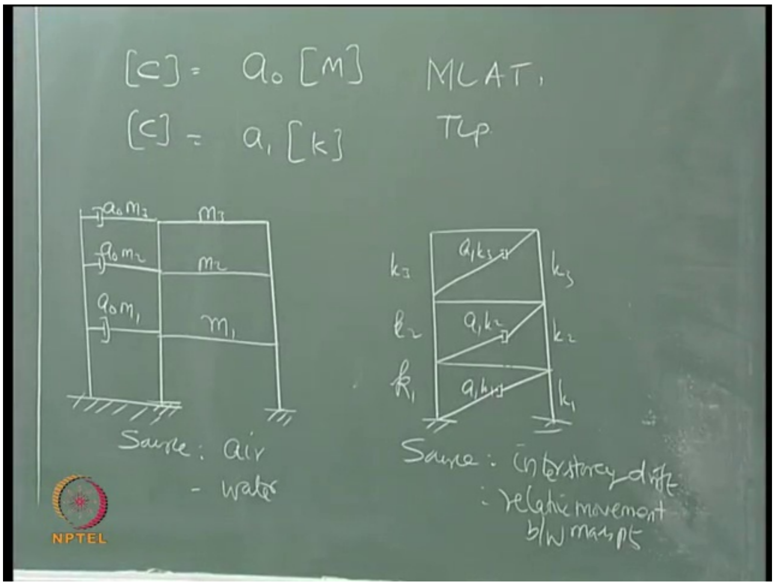

For example, my damping value can be a proportional value with respect to mass or it can be a proportional value with respect to stiffness, because essentially the dissipation happens either from the mass interference inertia interference or from the restoration interference from the elasticity of the material or the member. We are trying to plot this graphically, let us see how does it look like, if I have a structural system like this I am just taking an idealistic modal this can be also similar to a jacket structure or a TLP anything, this discussion does not hold directly related to a TLP. Now, I am just off lining from TLP of discussing a classical damping theory first, and then applied to TLP and show how this can be done. So, I have got let say a three degree freedom system modal which is a conventionalist model.

So, the damping can be graphically represented like this. So, there is pseudo frame which is fixed to the ground or fixed to the foundation and I have a damping applied, which dissipates the energy caused because of the vibration on this modal which is proportional to the respective mass, which is lumped at the respective locations. Now, where does this come from, the source for this modal, if the structure is not offshore it is onshore let us say then the source comes from air the damping has to come from air actually is it not.

So, this can be relatively low, if it is in air right. So, this modal of proportional damping’s for to mass inertia force, if the modal is in air or maybe in onshore then this can be relatively low, in our case this is relatively higher because majority of the structure is retaining in water. For example, gravity structures it may not be true for a TLP because partly it is in water, but majority of the structures like gravity structures AT's etcetera guide towers they are in water they have got phenomenal representation of this, so this can be applied.

Alternatively, we look at the stiffness proportional modal of damping it looks like this, let us say I have again a three story framed system, whose stiffness are indicated as k 1, k 2 and k 3 with respect to the inter story displacement. The relative movement of each floor or each mass position with respect to the relative movement of each mass position, you get damping, which is a 1 k 3, a 1 k 2 and a 1 k 1.

So, this depends on the inter storey drift what I call or the relative movement between the mass points, the relative movement between the mass points because these are the points where the mass is lumped this is true. Because, if I take for example, a multi legged articated structure, a TLP you will find that the mass is being lumped between the stiffness elements at different location, and they are allowed to move independently because of the hinge in between them.

So, we can use this modal now, both of the modal relatively applied to an offshore structure, but let us see what are the difficulties? If we apply alone mass proportion alone or k alone. Let us see, what happens to this and why Rayleigh is invoked, Rayleigh modal or Rayleigh damping is not, Rayleigh damping is neither mass proportional nor stiffness proportional a combination of these two, let us see what is the difficulty when I use this independently.

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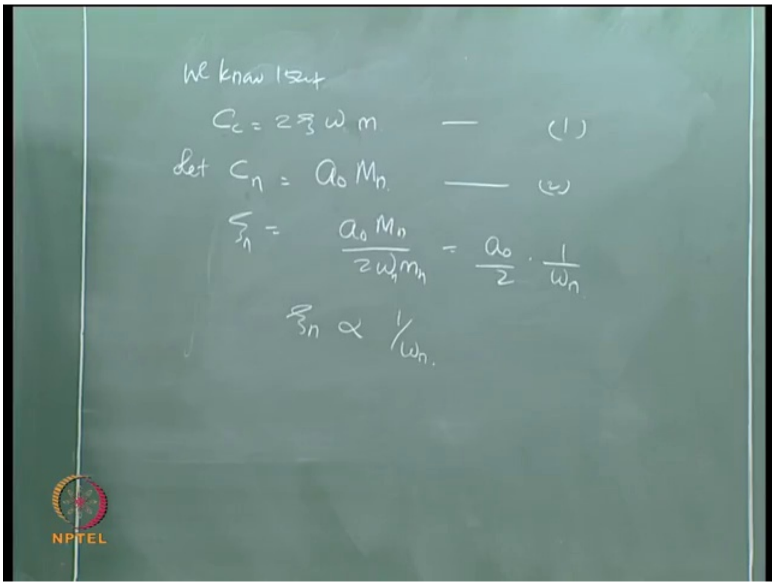

So, we know C c is 0 zeta omega m is it not fundamental thing which we already know equation one we also know let C n, I am instead of saying critical damping I am saying C n which is now taken as a 0 M n. So, zeta will be a 0 M n by 2 omega m n and omega n where I say zeta n which will amount to a 0 by 2 1 by omega n it means zeta n is inversely proportional to the frequency, it is inversely proportional to the frequency, if I try to plot this.

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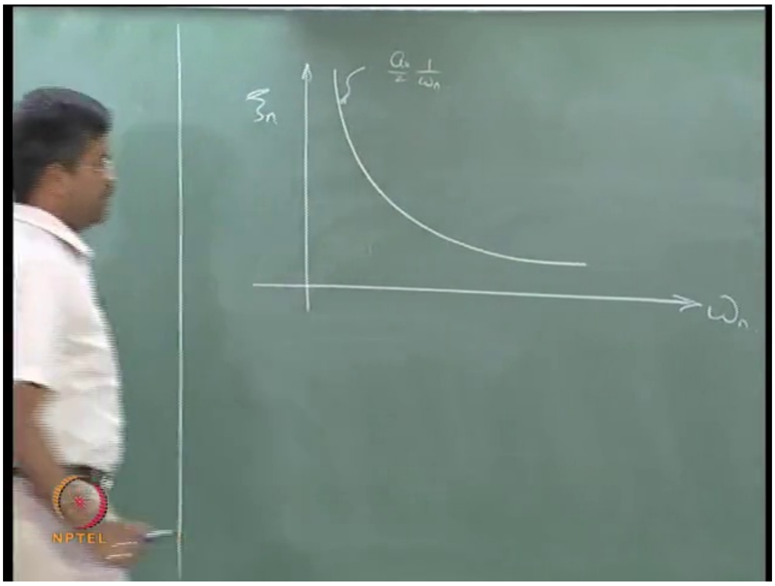

Zeta n it will be gone linear the circuit going to look like. So, what say we say is a 0 by 2 1 by omega n it is actually mass proportional damping.

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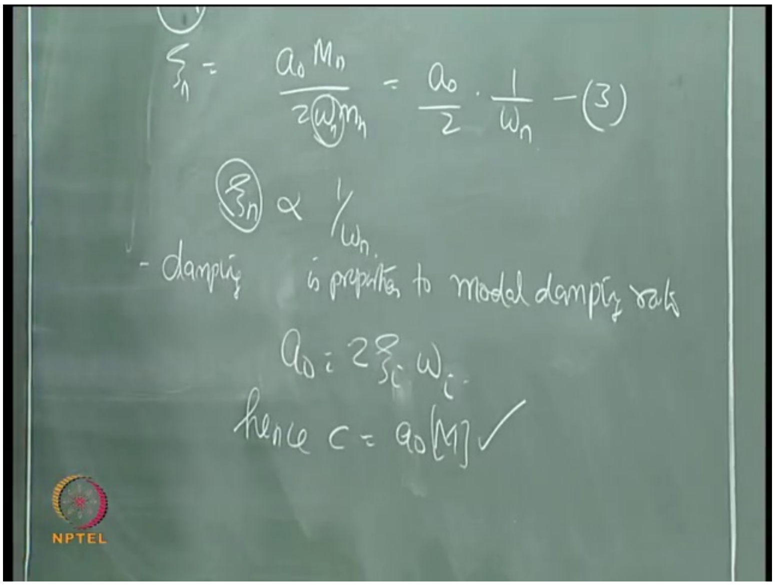

And, we can also note the damping ratio is proportional to or we can say, the damping is proportional to modal damping ratio, see is proportional to a specific omega and zeta is modal proportional a modal damping ratio. So, I call this equation 3 therefore, a 0 will be 2 zeta n omega n or let us use a different suffix a here, instead of saying natural frequency any mode i'th mode will give me a naught. So, once a naught is determined one can easily find c, hence c which is a naught m can be computed because a naught is known.

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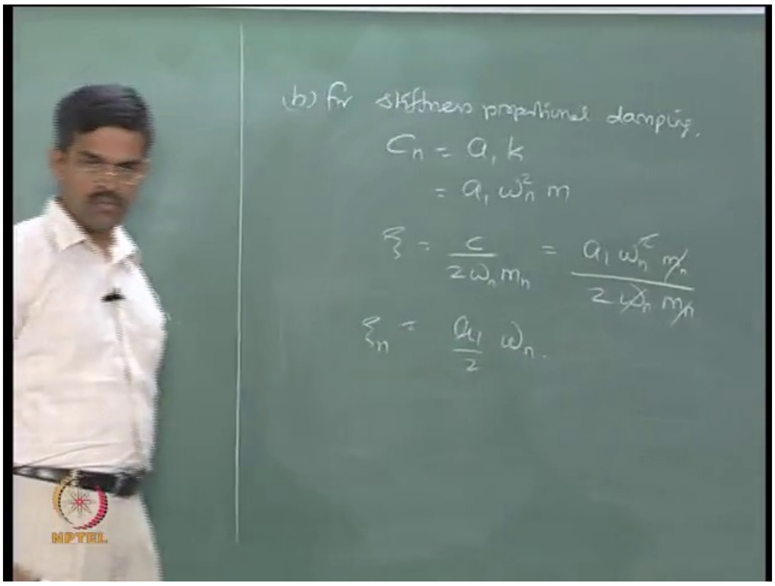

Similarly, let us take the c for stiffness for stiffness proportional damping C n is a 1 k which is a 1 omega square m and c value known is 2 zeta omega n, c is 2 zeta omega n. So, zeta which is c by 2 omega n which is a 1 omega square m n by 2 m n this is nothing but a 1 by 2 half omega n. So, it is directly proportional, this is again linear.

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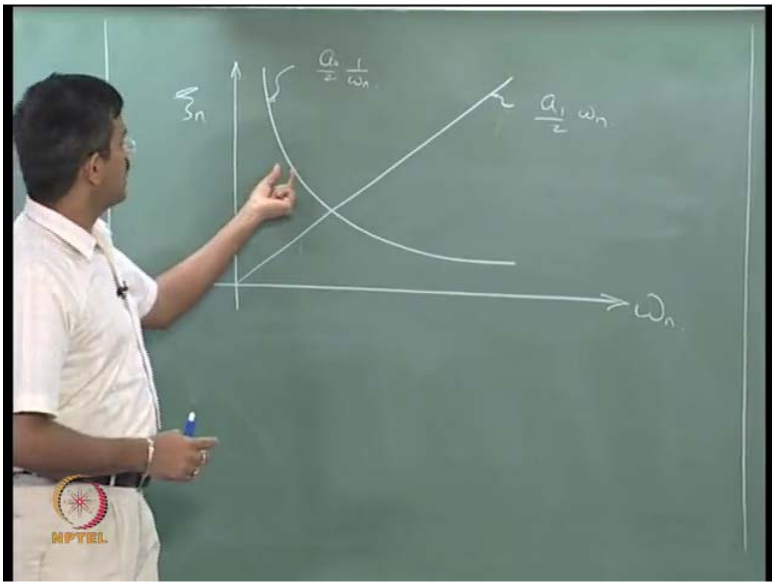

So, this is my stiffness modal which I get, this as a 1 by 2 omega n. So, very interestingly if you pick up a mass proportional damping for a higher frequency I get lower damping, if you pick up stiffness proportional damping for higher frequency I get higher damping.

So, it has been seen in the experimental studies the literature, which you give the reference as zeta that neither of this modals hold good for experimental determinations.

Because, we have estimated the damping ratio, by conducting physical modal studies on different structural members and try to apply free vibration test and find out the damping ratio from the response of the structural system and compact that. And they have seen that neither mass proportional nor stiffness proportional alone, will qualify the results what you have got experimentally.

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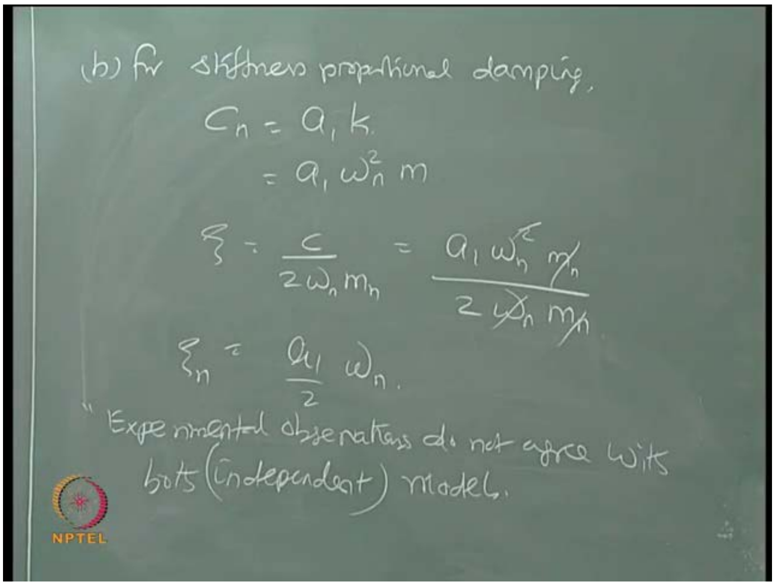

So, the critical comment here is experimental observations do not agree with both independent modals. So, Rayleigh's said let us combine this.

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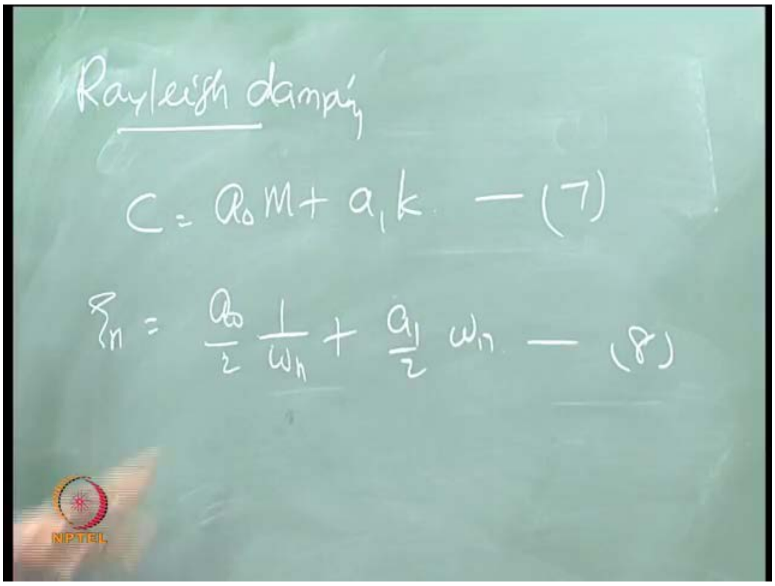

Which is going to be a combination of mass and stiffness, this equation 7, 6 I think I missed out somewhere therefore, the damping ratio can be given by a 0 by 2 half omega n a 1 by 2 half omega n. Now, the problem is how to get this a 0 and a 1 or how to get select the specific omega. So, now; obviously, if you try to (( )) this for different values of omega n versus zeta, the curve comes like this.

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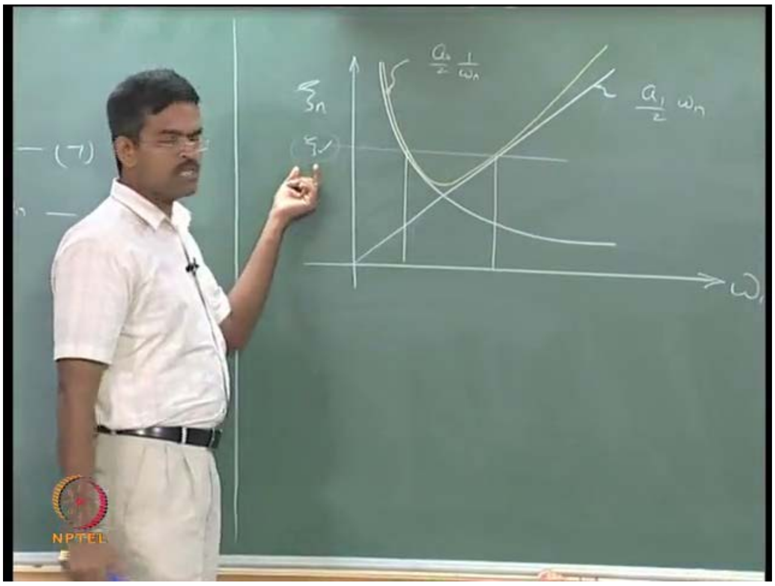

It closely agrees with the mass proportional then it starts agreeing with stiffness proportional, it means at any specific value of zeta I must have natural frequency with the system of two values which agree for the same zeta. So, I must pick up those frequencies, in my given system, that agree for the same zeta. So, it is an iterative thing or it is a guess select for example, zeta 5 percent and see whether all frequencies do agree or match in the closer range of zeta is equal to 5 percent.

So, it is very interesting that we must first find out omega's for a given system, pick up zeta and pick up any two frequencies to find out them a 0 and a 1 because there are two equations here the two unknowns and two equations pick up them, find a 0 and a 1 which I will give you the equations. Now, we will derive them I get a 0 and a 1 I get c then I apply that c or zeta for the third frequency which you have not picked up, so far because my degree has got 3 degrees of freedom let us say my modal.

I have picked up two frequencies I saw that I agree, I want to check will they agree for the third one also, if they agree in the closer range of zeta then this modal can be applied easily for my structural system. Now, a question may come, if you have got infinite degrees of freedom modal let us say very large number how do I pick up a zeta for a specific value. Then we can apply the third concept what we are going to discuss later I can pick up damping ratio to a different modes separately and use modal super question for damping I can do that.

But, unfortunately in our problem of TLP or in offshore structures, we have a limitation of let say 6 degree freedom system all for rigid body. We do not have a large number of omega's to be discussed, but one can do this also which I will discuss in the later part. So, argument here is I must pick up omega i and omega j such a way that they agree for a constant zeta to get a constant zeta or equal zeta different omega's.

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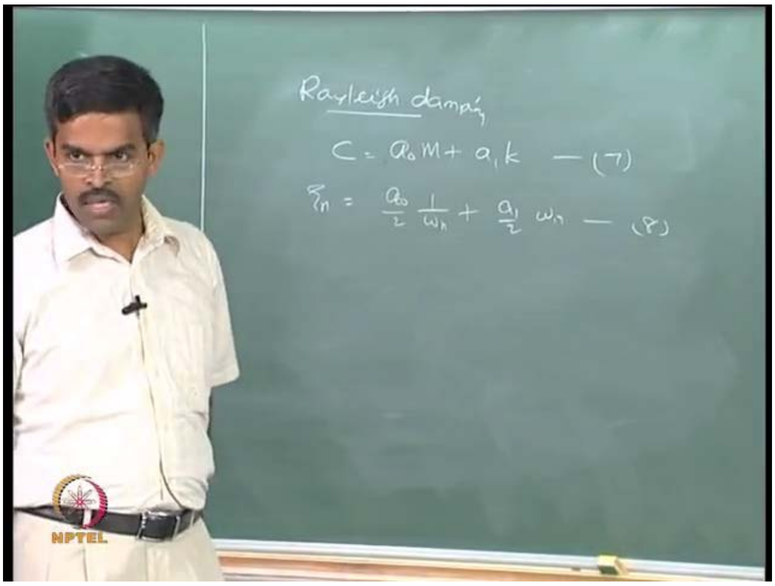

Let us try to find out what are those governing equations which will give me a 0 and a 1 because if we know a 0 and a 1 using equation 7, I can find c is it not because m and k are known to me is that clear. So, I must have a governing equation now, to find out a naught and a 1 for a zeta which agrees for omega i and omega j is that clear, this what a real damping is now let us get these equations I will take away this.

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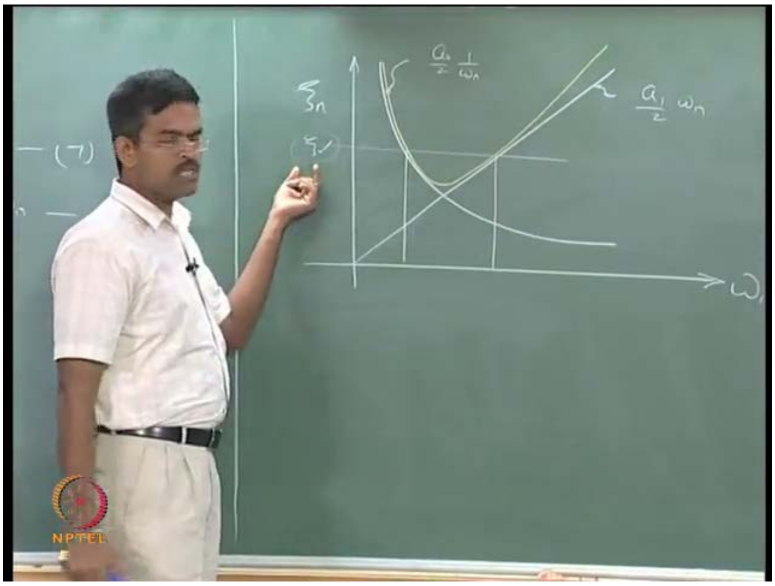

You must select closely spaced or widely spaced or enough spaced omega such that, your zeta has a common agreement between two omega's of your choice, in a given 6 degree of freedom omega 1, omega 1, omega 6 can pick up any two omega's such that zeta is agreeing for that, I will demonstrate an example here and show, how they agree and how they do not agree.

So, you have to keep on changing the zeta, such a manner that they agree for almost all frequencies which you are interestingly to contribute, but in TLP you have greater advantage you have got only two set a frequencies, one is either very high or one is either very low. So, I can easily handle this modal of i and j in my problem for a TLP because I have divided the system in such a manner I have got two groups of frequencies, either very high or either very low either very high either very high or very low. Probably if you agree on a specific zeta, we do not know actually we have got to try that.

That is guess, given omega 1, 2, 3, 4, 5, 6 should I say 1 and 3, 2 and 4, 1 and 6 that is guess that is guess we have got to try that which 2 degrees, generally you should not pick up closed values just for agreement omega 1 and omega 2 are very close pick up these two then you are missing. The whole idea is the zeta should represent larger range of frequencies that is the idea that is the idea is not. Suppose you do not have larger range of frequencies at all the whole frequency of a structural system is focusing only on a small zone, then I want to include zeta of all the frequencies then also we have an option, but not using Rayleigh. Rayleigh's idea is you must spread this for entire structure that is the idea. So, I want to get the governing equation for a 0 and a 1 let us strictly do that now.

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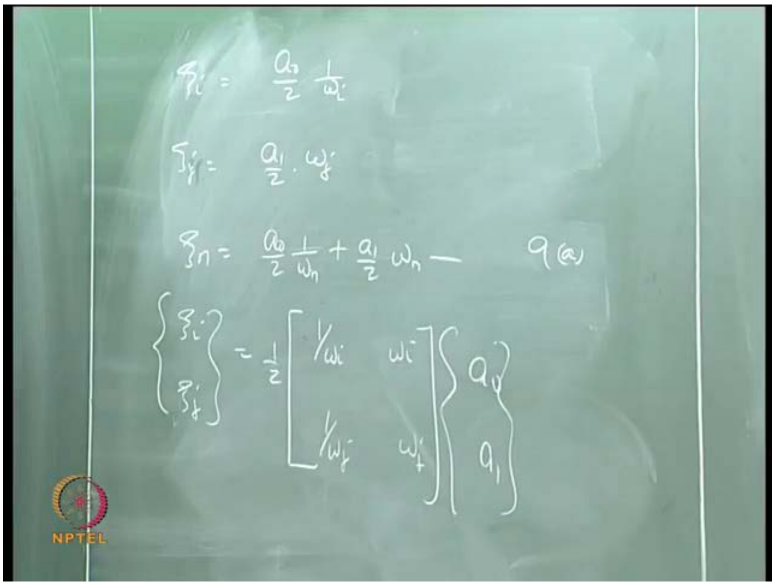

So, let us say zeta i I am expanding equation 2 now, zeta i is a 0 by 2 1 by omega i and zeta j is a 1 by 2 omega j is that and we all know that zeta n is nothing but a 0 by 2 1 by omega n plus a 1 by 2 omega n let me call this as equation number. Let us say 9 a let us expand it zeta i zeta j half 1 by omega i omega i 1 by omega j omega j of a naught a 1 is that I am just writing in a matrix form. I call this as to be matrix A, I want to find a naught and a 1 I will invert this matrix multiply with zeta get a naught and a 1 get the inverse of this matrix here.

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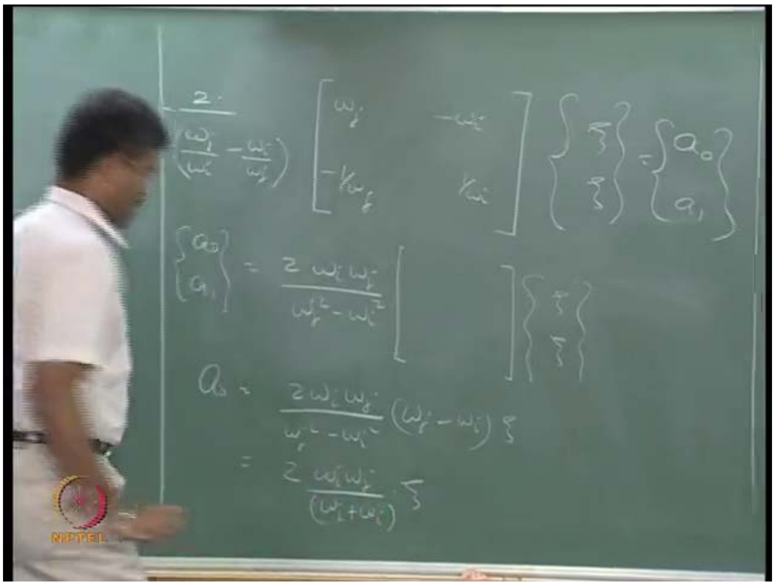

So, twice of determinant of omega j by omega i minus omega i by omega j of omega j minus 1 by omega j no sorry minus omega i is it I will remove this write it here, minus 1 by omega j minus omega i 1 by omega i that is my inverse, this is j this is also j of course, I think I will write it with more clarity this is j. So, omega j minus omega i minus 1 by omega j and 1 by omega I of.

Now, I have 2 zeta's which are same I can put zeta and zeta to get a naught and a 1 is that. So, can you give me the equation for a naught, simply multiply this in a proper format. So, that is going to be equal to, we can write this as a naught a 1 is given to be equal to 2 omega i omega j by omega j square minus omega i square of this matrix of zeta, zeta. So, a naught will be equal to 2 omega i omega j j square i square of omega j zeta minus omega i zeta is that. So, I simplify this I get twice omega i omega zeta by j minus i of zeta I call this equation number 10.

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Similarly, a 1 will be 2 omega i omega j by omega j square minus i square of minus 1 by 1 by omega i minus 1 by omega j of zeta, which will be 2 omega i omega j omega j square minus i square of omega j minus omega i by omega i omega j of zeta and this can be again cancelled I will get this as omega j plus i of zeta is that, that is my a 1 I call this equation number 11.

Now, from equations 10 and 11 we know a 0 and a 1 let us see how, if I know two frequencies j and I, if I assume zeta any value varying from 2 percent to 5 percent any value find a 0 and a 1, if I know a 0 and a 1 I can find C as simply this, so that is what my idea is to get that being matrix. Now, what are the problems assigned to be this particular method any doubt here, any doubt any doubt. So, we will be able to get C matrix because I know a, and m and k already I have computed a 0 and a 1, so I can find C matrix.

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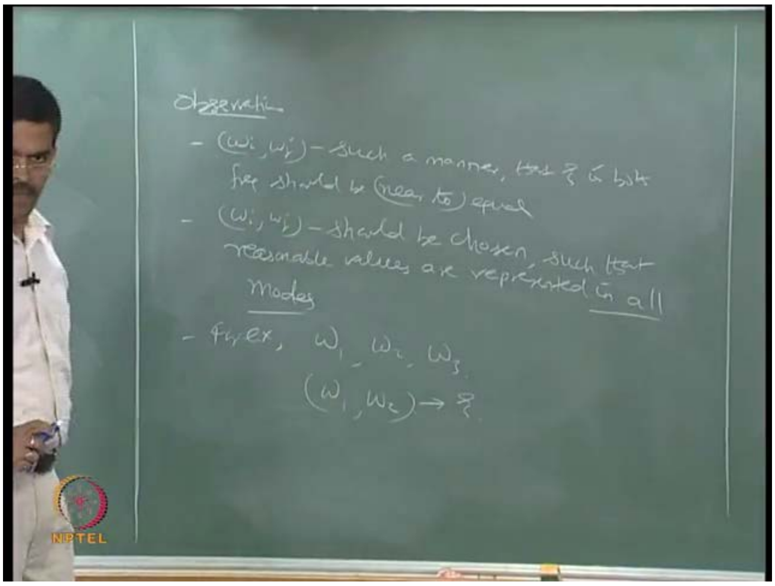

So, there are some specific observations of this method, one you must select omega i and omega j in such a manner that zeta in both frequencies should be near to equal. Further omega i and omega j should be chosen in such a manner, reasonable values are represented in all modes that is very, very important, you cannot simply pick up any arbitrary value of omega. And say, we are distributing it for the entire structure, you must check whether other modes also have the relative zeta value or not. So, how to do that, let us say for example, I have omega 1, omega 2 and omega 3 in my given problem, I pick up omega 1 and omega 2 for a specific zeta. So, when I pick up omega 1, omega 2 for a specific zeta I fix.

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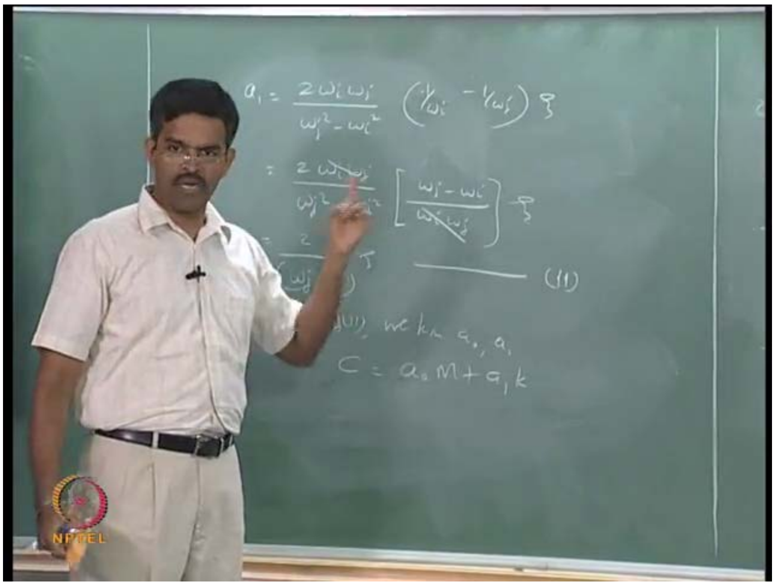

Omega i omega j and zeta I get a 0 and a 1 and I get C.

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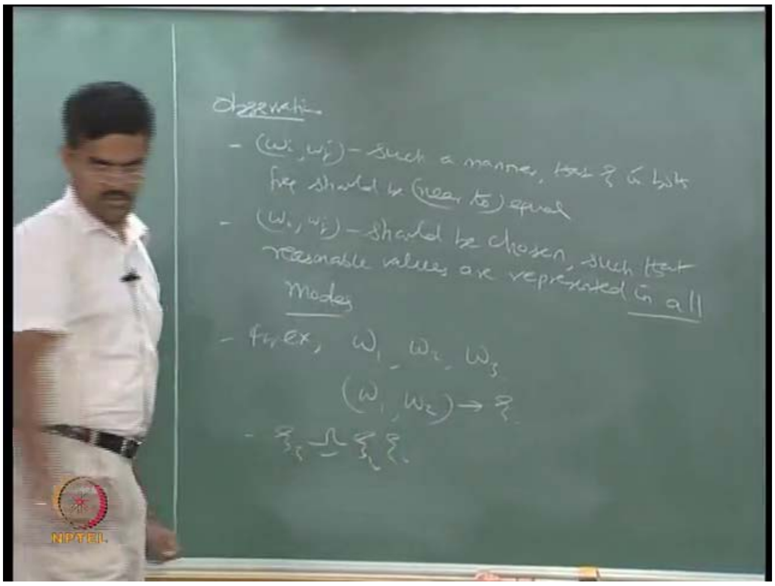

Then check back whether zeta 3 is also approximately equal to zeta 1 and zeta 2 that is the zeta in third mode is equal to 1 and 2 or not you just check this, that is how we can satisfied, if it does not change 1 and 2 to 1 and 3 and keep on iterating it and finding it out, it is very easy to look at the site here. We will take an example quickly, we will solve this in 10 minutes time let us say, it is a long problem I have got to find m, I have got to find k, I have got to derive the equation on motion, find omega's, find phi, find zeta and compute C and check at zeta 3 also, all has to be done in 15 minutes time for a three different system modal, we will pick up this problem.

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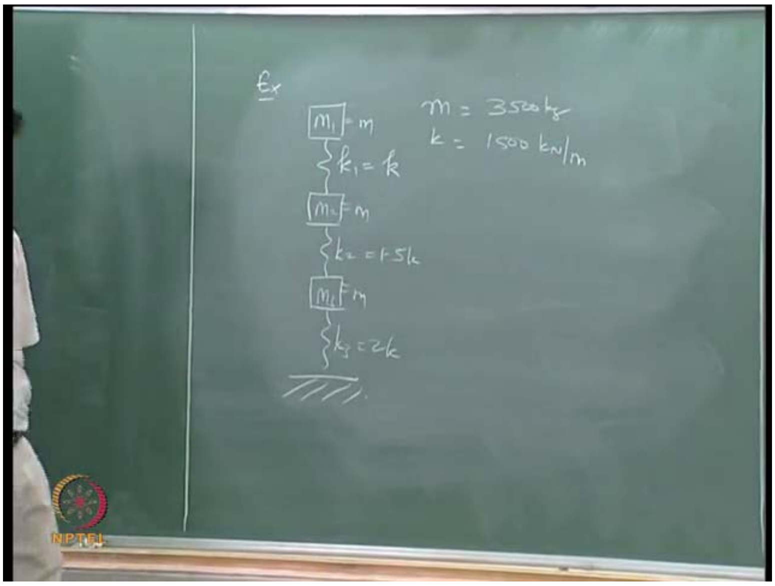

So, let us have an example, I call this as m 1, m 2 and m 3 which are nothing but simply same mass, I call this as k 1, k 2 and k 3 where as k 1 is k and k 2 is 1.5 k and this is 2 k, where m is 3500 kg and k is 1500 kilo Newton per meter. So, let us use third law to find out the fundamental frequency and more chief words. So, you are an experts in third law method. So, give me a reply or an answer for omega and phi 1 quick.

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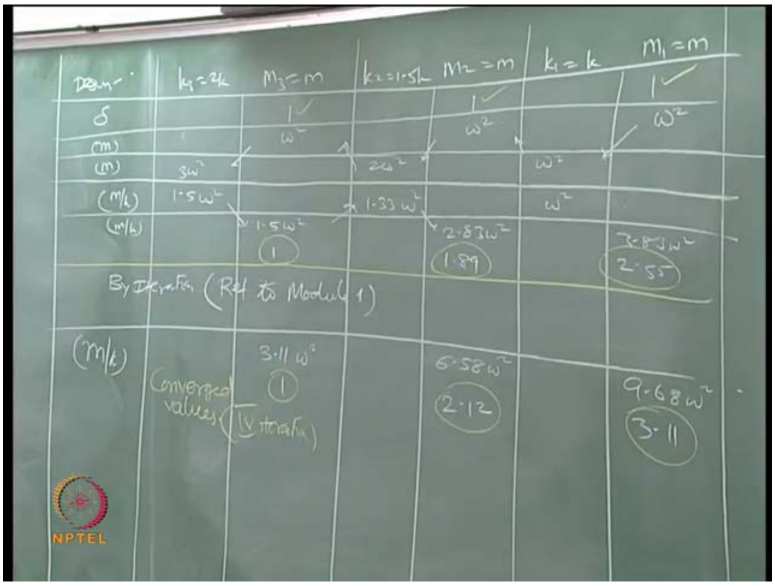

So, m 1 is m, m 2 is m, m 3 is also m, k 1 is k, k 2 is 1.5 k, k 3 is 2 k here description. So, I assume the deflection at mass points, then successively I agree these values omega square, omega square and omega square then omega square, 2 omega square of course, m is common here, m is common here then a dividable spring force I got 1.5 m by k omega square, 1.33 omega square, omega square cumulative deflection 1.5 omega square m by k common 2.83, 3.83. So, I get the ratio of 1, 1.89, 2.55.

So, that is the first band, these are the values we started with 1, 1 and 1 we got some other value, so they are not converging. So, iterate kindly do this and tell me what is the convergent value, quick, I will give you 3 minutes time, otherwise write the answer here. So, by iteration, I think we should say, refer to lectures of module 1, I have the converged values like this m by k multiply arrow you please check whether I am right or wrong 3.11 m square sorry 6.58 and 9.68. So, I have the multiplier as 1, 2.12, 3.11 that is my final answer, these are converged values at 4’th iteration. So, let us quickly work out omega and do it here.

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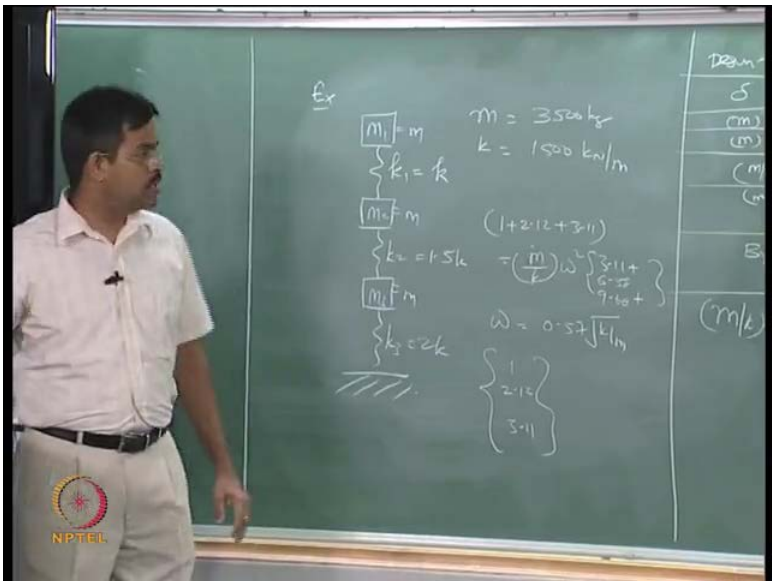

So, 1 plus 2.12 plus 3.11 is equal to m by k of omega square of 3.11 plus 6.58 plus 9.68 this gives me omega as 0.57 root k by m and phi 1 becomes 1, 2.12 and 3.1. I will give you another 5 more minutes, you can try to work out omega 2 phi 2, omega 3 phi 3, using influence co-efficient technique that may take some time you can verify it later I will give you the answers now. Do you all agree with these answers? At least is there anybody who has done these two answers or not, yes or no, yes or no, you try this.

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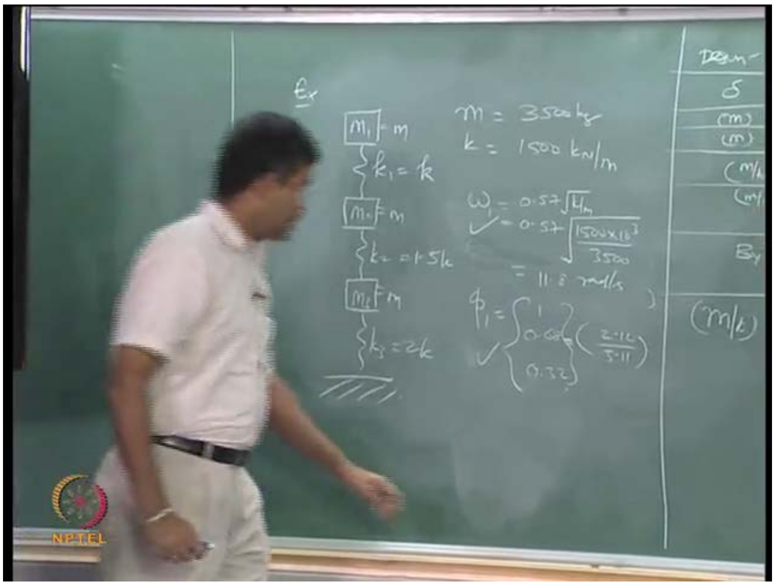

So, now, I am writing omega 1 phi 1, omega 2 phi 2, omega 3 and phi 3. So, omega 1 was equal to 0.57 k by m which will be equal to 0.57 of 1500 into 1000 that is Newton per meter divided by 3500 kg I get this as, so many radians per second which comes to 11.8 and phi 1 I am just slightly rewriting it in a different fashion because I always wanted, the top deflection should be the unity with respect to which, what will be the proportion of this that is an order generally given because I am like always speaking at the tip deflection in any system.

So, but the phi what they had the tip deflection was 3.1 where as the bottom was 1 I am just reversing the proportion. So, it will become now 1, so 2.12 by 3.11 which is coming to be 0.68 which is nothing but 2.12 by 3.11 I am just writing it here for your understanding and of course, 1 by 3.11 0.32 this my phi 1 omega 1. Now, similarly for phi 2 and omega 2 and phi 3 and omega 3.

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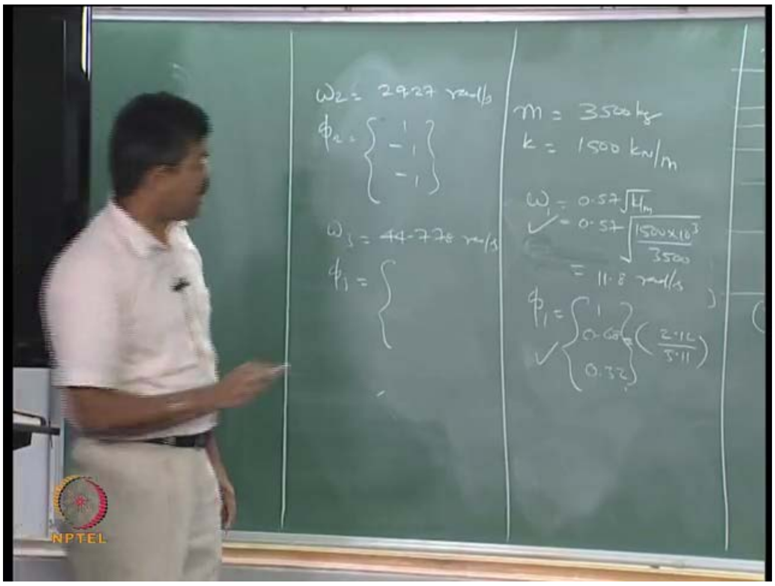

I am writing the values here 29.27 radians per second and phi 2 was 1 minus 1 minus 1 there is only one 0 crossing here, from positive to negative it remains negative later. So, there is only one 0 crossing. So, second mode phi 3 was omega 3 was 44.778 radians per second and phi 3 was 1 minus 3.68 plus 4.6820 crossing per mode now, the mass matrix I will remove this.

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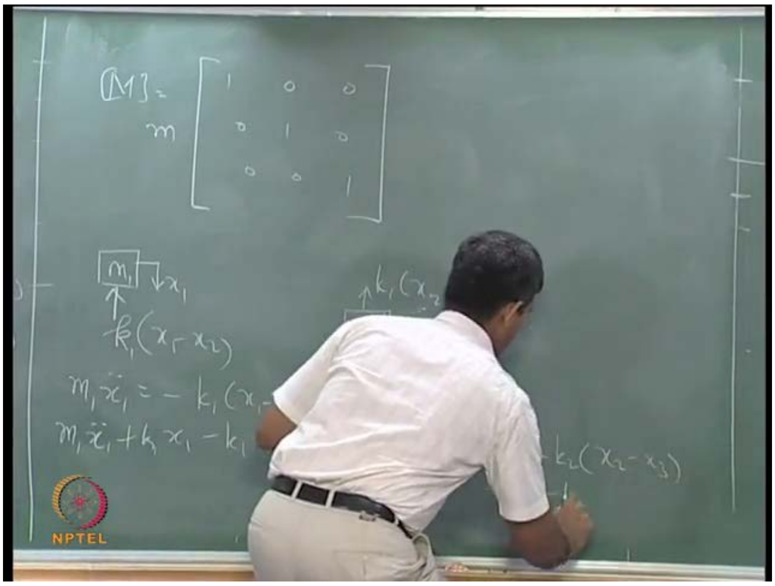

The mass matrix, if your diagonal matrix because my d is a freedom on measured on the point where the mass is lumped which is equal to m of I want to write the stiffness matrix. So, let us write the equations of motion and write this stiffness matrix for this. So, let us pick up the first mass m 1, which is acting at x 1 trying to pull it down and this spring which was k 1 will push it up, but of course, with the relative displacement.

So, I must say m 1 x 1 double dot is minus k 1 of x 1 minus x 2 which will give me m 1 x 1 double dot plus k 1 x 1 minus k 1 x 2 is 0 that is my let us say first equation of motion. Similarly, the second mass which is m 2 is again equal to m moving downward way x 2 whereas, this spring is trying to push of x 2 minus x 3 as well as x 2 minus x 1, we already said, you must use the first co-efficient at the point where you are measuring and marked arrow direction accordingly. So, equation of motion in this case will be m 2 x 2 double dot will be minus of k 2 of x 2 minus x 3 minus of k 1 of x 2 minus x 1 rearranging I will remove this.

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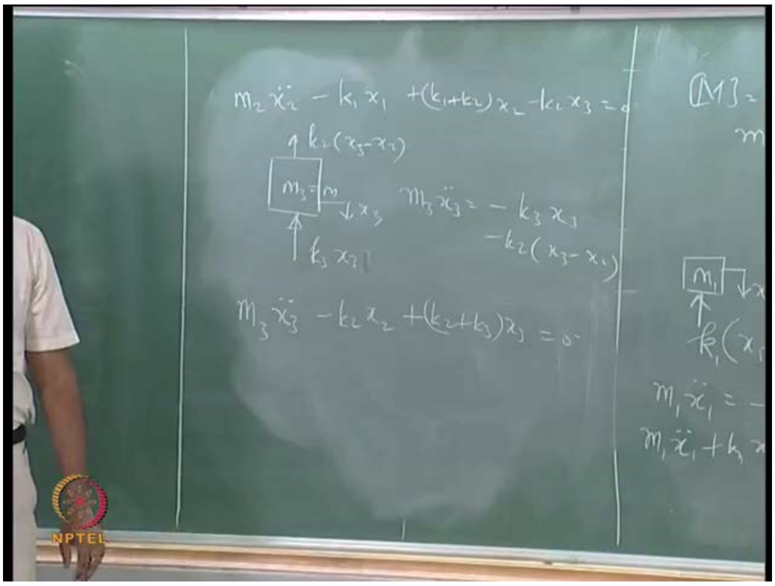

Minus k 1 of x 1 plus k 1 plus k 2 of x 2 minus k 2 of x 3 is 0 that is my second equation of motion. Similarly, third mass m 3 which is also equal to m I am pushing it down by x 3 this spring which is k 3 of x 3 will push it up where as the second spring here, which is k 2 of x 3 minus x 2 will also push it up. So, m 3 x 3 double dot minus of k 3 x 3 minus of k 2 of x 3 minus x 2, rearranging m 3 x 3 double dot minus k 2 of x 2 plus k 2 plus k 3 of x 3 will be 0 that is a third equation of motion. So, pick up only the k terms let the k matrix I am doing it here.

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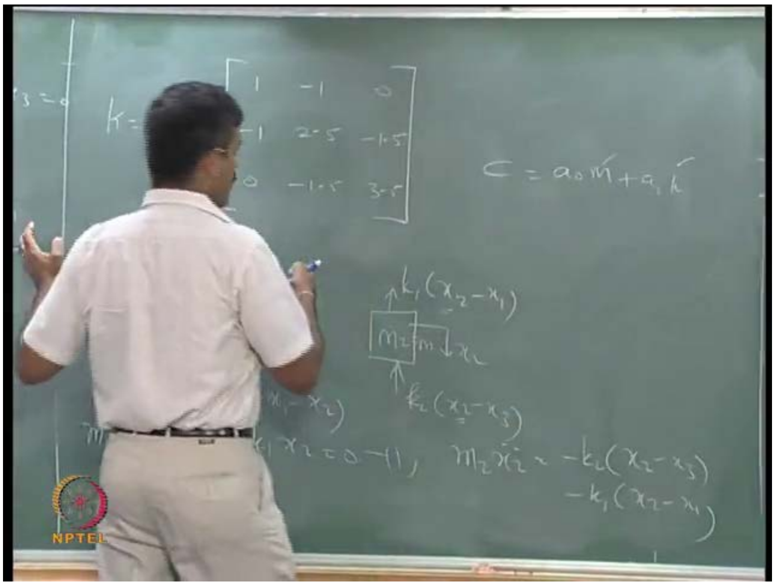

So, k times of with x 1 it is 1 with x 2 it is minus 2 and of course, x 3 it is 0 is that right, look at the second equation of motion which is here, with x 1 it is minus 1 and already we know k 2 is 1.5 times of k 1. So, I should say plus 2.5 and this is minus 1.5 is that. Look at the third equation with x 1 there is nothing, with x 2 it is minus k 2 sorry, I should say minus 1.5 with x 3 it is k 2 plus k 3 1.5 plus 2.5. So, nothing but 1.3 and 2, so 3.5 this is my k matrix, why I am interested in finding out m and k separately because to find c I must have a 0 m and a 1 k. So, I must have these two otherwise I cannot find c.

So, I have got m and k now, I have got governing equations for a 0 and a 1 I have got three, omega's 1, 2 and 3 pick up any two for the specific zeta value assumed by us and check whether the zeta is confirmed third frequency also or not, that is all we reef the Rayleigh damping. So, let us do that quickly, we have only just about three minutes time, but still I want to complete this problem I will complete this because any doubt here, any doubt, so we have got equations for a 0 and a 1.

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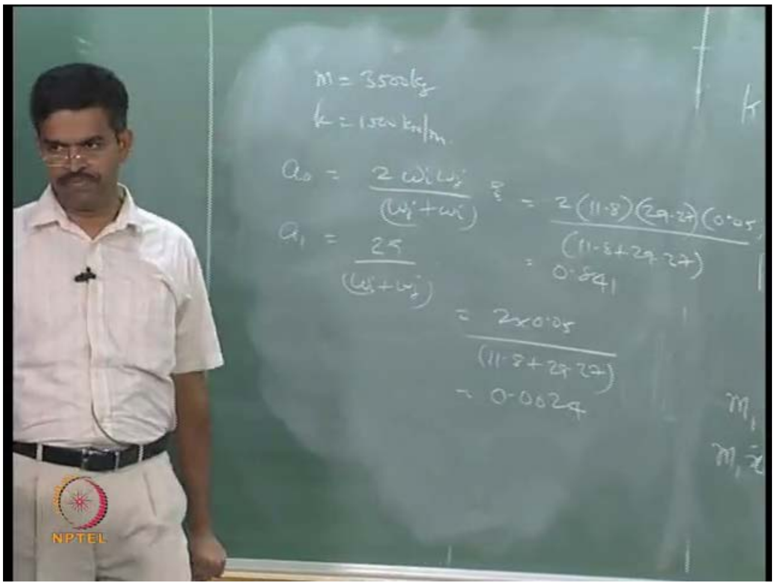

We already know m is 3500 k g and we know that and k is 1500 kilo Newton per meter we know this and a 0 and a 1 the equations were 2 omega i omega j by omega j plus omega i of zeta. Whereas, this is 2 zeta by omega i plus omega j is that i am substituting here, twice of omega 1 was 11.8 and pick up one and two 29.27 I am working for 5 percent damping 0.05. And of course, divided by 11.8 plus 29.27 this will give me a 0 as 0.841 you can check up that later a 1 will be twice of 0.05 by 11.8 plus 29.27 which will become 0.0024 very marginal contribution from stiffness, very major contribution from mass. So, I have a 0 and a 1 I have m and k I can find c I want you to do that, but since we have no time we take an advantage of that and give you the answer of c quickly.

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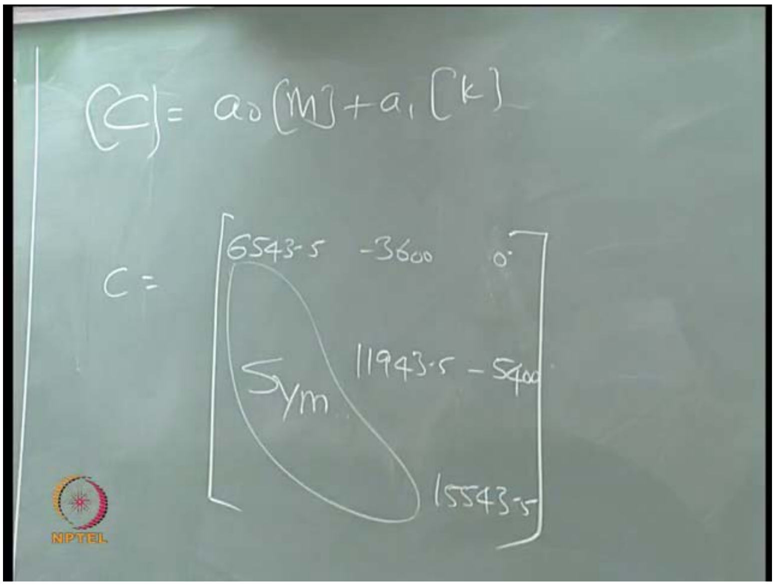

So, c will be a 0 m plus a 1 k a 0 and a 1 are known to me I have m and k I get c matrix as 6543.5 minus 3600 0 1194, sorry 943.5 11943.5 minus 5400 15543.5 this is my c matrix using Rayleigh damping. Now, to check zeta 3 now see my whole argument here I will rub this.

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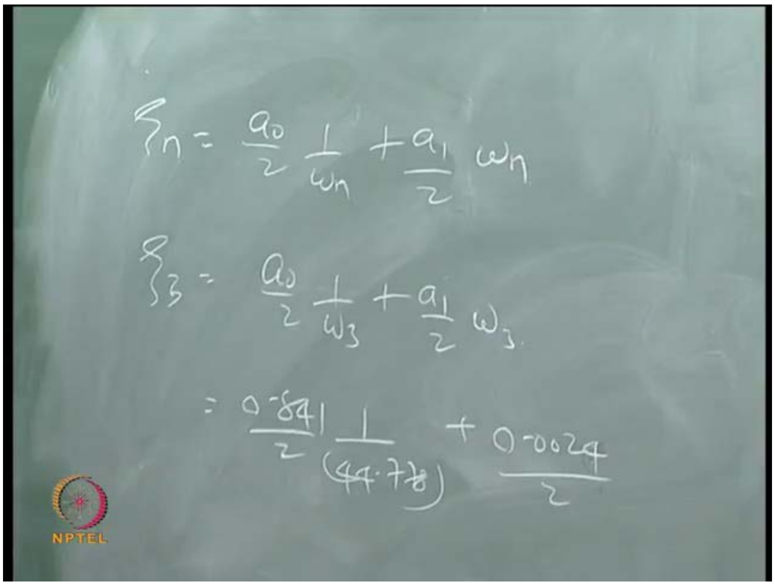

We already know zeta n is given by this equation. So, let us find zeta 3 for the known a value for omega 3 which will be 0.841 by 2 of 1 by 44.778 plus 0.0024 by 2 of 44.778 which gives me 6.3 percent which is agreement phi. So, I can apply Rayleigh damping for this problem. That is how we will check, whether we have taken the correct value, if I does not agree keep on changing zeta, keep on picking different omega's and c because idea is you may distribute the modal damping ratio for the entire structure.

So, hypothec ally you pick up either loss space frequency or closed space frequencies depending upon what zeta you want. So, now, this answer has a major problem that for a given such a system for a known mass for a known stiffness I can accuse data value, I can find omega's first I can find c matrix, which is a combination of inertia damping and stiffness damping together.

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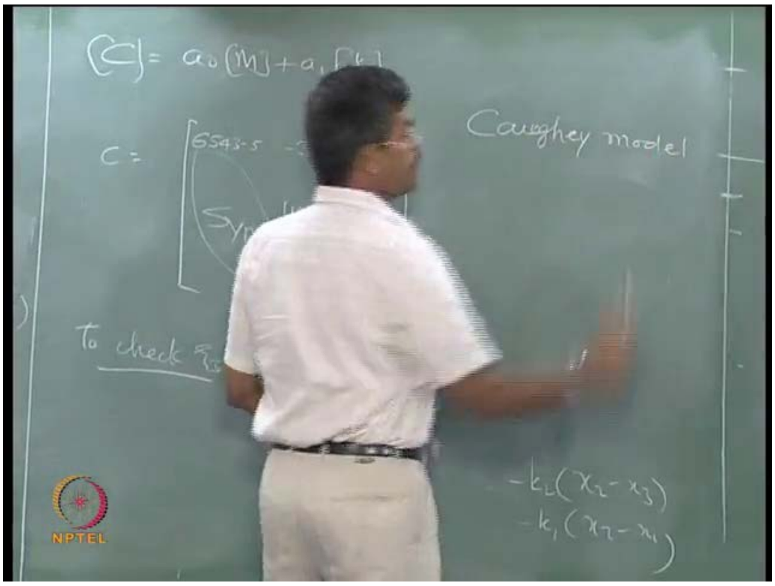

The next modal what we discuss, will also be applicable to FSI values which is Cauchy model, Cauchy model is an improvement of Rayleigh damping it says that let us add a 0 k a 1 m plus something different, why that is added, how it is added how it is improved by Rayleigh we will see in the next lecture.