Dynamic of Ocean Structures
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Lecture - 7
Iterative Frequency Domain I

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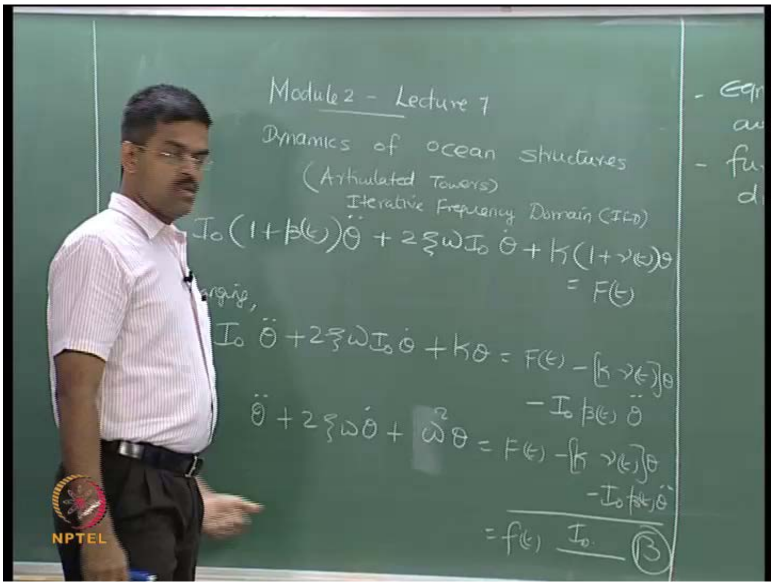

So, in this lecture we will continue with the discussion what we had in the last class. This lecture will be lecture on 7 on module 2, where we are focusing on dynamics of ocean structures and example application to articulated towers. We picked up an example in the last lecture, we wrote down the equation of motion. So, let us rewrite the same equation once again and see what would be the scheme of solution what you want to handle for this specific problem which we call as an iterative frequency domain. This was the equation what we had as equation of motion in the last lecture. I will rearrange this back again which we already did. So, further we rewrite this equation slightly in a different manner. See, I call this as f of t, which I write as equation number 13.

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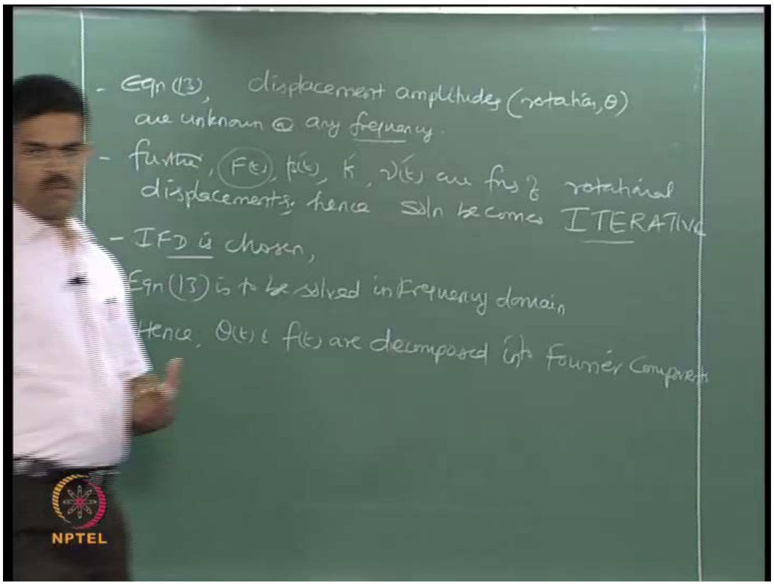

So, in this equation number 13, the displacement values, the displacement amplitude basically in my case displacement are rotation and the base of the tower are unknown at any frequency, there are unknown at any specific frequency. Further, the F of t, beta t K, and nu of t are functions of rotational displacements. Hence, the solution becomes iterative. Why iterative, because the right time of the equation of motion F of t is a function of the left hand say the equation of motion which is displacement. So, you do not know actually because as long as you do not know the displacement you will not able to solve this equation of motion.

So, let us assume certain value and K point iterating. In addition all these functions are also being shown in the last they are functions of displacements which are none linear as well. Therefore, we follow iterative frequency known that is the reason why we follow this procedure. The moment I say is chosen then, equation 13 is to be solved in frequency domain. Equation 13 is the rearranged the equation of motion actually. The moment we agree that, I must solve equation 13 in frequency domain then, I must decompose the task and F in Fourier components because I want to look for the frequency content of the loading. Therefore, theta of t and f of t are now decomposed into Fourier components. The moment I say Fourier components then, they will have cosine and sin value and fix it amplitude. So, let me write those values here.

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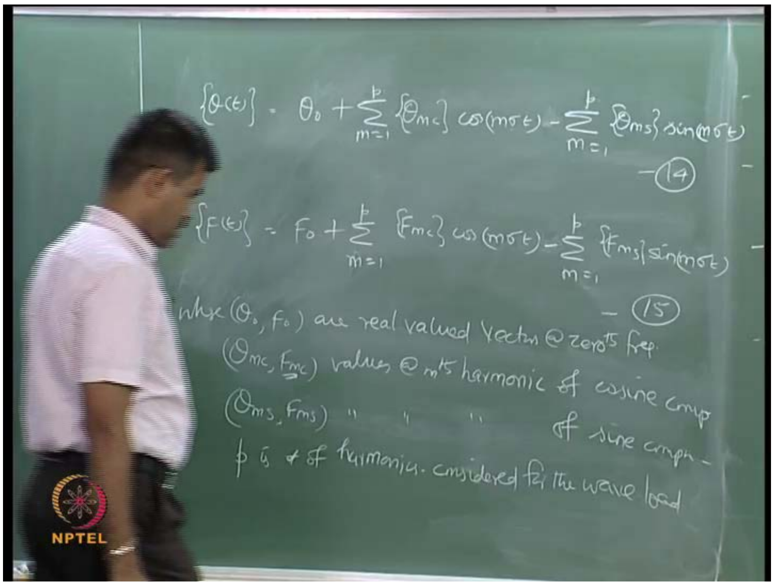

So, theta of t and F of t can be written as theta 0, F 0 summation m equals 1 to p of the sin components and the cosine components. So, I say theta m c cos m sigma. So, here I write F m c cos m sigma t, this is negative component. In Fourier component you will see this is going to be negative, because of the coefficient become negative. Theta m s and F m s, sin m sigma t sin m sigma t, I call this is equation 14 and 15. So, where, theta 0 and F 0 are real valued vectors at 0-th frequencies. Whereas, theta M c and F m c are values at m-th harmonic that wise, this M is there of the cosine components whereas, theta m s and F m s are values of sin components. Let the summation p is the number of harmonics you are considering in your analysis summation p. Consider for the wave load.

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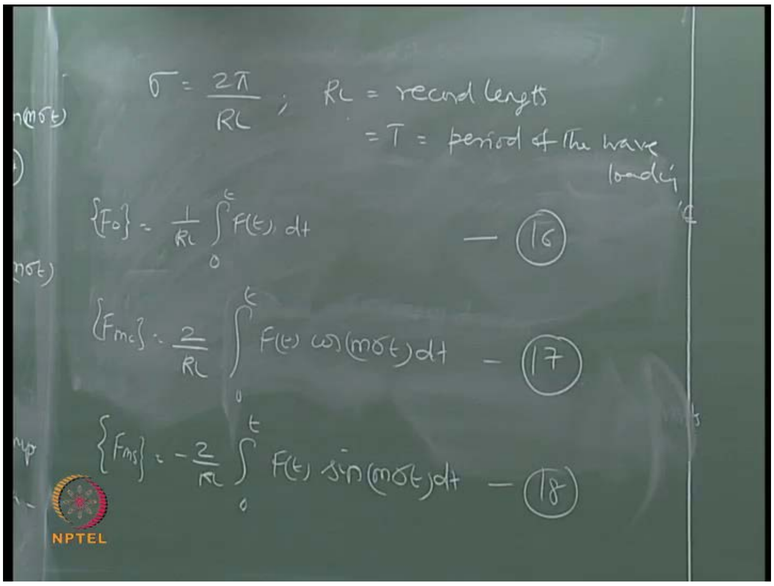

And of course, sigma is 2 pi by R L, where R L is call record length or sometimes is also called as T, which is period of the wave loading. I can also find F 0, F m c and F m s in further integrable form because they are in summation series, I can easily find out that as F 0, F m c and F m s can be given as 1 by record length of 0 t of F of t. These 2 way records length of 0 to t dot of F of t cos m sigma t d t, whereas F m s will be minus 2 be record length of 0 to t F of t sin m sigma t d t.

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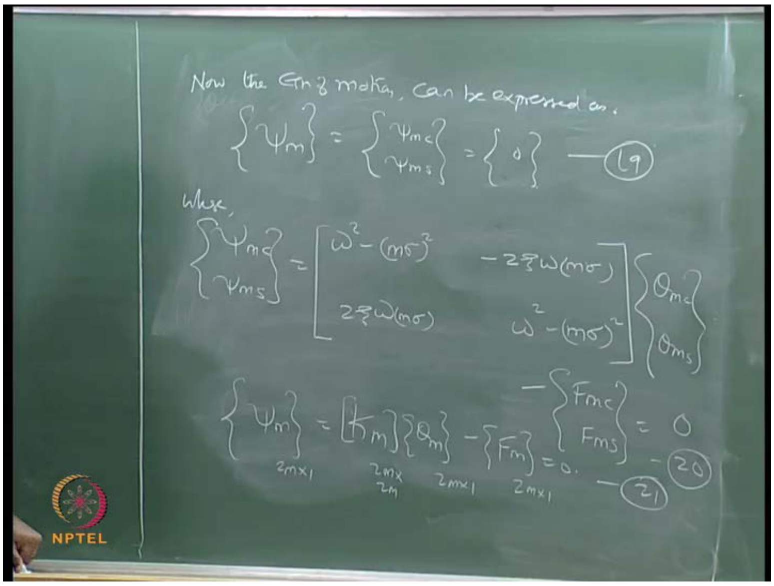

Let me call this equation as 16, 17, and 18. Now, the equation of motion can be expressed as let us say in simple form, I say psi of m psi m c and psi m s, this is equation number 19. Where, psi m c and psi m s can be given as a matrix omega square minus m sigma the whole square minus 2 zeta omega m sigma 2 zeta omega m sigma omega square minus m sigma the whole square of theta m c theta m s which is equal to, sorry minus because I need the minus value also as well, minus F m c and F m s which can be not set to 0. That is RHS, equation 20. In simple terms equation 20 can be rewritten as psi of m, K m theta m minus F m is 0, I call this as 21. So, these are all vectors of size 2 m by 1 whereas, this is size 2 m by 2 m.

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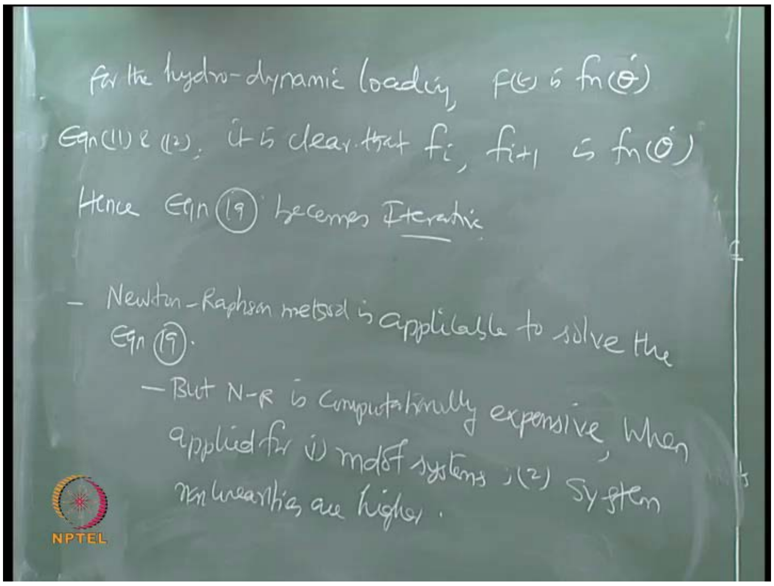

Now, let us look at the hydro dynamic loading part. Now, for the hydro dynamic loading, we already know that F of t is a function of theta dot. You just see equation number in your literature 11 and 12. If you look at equation number 11 and 12, it is clear that f of i or f of i plus 1 is a function of the angular velocity of the tower. And therefore, hence equation 19 becomes iterative. Equation 19 is the modified version of equation of motion in frequency domain. Now, how will you solve an equation which is iterative in nature in frequency domain. One can use the classical Newton-Raphson technique equation 19. But, the Newton Raphson technique is computationally expensive when you apply for 1: high multi degree freedom system models, 2: when the system non linearity is high.

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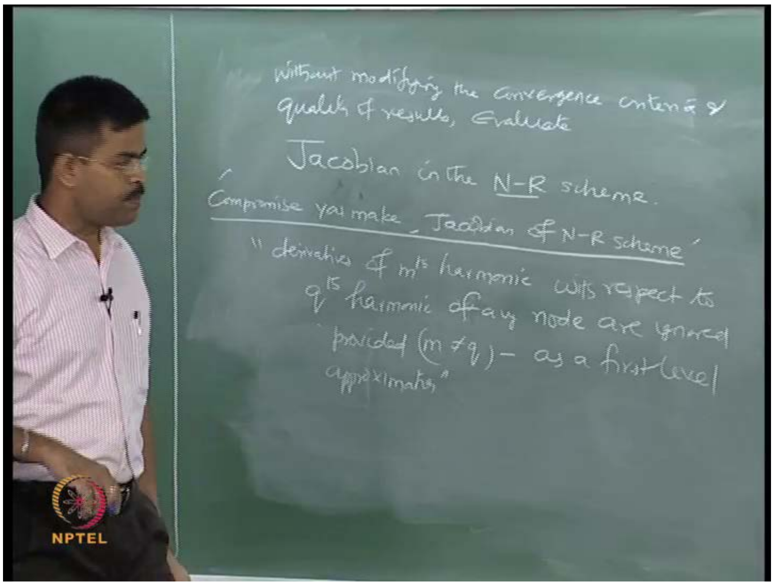

So, what is alternate solution for this people use Jacobian so, without modifying so, without modifying the convergence criteria and quality of results, one can evaluate so, evaluate Jacobian in the scheme. N-R stands for Newton Raphson scheme. So, now the question is what is the compensation you make? Or what is the compromise you make when you evaluate Jacobian of N-R scheme because we must know the limitation of this problem we make certain compromise on this. Let us clearly state what is the compromise we have to make? Then, follow a simple way solution of the nature here. The compromise is, we already know that the f of t is the function of theta dot so the derivation of the displacement.

So, if want really find F of t at i is equal to 1, you must have the derivative of the displacement at i equal to 1 because, it depends on the theta dot at the deterioration. What I will do here is, I will take the theta dot of the previous iteration and use it for the next iteration ok. That is the compromise I am making here. The F of t should be function of theta dot of the deterioration itself but, what I am trying to do here is, pick up the theta dot with the previous iteration use it for the next iteration. That is the compromise I am making. That way the solution can become slightly simple.

So, I write the derivatives in plural because there are theta dot and theta dot and both are available in F of t. So, the derivatives of m-th harmonic of mth harmonic with the respect to qth iteration or qth harmonic of any node are ignored. Provided m is not equal to q.

This is a first level approximation. This is the compromise I make in evaluating Jacobian for the Newton Raphson scheme.

So, proceeding further let us set the equation for any let us say rth iteration and see what happens. Any doubt here? So, I leave a question here, that you study some material an advance mathematics or applied mathematics and try to understand for a given function how to evaluate Jacobian’s they are available in standard literature. If you want I can do a small problem in the next lecture just to take up function and evaluate Jacobian for that function. You must know how to evaluate Jacobian for a given scheme. Any question here so far?

So, we moved on to equation number 21 where, I have set the equation of motion of in the frequency domain which will be obviously iterative. Since, it is the iterative and the non-linearities are higher, I am not using the exact N-R scheme for this problem. I am modifying it by evaluating the Jacobian of the N-R scheme where, I compromise saying that F of t depends on the derivative of the iteration itself, but I am picking up the previous value and keep on iterating. That is what I am trying to do. So, there is the small it could create certain error but, this will have a faster conversions and it will become it is proven to be comptetional less expensive compare to the original scheme or N-R.

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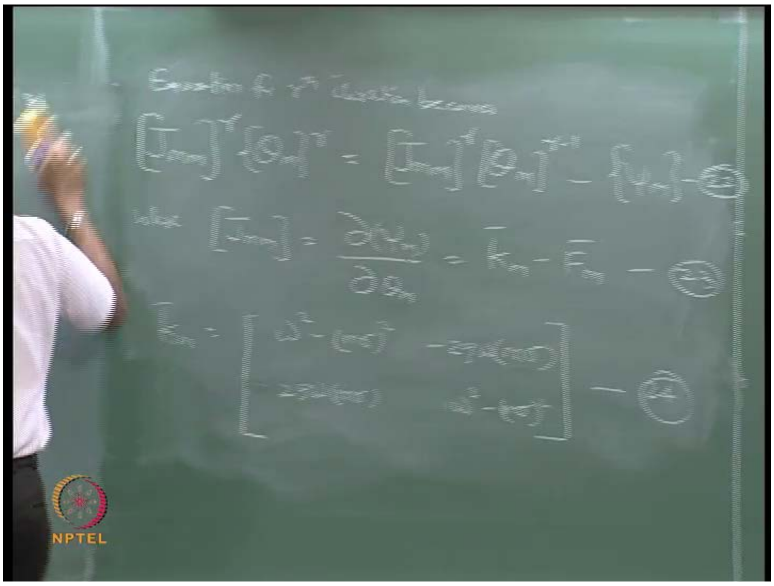

Therefore, the equation for rth iteration becomes. Now, you would not have the governing equation for the iteration scheme where, I will now use the Jacobian that J m m of r, that is a Jacobian, theta m of r is equal to, these are all r they are not mu rth iteration some iteration in the scheme. So, simply equal to J m m of r theta m of r minus 1, that is what I am doing here r minus 1 the previous 1, minus of course, the psi values that is my frequency components of the 4 c function so 22. Is the number running right? Whereas, J m m is given by duo psi m by duo theta m that is a Jacobian which is alternatively K m bar minus F m bar. 23.

So, partial derivative of the frequency component of the forcing functions with the top of the displacement because I am actually finding out the solution on the displacement that is why the denominator in this case is displacement, where K m can be written as K m bar can be given by the original equation what we wrote earlier let me repeat it again here. Omega square minus m sigma the whole square minus 2 zeta omega m sigma 2 omega m sigma omega square omega sigma whole square minus m sigma the whole square which is K m bar. Though I am numbering it again but, already we wrote this equation. We already have equation with us.

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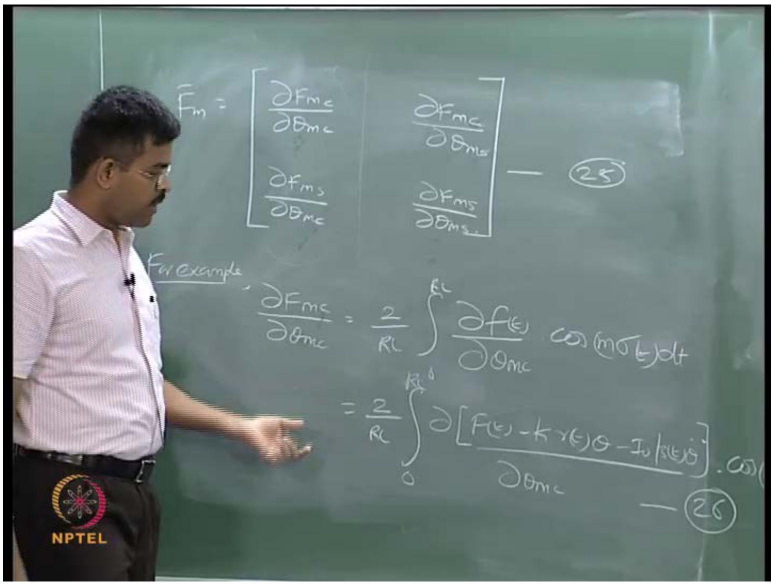

Now, F of m is tricky that were the problem starts. This is given by again 2 by m by 2 m matrix where, I say see here how am I writing it m c m s this also the m c this rho of m c this is rho of m s this is rho of m s. This is m c and this is m s.

So, this column this column is a partial derivative of the forcing function of the frequency components cosine and sin with respect to theta of the cosine component only.

Whereas, this column is the partial derivative the forcing function of cosine and sin components respect to the sin components of the displacement that how it is written. This is equation number 25.

Just for a sample we will pick up one derivative and see how we can evaluate this one derivative one value. So, let us say for example, I want to evaluate m c by theta m c that is duo F m c by duo theta m c is nothing but, 2 by the record length of 0 to record length of duo the forcing function with respect to theta m c of the cosine component because I am talking about c here, which is cos m sigma t d t, which can be given as 2 by R L of I m expanding this F of t already we have F of t with the as I wrote in equation number 30.

I am re-writing it again so, 0 to record length of derivative of F of t minus k mu t theta minus I knot theta of t theta double dot by duo theta m c multiplied by cos m sigma t d t. I write it here I write it here cos m sigma t d t that is my derivative. I call equation number 26. So, we are now setting an equation for an rth iteration which involve certain derivatives of the conveyance that I am explained those derivatives here from the original equation of motion rearranged in the frequencies that is what I am trying to do here. Now, here the interesting part is if you now, look at the ith node of F of t, how it will look like because this is not and F of t of the whole problem is node by node hour to travel is it not. I divide the tower into the n number of nodes. I also divide the wave loading into n m number of harmonics that is different that iteration is separate.

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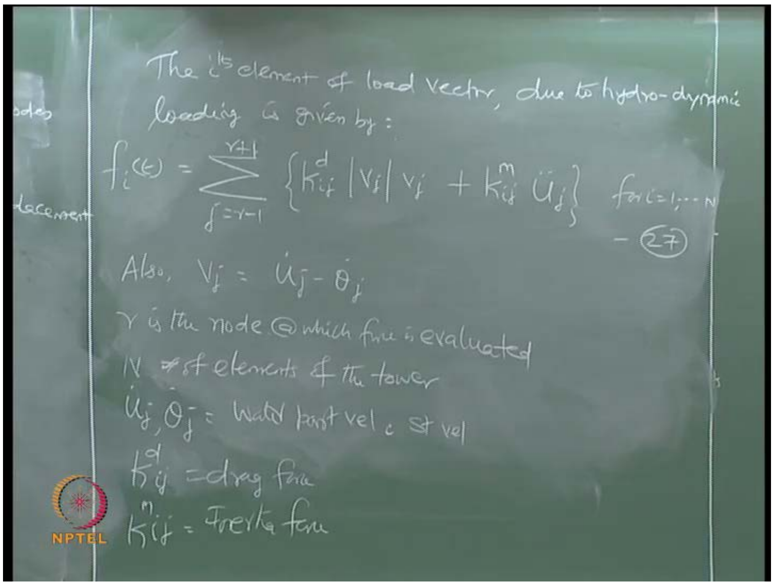

I am dividing the tower n number of node i i plus 1 i plus 2 and so on. Let us look at ith node how is F of t looks like. So, the ith element of the load vector the ith element of the load vector due to of course, I am taking only hydro dynamic loading I can also do this for error dynamic loading, I am in this lecture we will discuss and focus only on the hydro dynamic part of it, is given by is given by can you attempt the write this equation let us see we already have it we already know the classical (( )) equation for this. Can you write what, would be F of ith of t, why I am saying F of ith small letter because it is per unit length?

So, it is going to be summation of j is equal to r minus 1 to r plus 3 nodes that is how Newton Raphson works. I rewrite it on different notation actually k i j of d, please follow this notation. I will explain this now. V j and b j that is the drag component that is the drag component plus k i j of m, I call it this is mass component or inertia component you double dot of j.

And this summation runs for this from r minus 1 to r plus 1, where, i vary from for i equal 1 capital N N is the number of divisions or elements in your tower also. I call this equation number 27. Also, we know that v j is the relative velocity between the water particle and the tower. r of course, is the node element. r is a node at which you are evaluating this force. r is a node at which force is evaluated. Is my handwriting legible from the back is it legible. Even if it is not legible I think you have got a because I am writing faster. And of course, n is the number of elements of the tower. Of course, u dot and theta dots are water particle velocity and structural velocity respectively. k d i j is a drag, is the drag force and k m i j is a inertia force of F of t.

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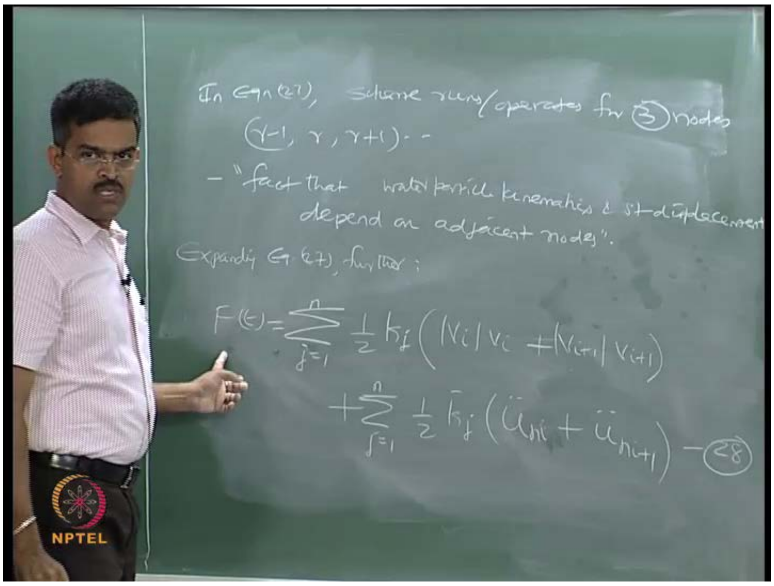

Now please note that is the equation 27, the iteration or the run scheme runs for or operates for 3 nodes that is, r minus 1, r and r plus 1. That is, how the scheme will work actually scheme needs 3 points. So, what is it mean is the scheme reinforces the fact that the water particles kinematics and structural displacements depend on adjacent nodes. That is a fact what is the re enforcing. So, let me explain equation 27 further.

Any doubt here yes or no.

Please go back few minutes see the whole scheme how we started the problem from iterate the field equation of motion and see few minutes; you must understand the flow, otherwise at the end of this lecture you will thoroughly understand it. Please go through it few minutes- where we started and where are we and what actually we are looking for. If you have any doubts you can ask me. In fact, I will give you tutorial sheet on this I will give you a tower I will divide this into 10 parts and you want to I wanted to solve this problem for a give reloading for a specific water depth and wave height and wave period. You write a program and try to solve this and show me the results.

Let us expand this f i of t slightly in a different manner. Well this takes care of the different nodes. We already saw that I want implement itself into this body so, I will rewrite it again so, expanding equation 27, further if I expand this because I want to take this iterative scheme of the different node dependency into the body of the iteration itself or into the body of the F of t. So, now I can write f of t now depend on you remember this is actually a summation, I am expanding it and getting F of t .

Depends on j is 1 to n half k j that i summation I am removing that is a node number which is V i of V i plus V i plus 1 of V i plus 1. Is that ok? That is what we also wrote earlier if you look at 2 equations again the numbers 11 and 12. I am writing it here back again and k j stops, that is my drag force, plus j is equal to 1 t n again half k bar j u double dot n plus 1 sorry n i plus you double dot n i plus 1, this is i. I call this equation number 28 where, k j and k bar j are given by the following. What I have done in equation 28 is that I have just imposed the summation of nodes into this and expand it on the base of equation 11 and 12 which we wrote earlier.

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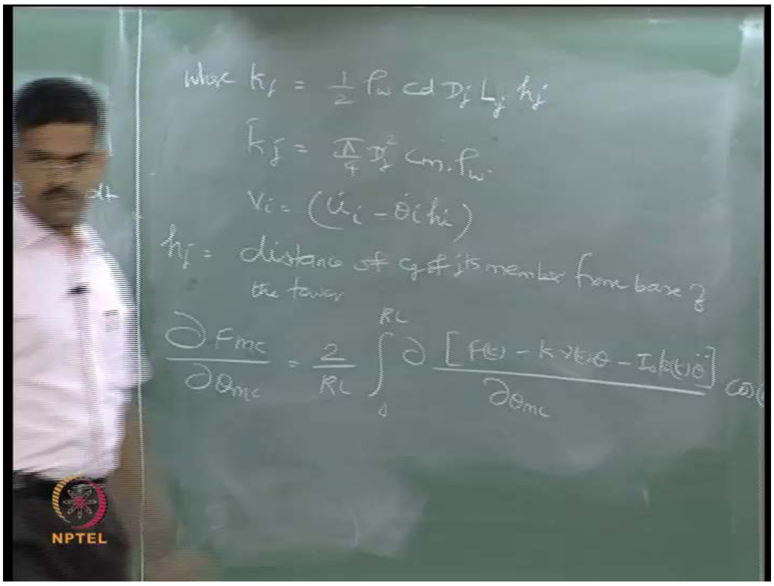

Now, let us see what this k j and k bar j looks like. Where, k j we all know that still half rho C d D j L j h of j. Now, why I am multiplying with L because I am looking for the capital F of t multiplying the length of the number, why I am looking for h of j multiplier here because I am looking for the moment of this forces above the base of the inch. I am not looking at the forces at all, I am looking at the moment of the forces because when degree of freedom is rotation not displacement. So, that is why the multiply h j is there. And k bar j obviously, I think you can write this which inertia component, which will be pi d square by 4 c m of course, rho w.

Nevertheless, we can write V i in this equation as u i minus theta dot, it is a relative velocity theta h of i. Can you tell me, what is h of j correct definition? Anyone what is f of j? This is not c this is L, I rewrite it again here this is L length of the member and this is subscript. Why I am multiplying with length because I am getting capital F for t small f of t was per unit length of the member. So, I am multiplying this with L.

What is h of j? H of j is the distance of c g of the member jth member from where, base of the tower. Now, let us look at the derivative of one example we have picked up one derivative from the Jacobian which is duo F m c by duo t times. Let us see how this looks like. Now, because I have expanded F of t, this will now become 2 by of record length of 0 to record length of duo of the derivative of F of t minus k nu t. This is now, r of t this is nu t of theta minus i beta the theta double dot by duo theta m c of d t which is expanded further because it has already return.

Now, the cos component and F of t has got again a cos component or a sin component depending upon what component looking in the Fourier. So, if you expand it further, let me do it here. So, I take a partial derivative of the cosine component of the m-th frequency and again there is a multiple already available in m sigma t and this has got 2 components 1, 2 and 3.

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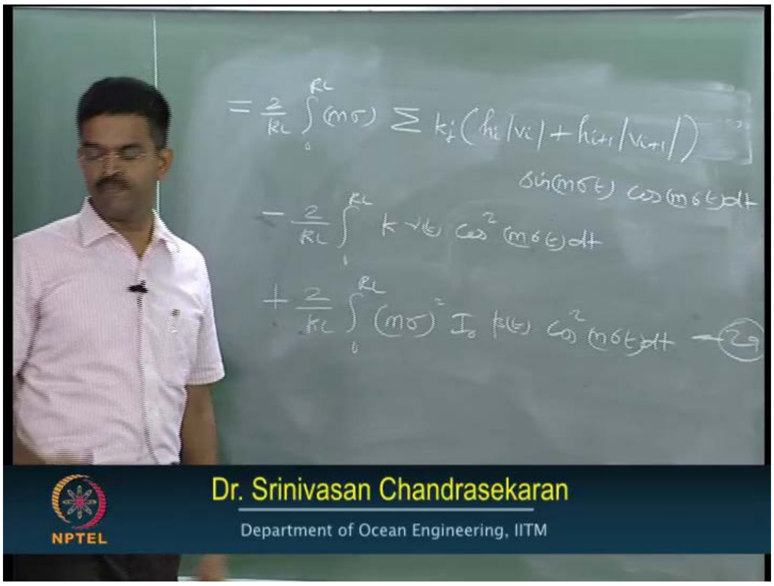

Let me do it here, that is going to be equal to 2 by record length of 0 to record length of so, I am taking a derivative there is a cos m sigma. So, m sigma summation of k j, I am just putting F of t, I mean the equation which I wrote earlier 28 back in the F of t here. Summation of k j h i V i plus h i plus 1 V i plus 1 of sin m sigma that is, a derivative cos m sigma t d t minus there is negative sign here minus k mu t 2 by R L, 0 to R L k nu t theta sorry, theta is now substituted, which is going to be cos square m sigma t d t plus is a negative sign here, the double derivative here and displacement are related by a negative sign, they are out of 180 degree. Is it not?

So, the minus will become plus here, 2 by record length of 0 to record length of m sigma the whole square because this having m sigma multiplier this is double derivative out that here m sigma the whole square are i naught beta of t cos square m sigma t d t. I think you agree for theta and theta double dot there are the same term defeated except the there is multiplier here. And there is a sign change. That is what I have done here. I call equation number 29, is the number ok? This is my duo F m c by duo theta m c 1 value of Jacobian, only 1 value. I would not rearrange this term. I think I will stop here, because there is only 2 minutes left over.