Dynamics of Ocean Structures
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Module - 2
Lecture - 6
Dynamic Analysis of Articulated Towers

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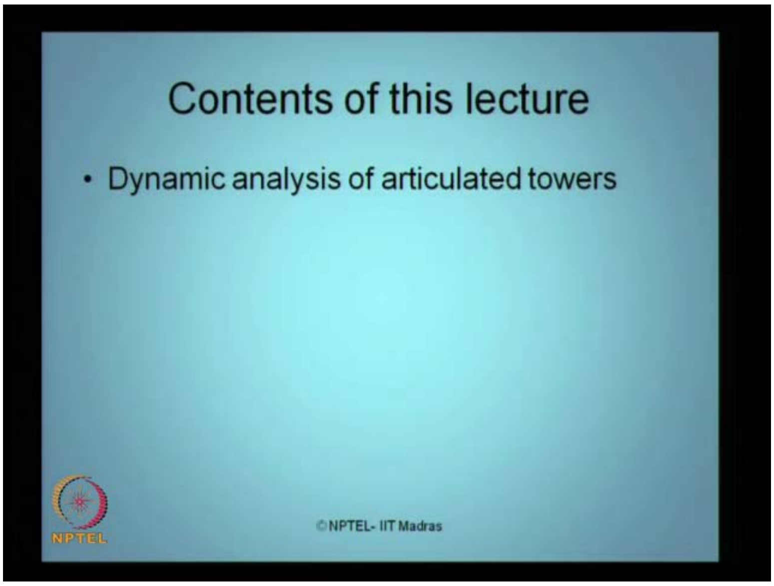

And now we have the 6th on the module 2. In this lecture, we will talk about the dynamics of one of the type of structure, which is articulated towers, we already discussed the dynamic analysis of fixed jacket structures.

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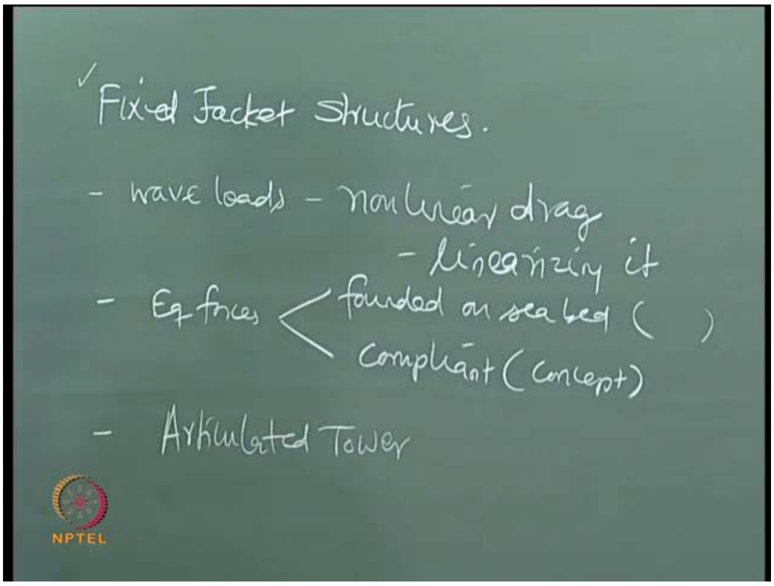

We have also seen how to handle the wave loads using the non-linear drag part by linearizing it. We have also seen how to handle the earth quake forces for 2 types of system, one is let us say founded on sea or resting on sea bed conceptually. We have of course, derived the equation of motion for this, whereas for the system, which is complaint, we have to discuss the concept. In the coming example, we will show you how this can be analyzed using the convention theory for a compliant system.

So, in this lecture, we will take up one article to tower, we have seen in the last lecture what are the application advantage of rotation tower where they have been used. In this lecture, we will about the dynamic analysis of this. Of course, we will show some results of the dynamic analysis and some of the important conclusion for this kind of compliant tower in this lecture.

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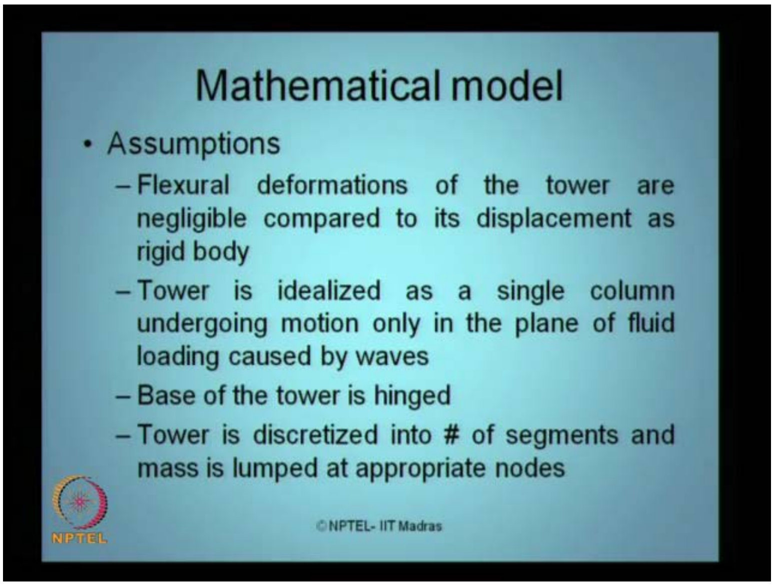

So, if you look at the mathematical model, we makes at an assumptions, that the flexural deformations of the tower are negligible compared to it is displace as rigid body. That's very important, because now we idealize this tower as a rigid body motion. So, the flexural deformations related from the tower members, because of the electro loading acting on the tower become negligible. They of course, exist, but they are very less in magnitude in comparison to the displacement of the tower, so rigid body. Therefore, the tower is idealized as a single column undergoing motion only in the plain of fluid caused by the waves. The base of the tower is considered to be hinged joint it cannot take any moment. The tower is discretized into number of segments and the masses lumped at appropriate nodes.

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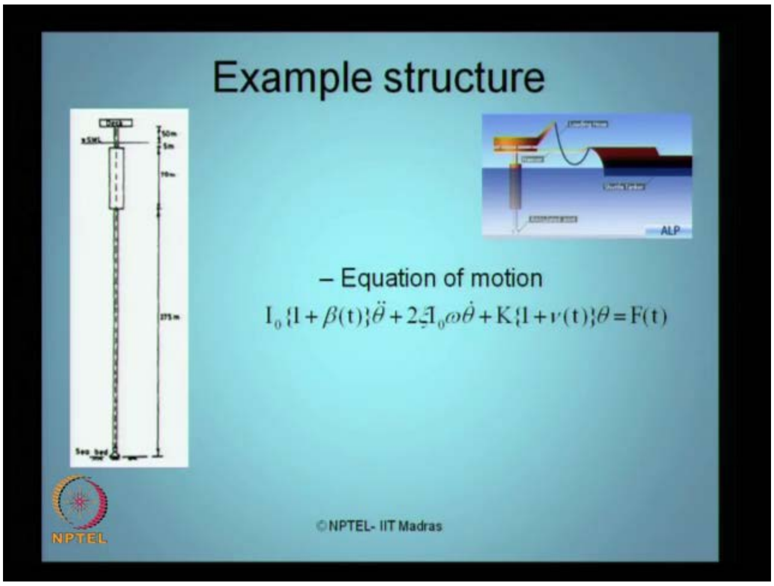

So, what we do is we pick up a tower, we see in the left hand side here and you can see, there are discretized points, which has been dividing these towers into various components. So, the height of the tower the physical dimension about 275 meters and then the buoyancy tank is about 70 meter deep, which is located close to M S L or the still water level.

And of course, a free surface or the free board at, which is depth take place with respect to the still water level is about 50 meters and the deck is having all conventional directions arrangements, which is not shown in this figure, but that will attract the aero dynamic loading on the structure. So, the equation how to see here is the equation of motion. So, we write the equation of motion of back again here.

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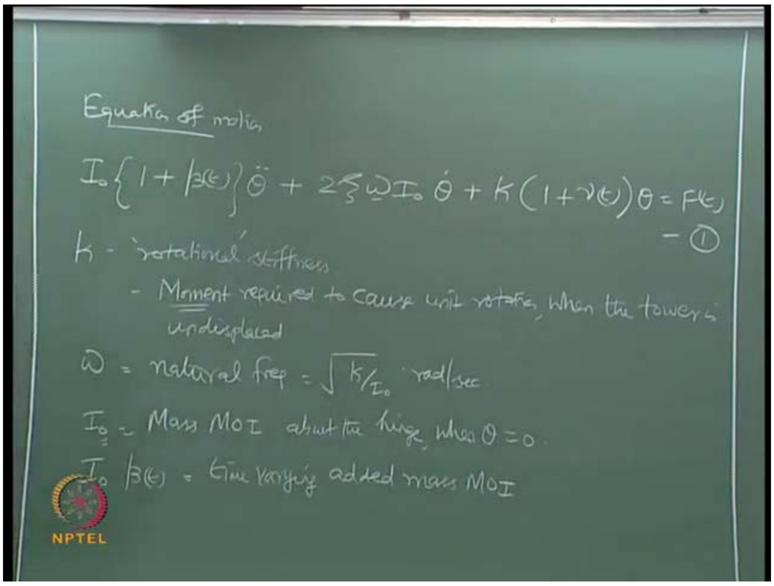

Then, I will explain each term of this equation of motion in detail, let me call this equations number 1, now in this K is the rotational stiffness the degree of freedom for this problem is theta, that is the based rotation at the hinged here. The degree of freedom is the rotation of this hinge here theta that is why the equation of motion is written in theta double dot theta dot and theta. K is the rotational stiffness is nothing but the moment required to cause unit rotation, stiffness is the force required to cause unit displacement.

Similarly, here it is rotational stiffness, therefore, we say this is the moment cause or required to cause unit rotation and the base of the angel or base of the hinge. When the tower is un displaced position, you can easily find out K by the conventional method. And of course, omega here you see is of course, the natural frequency of the tower, which is square root of K by I naught in radiance per second. And I naught is the mass momentum inertia mass moment of inertia of the tower about the hinge, when theta is 0. That is the undisplaced question as to see in the figure, that is why it is called I 0 I naught is to see mass momentum of inertia of the tower about the hinge, when theta is 0. I naught beta of t is the added master is the time varying added mass moment of inertia, I will come to that, how to compute this is time varying added mass moment of inertia.

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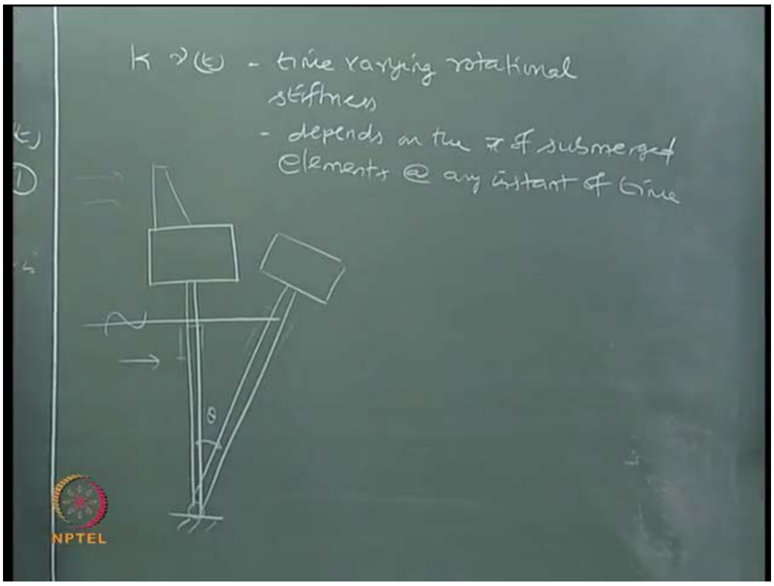

Whereas K nu T is again a time varying rotational stiffness, it is a times varying rotational stiffness, this depends on the number of submerge elements at any instant of time. Whereas K is the rotational stiffness at undisplaced position, I think you will understand at now, when I apply lateral force to this tower, when I have a tower this form is single tower hinge at the bottom subjected to wave action.

There is a lateral force, of course there are risk etcetera, there is a wind over acting, we will come to that figure conceptually later. So, when I try to apply the lateral force to this tower will have a tendency to rotate. So, I call this as theta. So, when I and there is a buoyancy chamber here, there is buoyancy chamber here. Now, depending upon the number of elements, which is submerged you work out K nu of T, which will be the time varying rotational component of this stiffness.

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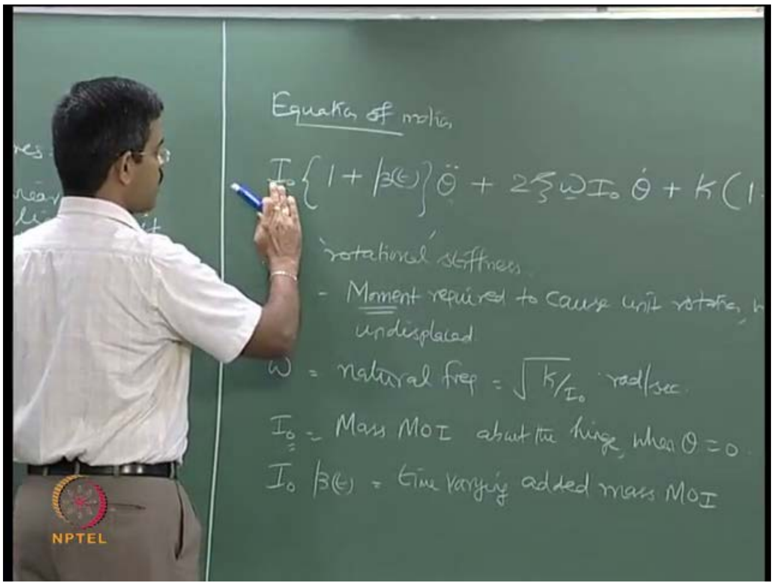

So, that is why you have got, you have 2 terms here K theta and nu of T F theta. Similarly, you have 2 terms here I naught theta double dot, which is conventional mass moment of inertia, when the tower is at undisplaced position, when theta is 0. And I naught beta theta beta T of theta double dot, which will be the time varying added mass moment of inertia, because during any instant of time, you will see the sum of elements will have a different volume of submersions, that will attract more mass moment of inertia. All these moment of inertia are computed above the base here.

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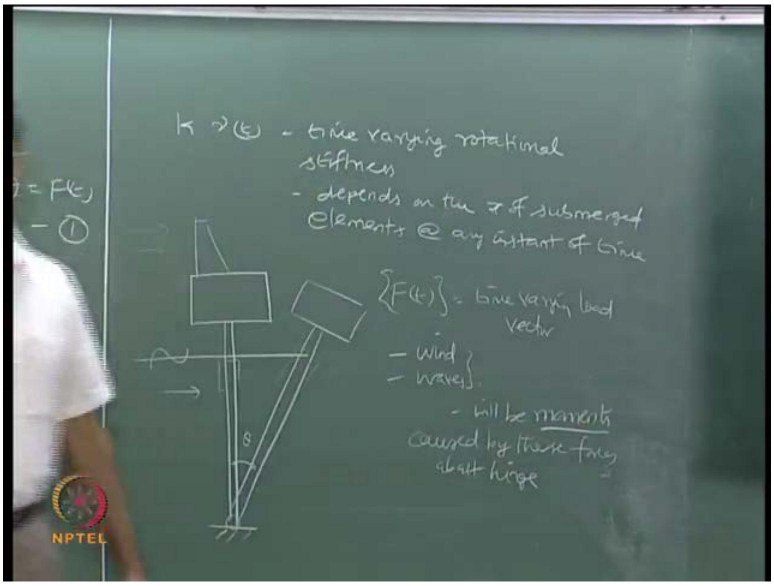

Now, of course, F of T, it is the time varying vector, time varying load vector, which in this case arise from wind and waves both are consider in this analysis. And in this essentially, F of T will be the moments caused by this forces about hinge, not the load actually, it is a moment. That is a moment caused various forces above the sea. So, let me re write this equation again here.

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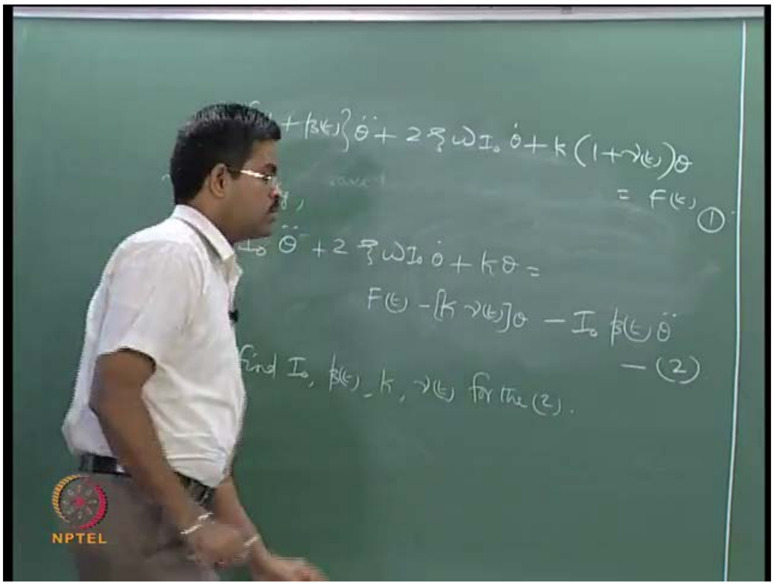

Let us say I naught, I call this equations number 1, I rearrange this terms simply as say rearranging I naught theta double dot plus 2 zeta omega yeah I 0 theta dot plus K theta as F of t minus K nu of t theta minus I naught theta t theta double dot calls equations number 2. Now, I want to find I naught beta t K and nu of t for equations 2, that is my objective law. To do that let us draw the initial and deflected position of the tower.

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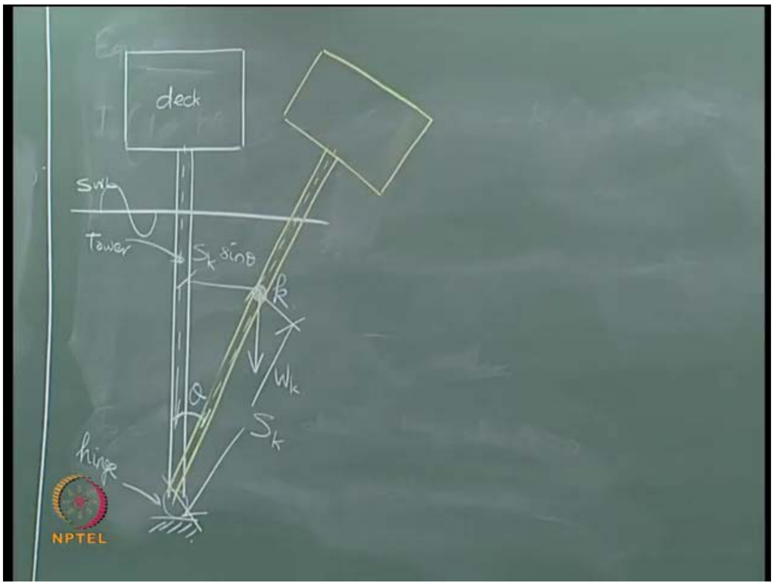

So, I pick up any element here, which is K, I call this length of the element from the hinge as S K on the element has W K as weight. Now, of course, we know this value will be S K sin theta.

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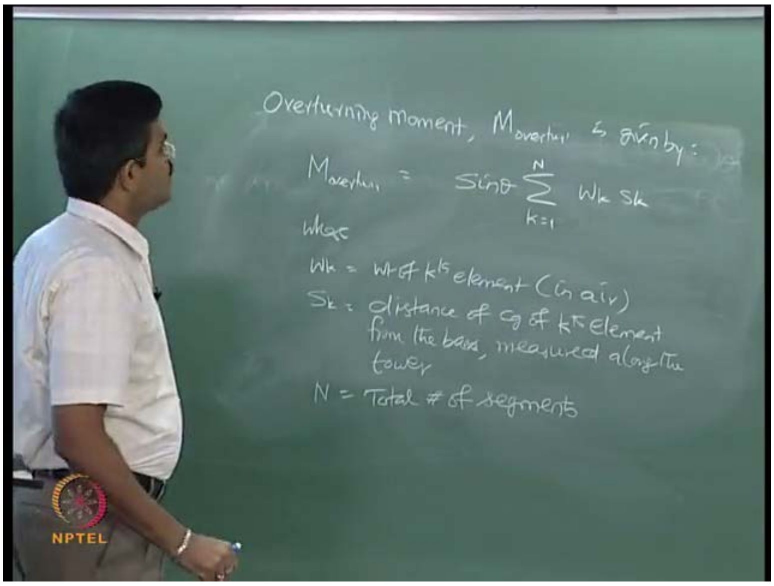

So, now from this figure, let us say the over turning moment, which I call us M overturning is given by. So, M overturning the overturning will be responsible for W K and this distance about this hinge. So, that is going to be sin theta of sum of W K S K, I have taken sin theta out, I am summing it for K equals 1 to N capital N where, W K is the weight of the K th element in air.

S k is the distance of the C g of the K th element is resistant of C g of the K th element from the base measured along the tower. Of course N is the total number of segments, you are dividing the N number of segments, total number of segments. Let me call this equations number 3 2 was the rearranging equations of equation of motion. And now this is the moment response for the overturning. Now, there is the restoring moment also, I will remove this can I rub this, because this difficult to move there, I will retain this figure rub this is that, can I remove this.

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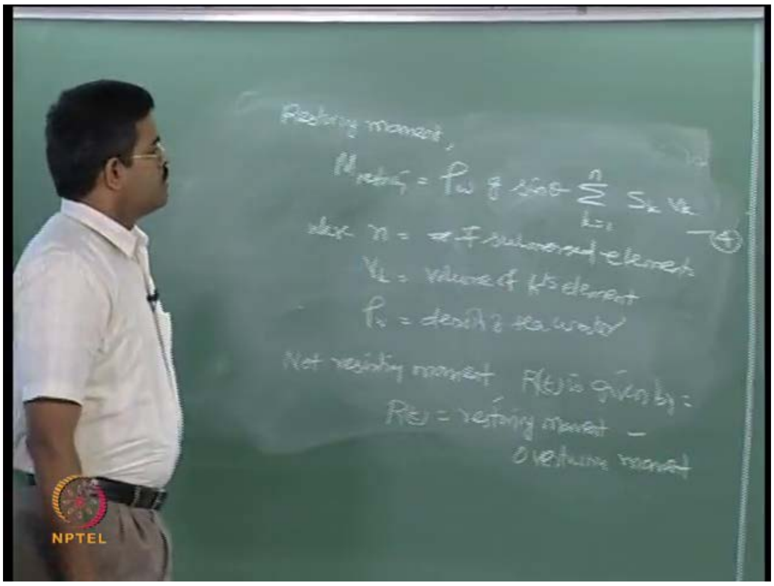

And the restoring moment, which I call is given by summation of K equals 1 plus small n rho W g, because I am talking about some merged volume, now of sin theta S K V K, where first of all n is the number of submerged elements and V K will be the volume of the K th element. Of course rho W is the density of sea water and theta of course, the degree of freedom where, the angle of rotation on the base of the tower. So, I have got 2 equations now, one is the overturning moment, other is the restoring moment.

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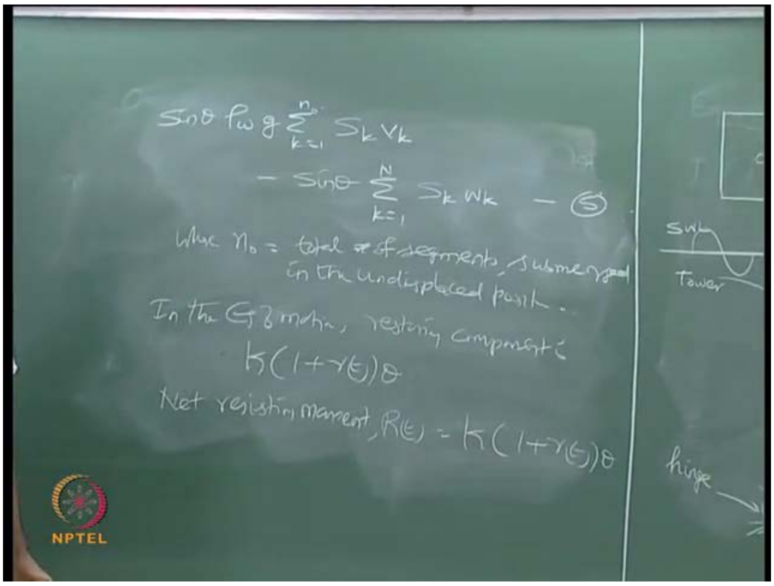

Making use of these 2, I find the net resisting moment, which I call as R of t is given by the restoring moment minus overturning moment, which is given by sin theta of rho W g of sum of K is equal to 1 to n, S k V K minus sin theta of sum of k equal to 1 to n S K W K. Whereas I change the substitute n naught where, I call this equations number 5 where, n naught here is the total number of segments of the tower submerged in the undisplaced position. Now, we all know the in the equation of motion, but the restoring component is actually in the equation of motion the restoring component is given by K times of 1 plus nu of T of theta is it not. So, I should say that, the net resisting moment R of t should be actually equal to K of 1 plus nu of T of theta, I will remove this.

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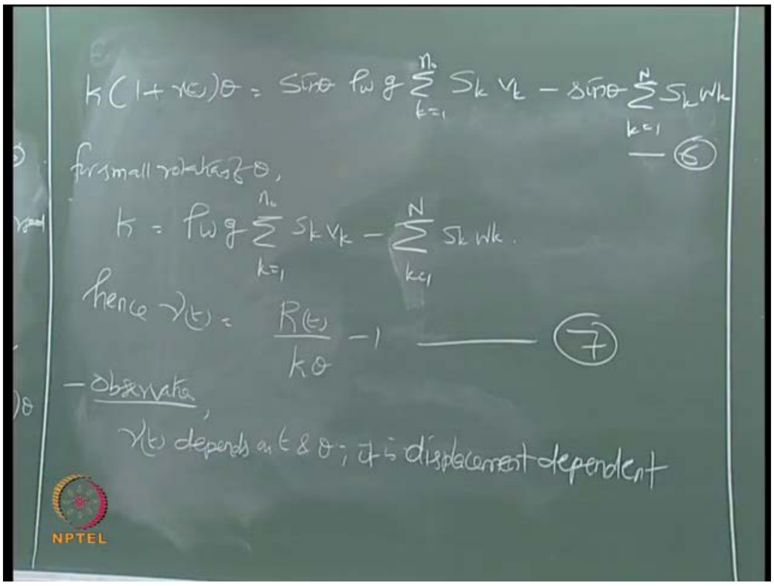

So, let me write that K of 1 plus nu of theta should be equal to sin theta rho W g K equals 1 to n naught S K V K minus sin theta K equal to 1 to capital N of S K WK. I call equations number 6. So, for small rotation is of theta, I can simply this equations as K will be simply equal to rho W g K equals 1 to N naught S K V K. And hence nu of t can be straight away given by R of T by K theta minus 1 known these equations. Now, what we see from here is an important observation, what see from this equations is let me call this equations number 7. And important observation what you see from here is the nu of t depends on time T and theta. So, it is displacement dependent and displacement is non-linear, therefore there is nonlinearity here in the system. So, we wanted to find nu of T N K, we found that. Now, let us talk about I naught, I will remove this any questions here, so far.

In the equation of motion, we have some unknowns, we are deriving them 1 by 1, I got K and nu T now. I will find beta of T and I 0 from now in the second derivation. So, I will have full equation of motion with me, I will talk about the right hand side of equation of motion F of t, I tell you how to solve this in what scheme, then I will show you the results how it has been solved. So, I will remove this now any question here. Now, I want to find the mass moment of inertia in the undisplaced position.

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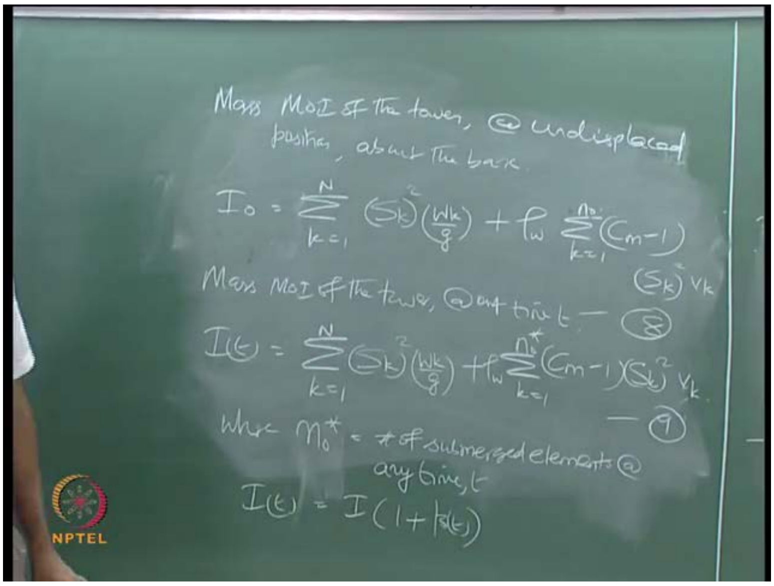

The mass moment of inertia of the tower at undisplaced position above the base, because always, we have to find the moment of inertia about the specific reference stratum. We said is the base of the tower a simply give by I of t, it simply say about the base at undisplaced position, we say I naught I naught is simply a sum of K equals 1 to N, that is my number of element at sigma, it is in the tower. It will be simply length of the member by it is mass plus rho of C, what I am using rho W rho W of the submerged elements. Now K equals 1 to n naught C m minus 1 of S K square V K, I write it here, S K square V K.

So, this is the dry weight, this is the submerged weight I am taking the volume here, multiplying with the density. And n naught is the number of submerged elements and 0 all stand for undisplaced position. So, I got n naught. I call this equations number 8. At any instant of time, I want to get beta t, which is the function of again, I naught, but that is going to give me the added mass variable submergence effect at any time t. So, I must get beta t.

Therefore, what will be the mass moment of inertia of the tower at any instant t at any instant time t, which will be given by I of t, which is again K equals 1 to n S K square W K by g, that is the mass plus of course, rho W multiplier out K equals 1 to n star, I will come to that, what it is C M minus 1 S K square V K. Where I call this equations number

9, where n 0 star is number of submerged elements at any instant time t. And we already know that this I t should be equal to I naught of 1 plus beta and so on.

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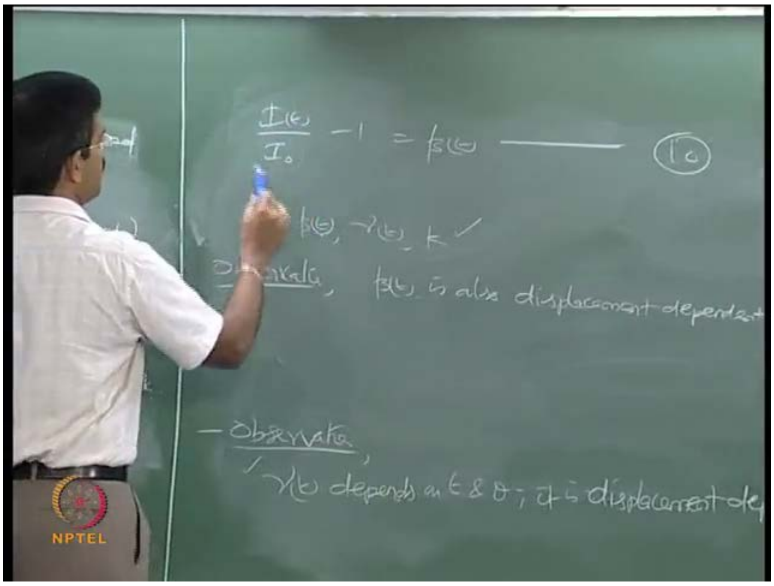

So, from, which I can find beta of t as I t by I naught minus 1 equations number 10, So, I t is instantaneous moments nodes of inertia of about the base whereas, I naught is the undisplaced position, the ratio of this of this give me beta of t directly. So, in my equation of motion, I wanted naught, I wanted beta of t of course, beta of t, I wanted nu of t and I wanted K and I got all of them now. Any doubts here, now, interestingly all of them are obtained. There is an important observation here. The observation is that beta of t is also displacement dependent how, I will come to that is displacement dependent because beta of t is actually, depending upon of t and I naught is a undisplaced positions.

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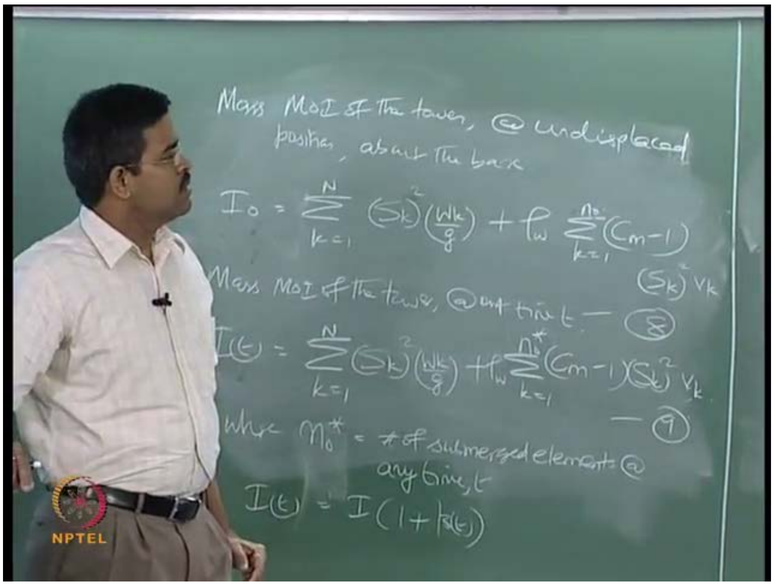

So, there is no problem, whereas I of t talks about number of n star, which are number of submerged elements, so beta of t is also displacements dependent again this is non-linear. Now, in the equation of motion, we have left out only with one term, which is F of t, which will now discuss any doubts here. We are formulating the equation of motion term by term then we will try to give you a scheme, how this is been solved, we will derive the scheme here, and then we will do a netalative operation showing the results directly and discuss the merits and demerits of this results. Of course, I will not be able to show the live solution of this equation of motion here, it is very risky it is not possible; I will show you the results directly, after we complete the formulation of the problem. Now, we are doing the mathematical development of the problem now I am focusing on F of t, any doubt here.

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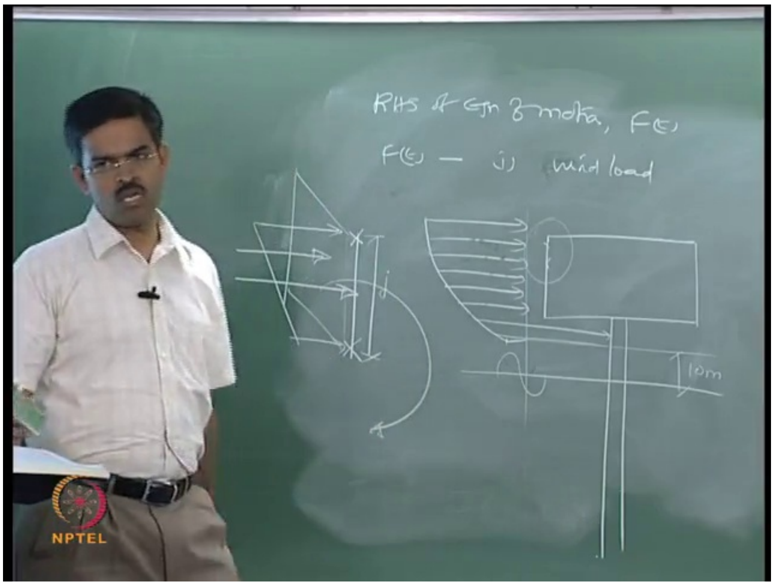

Let us talk about right hand side of equation of motion, which is F of t. As I said F of t considers 2 components, one is though wind blow, we already know how to compute the wind blow, we have the governing equations with us. I will just explain schematically, I am not giving the equations, because they are available in module 1. So, this is my super structure above the still water level at the reference datum of 10 meter at the reference datum of 10 meter above the till that value the pressure is constant. After the pressure varies let us say some value of Z naught, then the pressure varies as a power law. I compute this pressure values at any instant of time, at any point I want.

So, I know the pressure from the pressure, I know the force, because the projector areas known to me, because I know the link of the node how. For example, I will i will magnify this, I have a member, this is my I th element, this is I plus 1th element, I know the length of the element, which is j, I know the pressure here and I know the pressure here, because it keep on decreasing.

So, I assume this is to be linear in between this, because the length of the element is So, small I can linearise, this in between this term. I can get the C g of this, I can get multiplier length of this, I know the projected width of this, I get the area. For that force now what is the distance of that C g from the base will give me the moment, which is F of t. I can do this for n number of elements and keep on adding them is that clear. Any doubt here. I am not explaining these governing equations, because already, we have this power law given to you, we already know I believed that, you have read this and you have understood how it has been computed, this is the scheme how it has been done.

So, I think there is no difficulty in finding out F of t caused, because of the wave load on any member in any element you want, because all these are not forces, these are moments above the base. Why we are talking about moments, because my displacement degree of freedom is not x, it is theta. So, I am talking about all moments, because I am talking about rotation is it clear. I will remove this.

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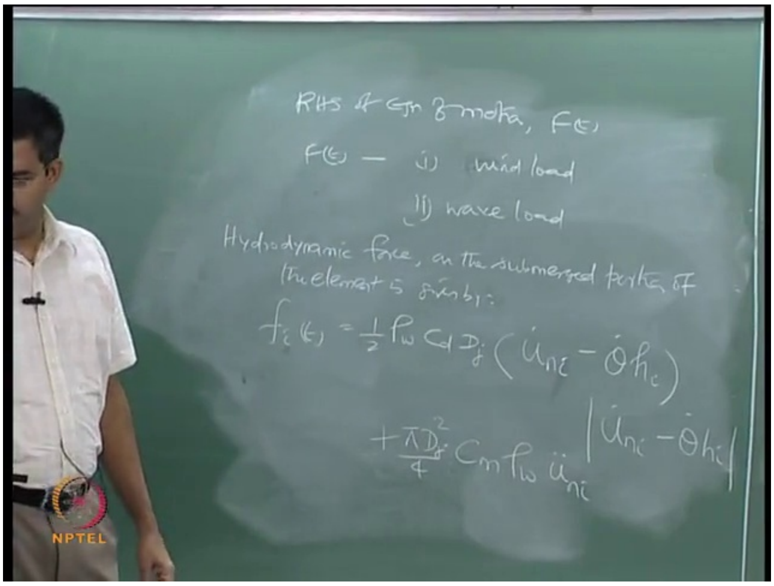

The second force of course, we have is the wave load. We will try to slightly expand this, because I need this to expand it for understand the scheme of solutions procedure for this problem. So, we already know the hydrodynamics force on the submerged proportion of the element is given by let us say f i of t, I am using, I here, I will explain what I is half rho C d D of the j th element is given by j th element.

U dot n j no n I, this should be n I, because there is node n i minus theta dot into h i i will come to this what it is multiplied by u n i minus theta dot h i mode value plus pi D square by 4 of the j th element, C m rho w u double dot of n i. I call this is equations number 11, 10 is here, we will quickly explain what it is but I want write 1 more equations, this is the i th node. Now, at i th node the force per unit length is given by here. I want to write same algorithm, for i plus 1th node. I am talking about 2 nodes, i th and i plus 1th node. So, the j element is between this 2 nodes.

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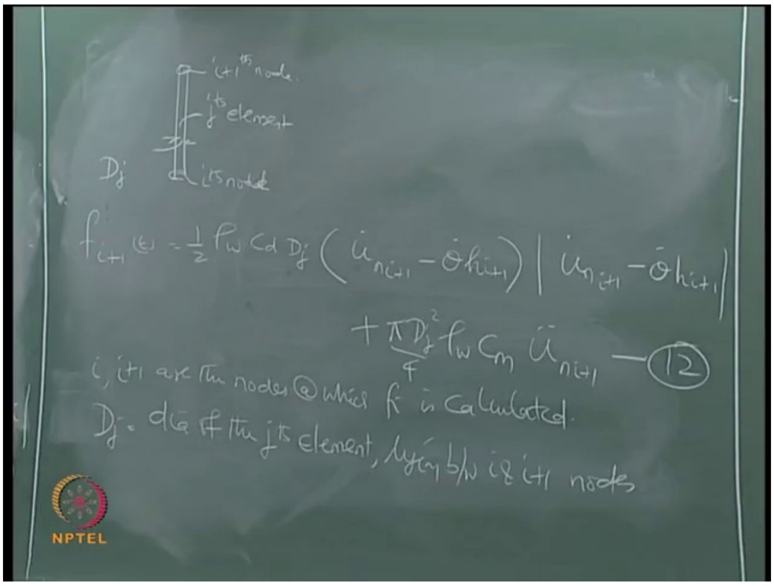

This is my element, this is my j th element, this is my i th node, this is my i plus 1th node and this is my diameter of the member of the j th node, that is what I am using here. And the velocity and the displacement, I mean the angular velocity is all at i. what is h i, I will come to that first let me write i plus 1th node. So, f of i plus 1 of t is given by simply replace all this with i plus 1 half rho C d dia well the dia will remain same, because is not the node, it is the member. U dot n i plus 1 minus theta dot h i plus 1 mode value of plus pi D square by 4 rho w C m, u double dot n plus 1, I call this equations number 12. So, in this 2 equations the set of equations 11 and 12, I will understand that, i and i plus 1 are denotes at, which f of i is calculated.

D j is the diameter of the jth element lying between i and i plus 1 nodes, I will rub this part. Of course rho w is density of sea water C d and C m are the dragger inertia of coefficient, we do not we know them. And you dot or water the particle velocities at i th and i plus 1th nodes, which I calculate from appropriate theory theta dot is the angular velocity at i th and i plus 1 th nodes of the tower, which is my displacement degree of freedom.

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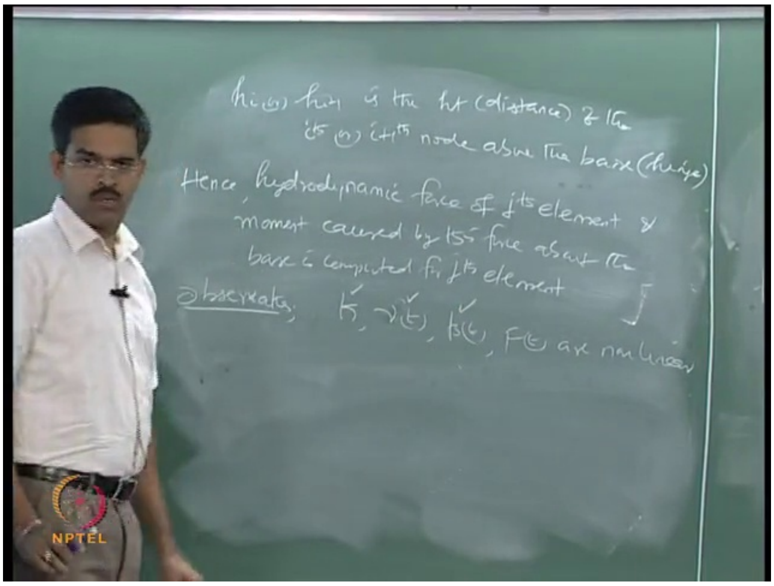

H i are h i plus 1 is the height, I should say distance of the i th are i plus 1th node above the base, which is the base hinge hinge is my base. U may wonder the why, I am multiplying this with each other, importantly remember, I am looking for the moment of this forces. That is why I am multiplying it here. Hence the hydrodynamic force of jth element and moment caused by this force above, the base is computed for j th element.

If you want do it for the entire tower, I should keep on adding this, for different number of elements. So, now my f of t comprising of wave and wind are clearly known to me, I know how to divide them in segments, the super structure part above the still water level find the hydrodynamic forces and the corresponding moments of these forces above the base of the tower. For the hydrodynamics part, I will again subdivide into different number of nodes the portion below the still water level and find out water particle velocity at these respective nodes, and the distances of those nodes from the base keep on adding them, I get the moments.

So, f of t can be easily obtain for the entire tower, which compress of later loading caused from hydrodynamic and aerodynamic. Now, the interesting part here is observation K nu of t beta t and f of t of course, are non-linear. I have already shown you how K nu of t beta of t are non-linear, I think you very well appreciate why f of t is non-linear, because of the drag term percent in the hydrodynamic forces.

U also nonlinearity from the drag term percent on the hydrodynamic forces, because if you see as a v square, they are also in pressure drag in hydrodynamics forces. So, there is a drag term percent in hydrodynamics as well as hydrodynamics forces therefore, f of t is also non-linear. When you have any of the system nonlinearity of this order percent in an any given dynamic system can be solve this in any methods. There are 3 methods available, one is what we call timed the wine method, other is frequency domain other is iterative frequency domain.

So, I will solve this problem, now with a new algorithm call I F D, that is iterative frequency domain, I will pick up this problem frequency domain and iterate it in frequency domain and solve. So, it is a new algorithm, it is a research based techniques, which is very commonly not applied for offshore structures. There is an advantage why, I pick up iterative frequency domain to solve this problem, I will show you at the end whence I do this problem.

So, what I will do here is at 1 iteration, I will show you the control equations. You have got to iterate that write a program iterates that and get the solution. Once we understand the control equations at 1 iteration then, we will directly show you the results on the screen here and we will draw the inferences from the results. That is what I can do as far as the classroom is concern. If you have any difficulty in the formulation of the problem till here, please let me know how we have formulated the whole problem, we started within ideally stubbier model like this, we tricked with the degree of freedom as rotation at the base is one of the approaches.

We divided this sub differ in the tower and the deck of course, that different number of nodes and we found out I wrote the equation of motion rearrange them, and then we found out each and every terms separately, then we understand hypothetically and mathematically how to compute the equation of motion and f of t on any node, I want or any member, I want ok. Now, I am looking for a scheme to solve this equation of motion, the solution of this equation of motion should give me the displacement degree of freedom, which is nothing but theta for this problem is it not. That is my solution; I must get theta, at the time history r as a power spectral density function, if I do it in a frequency domain.

So, time is not there now, because this will take another 1 hour. So, already we are in a equations written in a black board today. So, I will not load you further, because I got around. Now, we have finished equations number 12. So, I have total around 36 equations. So, it will take time. So, I cannot do it, I do not want to hurry it, because 10 46, now will stop it here. If you have any questions, I think we can spend couple of minutes in explaining, you those answers, if it is quickly as possible, if you if you have no question [laugh] all I think, I am very happy that in 1 instant, I am able to understand the analysis, I am really, I think there is the magic happening.

So, there may be the large amount of audience, which may feel the (( )) as you are as long as you are writing on the board, as long as you see it in this screen, as long as you are promising as the results. We have absolutely no problem at all because we agree whatever you says the results are 5 equation of motion is agreed, because I see some 2 zeta omega terms.

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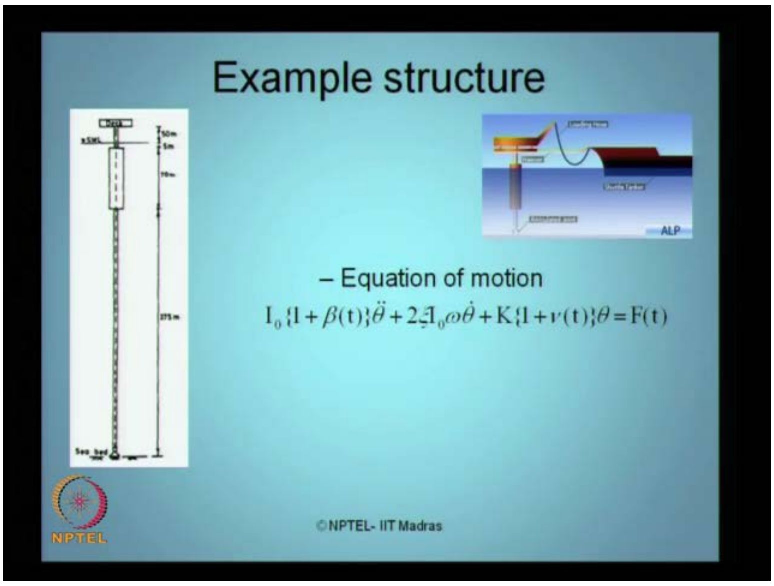

There I understand c c as a ratio, so all these are appreciable, I am able to get those areas and I have explained some complex term as beta of t nu of t in the derivation here, though I am not understood completely, but still I am trying to understand them, but I think they are here with me, I can read it. So, iterative frequency domain is a very interesting solution scheme, it is a new it is conventional law done in dynamics of ocean structures. So, it is a new scheme, it takes some time for us to understand. We will go slowly, in the next lecture possibly on Monday morning. So, you will be also remaining fresh at 8 O clock. So, I will load you with more equation and understand the scheme and I will write the control equations for 1 iteration, for the whole scheme. Then of course, we understand that, you will write the program for this and try to solve this problem of course, I will give the data for this problem, it is available here.

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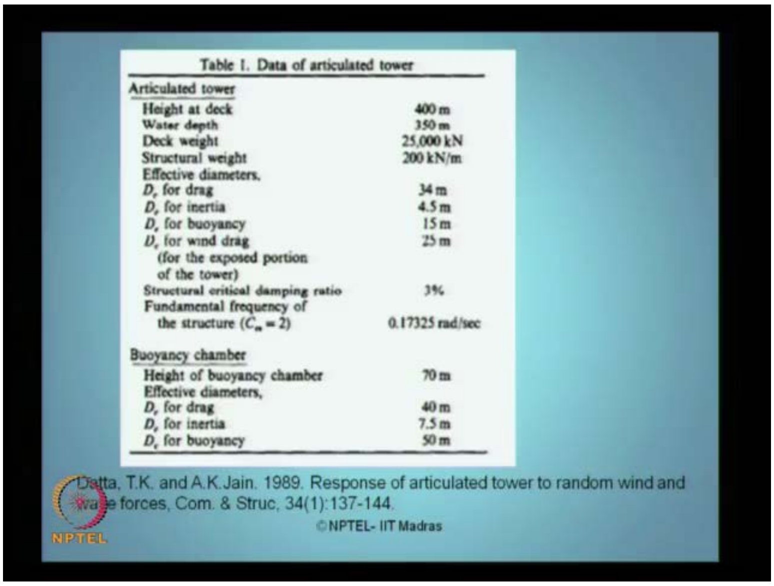

Table 1. Data of articulated tower

Articulated tower	
Height at deck	400 m
Water depth	350 m
Deck weight	25,000 kN
Structural weight	200 kN/m
Effective diameters,	
D_e for drag	34 m
D_e for inertia	4.5 m
D_e for buoyancy	15 m
D_e for wind drag (for the exposed portion of the tower)	25 m
Structural critical damping ratio	3%
Fundamental frequency of the structure ($C_m = 2$)	0.17325 rad/sec
Buoyancy chamber	
Height of buoyancy chamber	70 m
Effective diameters,	
D_e for drag	40 m
D_e for inertia	7.5 m
D_e for buoyancy	50 m

That is the data, which I am going to use, which is taken from this specific paper. So, this is what you are going to use.

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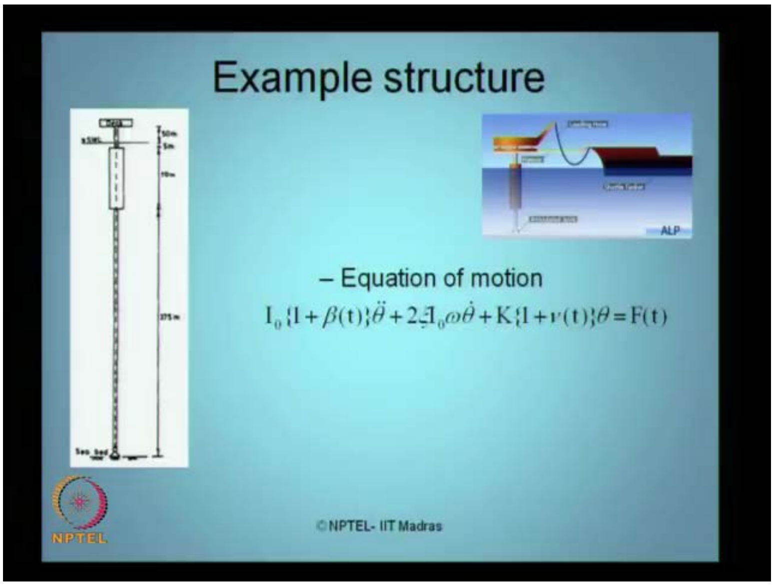

So, all the data are available with me, hence all this equation of motion and I will give you the results directly, that is the idea so any questions.

(( )) Outcome of the dynamic.

That is interesting question, we of course, when we solve this equation of motion the variable of equation of motion, I think we agree S theta, which is the displacement degree in this case is the rotation. So, either I must say the time is S theta of t for any instant of time, what will be the rotation of the base of the base at the hinge, I can find or we do a frequency analysis, I will get power spectral density function for a wide range of frequency of theta. Now, from theta since, you know the tower dimension, I can always find x, which is nothing but the tower length multiplied by the sin angle of the, I can always find I can find the maximum displacement of the tower, I can find the settlement of the tower, if it happens and so on, if you know theta.

I will check whether the maximum displacement beyond permutation value is running in kilometers and so on. From this I can easily connect this to my stability of the model also, which I am not touching here can be done. So, the outcome of this equation of motion of the solution of this equation of motion to be precise will be time history of the rotational degree of freedom are power spectral density function of theta.

That of course, in my procedure, I will get p as d f, because I am doing it in frequency, I am calling iterative frequency, because I will show you how I am iterating. So, it is I F D it is not time domain, it is iterative frequency domain. So, that is what it is so we will get theta. Of course, using for your transform you will get P S D F converted to theta of t also I will not show that here, I will directly look at P S D F of theta S.

So, it is a simple idealized problem that is why we picked up this example for us to understand how Ii can do dynamic analysis for a a t. Of course I will extend this for multi legged articulated tower in the next successive lectures.. There I will talk about not the solution procedure talk about control mechanism of m l a t s. How do you control the response, because I know how to get the response, now I am interested in controlling them?

So, what mechanism do I do, I will use tuned mass dampers, that will be a secondary system of dynamic wave vibration control, the response of the tower. That is what I will discuss in the next part of this problem, after we complete the results of this problem that is about 80. So, the third class of problem will be A L T P the 4th will be a new generation of offshore structure, which will be a trisore at top.

Again, we will derive the mass stiffness matrices completely here, we will not solve it here, we will solve solution leave it to you, but you can refer the journal papers to understand them. We will discuss only the summary of results. So, that will end the second module completely. So, first modules spoke about basics, second module is speaking about applications, third module talk about advance application in a stochastic dynamics that is what the plan is, so any doubt except one anybody else.