Dynamics of Ocean Structures
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Module - 1
Lecture - 24
Stodla, Rayleigh-Ritz and Influence Coefficient Methods, Dunkerley

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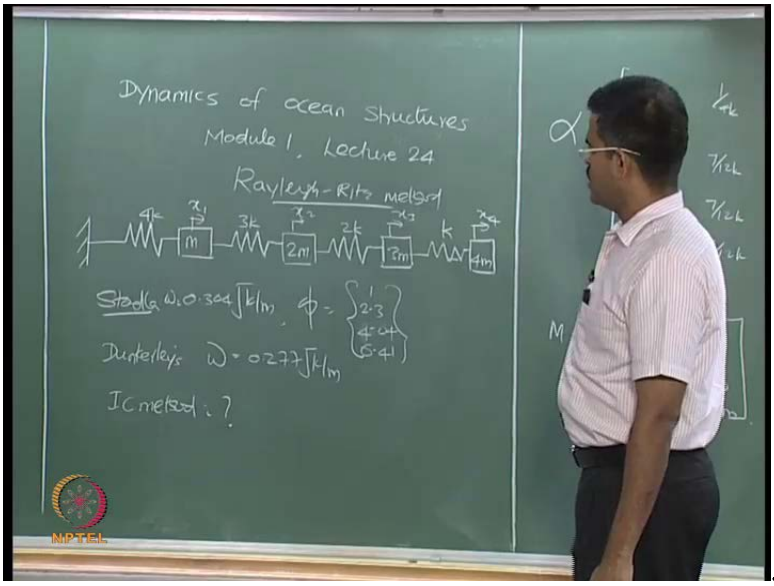

Now, we had this example in the last class. We had a four degree freedom system problem, where there are 4 mass points: 4m, 3m, 2m and m lumped at (( )) degrees of freedom respectively as x 4, x 3, x 2 and x 1 as shown in this figure. These mass are connected to the continuous support here by different springs of stiffness: 4k, 3k, 2k and k as we have shown. However, we wrote the influence coefficient matrix yesterday in the last lecture, and we found out the alpha matrix is nothing but the forces for the displacement values or the influence coefficients by giving unit force for different locations at different degrees of freedom.

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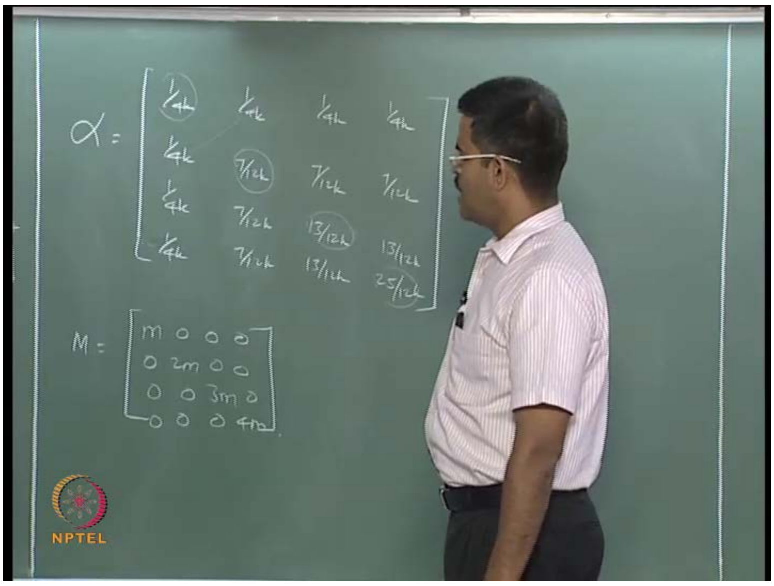

Column wise, we derive these matrices. We already said that, this is completely symmetric and so on and so forth. We also used the Dunkerley's technique, picked up the diagonal elements of this and picked up the respective mass values. We have found out the frequency as 0.277 k by m. We of course found out the iteration scheme, the Stodla's frequency, which is a fundamental frequency; I say omega 1 and phi 1; which is 0.304. They both match closely, but we have got an eigenvector, which is nothing but the first mode shape of this. And we have already said that, we will show a comparison between these two methods with that of the influence coefficient method, which we will do it now.

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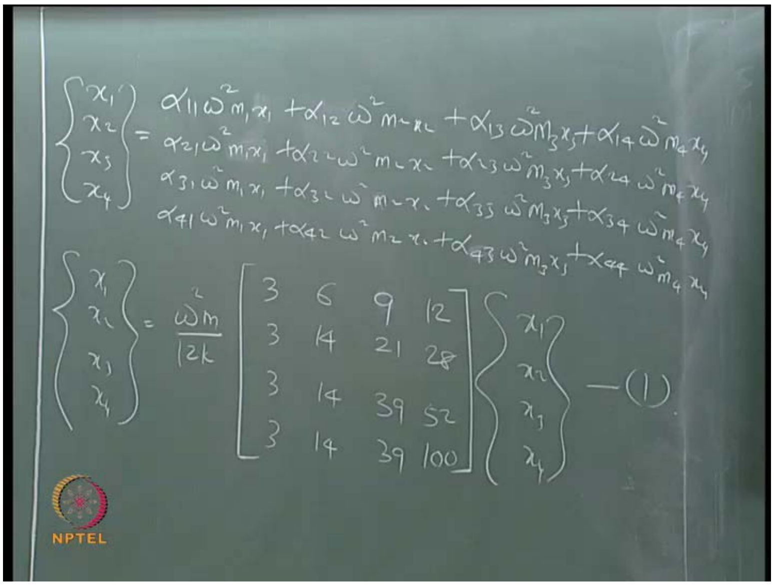

We already said for the influence coefficient method, the control matrix, what we will do is… We should say omega square everywhere. And that is why I am eliminating the negative sign here. I should always say this is as m 1 x 1 double dot and so on; omega square x 1; m 1 x 1… This is m 2; this is m 3; and this is m 4. So, I can write this in a vector; you will find like this; which is x 1, x 2, x 3, x 4, which is going to be the alpha matrix. I simply say omega square m by 12k. I have a common denominator here; I take it out. I have got integrators joined the alpha matrix with that of the corresponding mass matrix. I have a mass matrix here. So, the first column is going to be…

So, this is going to be 6. How do we get this? This is very simple. I have taken 12k denominator here; I have got 4k here. So, there is a three multiplier; I have got 2m here; becomes 6 and 14. How do I get this 14 is very simple; I have got 7 here; I have got 2 here; so, 14. Similarly, 14 and so on. The third column again going to be… The first value is going to be multiplied by 3, because I have got 12k at the denominator; I have got 3m here. Therefore, it is going to be 9, 21, 39, 39; 12, 28, 52, 100 of x 1, x 2, x 3, x 4. Let me check this. So, this is the iteration scheme now.

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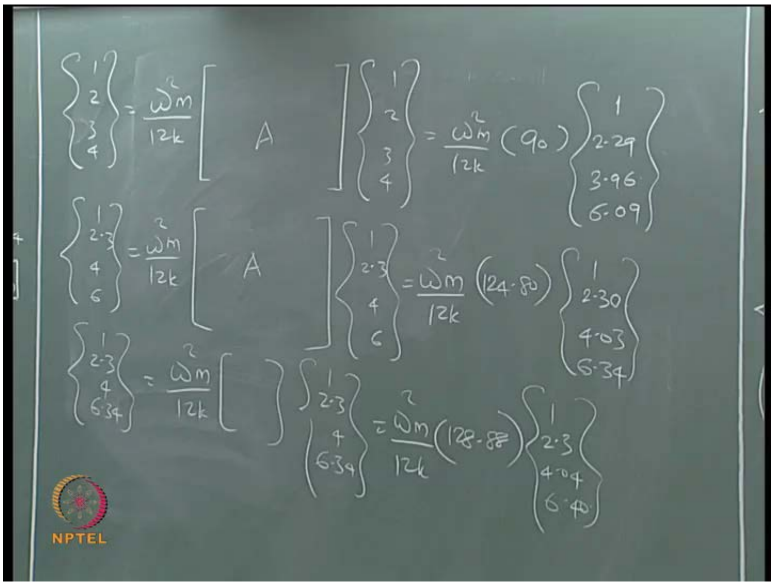

Let me remove this matrix. So, let me start with 1, 2, 3 and 4 equals omega square m by 12k of this matrix of 1, 2, 3 and 4, which will be omega square m by 12k of some multiplier of some ratio. Quickly tell me what is that ratio? So, this is going to be 90 and this is 1. So, 2.29, 3.96, 6.09. So, obviously start with this next iteration: 1, 2.3, 4 and 6 as omega square m by 12k of this matrix. If I call this as A, this is A… 124.8… 1, 2.30, 4.03, 6.34. The next iteration – take it as it is; 1, 2.3, 4 and 6.34 and so on. So, what is the value you are getting? So, I get 128.88. And the ratios are 1, 2.3, 4.04, 6.40.of it so.

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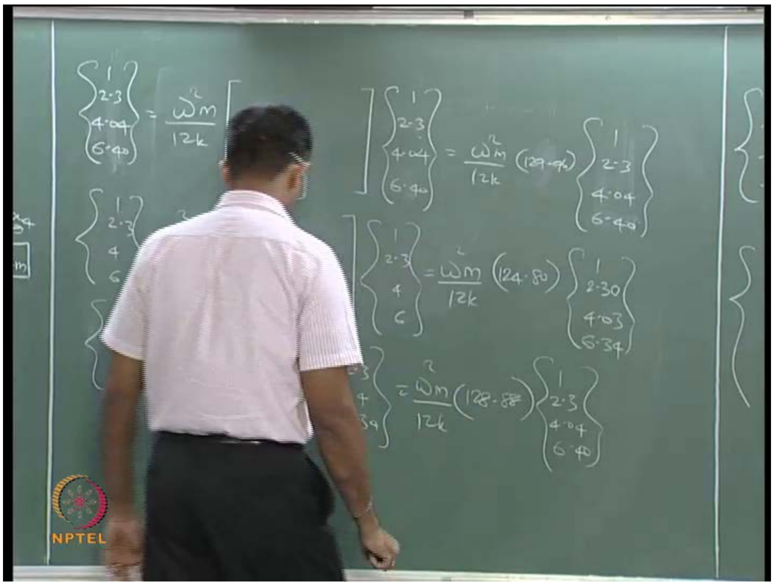

I pick up this. 1, 2.3, 4.04, 6.40; 1, 2.3, 4.04, 6.40; I get 129.96. And the vector is repeated.

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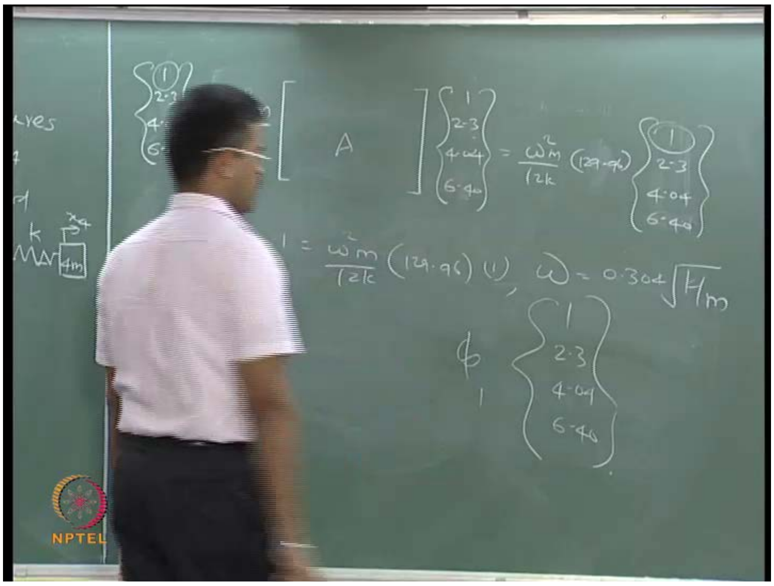

Remove all of them. I can equate the first value with that of the first here; which gives me omega as… 0.304 root k by m; and phi – 1, 2.3, 4.09, 6.40. Is that all right?

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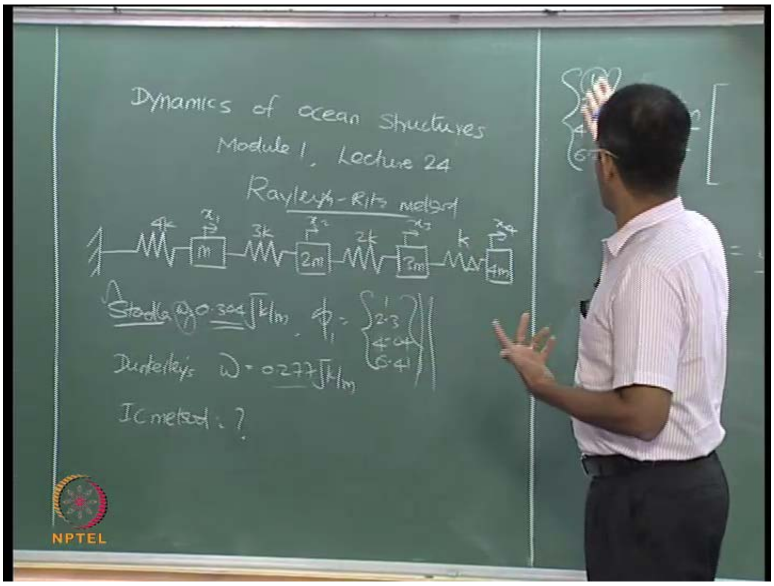

So, I get the same value here back – Stodla; 304 phi; which I got exactly… So, this method is very fast and converging and very confident. And it will give a realistic values that can compare the Stodla. The Stodla is on the other hand is also very good in finding out the fundamental frequency. And I have taken here the initial values as 1, 2, 3, 4. They are not proportional to the stiffness in any form; they are not proportional to the mass in any form, but ultimately, landed up in 1, 2, 4 and 6; which are again not proportional. Therefore, one can start with all positive values, because I am looking for the first mode, which has got zero crossing. So, all these three methods including the eigensolver method – the fourth one, which I have not demonstrated for this example, will be able to clearly tell you how to find out the fundamental frequency and the mode shape.

In many of the cases, you are interested only in these two values. But, eigensolvers and inference coefficient method can give you all the values of omega and corresponding frequencies and mode shapes. So, it is very interesting. We can write a simple program for this. Computer control algorithm – it is an iterative scheme and you do not have to invert a matrix at all. We already inverted it by an influence coefficient matrix. So, it is very simple process.

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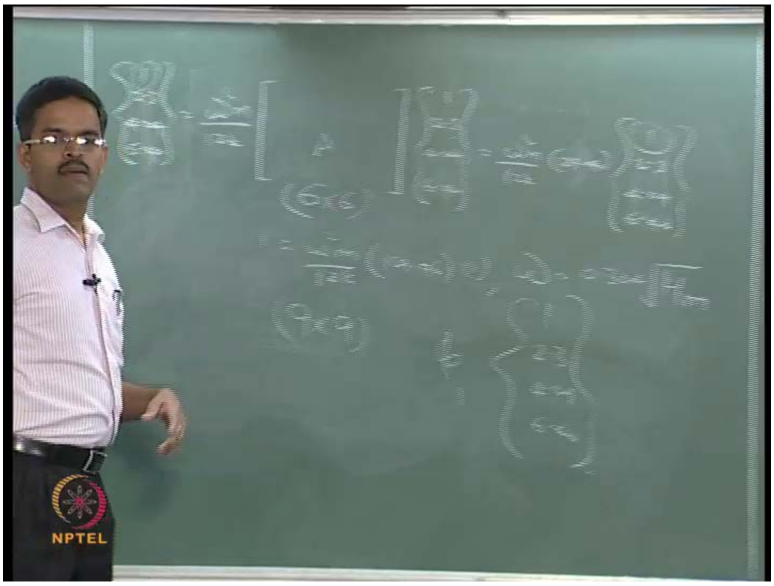

And most of the cases in offshore structures… A matrix – obviously would be a size of 6 by 6 in most cases. It is not very large. Or, in special cases, it can be 9 by 9. So, we should be able to do it by hand, because we have demonstrated by 4 by 4; we can do it by 6 by 6. They are not very difficult. So, one should be able to use this method usefully by hand calculations for finding it out. Any question?

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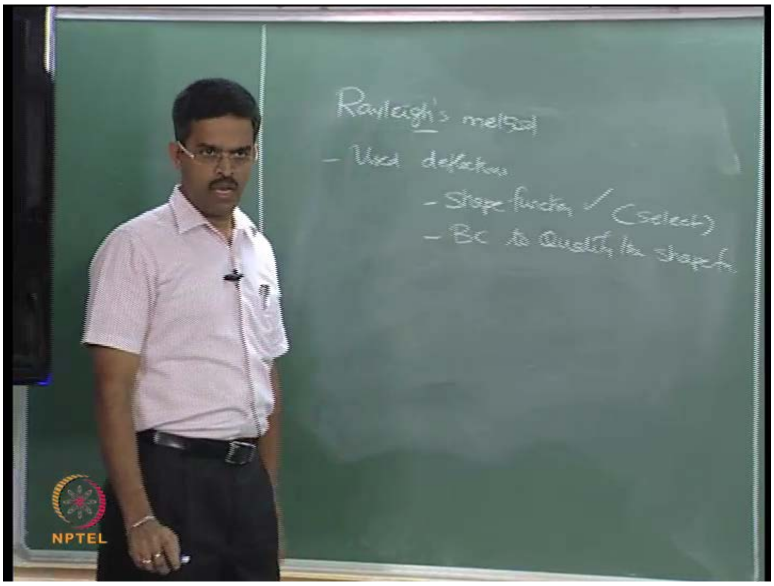

We move to the next method which is Raleigh-Ritz technique. This will be the last lecture for module one, which I will complete today with one or possibly two more examples. One example certainly I will do; second example I will try to do. Then we will give a very brief summary what we so far studied in module one. Rayleigh's method is again… It is a very popular application with classical structural mechanics. Rayleigh's method has been very popularly used for finding out deflections. And we all understand… If you really look at the basics of structural mechanics classical theory, you will see that, Rayleigh's method will basically depend on assumption of a shape function. You have to pick up a shape function; select a shape function – may be linear, may be non-linear. In non-linear, may be quadric, cubic, etcetera. Then apply the boundary conditions to qualify the shape function. I will exactly do the same thing here and I apply it for dynamics.

As I said, I am using the unit load method, dummy load method of classical structural mechanics to find out frequency of vibration of a thrust problem, which we did somewhere in the module one in previous lecture. So, I am using the same fundamental principle back, but I will use this method for finding out the omega – frequency of the system. See how I am doing it. So, again in iterative scheme, I will do this problem in two ways. One I will do it analytically; I will do it numerically. First I will do the numerical problem; we will do the iteration scheme. I will do the same problem or similar problem in analytically method, because analytical method we will take it later towards the lunch time, because then you will clearly understand that method easily. When we move it towards the lunch time… because there is a saying in Tamil people say. I do not know how many of you know Tamil; how many viewers know Tamil, but still let me quote this. In the Tamil, there is a saying; people say that (( )). It means that, when you have fulfilled your thoughts and gains and knowledge for your brain, then you can think of feeding your stomach. So, let me give you this example of the analytical exactly at the lunch time. So, we see how many of you are really going for lunch.

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So, let us take a simple example of a three degree freedom system. I will also simulate that – what I mean by a three degree freedom system in corresponding to dynamics of structures in the second module. I will explain all these things. What is the relevance of assuming a model like this and how would it really in terms of model in structures in offshore. I will talk about that in the next module. All we have to understand in this module is that, these are all idealized models. These are all idealized models. If I ask you a question, how many of you can really know the answer? Please raise your hand. If you say why I am not putting damping in this, how many of you can answer this? Why I am not putting a dash pots in these connections? Why I am always showing only a spring mass system? How many of you know the answer for this? Why I am not including c in my problems here? How many of you know the answer?

Nobody knows? only one; that too he is raising the hand only half of it; I think he knows half of it, but he is not sure. Remaining all are keeping quiet, because they do not know, they would not want to take a chance actually. So, I am looking for free vibration problem; undamped frequency I am interested in. You may wonder that, why I am not looking for a damped free vibration frequency; why I am not looking for omega D? Why omega? Why not omega D and why omega? I think now you will all be able to understand what is the difference between omega D and omega, which is connecting by zeta, which is nothing but the damping ratio, which is C by C C. So, why we are not talking about the omega D and why we are talking about omega? I am looking for a free vibration analysis; I am looking for the worst conservative frequency. I am seeing whether this can match the excitation frequency for near resonance case.

And, you will appreciate that for a zeta value of very low – maybe 2 percent – 0.002. Or, let us say 1 percent – 0.01 value. You will see that omega D and omega do not vary much. And zeta cannot be obtained analytically; it is very difficult; it is only done by experiment. Even to perform an experiment on this, I must have a fair idea – where is the natural frequency; otherwise, if we excite the model to that value, model will get damaged. So, I must have a rough idea at least. So, omega is not bad to find out that. Therefore, dash pots are not included in my discussion.

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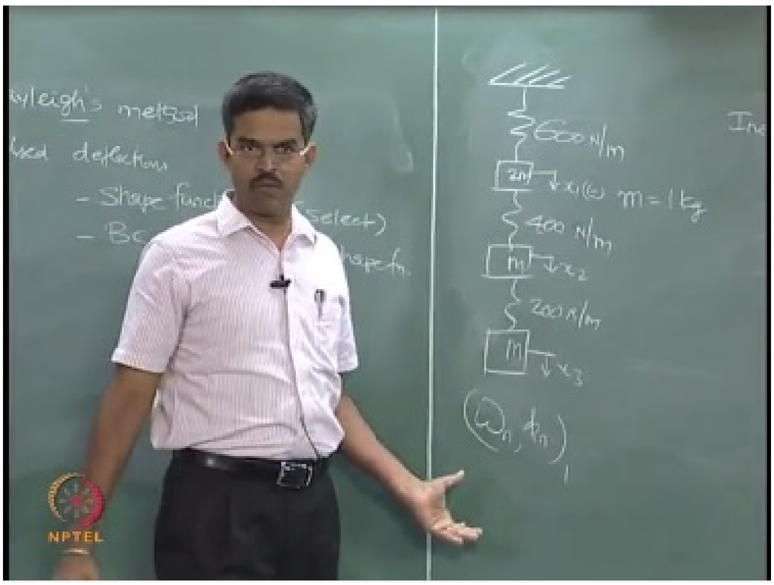

So, let us say this is 600 newton per meter; this is 400 newton per meter; this is 200 newton per meter; this is 1, 1 and 2. Or, let us say m, m and 2m; where, m is 1 kg. And this is x 1 of t of course; this is x 2 and this is x 3. I want to find omega n and phi n corresponding to the first mode. I am using Rayleigh's method. So, in Rayleigh's method, inertia force will be dominating. Can you tell me why? I am looking for a free vibration. In free vibration, there is no external agency, which can vibrate the system. But, if the system does not vibrate, I cannot find the frequency of vibration. Therefore, I am looking for inertia force to vibrate the system. So, let me open up a table.

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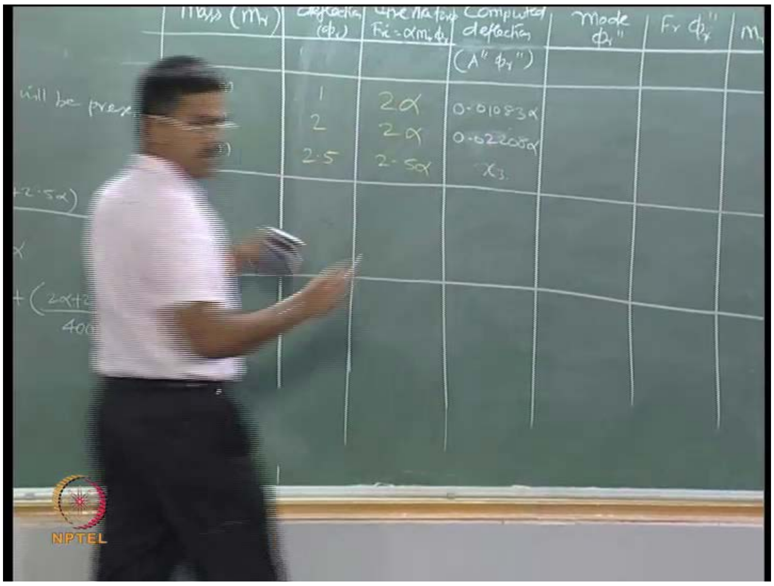

I will do the calculation on the left-hand side and open up a table here. I am using m r subscript instead of i (( )) I will refer to me degree of freedom; r is something different; I will explain you what is that. Deflection of the mass point; I am calling this is as phi r; you may wonder why I am calling as phi r, because phi r is a term related to mode shape. We are always worried about physical terms. Mode shape is nothing but the relative displacement of the mass points only. So, I can call this as phi r. Inertia force – I call this as F r i, which is alpha, m i, phi i – m r and phi r; alpha m r and phi r. Computed deflection and mode shape; mode shape. This has A double prime phi r double prime. This is simply phi r double prime. There are standard notations recommended in the literature. So, we are using it as it is. So, this is F r phi r double prime. It is a product. This is m r phi r square. This is part of this.

Let us say it is the first iteration, second iteration and so on. So, mass points – 2m, m and m. I am assuming a deflection, which is 1, 2 and 2.5. We can take any value. Should be all positive; and I am looking for the fundamental frequency. So, alpha is the influence coefficient or mode shape value – m r and phi r. m r I have; phi r I have. I should say 2 alpha, 2 alpha, 2.5 alpha. Of course, m is a multiplier available here. Now, if you want to compute this value, let us say x 1; you want to compute this deflection.

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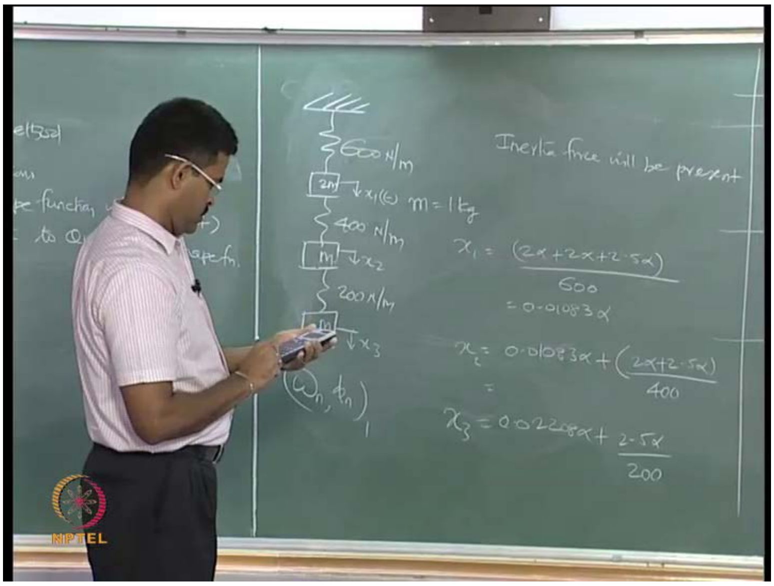

x 1 is nothing but 2 alpha plus 2 alpha plus 2.5 alpha, that is, the inertia force of all divided by this spring – 600, because that is the deflection of the first spring here; which will give me 0.01083 alpha. I am writing it here – 0.01083 alpha. If you want to find x 2, let say, this value; x 2 is nothing but.. It is a cumulative value. The deflection of the spring one or the mass one will get added to 2. So, I should say, the 0.01083 alpha plus these two displacements – inertia forces by the spring; so, it is nothing but 2 alpha plus 2.5 alpha by the stiffness of this spring, which is 400; which will give me 0.02208 alpha. The third one if I say x 3 – this value; which is nothing but 0.02208 alpha plus the last inertia force, which is 2.5 alpha by the corresponding spring, which is 200 newton.

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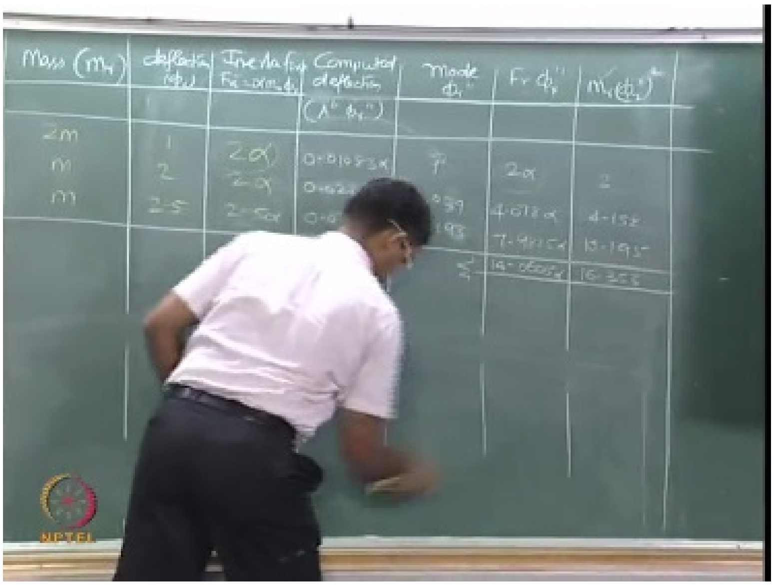

I rub this… 458 alpha. So, it is nothing but a computed deflection; I can take a proportional value of this. I divided all of them by this value; I get this as 1; I call the division value as A double prime; which is nothing but in my case, 0.0108 alpha. So, this becomes 2.04, 3.19 – 2.039 and 3.193. So, this column is nothing but the multiplication of this column with this column. So, it is going to be 2 alpha. This is nothing but multiplication of this with this. Let us sum this value. And this column is mass with phi r square; phi r is here – square of this. Anyway this is going to be again 2m. m is of course 1 kg. So, I can remove this m also. We can simply say 2, because m is 1 kg; 4.158; 10.195. Sum of this will be 16.353.

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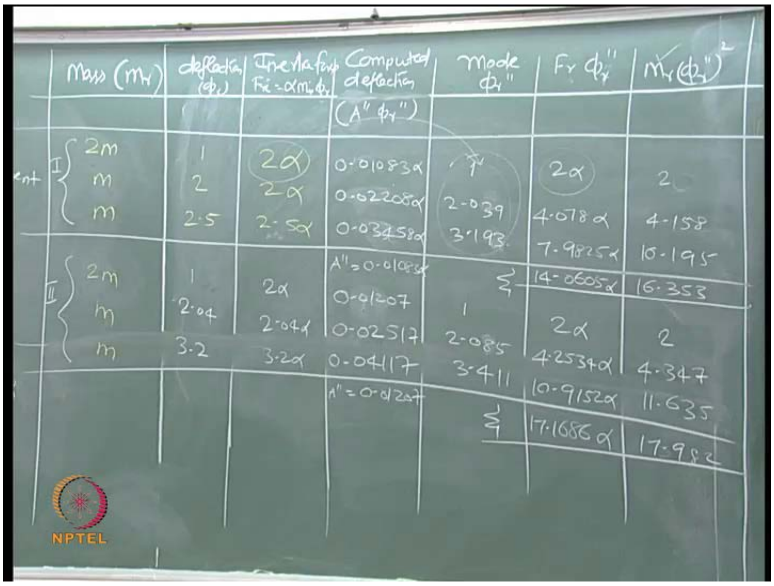

I will start the second iteration. I will pick up this value with minor change; so, 1, 2.04 and 3.2. So, can you fill up the second row or second set of values. This is the first iteration. This one is the second iteration. So, this is going to be 2 alpha, 2.04 alpha, 3.2 alpha. So, if you want to find x 1 again here; let me once again explain.

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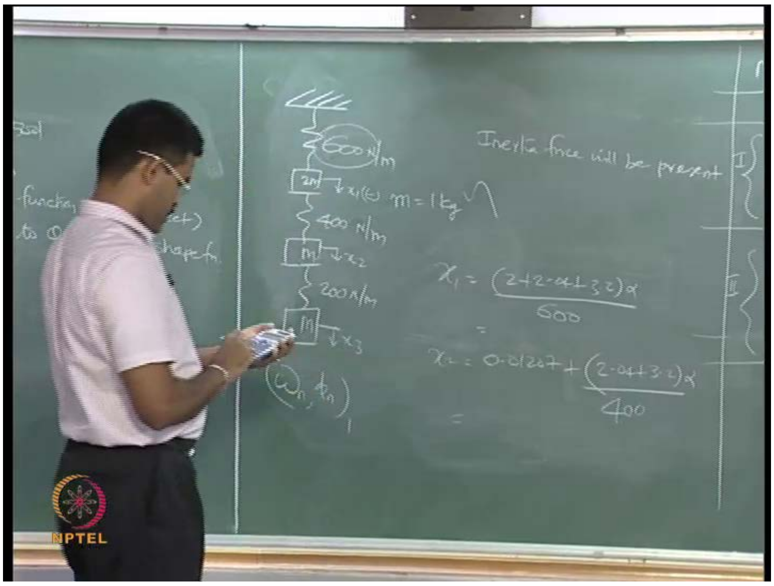

If you want to find x 1, this value is nothing but 2 plus 2.04 plus 3.2 alpha by this spring stiffness, which is 600; which is 0.01207; I call also that value as A double prime, which is going to be multiplied by mode shape. Therefore, this will become 1. If you want to find x 2; can you give me this value – x 2? Not the answer; the procedure – what shall I write in the numerator and what is the denominator? Louder.

Student: 0.01207 plus bracket 2.04 plus 3.2 alpha divided by 400.

It is going to be 0.04117. Actually, this is 3.411; this is 2.085; please make this change. I am looking for the (( )) So, this is 2 alpha again. So, this is going to be 4.2534 alpha…

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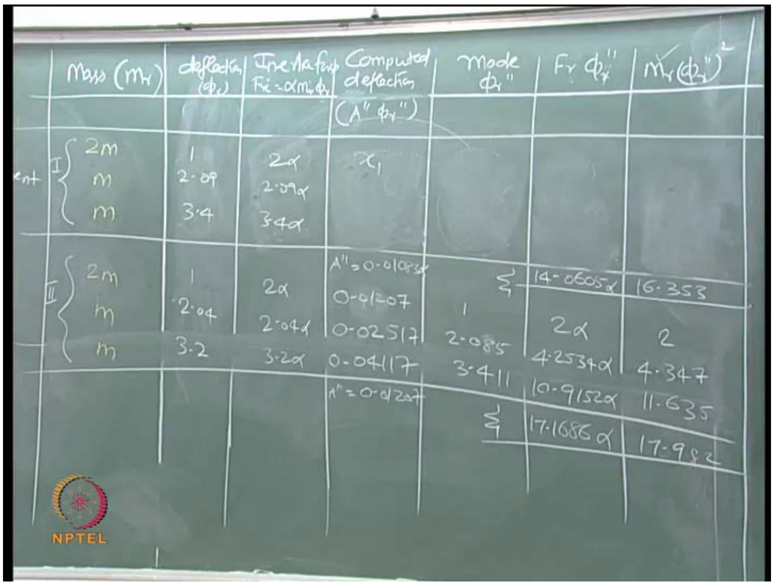

So, third iteration – I will just replace these values here. So, I am using 1, 2.09 and 3.4.

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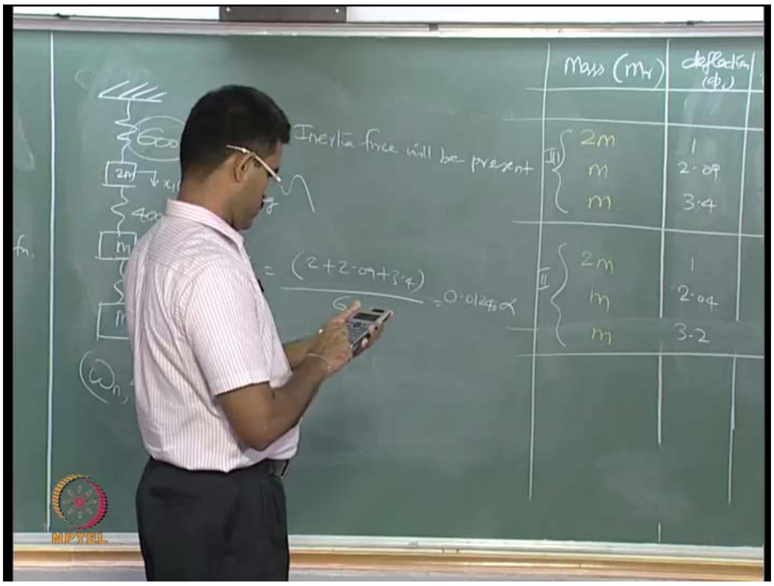

See if you want to find x 1; x 1 is nothing but…

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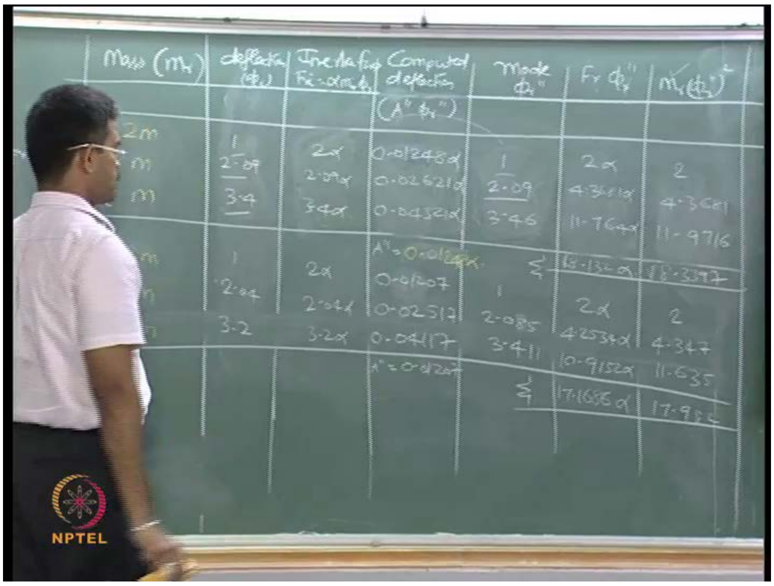

So, it is a third iteration. So, let me change this as… So, let us say that, it is converged, because 2.09, 2.09; 1 and 1; 3.46, 3.5. I can do one more iteration, but still…

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So, omega square will be given by this value divided by A double prime, that is, this multiplied by this, because that is the proportional deflection. And phi 1 is this value. I should say rather 3.5; I do not know where they will converge, but this is what it is. You can also check whether the utility scheme is collapsing in between. How do you check that?

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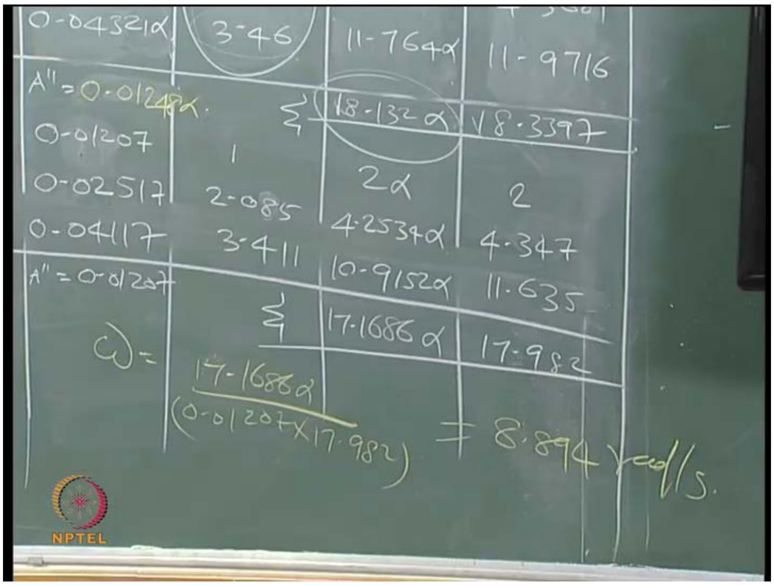

Let us work out omega here; see what happens. Can you work out omega here at this step? How do you get this? This is nothing but 17.1686 alpha by 0.01207 multiplied by 17.982. Say it is actually improving – 8.89; it has come to 8.901 and so on. Had you checked with first iteration also, it would have been around 8.87 something or other. So, it is progressing. When you find that, this value is dipping somewhere, the trend should follow; where it will decrease or increase. You can check. So, this is a very interesting method by which numerically, I can find out omega and phi using Rayleigh’s method. So, this is another method by which fundamental frequency can be determined. Why fundamental frequency? There is a zero crossing of this mode shape. The basis for this method is inertia force will be present. Generally, if you look at the Rayleigh’s method in general, it depends on shape function. How we are going to relate shape functions to dynamics is what I am going to discuss now.

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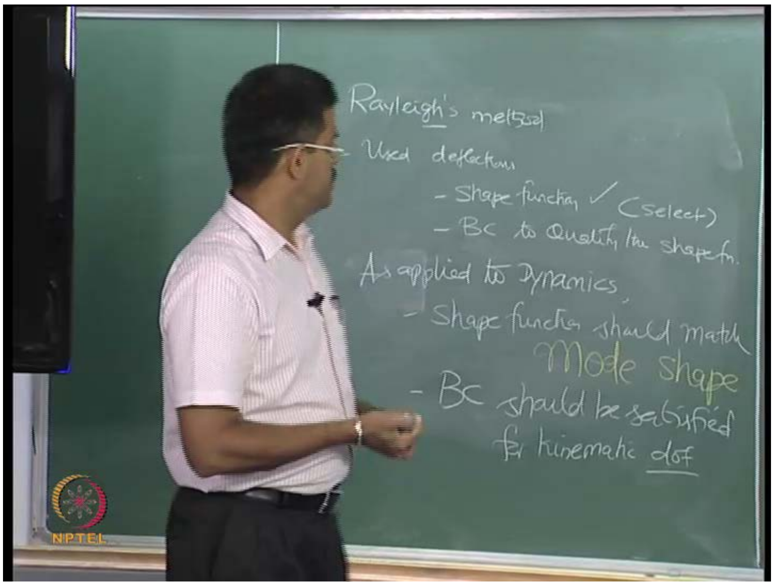

Rayleigh’s method as applicable to dynamics will say that, a shape function let say as applied to dynamics; shape function should match mode shape. Boundary conditions should be satisfied for kinematic degrees of freedom. What do you understand by kinematics and statics? Kinematic degrees of freedom – what do you understand by this? What are static degrees of freedom? Static degrees of freedom are addressing the boundary condition with respect to support, which are armed forces. Kinematics are degrees of freedom on displacements and iterations. Mode shape is a displacement – relative position of mass. So, if these two conditions satisfied, the same algorithm of Rayleigh can be extended to find out omega as I did here, because the shape function is proportional to mode shape of the mode.

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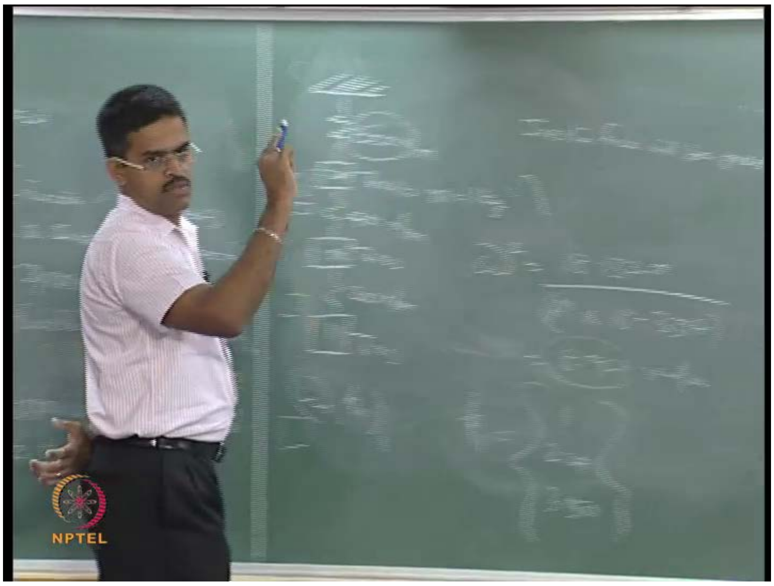

Linearly, if this mass moves by let us say by one unit under the whole action, this mass will move by 2.09; and this mass will move by 3.5 when the mass is vibrating or when the unit is vibrating at this frequency. That is a physical meaning of this. There are many examples in analytical methods, where Rayleigh-Ritz procedure can be applied to analytical problems. I think we will definitely not touch this now. There are two reasons for this. I have got only two minutes left over. Secondly, the lecture is followed by, let us say the most important task of human being in life as we understand; I do not include that in that list my name as human being understand. Therefore, I will not be able to take it forward. And we will have no time to spend this on the next lecture, because I am going to module two, where I will talk about dynamics of structures on motion structures applications. So, I will take up examples and talk about geometric form of different structures.

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So, what we quickly saw in this module is what we are interested in; a very quick walk through of 24 lectures in 2 minutes. We first asked a question that, why dynamics is important for offshore structures. Dynamics can be applied to any system. Offshore structures are having phenomenal mass. The mass may not come from the platform, but from pay load, of course the platform way; and more interestingly because of the variable submergences or buoyancy. And stiffness of the platform or the structure comes from the members, comes from the material, comes from the arrangement of members. I will talk about this.

Both together, I will call this as form dominated design. It is geometric form dominated design, because same constant form of fixed jacket structure was never able to use in deporters for oil explosion, because a very expensive, because mass was very high; the structure was very stiff. The moment you say stiff, I understand it attracts lot of load. When it attracts lot of load, member dimensions will become larger. When member dimension becomes larger, it attracts more forces; mass becomes increased; and installation cost goes high and so on and so forth. Cost goes enormously high; I cannot use the same form as I did for shallow waters to the deeper and ultra-deeper waters. I (( )) different forms. The moment I select the form innovative application, which I say it is innovative; I am looking for a very fancy form, which can be flag of a specific country. Let us say I am looking for that plan; I am not joking.

Geometric form comes from this kind of inspirations only. I will look an innovative form. The moment I say form, I am looking for an arrangement of the member and shape and size; I am looking for these two properties undamped state; I am looking for the fundamental frequency of this system. Why? because I do not want this to match at any cost with the omega band of the excitation force. The excitation force comes from many sources, which we have discussed.

So, after I arrive at the form, I am interested in not only in omega, but also associated property along with this, which is the mode shape; which gives me the relative displacement of mass positions at any frequency of vibration, because this will tell me whether the maximum mass point is going beyond the allowed deflection. Why, because if a platform is having a proportion of 1 at the bottom and somewhere around phi; it means if mass point – 1 is moving the unit dimension; that one dimension can be 1 meter, can be 1 millimeter and so on; the depth may move away by 5 meters – 5 times of that. Therefore, we are not interested in mode shapes in the design.

We will talk about response amplitude operators. These are nothing but mode shapes in different format. They are proportional values actually. But, here the denominator is not proportional to the mode shape of mass position, but to the amplitude of the exciting force. So, mode shapes become very important for me. So, I must have techniques of finding out these two. And when I use different material; when I use hydrodynamic damping available in the material, because of the water body present; I am talking about the effect of damping matrix; then we subdivide the whole argument into free vibration and forced vibration. In free vibration, with and without damping, what are the qualitative understanding of these models; and in forced vibration, at resonance what happens. And we already said, when we have got damping implemented, the upper bound stays at 1 by 2 z.

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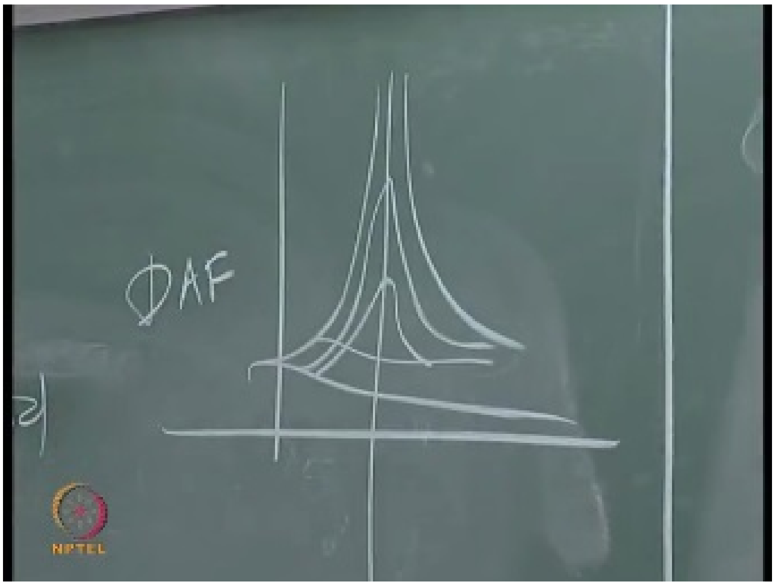

And then we also saw, what is the dynamic amplification factor; how do we keep on moving and so on. What different zeta values even at omega equal to 1; that ratio – omega, omega bar is 1; we are talking about DAF factor. So, we studied all. In addition, we have already seen what are the famous innovative geometric form; people attempted in yester years from 20s till 2012 for using it is as an oil explosion and drilling in production platforms in terms of floating, in terms of particular destructors, in terms of guide towers, in terms of fixed jacket structures, in terms of steel as material, in terms of concrete as material, we have seen all of them. That was the first summary what we had in the first module.

The next module with this understanding, we will get back to the different forms existing and how to do dynamic analysis. So, we will try to see that inside, what are the different research papers available; why they have arrived at these forms; what are the advantage of the form first; then how did they do this; and then of course, how to do the dynamic analysis in total using this. Of course, we will have to take the help of software and some results from papers directly, because I will not be able to do the demonstration of every dynamic analysis in one class, because dynamic analysis of a platform has been in a (( )) elsewhere about three years; I cannot demonstrate in just one hour; I can only show the results. If you do not believe it, do it again; if you believe it, try to do it again. Any question?

Thanks.