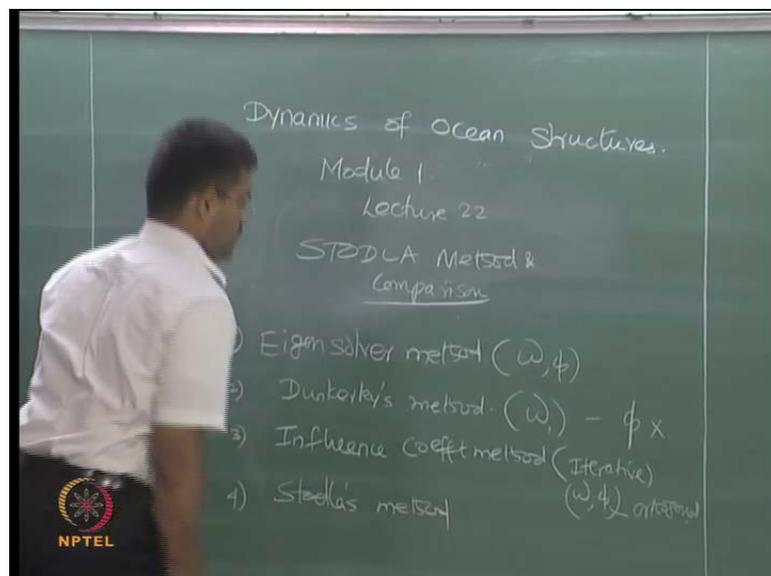


**Dynamics of Ocean Structures**  
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**Module - 1**  
**Lecture - 22**  
**Equations of Motion**

We will today discuss about another method; where, we can solve the natural frequency mode shapes of the multi degree freedom system models.

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So, let us quickly see in the last few lectures, we saw the classical eigen solver method, which can give me omega and phi. We already understood that for larger sizes of matrices of mass and stiffness, this method will be slightly cumbersome and computationally expensive. So, we look forward for some of the approximate methods or numerical methods, which can give me the omega and mode shapes of multi degree freedom system. We started with the oldest method, which is Dunkerley's method, which can or which is capable of giving only the fundamental frequency; let me put it as omega 1 here. We did not speak anything about the mode shapes; it was not capable of computing the mode shapes.

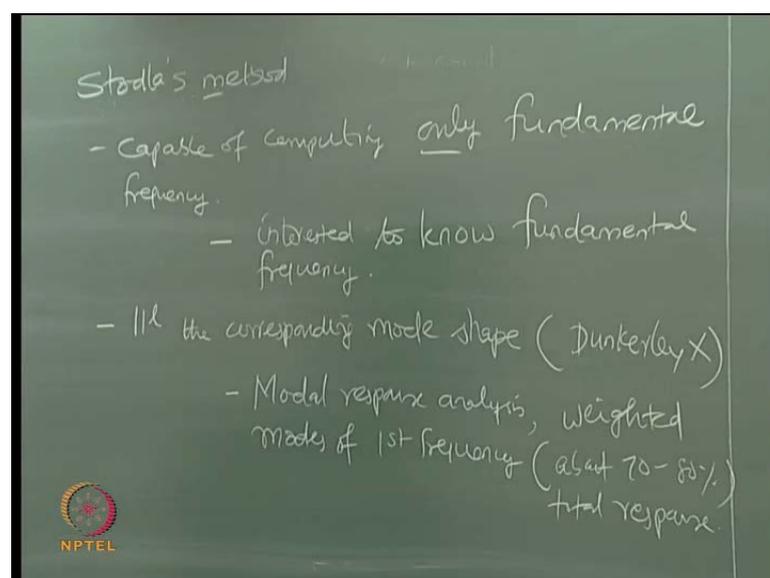
Then, the third level of interesting method, where we discussed to find omega and phi is the influence coefficient method, which is an iterative scheme; which gives me omega

and phi. And, phi – another advantage is they also become orthogonal. There were some relative difficulties of this method compared to that of the eigen solver methods, because the error what you commit in this calculation can become cumulative that can affect your ultimate results. And, it is very difficult to find out the sequence of the omega, whether the omega what you find in the first is the fundamental frequency or not.

What we have got to do is we have got estimate the mode shapes corresponding to the frequency and try to see if the mode shape has a zero crossing or one crossing and so on. We can appropriately map the omega as the first frequency or second frequency and so on. But, of course, this method has a capability of giving me the pair of the values, which is eigen values and eigen vectors or frequency mode shapes together in an iterative format.

Of course, this method can be easily programmable. Therefore, this method is faster and it is converging. We demonstrated two problems in this method. We also gave an exercise in the last class to show you how this can be solved with this method. And, we have also compared the answers what we got from this method with that of the Dunkerley with the fundamental frequency. And, we agreed that, there is a close matching between the fundamental frequency values, what we get from the influence coefficient method compared to Dunkerley's method. Now, today, in the class, we will discuss one more method, which is called Stodla.

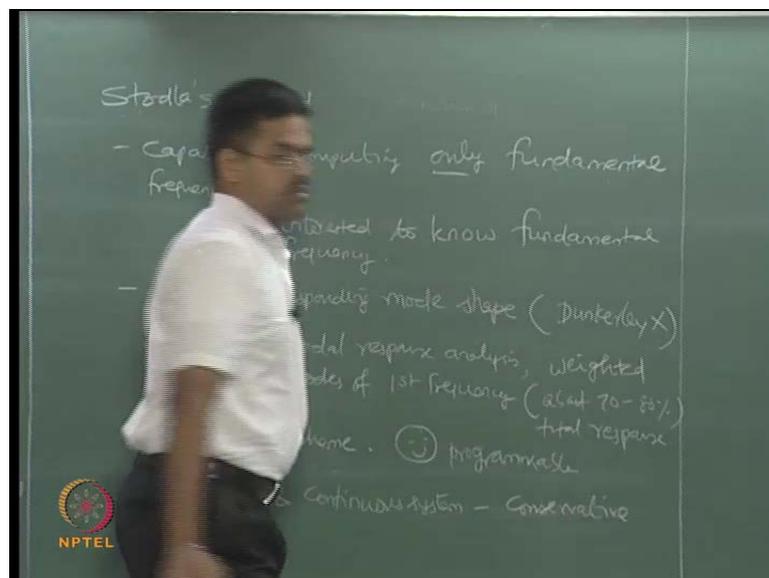
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The method is capable of computing only the fundamental frequency. Now, you may wonder that, why this method is popular, because in most of the cases of multi degree freedom systems vibration problems, we will be interested first to know the fundamental frequency. It is of high importance to us. So, since the method addresses this, this method is popular. This method also gives me parallelly the corresponding mode shape, which Dunkerley could not give. This method gives me also the parallel mode shape associated to this frequency, which Dunkerley could not give.

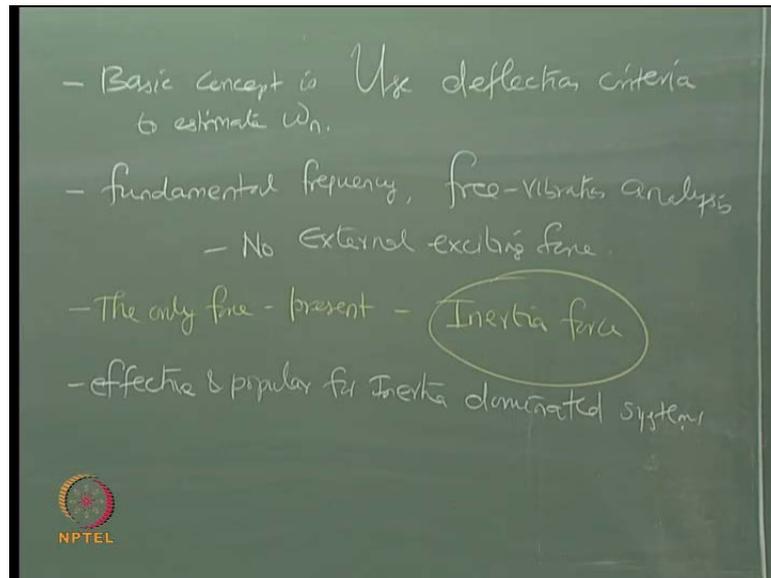
So, if I know the mode shape, the great advantage is I can do what is called modal response analysis in doing modal response analysis, which we will discuss in the second module. You will be generally considering the weighted modes of first frequency, which usually contributes to about 70 to 80 percent of the total response. So, if you have the first mode and the first frequency of the vibrating model, I think you are closely getting the answer of the response of the model; which other ways need not be solved using an influence coefficient method, where I will get all omegas and phi's.

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The other advantage method has is... This method is again an iterative scheme. Therefore, easily programmable; can easily write a program for this. This method can be applied to discrete and continuous systems. But, the system should be conservative; means there should be no energy loss in the system.

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The basic concept of this method is use deflection criteria to estimate frequency. Now, once we talk about fundamental frequency, we are talking about free vibration analysis. The moment we talk about free vibration analysis, I have no external exciting force. To cause a deflection, you need a force. But, the system is talking about free vibration; and therefore, there are no forces. Now, what is that fictitious force present in the system? The only force which can be present in such situation is the inertia. So, the method circles around the inertia force of the problem. Therefore, the method is very effective and popular for inertia dominated systems.

There may be inertia dominated systems in practice, for example, of the structures; which are having a very heavy mass are generally inertia dominated systems. So, the representation value of the inertia force in case of fixed structure, because  $m \times \ddot{x}$ , because  $m$  is very high though  $\dot{x}$  may be very low. In case of complex structures, they are again  $m \times \ddot{x}$ ;  $m$  may be low, but  $\ddot{x}$  is very high. Therefore, the representative value of the inertia force – if it is dominant, then this method of estimating  $\omega$  and  $\phi$  will work successfully. Let us quickly outline this method and see a problem how I will get  $\omega$  and  $\phi$  for a specific vibrating system, which is multi degree, which we gave you in the yesterday's class; we will solve the same problem. So, I would like to solve the same problem again with influence coefficient technique and compare the results with Dunkerley as well and show a clear comparison

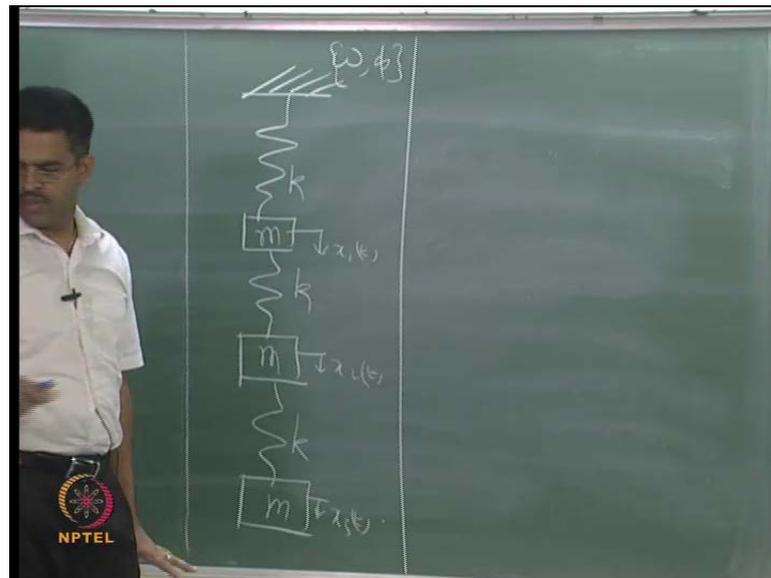
between all the three methods like Stodla, influence coefficient and Dunkerley – at least the first frequency and see are these methods map in the same value or not.

Though I have given you the problem yesterday, you must have solved it by influence coefficient method by this time. But, if you have not understood for the benefit of those people, who are not clear, we estimate the influence coefficient matrix. We will once again do it for this problem, because in the last example, we did by a slightly different method. Here I will again give you a shortcut technique of estimating alpha matrix. So, once you are able to estimate the alpha matrix or the influence coefficient matrix, simply invert it, we can also get the stiffness matrix. The moment I get the stiffness matrix, I have the mass matrix; I can write the equation of motion also, where I am interested in.

So, there may be a small catch here how to write the alpha coefficient matrix for the given multi degree freedom system. We have already seen in the previous examples, how to write the stiffness matrix directly by giving unit displacement and get the forces. We have done couple of problems. Here we are applying unit force and getting the displacements; that is, the influence. So, we are trying to find out this by slightly a different technique today. So, closely follow this. If you are able to follow this technique of estimating influence coefficient matrix, it will be useful for you to solve the problem for omega and phi as well as the equations of motion and then solving the same problem using classical eigen solver technique.

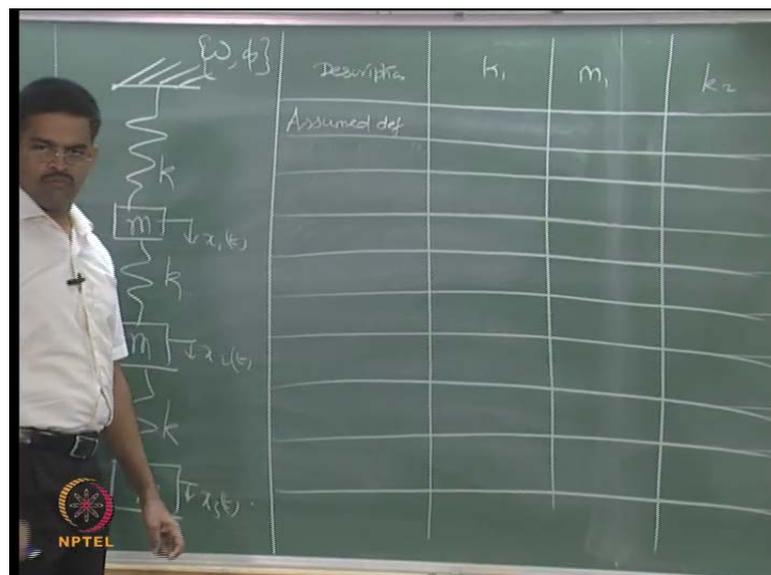
Because you are solving the problem using the classic and eigen solver technique, you need k and m; you do not need alpha matrix. So, you must know how to find the k matrix from the alpha matrix. The very problem or difficulty in all the multi degree freedom system exercises is that, how will you mathematical model the system and how will you find the first step of omega and phi for a given system, because to start the problem, you must know the dynamic characteristic of the system. So, free vibration response is a very important and vital landmark estimate of any MDOF systems. Therefore, I must at least know how to estimate MDOF system characteristics for the given problem. So, we will take up the Stodla's exercise now – the same problem. Solve it; then, compare it quickly with Dunkerley and influence coefficient method. Surprisingly, each method will at least take an hour. But, we will do it faster. Let us see whether we will be able to do it all the three in one shot. So, let us say, the problem, which we had yesterday.

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This is the problem. All were mass matrices  $m$  and all were  $k$ . And,  $x_1$  was marked here;  $x_2$  was marked here; and,  $x_3$  was marked here. So, what I wanted is to find  $\omega$  and  $\phi$  on the system. There are three lumped masses. There are three points of relative displacement. Therefore, it is a three degree freedom system model. Therefore,  $\omega$  will have three values in the corresponding vector, will be 3. So, let us outline the method of Stodla.

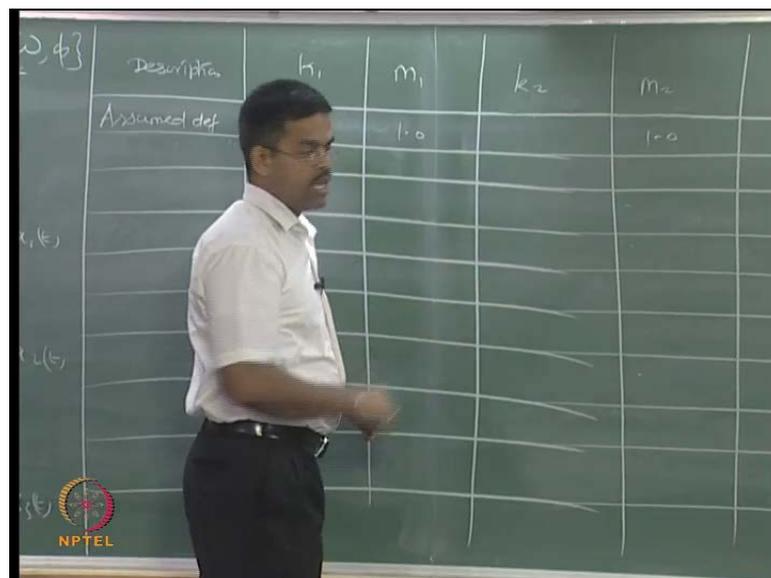
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So, I may say description. It is a tabular method. So, please open a table in the landscape format. Then, you say  $k_1, m_1, k_2, m_2, k_3, m_3$ . Please follow this method. It is a very interesting and an easy technique. And, remember very carefully, this method is not followed here. I am guaranteeing you that, there is no even a single text book, which will give you this method in the same style what I am discussing. It is not available. Not even a single text book will tell you the same way I am addressing this problem. If you do not follow this, you have to suppose to follow only this from me; no other method will help you this. So, please carefully watch this. This is a very interesting technique, but very simple.

But, once I fill up the table, you will never be able to make up how this table was filled. So, do not try to wait; keep on doing it parallelly how I am doing it; otherwise, you will not be able to do it. So, let me first draw some lines of table so that... So, what I am going to do here is I am going to assume the deflection; I do not know the deflection; I am working about the deflection. I already said that. Use the deflection criteria. So, I do not know the deflection of these points. I assume the deflection. So, the deflection will occur at the mass points; or, the deflection will occur at the coordinates, where you have marked  $x_1, x_2, x_3$ ; which are in this problem fortunately the mass points.

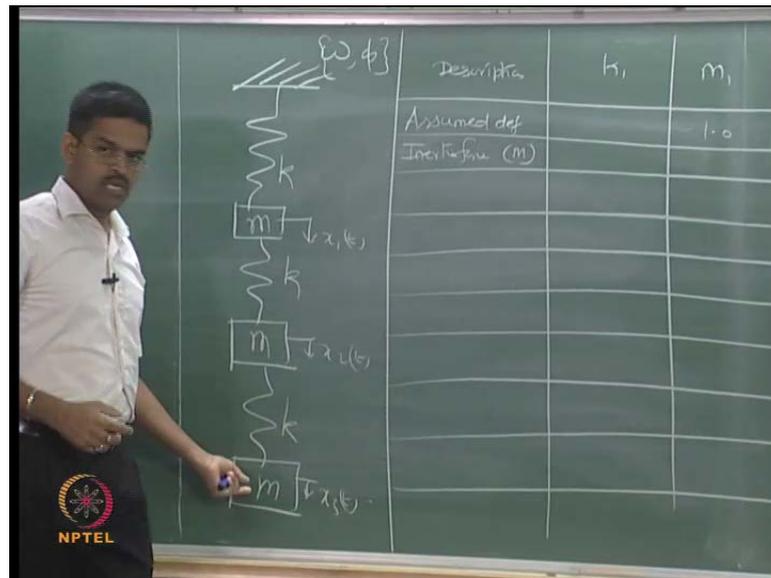
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So, I must write those values of 1.0, 1.0 and 1.0 at the mass columns. Only one said you have to always start at 1 1 1; I already said Stodla method will talk about the

fundamental frequency in the fundamental mode shape. Fundamental mode shape was the first mode shape, will always have the positive value; there is no zero crossing. Therefore, I start with all positive values. You may say in this case, mass are all 1 1 1. If it is 3, 2 and 4. How this will vary? I will take another example. Vary these values and see how I am assuming them. But, the interesting part of this method is you assume any value; it will converge automatically. I will show you that in the next problem.

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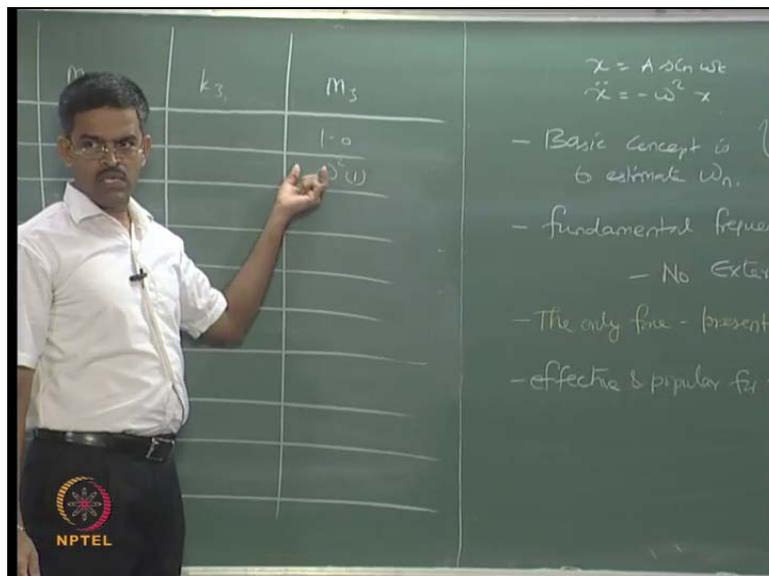
Once I do assume deflection, then I calculate the inertia force, which is nothing but,  $m \times$  double dot. So, I take  $m$  out here and just the multiplier; I am taking it out. You may wonder, sir, for example, if this is  $4m$ , this is  $2m$  and this is  $m$ . Express all of them as proportion of  $m$  and still take  $m$  out. So,  $m$  can be a common multiplier. Even the  $m$  values given as 1000 kg, 50,000 tonnes; do not substitute them here; convergence will be a problem. Keep this as  $m$ . ultimately, the answer can substitute this and get the omega and phi. So, I should start from the third degree of freedom.

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I should say omega square 1, because m is 1.

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Why I am writing omega square? It is very simple; we remember that still; let us write. If  $x$  is let us say  $A \sin \omega t$ ; we understand  $\ddot{x}$  is minus omega square of  $x$ . So, omega square multiplier will be there with the inertia force. So, I am using omega square for the inertia force here. So, omega square – this one corresponds to the value of mass; not this and so on.

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So, omega square again and omega square again. There is a multiplier m here. Once I do this, I keep on commutatively adding this from the bottom. See how we are doing it.

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Let us say I transform this value here. This is omega square. I transform this value further here. This becomes 2 omega square.

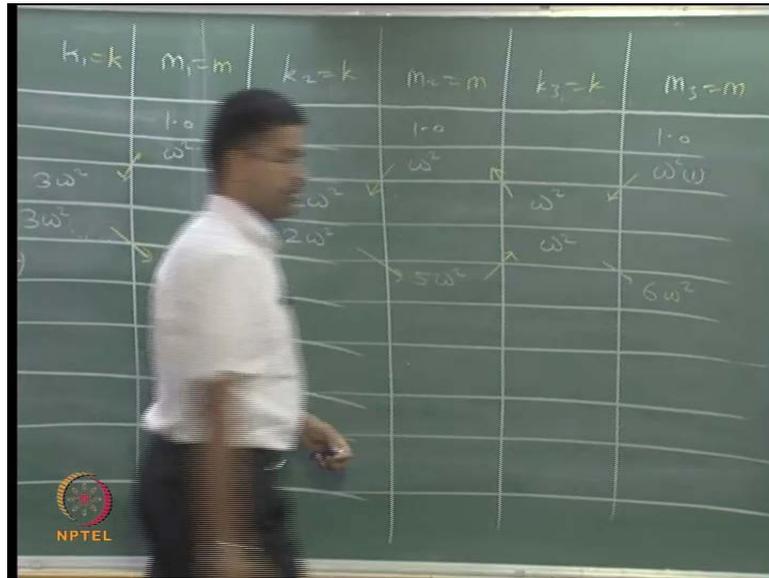
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Description	$k_1=k$	$m_1=m$	$k_2=k$
Assumed def		1.0	
Inertia force (m)		$\omega^2$	
Spring force (m)	$3\omega^2$		$2\omega^2$
Spring deflection (m/k)	$3\omega^2$		$2\omega^2$
Calculated deflection (m)		$3\omega^2$	

I transform this value further here. This becomes 3 omega square. This is what we call as spring force. So, m is multiplied here again. Now, I work out spring deflection, not stiffness; stiffness already given. This is spring deflection, which is again a multiplier of m by k. Spring stiffness is k 1. Let us say, for example, I take in this problem, k 1 is again k; m 1 is again m; k 2 is again k; m 2 is m; k 3 is k; m 3 is m. That is what the problem says.

So, if I know the force, if I know the stiffness; I can divide this to get the deflection of the spring, which will be... In this case, it is going to be 3 omega square m by k; I am taking m by k out. So, it is going to be simply 3 omega square. I will rub this. So, it is going to be only 3 omega square. Similarly, 2 omega square; similarly, omega square. These values are written on the stiffness columns, because these are spring deflection, not the mass point. Once I get this, I work out what is called calculated deflection. Now, when you do that, again, it is a multiplier of m by k here. Now, the deflection of this part will add to this – subsequently, will add to this. So, I start from here. So, this is going to be 3 omega square. I am entering these deflections at the mass points now, because these were the points, where the deflection actually will occur.

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Then, cumulatively, add 5 omega square; cumulatively, add 6 omega square. I take a ratio.

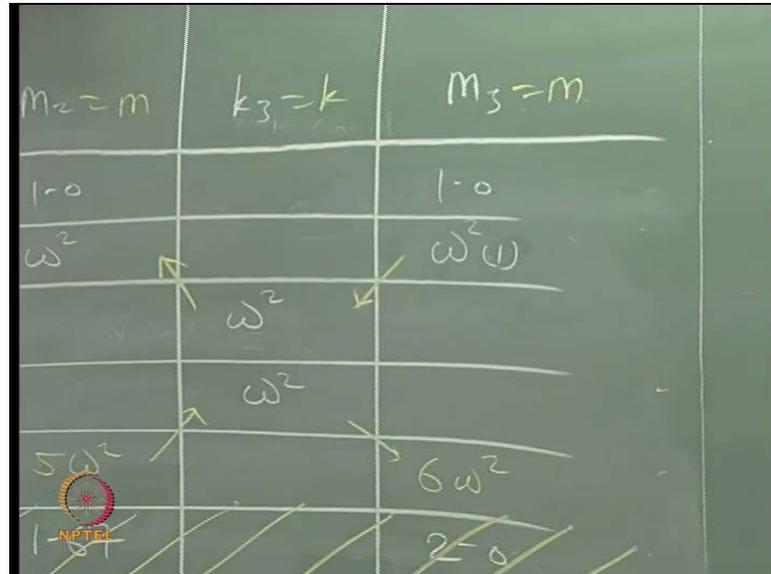
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Let us say divide all of them by three omega square; this will become 1; this will become 1.67; this will become 2.0. It is a ratio. I started with an approximation of 1 1 1; I got 1, 1.67 and 2; it means I am not converging. Let me start the second cycle. So, I should say the assumed deflection; let us say m; I use the same value as 1, 1.67 and 2. This is what we call banded value. And, once the banded values attain, one cycle is completed. So, let

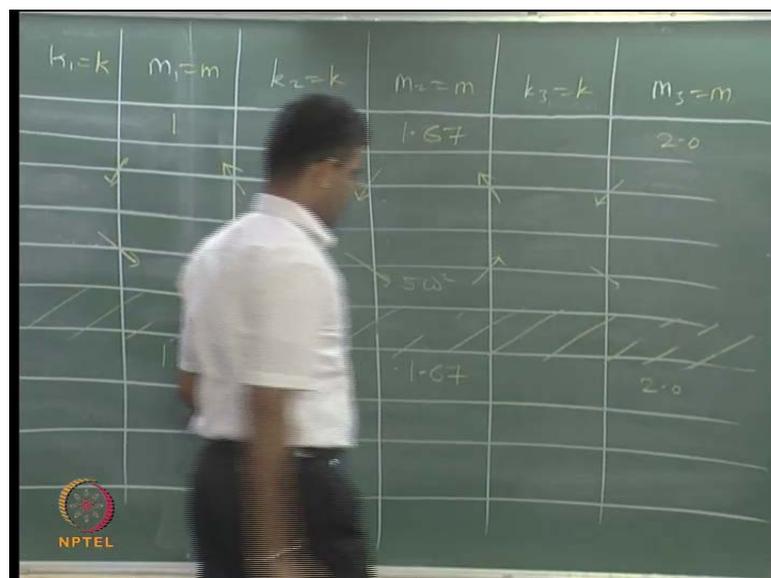
us repeat the same steps again. I do not have space below; I am going to rub this and rewrite it on the top. But, you will not do that. I am removing all the values.

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I will once again do it for you – how this is done. So, it becomes easy for you. You follow this for one more cycle; then, it will be easy.

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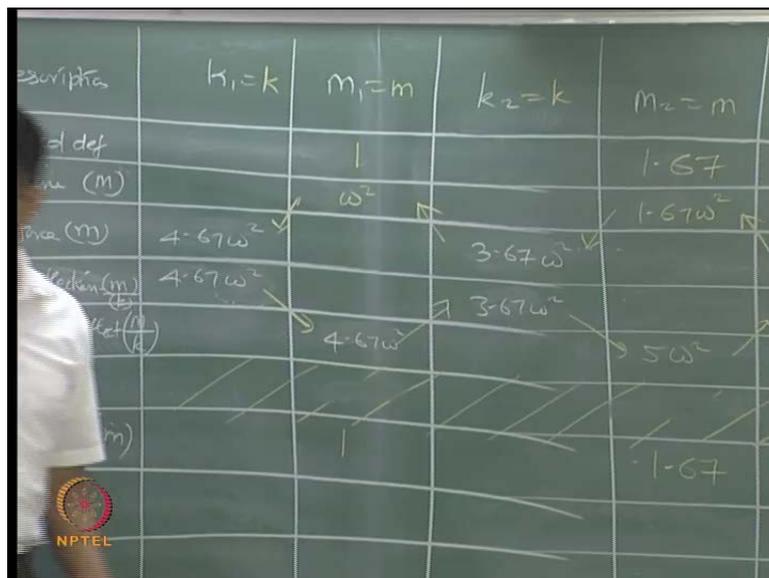


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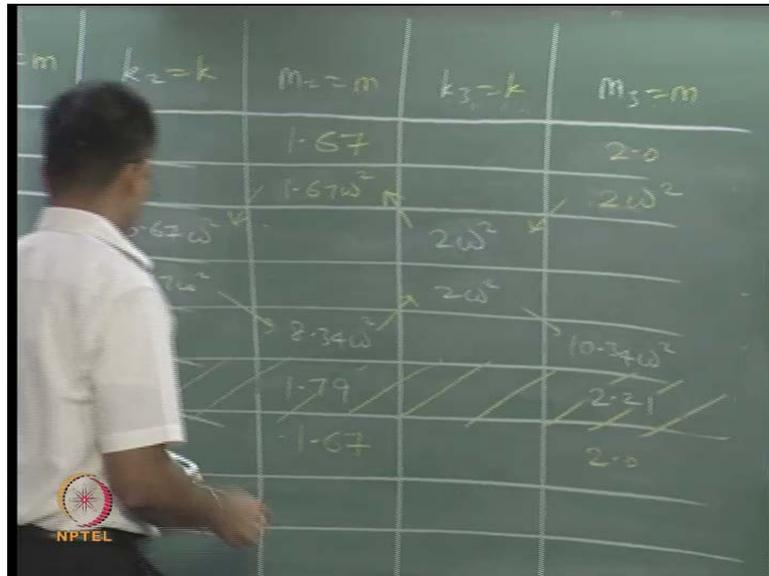
So, I am just rewriting these values back here for my calculations. So, I am writing this as assumed deflection 1, 1.67 and 2.0. I am supposed to do this here; since there is no space here, I am doing it on the top here. So, this becomes 2 omega square; this becomes 1.67 omega square; this becomes simply omega square with an m multiplier out. So, adding, this becomes 2 omega square.

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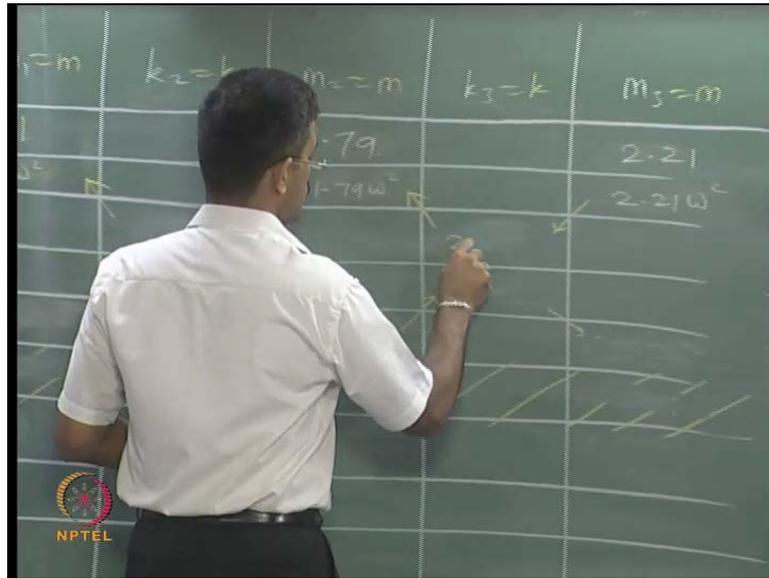
This becomes 3.67 omega square; this becomes 4.67 omega square; again, m out. I divide this with the stiffness; I get 4.67 omega square, 3.67 omega square and 2 omega square. I transform in the same arrow style. So, it is 4.67 omega square plus and so on.

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Let us do that. So, 8.34. Let me rub this; 10.34 omega square. So, I divide this to get a proportional value. I say this is 1. So, this will become... which is 1. – let us say 79; which become 2.21. I started with 1, 1.67 and 2. I have got 1, 1.79 and 2.21. You can now visualize the convergence of the problem. Now, what I will again do is, since it is not converging, I will copy these values back here and do the same third cycle. I have now finished second cycle; the second cycle banded values are this – 1, 1.79 and 2.21. So, I will remove this to avoid confusion. So, the method goes in the same fashion. So, I want you to quickly do the third cycle. I will do it here. I will rub this.

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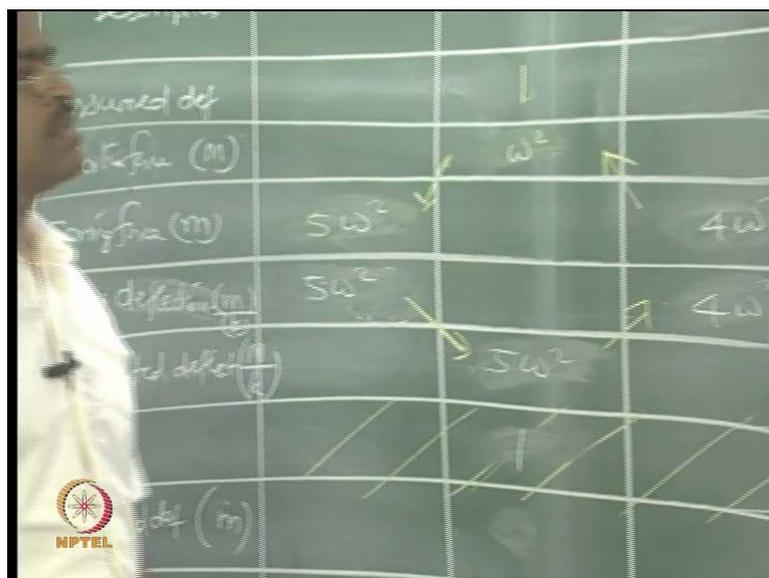


So, I start with 1, 1.79 and 2.2... What is that value here? 21.

Student: 2 1.

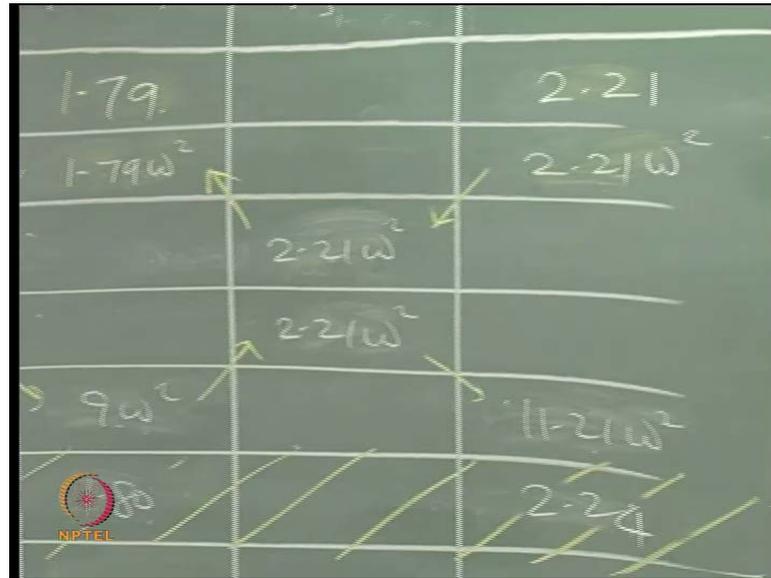
So, it is going to be 2.21 omega square, 1.79 omega square, omega square. So, 2.21 omega square, 4 omega square, 5 omega square. So, 5 omega square m by k, 4 omega square m by k, 2.21 omega square m by k.

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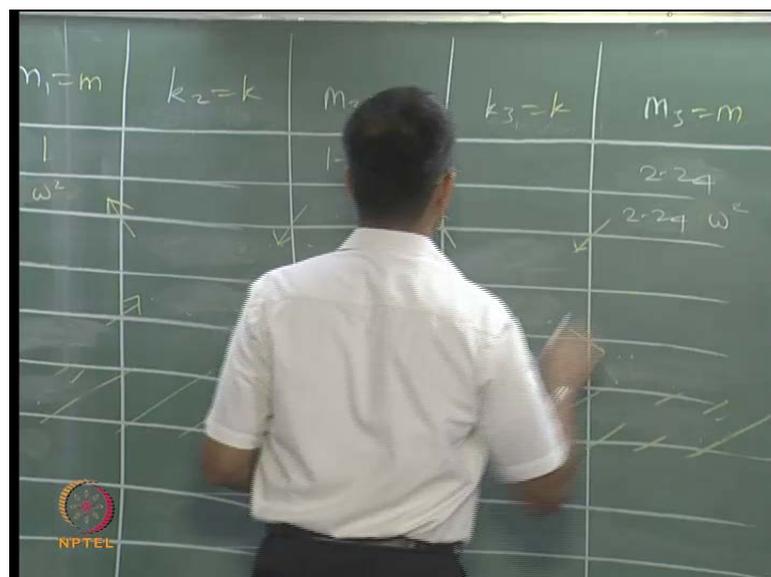
So, 5 omega square, 9 omega square, 11.21 omega square, 1.

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1.80, 2.24. You can see already second mode is already converging here. But, still I have got some discrepancy in the third value; let us again repeat it. So, I have finished the third cycle. These are the third cycle values, which are banded values of 1, 1.80 and 2.24. Still they are not converging, because the values are 1, 1.79 and 2.21. I have still some marginal discrepancy. I can again iterate it. So, let me write down these values.

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So, I should use 1, 1.80 and 2.24. And, do all the calculations back again quickly. So, 2.24, 1.80 omega square. So, we are doing the fourth cycle. So, I start up with 1, 1.80 and 2.24. So, adding them from the last – 2.24 omega square plus 1.80.

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Assumed def			
Interfer (m)		$\omega^2$	
Spring force (m)	$5.04\omega$		$4.04\omega$
Spring deflection (m)	$5.04\omega^2$		$4.04\omega^2$
Calculated deflection (m)		$5.04\omega^2$	
Assumed def (m)			

4.04 plus 1; 5.04 – m multiplied as common again. I say now, by k. So, 5.04 omega square m by k, 4.04 omega square m by k and 2.24 omega square m by k. If you want to assume the calculated deflection, I must start from the first degree of freedom or the first displacement degree. So, this becomes 5.04 omega square.

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	1.80		2.24
	$1.80\omega^2$		$2.24\omega$
	$0.4\omega$		$2.24\omega^2$
	$\omega^2$		$0.4\omega$
	$9.03\omega$		$11.32$
	1.80		2.25

9.08, 11.32; I take a proportional value and find out this. This has become 2.24; it rather becomes 2.25. But, I am just mentioning it as let us say 2.25. This is 1.795; I can call it as 1.8; this is 1. So, I can assume that, we have converged to the value what we started. So, once the convergence is reached, I am interested to find the omega and phi from this. How do you do that? Any questions for anybody? How we get this table filled up? So, we have four cycles of iteration in this case and we filled up the values and we got the proportional displacements of the mass points for the corresponding frequency, which I am interested in estimating. Any difficulty for anybody? How to fill up this table? This concept is purely based on deflection criteria; can be used for continuous and discrete systems. It will give you the fundamental frequency and the corresponding mode shape. The mode shape for the fundamental frequency will be always positive, because there will be no zero crossing; I started with all positive values here.

Since I am talking about the free vibration analysis, I am interested in not applying any external excitation. So, the force only present in the system will be inertia. So, that is what I stated with the inertia force in the system. Very simple, very fundamental algorithm, easily programmable, iterative scheme; definitely converging. So, let us see how we get this value. Any doubt for anybody here? Though I have rubbed and rewritten all the values, I am supposing that you have not done this. And, you must understand there is a specific reason why the flow occurs like this when you calculate this spring force. When you calculate the deflection back again, the flow is reversed. You must understand why it is so. This is very important catch in this problem.

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$$(1 + 1.8 + 2.25) = (5.04 + 9.08 + 11.32) \omega^2 \left(\frac{m}{k}\right)$$

$$\omega_1 = (\omega_n)_{\text{non-ol}} = 0.446 \sqrt{k/m}$$

$$\phi_1 = \begin{Bmatrix} 1 \\ 1.8 \\ 2.25 \end{Bmatrix} \quad \text{mode shape}$$

Answer given  $\omega = 1.8 \sqrt{k/m}$

$$\phi = \begin{Bmatrix} 1 \\ 3.65 \\ 5.125 \end{Bmatrix} \quad \text{WRONG}$$

So, I pick up this; I write 1 plus 1.8 plus 2.25 is the proportional value, which is supposed to be equal to 5.04 plus 9.08 plus 11.32 of omega square m by k; that multiplier is there. Omega square is anyway here; I have missed it here. I have written it here; and, m by k is available here anyway. So, from this, I can easily find omega 1 or omega natural of the first degree. How much is this? 0.446; is that right? Is that right or wrong? The corresponding mode shape what we get, which I should call phi 1 is 1, 1.8, 2.25. So, all are positive value, no zero crossing, first mode, first frequency. We want to check this quickly with the influence coefficient matrix and Dunkerley's; then see where do we stand in this method? Any doubt here? So, the answer what was given by the student in the last class was the following. The answer given – (( )) omega was 1.8 root k by m; and, phi was 1, 3.65 and 5.125, which is wrong.

Now, let us quickly see how we can solve the same problem using influence coefficient method and Dunkerley and compare and let us see where do we stand by comparing all these three methods. Once I understand this, I will ask you to do the same problem using classical eigen solver and compare all of them. And, you should be able to write the program. And, I will give you two more methods in this class. You must write a program of all the six methods comparing any problem of the system. You should be able to derive the omega and phi – all of them; and, compare all of them; give me the percentage error in a plotting format. Which method you will give me? The percentage of minimum error. And, which is fastest in terms of its CPU time? So, those who give me the

program, which will run here for the data what we audience give – they will get additional marks.

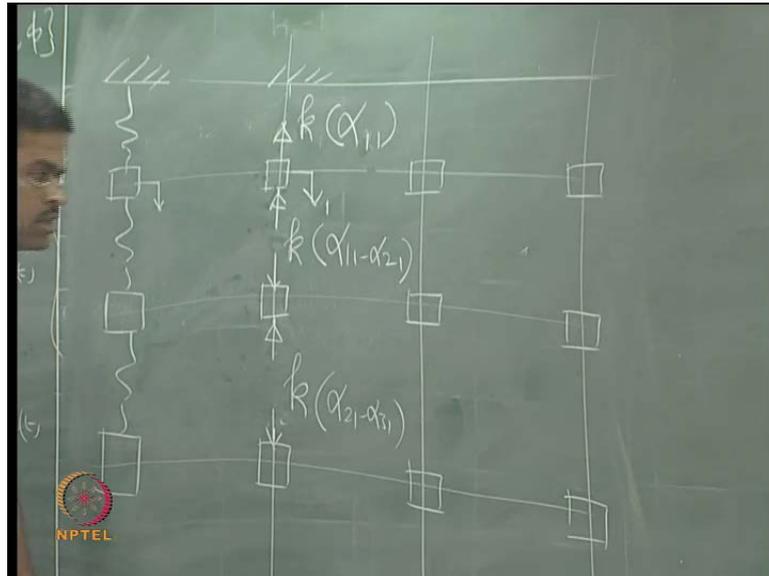
Now, I will remove this. Are there any questions here for this method? It is interesting and surprising that how people are able to understand this by single shot; I do not know. But, still if it is understood, I think it is very good. If it is not understood also, it is again very good, because as long as you have ambiguity in the class, then the teacher value will be keep on increasing. Let it be there. You should not understand all of them, because in one shot, you cannot understand all of them; I do not know whether you have understood or simply shaking your head. Anyway I will take it granted that you have understood. Now, I will remove all these stories and use it in a standard conventional form, which we have been solving in the last class.

Now, the difficulty to solve this problem using influence coefficient or Dunkerleys, both the methods will demand the alpha matrix. So, we need to find out the alpha matrix. So, there is a very interesting short cut method to find alpha matrix for this. Let us quickly and carefully watch this demonstration, so that if I understand this demonstration, I am 100 percent guaranteeing you that, you will never make any mistake in finding omega and phi and k and m matrix in your life ever. If you do not understand this, take it again granted; you will never do any of dynamic problems in your life. It will not be possible at all. You can simple wash off dynamics from your life. You cannot do it. Follow any book on earth; you will not be able to do it. This is a very interesting technique, very simple. Watch it carefully how do we do it, because this is the place where 99.99 percent will get confused. The moment you stop working from here onwards; forget about dynamic analysis. What people generally do is since they cannot do this particular part, which will demonstrate now; effectively, they resolve in software solutions. And, they give unfortunately odd observed answers.

Without understanding what the time period is in frequency of any given system, they will quote values in either two digits – the answers must be in three digits; or, three digits – answer must be in single digit without understanding what actually they are conveying. It is because of the nightmare of the software solutions. You do not know actually the problem – how it has been solved. Now, by Dunkerley and Stodla, you have n number of degrees of freedom problem. You will be able to easily do at least one frequency mode shape behind and check whether the answers obtained in the software is correct as far as

you are concerned in accuracy of about 5-10 percent. So, this is a very important stage in dynamic analysis understanding that, I must be able to solve all of them by hand first. And, write me one program, which is an analytical technique. See how we are drawing or deriving the influence coefficient matrix for this.

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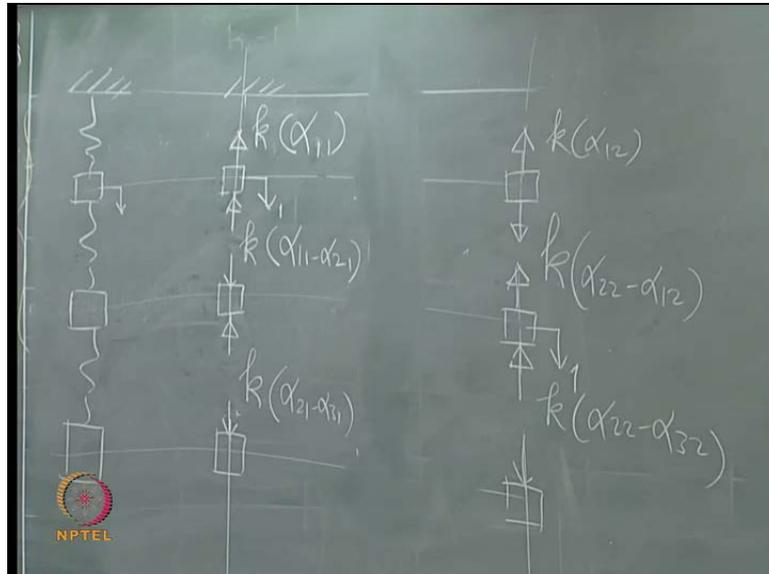


I am drawing the same figure back again here; same figure. I am drawing it three times, because I need... So, let me put the mass points here. The mass values may be  $m$ ,  $3m$ ,  $4m$ ; we are not bothered about that. What I am going to do here is take the first degree and apply unit force on it. So, I am working out the influence coefficient. So, it should be unit force. And, I am looking at the deflection for this. So, this will be compensated. Once I try to pull this mass down, this spring will try to push it up. So, I should say  $k$ . I am taking this as  $k_1$ ,  $k_2$  and  $k_3$ . Then, I will use it accordingly. Or, if you wish, let us keep the same way. I think I will remove the subscripts here. Let us keep it as  $k$ . We will do another problem with different case.

So, let us say  $k$  of  $\alpha_{11}$ . That is going to be the force here. When I try to push this mass or pull this mass down, this spring will counteract this like this. And, this value will be stiffness of this spring multiplied by... You are starting it here. So, you say simply  $\alpha_{11} - \alpha_{21}$ , because it is a relative displacement. Then, same way here all these arrow directions are clear. So, this is going to be the stiffness of this value. That is

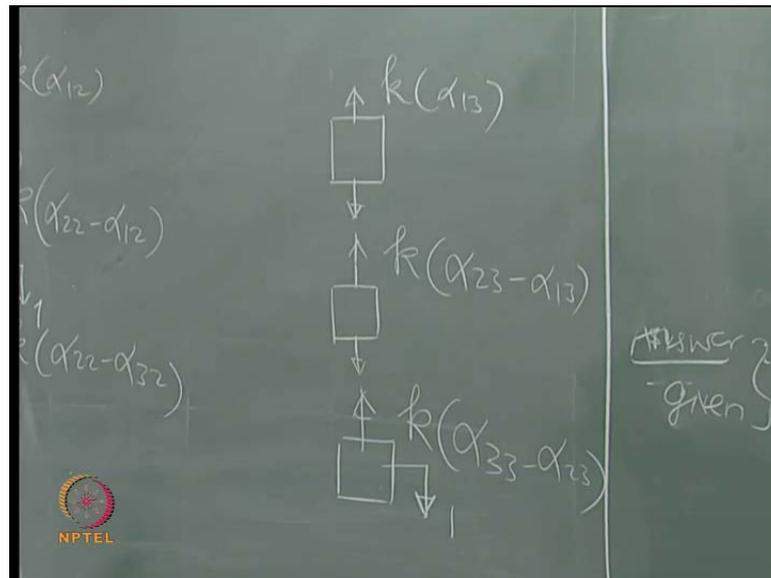
the third one – this one multiplied by... started from here. So,  $x_1 - x_2$ . So, let us check this. I have to do this. Let me do it for the second degree.

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I give the value of unity here. That is the second displacement degree; it is here. When I try to pull this mass down, this spring will try to push it up. So, that value is going to be  $k$  times of... That  $k$  is corresponding to this value. So,  $k$  times  $x_2 - x_1$ , because this spring is connecting these two coordinates. So, the first value is what you write here; the second is what you write later and put the reverse condition here like this. When this mass moves down, this spring will again pull it up. So,  $k$  of simply  $x_1$ , because there is no relative displacement of this. When this mass is moving down, this spring will try to oppose this. So, this value will be  $k$  times of  $x_2 - x_3$ .

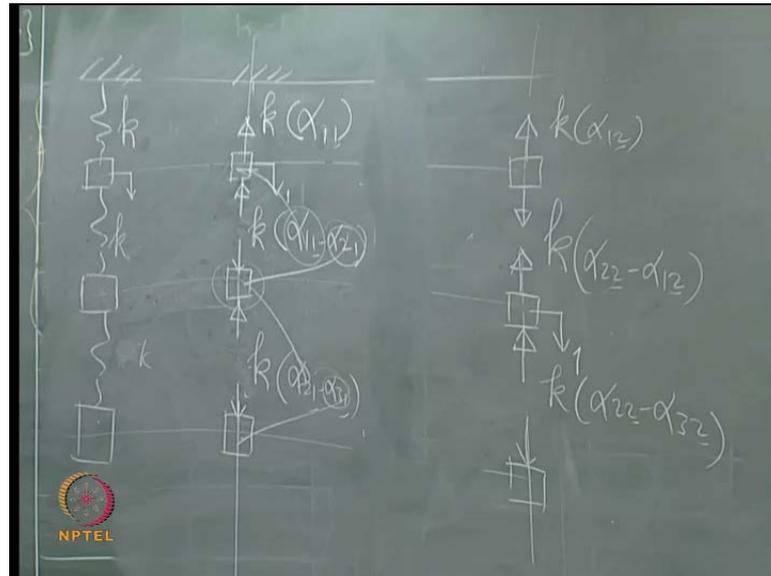
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So, let us quickly do the third one. So, I am giving unit value here – this is unity. So, when I pull this mass down, this spring will try to act up, which is going to be  $k$  times of  $\alpha_{33} - \alpha_{23}$ , because I am starting from here; and, this is in opposite direction. Similarly, when this mass moves down because of this force, this spring will try to take it up; which will be  $k$  of  $\alpha_{23} - \alpha_{13}$ , because this is connecting 2 1 1 and opposite. And similarly, this will be going to be  $k$  of  $\alpha_{12}$ . So, in this, very interestingly, all the second subscripts what you see here – all the second subscript what you see here will be related to the point where you are giving the unit value. All the first subscripts will be at that point where you are measuring it.

Now, where the confusion of this will start? There are many places where it will start. It will start from the right from the beginning that, I do not know anything; I have not understood. Excellent. That is also a very good conclusion. If you do not understand anything, it is better; if you understand partially, it is very dangerous, because then you will not be able to clearly understand it. So, what I did is, I just replaced the free body diagram here; picked up this mass; gave the unit force here applied here as the first degree. And, I may try to pull this mass down; this spring will try to push it up. So, I have marked this arrow.

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When I say this is  $k$ ; if I say, for example, this is  $k_1$ ; this will be  $k_1$ . And,  $\alpha_{11}$  is this displacement coefficient or the influence coefficient. When this mass moves down, this spring will try to oppose it. So, I am marking this arrow. If I say this is let us say  $k_4$ , this will be  $k_4$ . And, this is connecting 1 and 2. Therefore, put the first coefficient here; the second coefficient of this here. And, mark the arrows. Now, this force will try to push the mass down. This spring will oppose it. So, put the arrow. And, this is let us say  $15k$ , for example. This will be  $15k$ ; same spring. And, take this coefficient first; this coefficient next. Why these two are connected? Because this spring is connecting these two coordinates; same fashion you apply for all the three points. First, understand this carefully; then only, we will be able to derive the influence coefficient matrix now quickly.

We have got only I think 1 or 2 minutes more. I will not be able to derive it now. We will stop. If you have any questions here, we can explain you, because I cannot derive it and then solve it. We will do it in the next class. Very very few text books will explain you the influence coefficient derivation like this. Do not think that I am discussing the name of the book. It is all taken from the research papers. If you have any; if you are able to find a good book, which can explain you the derivation of influence coefficient matrix similar to this or better than this, please advice to all your class students as well as to me.

I can do this for n number of degrees very comfortably. I am using only a simple free body diagram. The confusion will be only with the arrow directions; or, you may remember the arrow direction correctly; you put these coefficients reverse. Once again I repeat. Start taking this; put these values first.

When you write here, put these values first. When I move here; when I am giving this value; put these values first. When I move here; when I am giving displacement; put these values first and so on. And, the second value will be respectively to which coordinate the stiffness of the spring is connected. And, the second subscript in all the derivations will be related to the node, where you are applying the unit vector. The first subscript will be corresponding to where you are measuring it. You may wonder; in this case, it is all right. You may wonder in this case, for example, this force. So, I am measuring this force in one.

Why I am not writing  $1\ 2$  first and  $2\ 2$  next? That question will come. The moment we get that question that, here I am writing this value. Therefore, I must get  $1\ 2$  here first and  $2\ 2$  here first. I think it is very comfortable; you have forgotten; you have not understood any of this. Totally, you are blank. That confusion should not come, because this is not governed by... Where are you measuring it? This is governed by this force and this is just opposite to this; that is all. If you remember in that fashion, it is simple. If you again get back and applying more and more thinking process that, it should be first  $1\ 2$  and  $2\ 2$ ; it should be  $2\ 2$  and  $3\ 2$ ; it should be  $3\ 2$  and  $2\ 2$ . I think you are completely lost. And, you will remain lost forever; that is for sure. It is very very important. So, I will do one more problem in the next class just to write the influence coefficient matrix only, because there is no time here now. We will do that in the next class. I am tailing back by four lectures; I must have finished it in 18 or 19 lectures; we are now running 22. It is too much.

Another two more classes, I must complete the first module and move on to the second module. The delay is not because I am not able to write or speak faster. The delay is I want to explain and that takes time. Is that clear? I want you to complete the influence coefficient matrix in the same fashion for all other remaining problems. This is a practice. And, see can you derive alpha matrix in the same style for all problems at least two degree, three degree and so on. It is very important. You must understand this

method. If it is understood, it is a foolproof technique; you will never make a mistake at all in finding omega and phi forever. It is for sure. And, the converse also applies.