Dynamics of Ocean Structures
Prof. Dr. Srinivasan Chandrasekaran
Department of Ocean Engineering
Indian Institute of Technology, Madras

Module - 1
Lecture - 21
Study of Multi Degrees-of-Freedom Systems

In the last lecture, we had a problem; where, we have arrived at the approximate natural frequency – the fundamental frequency using Dunkerley's method for this problem, which is three degree freedom system problem; where, there are three displacement coordinates, which measure the independent displacement of mass when they are connected or suspended by a system by spring stiffnesses as 3k, k and k.

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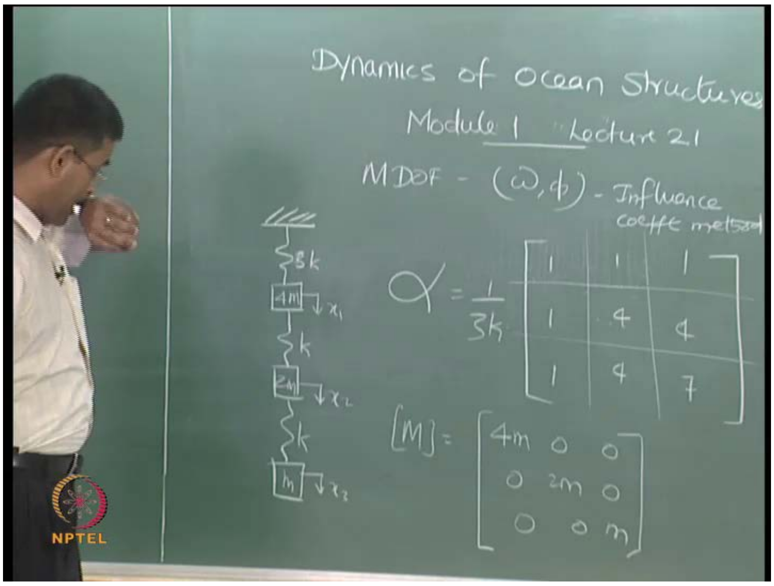

We also explained how to derive the influence coefficient alpha in the last lecture. And, this influence coefficient matrix, which we have derived in the last lecture; and, we already know that, since the displacement coordinates are measured with the same points, where the mass is lumped; the mass matrix becomes diagonally dominant. And, the off diagonal terms in this case becomes 0. Our objective is to find omega and phi; Dunkerley's system or Dunkerley's method will not give me the mode shape. But, I am going to derive or use this method, which is called as influence coefficient method to find out parallely the pair of omega and phi. That is natural frequency and the mode shape. Mode shape is actually the physical representation of relative question of mass with respect to the other degrees of freedom when the vibrating system is excited or vibrated with specific frequency, which is a pair of the mode shape.

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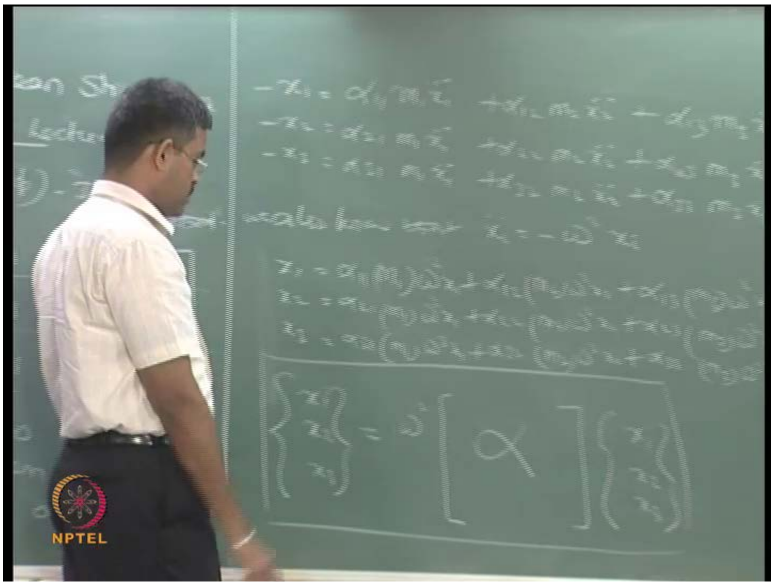

We intended to write the control equations. And, I already said this is an iterative scheme. So, there is a specific method by which you can write this equation. So, what I should do here is alpha 1 1, 2 1, 3 1. So, these are nothing but the values of (Refer Slide Time: 02:01) 1 1, 2 1 and 3 1 as it is; column-wise I have written. After I do this, I simply say m 1 x 1 double dot and so on. We also know that, x a double dot is minus omega square x i for x i being a sinusoidal function, which has been assumed. So, substitute back in this equation; I get… So, m 1; similarly, m 2… So, the minus sign in this case will get cancelled with this negative sign. So, I can remove this negative signs here. Simply say omega square x 1. I can transform this in a matrix format. Omega square of the alpha matrix, which is nothing but the influence coefficient matrix; that is what I will get here; multiplied by the same vector.

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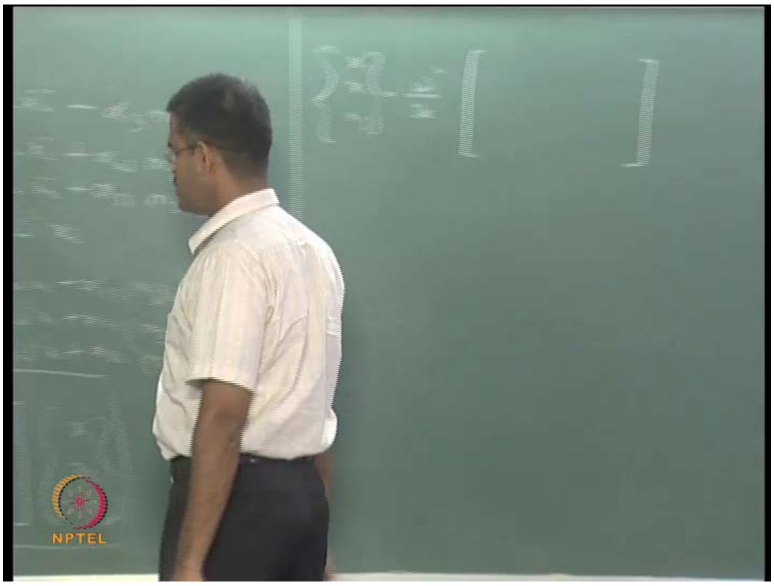

Now, we can clearly see there is an iteration scheme being set here, because I assume the left-hand side of this equation; get the right-hand side. And, I can substitute back and compare the value what I have trying with the value what I have substituted if they match; I say it is converged, non-matching; keep on iterating. So, I had an iterate scheme here now. So, let me do this scheme now here. So, my control equation is going to be x 1, x 2, x 3 omega square by 3k, because alpha has a constant here 3 k (( )). So, I should say omega square by 3k of…

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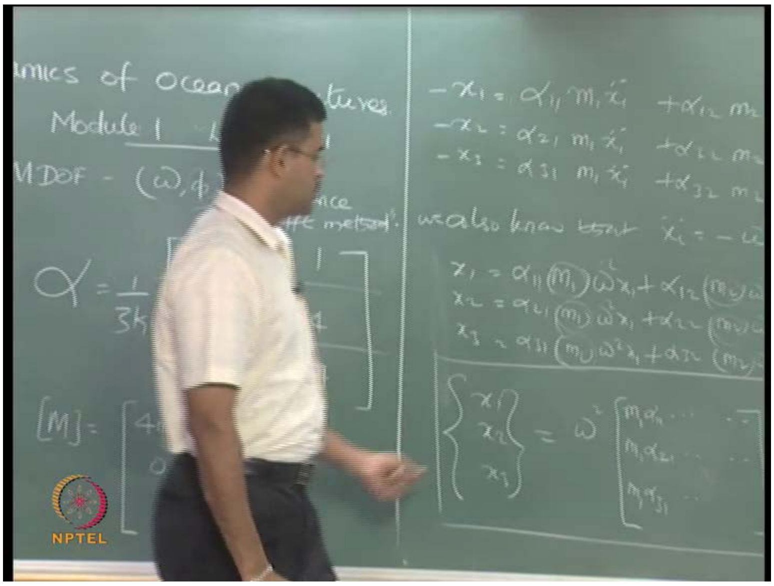

So, this alpha matrix (Refer Slide Time: 05:09) slightly modify, because this is not going with the same matrix here. I am multiplying here with m 1 – the coefficients with m 2. So, I should say this will be let us say m 1 times of alpha 1 1, m 1 times of alpha 2 1, and m 1 times of alpha 3 1 and so on. So, all these values will be m 2's; all these values will be m 3's. So, I will substitute back here. m 1 – I already know it is 4 (Refer Slide Time: 05:39). So, substitute back this multiplier value here.

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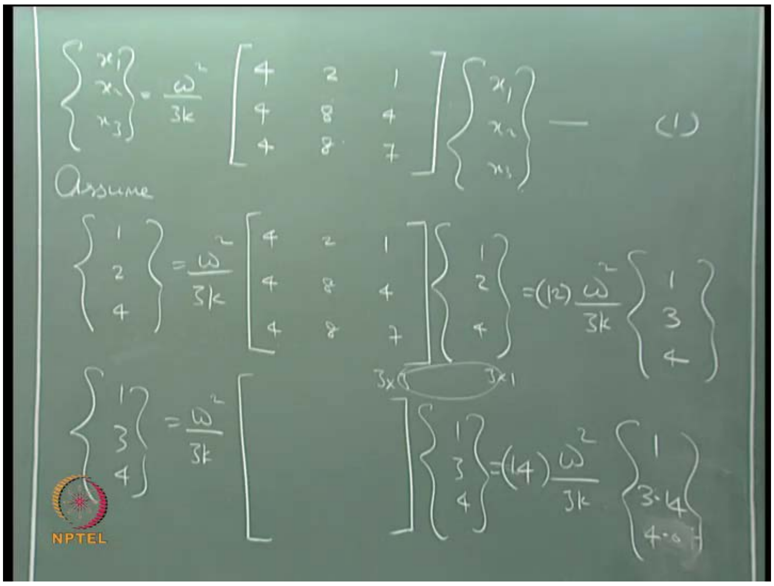

I get this matrix as… 1, 4 and 7 of x 1 x 2 x 3. So, this is the control equation 1. So, we already know for the first mode shape, I will have zero crossing; all will be positive. So, I assume – 1 2 4; you can even start with 1 1 1; you can start with 1 2 3 – any number. But, only thing is since we are looking for first mode, I understand them – all of them should be positive. There will be no zero crossing. So, I do this and see what happens. Multiply with again 1 2 4; I get omega square by 3k. So, there is a small correction here. Please see this how we are doing it. Let us multiply this – the first row with first column, because this is the 3 by 3 matrix and this is 3 by 1. I will ultimately get a 3 by 1 vector, which I am getting here. So, the first value will be 4 plus 4 – 8 plus 4 – 12. So, instead of writing 12 here, I am taking this 12 out; I am putting 1 here. Similarly, the second will be 3 and 4. So, I started with 1 2 4, but I am getting 1 3 4; it is not conversed. So, let us repeat it. I am going for two decimal convergence. Last one is wrong. So, it is 4, not 4.67; it is 4. So, again it is not converging, because 3.14; keep on repeating and tell me what is the final value you are getting.

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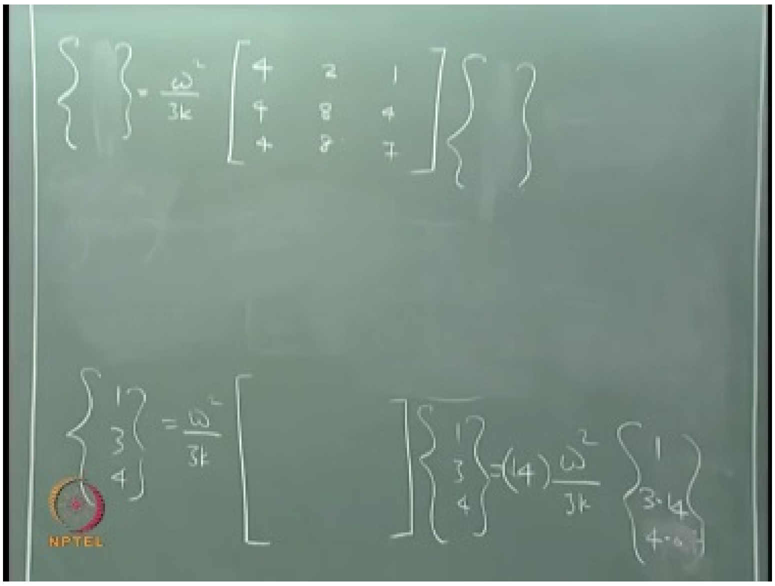

I have to unfortunately do it on the same space of the board, because I need this as well as I need this. The worst by worst, I will just rub this and keep on doing this. I am not going to do that. You will tell me the final value after convergence. So, I will remove this. And, tell me the final value you are getting. So, keep on doing the operation again and again.

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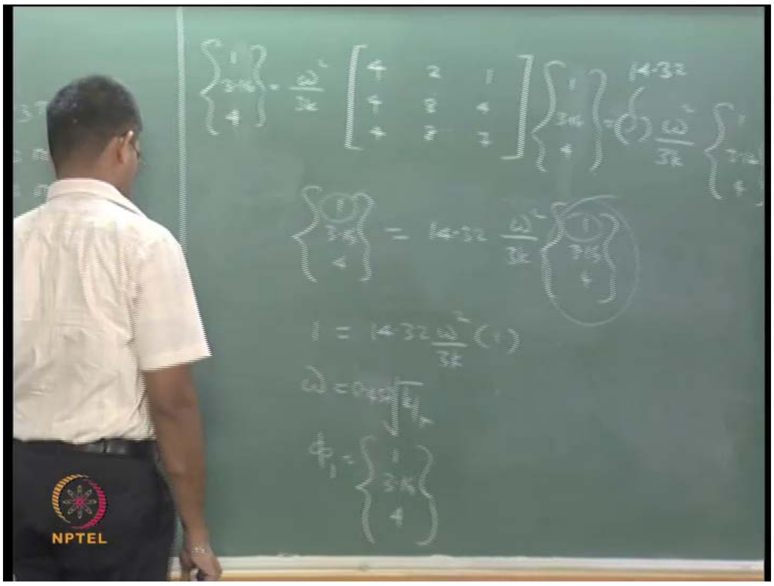

After the fourth step of iteration, I am getting this as 1, 3.16 and 4 matching with 1, 3.16 and 4. And, the multiplier is 14.32. This multiplier is 14.32. This is what I will get after the fourth iteration. So, how to get omega from this? So, I already know that, 1, 3.16, 4 is same as 14.32 times of omega square 3k of 1, 3.16, 4. That is what I am getting from the iteration scheme. So, equate the first row. So, I can rewrite it separately. 1 is equal to 14.32 omega square by 3k of 1. I can get omega from this as root k by m. The value will be 0.457; whereas, we compare this with Dunkerley; it was 0.4. So, the corresponding mode shape, which I call as phi 1 is this one; 1, 3.16, 4; will remove this.

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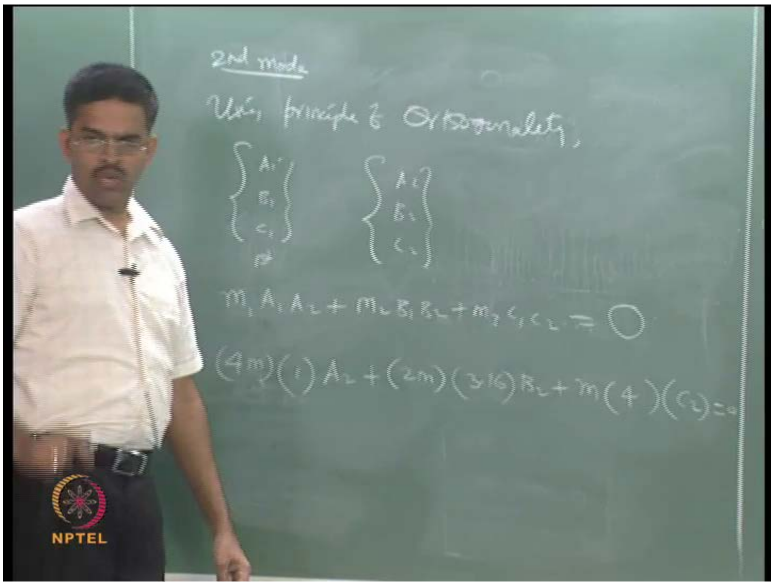

Now, I will use the principle of orthogonality to get the second mode. We already know the first mode. Let us say if we have first mode as A 1, B 1, C 1. This is the first mode. And, the second modes A 2, B 2, C 2. Then, according to the principle of orthogonality, I should say m 1 A 1 A 2 plus m 2 B 1 B 2 plus m 3 C 1 C 2. When I am crossing the modes, what should be this value? Should be 0. So, m 1 from the problem is 4 m; A 1 for the problem is 1; A 2 – I do not know; that is the second mode

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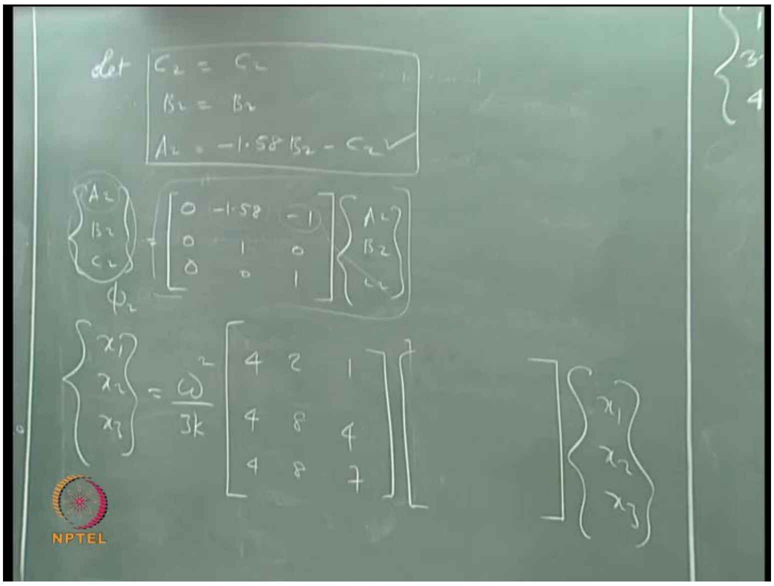

plus m 2 for my problem is 2 m; B 1 is 3.16; B 2 – I do not know. m 3 – simply m; C 1 is 4; C 2 – I do not know. So, I have only one equation, where I have got three unknowns – A 2, B 2 and C 2. How do we get it? I will remove this. Remember mode shapes are relative displacements of the mass position when the system is vibrating on the specific frequency. In this case, let us say, for example, omega 2. So, what I do is let C 2 be called as C 2; and, B 2 be called as B 2. Let us remain as C 2 and B 2. I find A 2 in terms of C 2 and B 2. That can be easily found from this expression. So, what is that you get for A 2? Minus 1.58 B 2 minus C 2. That is what you will get when you solve this. I write this in a matrix form. So, I say that A 2, B 2, C 2; which is my second mode. is nothing but… So, you can read it A 2 will be equal to minus 1.58 B 2 minus 1 C 2.

That is what I am getting here. B 2 is nothing but B 2; C 2 is nothing but C 2. So, all these equations are transformed in a matrix form here. So, instead of calling them as A 2, B 2, C 2, let me call them as again the same format of x 1, x 2, x 3 to get the iterative scheme. I will borrow the same control matrix of omega square by 3k (Refer Slide Time: 17:27). So, omega square by 3k. I have the same control matrix of influence coefficient as this value, which is 4 4 4; 2 8 8; 1 4 7. I multiply this matrix with this matrix and call this as x 1, x 2, x 3, because the new x 1, x 2, x 3. As we wrote here in the algorithm, is now modified as a multiplier of this matrix on x 1, x 2, x 3. That is what I am getting here. So, instead of this, I am substituting this here. So, let us do that.

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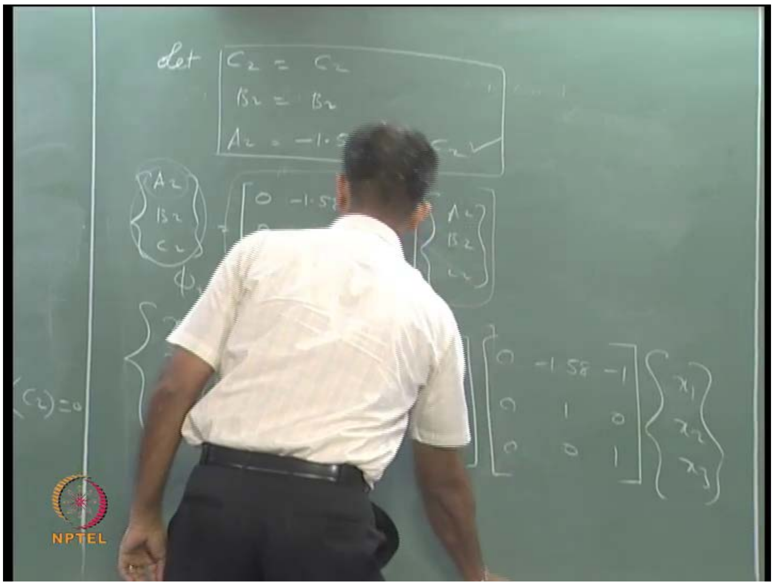

0 0 0; minus 1.58 minus 1 1 0 0 1 of this. So, I simplify this; I will get a control equation now, which is going to be the iterated to get omega and phi; which will be the second degree of freedom.

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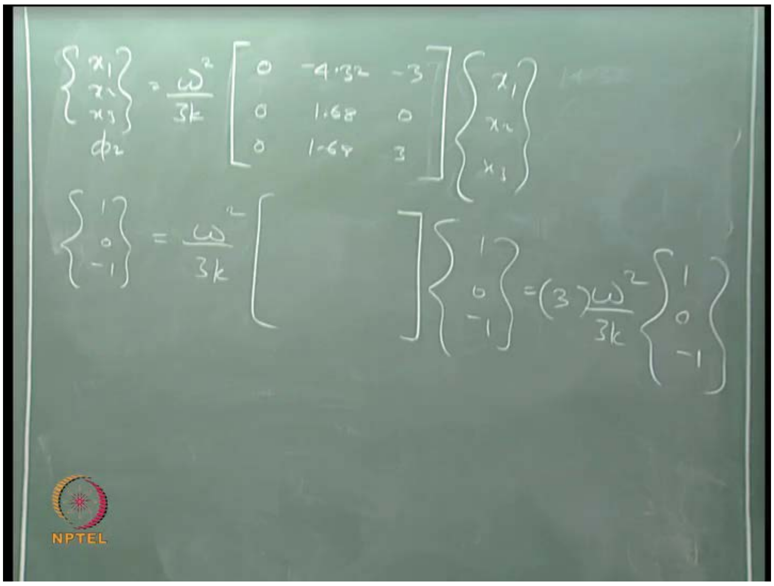

So, I should say here – x 1 x 2 x 3 of… This is actually the second mode, which is going to be omega square by 3k of this matrix of x 1 x 2 x 3. So, multiply these two matrices and get me this matrix, which is the control matrix now. Of course, the first column of this matrix will be 0 because it is multiplied by a zero-th column here. So, 0 0 0; minus

4.32 1.68 1.68 – second column; 0 3; thank you; minus 3 0 3. So. I am looking for the second mode; I will have one zero crossing. Let me do that by assuming the vector, which is 1 0 minus 1. It is a vector with only one zero crossing – omega square by 3k of this matrix multiplied by 1 0 minus 1 will give me omega square by 3k of some vector with a multiplier. What is that multiplier? I am getting a multiplier as 3; and, I am getting the same vector back. Do not copy; otherwise, you will never understand this problem. You have to do it by simple calculation; use the calculator and try to find out these answers. So, I got back the same vector. So, what can we say about this step? It is already converged.

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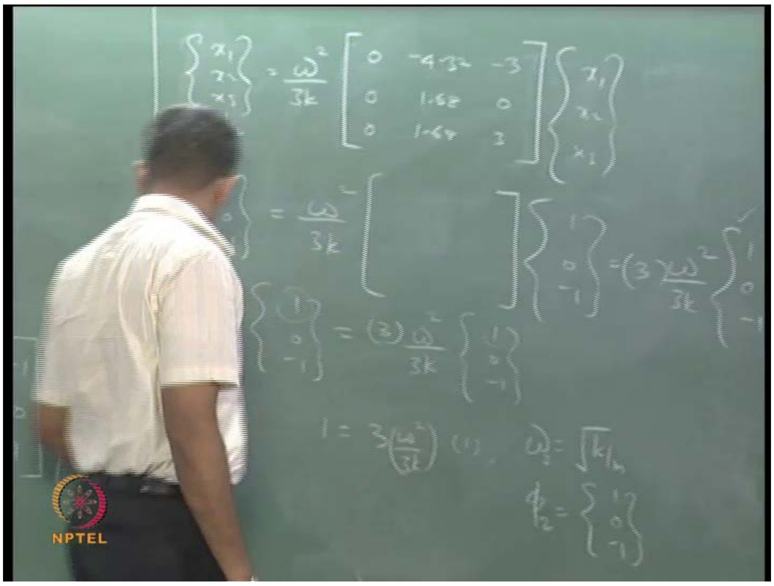

So, let us pick up the first row of this. So, one – that is, 1 0 minus 1 is equal to three times of omega square by 3k of 1 0 minus 1. So, pick up the first row and equate with the first row. I should say 1 is equal to 3 omega square by 3k of 1, which will give me omega as simply root of k by m, which is the second frequency; I can call this as omega 2; and, the corresponding mode shape is 1 0 minus 1.

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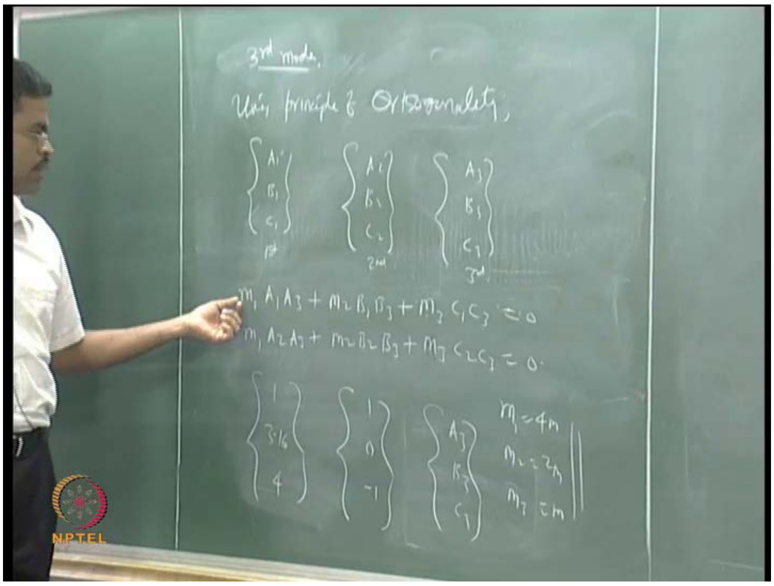

So, let us try to do the third mode. So, again, we will use the principle of orthogonality; I will have three modes here – A 3, B 3 and C 3; the first mode, the second mode and third mode. So, I want to write the orthogonality equations; I should say m 1 A 1 and A 3; I am crossing 1 and 3 plus m 2; B 1 and B 3 plus m 3 C 1 and C 3 is 0, because I am crossing i and j. Similarly, m 1 A 2 and A 3 plus m 2 B 2 and B 3 plus m 3 C 2 and C 3 is again 0, because I am crossing 2 and 3; i and j are not equal. So, m 1 I have; m 2 I have; m 3 I have. A 1 B 1 C 1; A 2 B 2 C 2 now I have. A 1 was… Let us say A 1 vector – the first mode plus 1, 3.16 and 4 if I am right. The second vector was 1, 0 and minus 1. And, the third vector I do not know; I call this as A 3 B 3 C 3. And, of course, m 1 is equal to 4m; m 2 is equal to 2m; and, m 3 is simply m from the problem. The substitute in this develops two equations. Let us see how do we get them. I will remove this. So, let us substitute back for m 1, m 2, m 3, A 1, B 1, etcetera; get me two equations.

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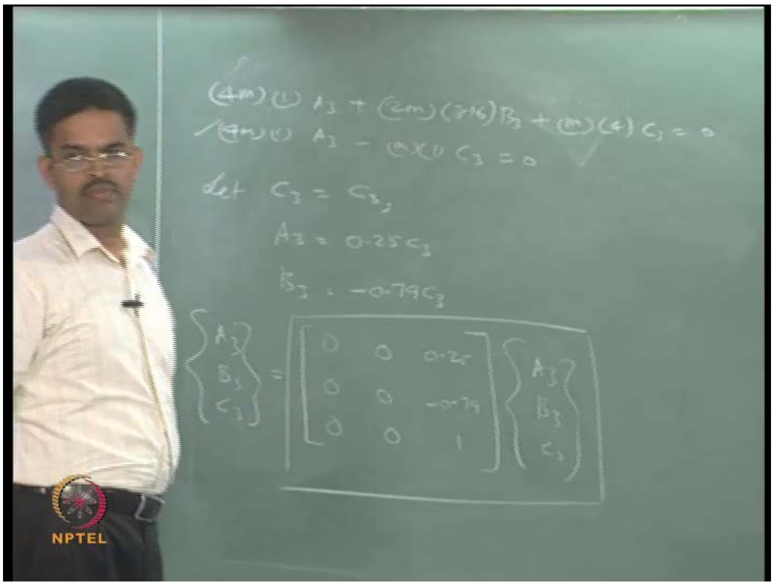

So, if you can do that, it is going to be 4m of 1 of A 3 plus 2m of 3.16 of B 3 plus m of 4 of C 3 is 0. That is the first equation. The second equation – 4m of 1 of A 3; B 3 term will not exist, because this is cancelling; minus m of 1 of C 3 is set to 0. So, I have got two equations. But, still I have three unknowns; I can set one. Let us say let C 3 be indicated as C 3 itself. This may be looking a very funny statement. But, we have got to assign this to solve these two equations. Now, what are A 3's and B 3's. A 3 will become 0.25 C 3. That is available in this equation straight. Substitute back in the first equation B 3 becomes minus 0.79 C 3; so write a vector now; so A 3 B 3 C 3, which I want to use for iteration.

Now, modified as two columns becoming 0; last column will be 0.25 minus 0.79 and 1 of A 3 B 3 C 3. Let us read it; A 3 is equal to 0.25 of C 3; B 3 is equal to 0.79 minus of C 3; and, C 3 is of course equal to C 3, which I am getting in the same equation. Now, a control algorithm will be… Make use of this matrix of phi 2 – this (Refer Slide Time: 27:22) matrix of phi 2 which we already had with one column 0's. Multiply this matrix with the new vector, which I am going to call as x 1 x 2 x 3 of phi 3. And, I am just rewriting it here. I am rewriting it here itself. So, I am making this as phi 3. This matrix already exists. And, this I am replacing with this matrix (Refer Slide Time: 27:51). So, I am just…

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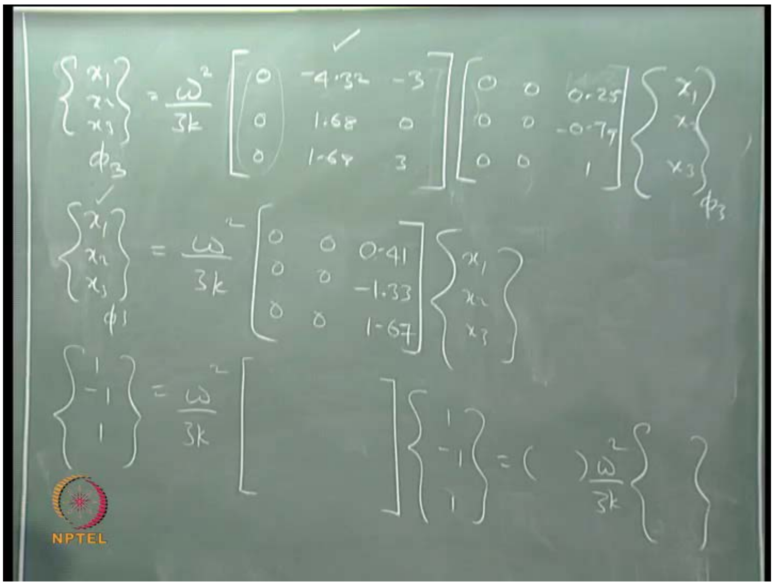

I am not writing it here; I am putting this matrix here with x 1 x 2 x 3 of phi. Just for completion sake… Now, I will get a new control matrix. I am saying x 1 x 2 x 3 of phi 3. I am getting a new control matrix, which will give me now. Do it sincerely; otherwise, this will be very difficult; do it sincerely. So, simple matrix multiplication; you should be able to do it faster. So, of course, these two columns will become 0, because I have a multiplier matrix, which is 0 in two columns; I am looking only for three values, which are 0.41, minus 1.33 and 1.67. So, now, I am going to assume the vector say third mode; I should have two zero crossing. So, I can start from a positive to negative back to positive. I will get two crossings now, which will be omega square by 3k of this matrix of 1 minus 1 1; which will give me a multiplier of omega square by 3k of some value. Let us see what is this multiplier after iteration. So, when you do with 1 minus 1 1…

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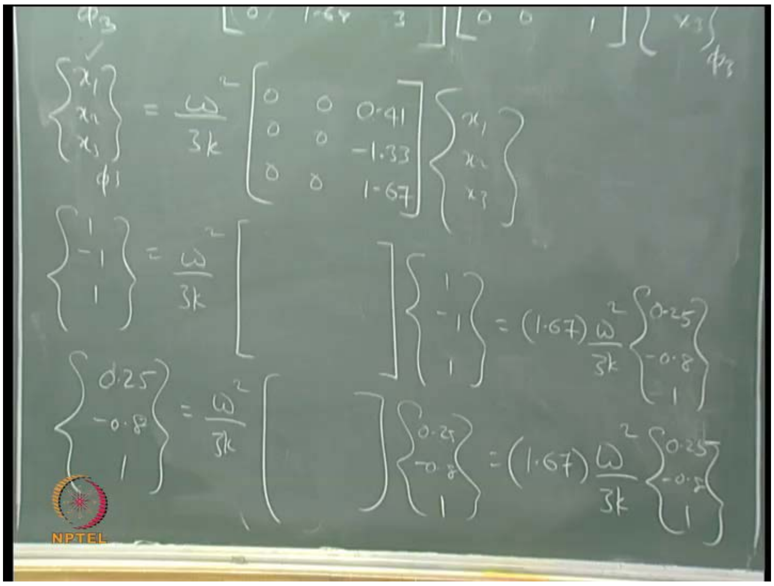

So, I got a multiplier of 1.67 and this is 1; this is 0.25 I should say; and, this is minus of 0.8 I can say. So, it is not converging. So, next step – 0.25, minus 0.8, 1 omega square by 3k – same matrix of same vector. I get a multiplier. So, this will now converge as… with the multiplier of 1.67 out; 0.25, minus 0.8 and 1. So, equating the first row; removing it here; or, equating the last row, because of unity there.

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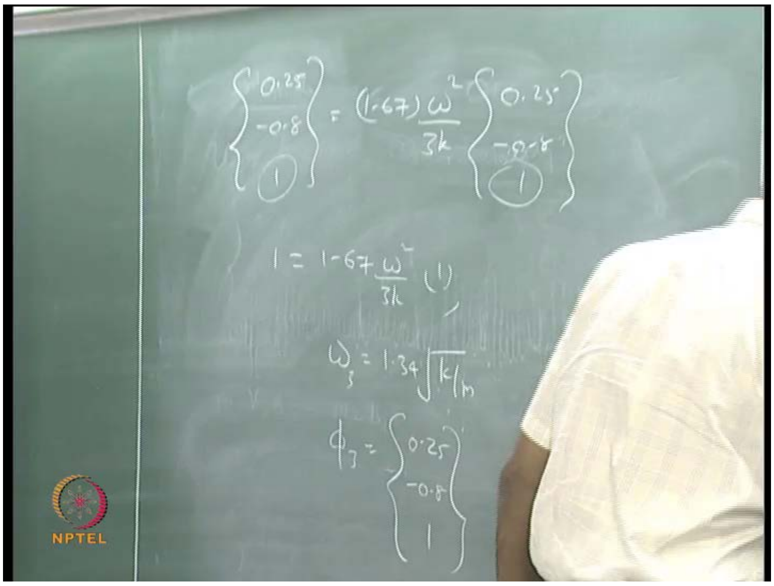

So, I should say 0.25, minus 0.8, 1 is equal to this multiplier of… Same vector getting repeated. So, it is converged. Looking at the last row, I can easily say 1 is equal to 1.67

of omega by 3k of 1. This gives me omega as 1.34 root k by m. And, the corresponding vector is 0.25, minus 0.8 and 1. Let us quickly see what are the observations or inferences we draw from this method and what are the… Let us see merits and demerits or limitations are worries of this method.

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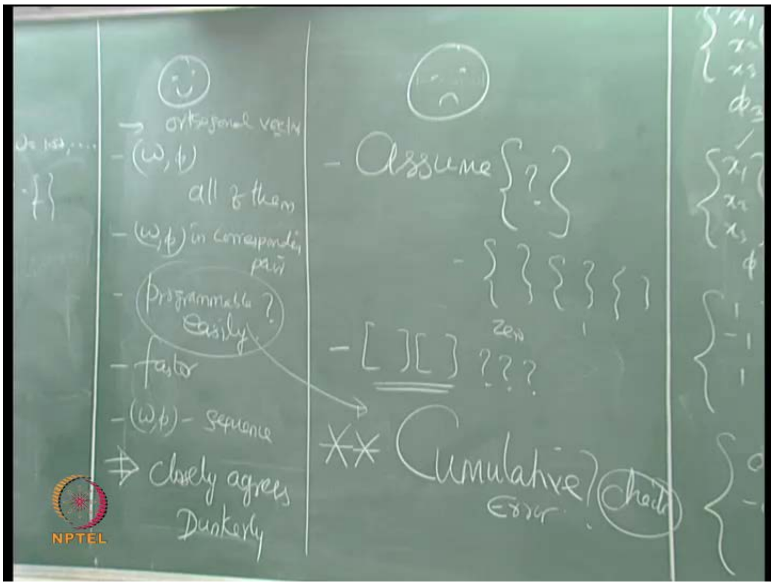

First, let us see the happy part of the method. Then, we will talk about the sad part of the method. So, this method has got many advantages and disadvantages. Let us see the happy part; then, we will see the sad part later. The happy part is it gives me omega and phi – all of them. It gives me omega and phi in a corresponding pair. So, that is the great advantage. I do not have to search for it. For example, it is a jumble issue. I have got omegas given as 1.57 something – three values; and phi’s as vectors. So, one does not know, which belong to which omega. We need not have to check that. It gives in a pair (Refer Slide Time: 33:49). The program… The method is easily programmable; how do we know the programming is a different part of it; we assume that, we know the programming – easily programmable. I should say easily.

Those who do not know this, then it shift to this next column also. But, we understand it is easy. Therefore, it is easily programmable. Relatively, this method is faster; there is no doubt. Most importantly, this method will give me omega and phi in a good sequence. First, I got omega 1; I got omega 2, and omega 3; 0.45, 1 and 1.34. Look at the values; it gives me a good sequence. And, more interestingly, people appreciate this method, because this closely agrees with Dunkerley. Dunkerley's method closely tallies with this; I compare it. It was 0.4; and, the first value is 0.45; not exactly cloning, but it is anyway agreed. Though I do not have space to write the happy part, I have one more point here; it gives me orthogonal vectors; eigen vectors are orthogonal, because I made them orthogonal. Why we are making them orthogonal? To make the matrix multiplication simple. That is the reason why we are making them orthogonal.

The sad part is the following. So, the greatest sad part is, how do you assume the vector? So, here I said 1 1 1; 1 0 0; 1 0 1; and, so on. So, how do you assume? The answer to the sad part is – can assume accordingly, zero crossing, first crossing, second crossing, any number; still it will converge. It may take long number of iterations. But, it will converge. The method will not diverge. That is the first worry. Second worry of course, you have to unfortunately multiply at least two matrices at a given time. The matrix size is larger. Then, again we have worries. How to multiply them? And, third part of course, is the programmable component. The most important worry is the error what you commit is cumulative. That is a very big worry here. If I have done any mistake in the first, because this matrix has been derived from the first algorithm; it keeps on calculating and transferring it. So, that is the problem here. So, we have to be careful.

Now, the question – once you know that it is cumulative error, how to check… The checking process is checking this with Dunkerley. That is how the method was developed. But, this method will not check the eigen vector or the mode shape; it will check only the eigen value or the frequency. So, check this with the frequency. And also, physically see the first mode should be all positive values or all negative values, whatever maybe. So, there should be no zero crossing. And, that physical understanding you must have. If you have these two physical understandings, this method can be easily handled. Now, do we have any questions related to these two methods? I will discuss two more methods in the next class: Storeland and Hallser. The matrix inverse – fifth method. So, all the five can be compared. And, whoever writes a program for all the five and show to me for the first, he will get 10 marks directly added to your quiz. I will accept only one program. The answer is wrong – zero; answer is right – accepted. I will give values from the audience; it should run for all the values.

If the program runs for all the values and compares all the five methods, you will get 10 marks. If it does not work, no problem; you will get no marks. But, unfortunately, you will also not allow to others getting marks. So, the best thing is give a wrong program immediately and block others opportunity. Write a wrong rubbish program and see that nobody else is getting marks and you also do not get marks. That is how people generally do. So, any difficulties we have here, because I am going to give a quick exercise to you. Now, it is 9 minutes left over; I will give you a problem; whoever does the influence coefficient matrix and the first eigen value and eigen vector, that is, mode shape and frequency first – only one; we will give them 5 marks. I have the answers here – first right answer.

Whoever raises first his hand, we will give him marks if it is right; otherwise, no. Other all will not get any mark at all. So, you can keep your hand now itself right. So, nobody else get any chance. So, it is 9 minutes; I will just describe the problem if you have no doubt in understanding the method. If you have doubt, then the only thing is I have got to repeat the method once again in the next class; I am not tired of this; we can do it. But, I think you can also try to do this yourself; it is a very easy algorithm; I am not going fast. My hand writing is excellent. So, there is nothing like worrying about it. You should be able to follow it. We are not going faster; of course, the method is faster. So, there is a problem here. So, we will do this problem quickly. I am giving a very easy problem. Let us see who is able to do this.

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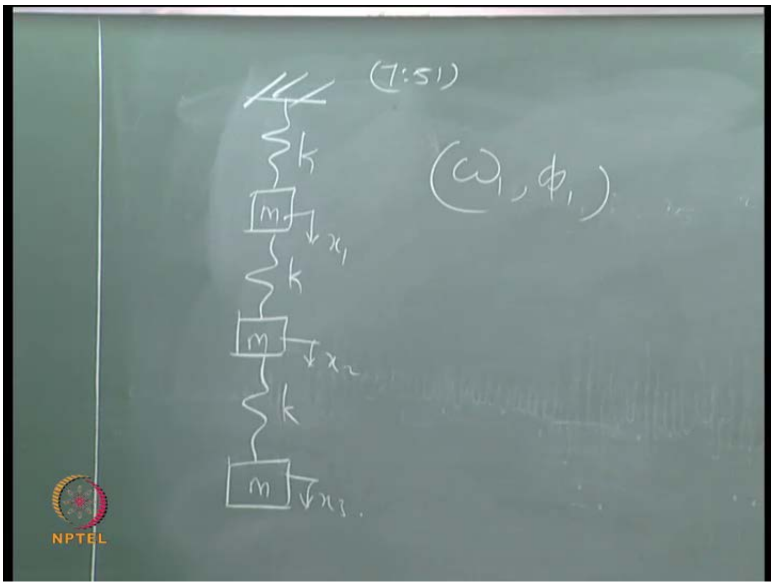

I am giving exactly the similar problem like we do in nursery schools. I am just changing the numbers; that is all. Just put them as k, k and k; m, m and m; x 1, x 2 and x 3. Your time starts now. You have got about approximately 8 minutes; I want only omega 1 and phi 1. Those who are going to read the method now, you can raise your hand now itself; you will not be able to do it; or, those who are very clever, try to raise your friend's hand. We will pick up the same problem in the next class; do it by some other method. Then, we will compare the answer. If the answer tallies, we will give the gentleman or the lady 5 marks. If it does not tally, whatever you have to give it to him or her, you can start doing it. So, we will close it here.