**Dynamics of Ocean Structures**

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**Module - 1**
**Lecture - 17**
**Numerical Problems in Single Degree - of - freedom systems**

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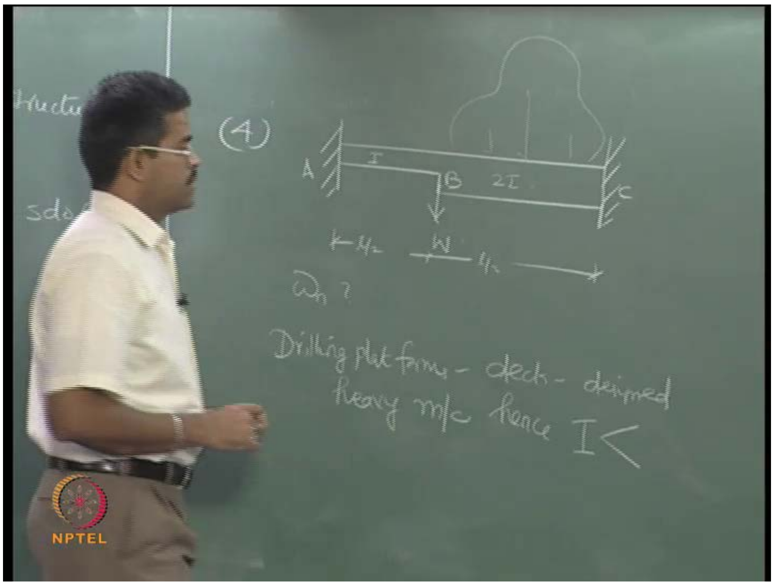


Now, we will continue with the application problem we have discussed yesterday and the last lecture three problems. Today, we will take up another problem where I have a beam with different moment of inertia and I have a load applied here, can be W. The cross-section of the beam is I here, whereas it is 2I here. And let us say, the length of the beam is equal on both the segments. What I am interested is to obtain the natural frequency of vibration of this.

Now, what is the practical significance of this problem? In drilling platforms the supporting deck shall be designed to equip for high machinery and hence sometimes the moment of inertia will be vary, it is not be constant. So, I have taken a very simple example of a fixed beam of span L where the moment of inertia of the cross-section is twice of the left half of the beam, whereas I have some heavy equipments or machinery located here. Therefore, the beam requirement is higher in cross-sectional dimension compared to this. I am going to apply a load to this beam here at this point, let us say W, and I want to find out the natural frequency of vibration of this beam. So, let me name this as A, B and C. So, we will use a classical mechanics for finding out this.

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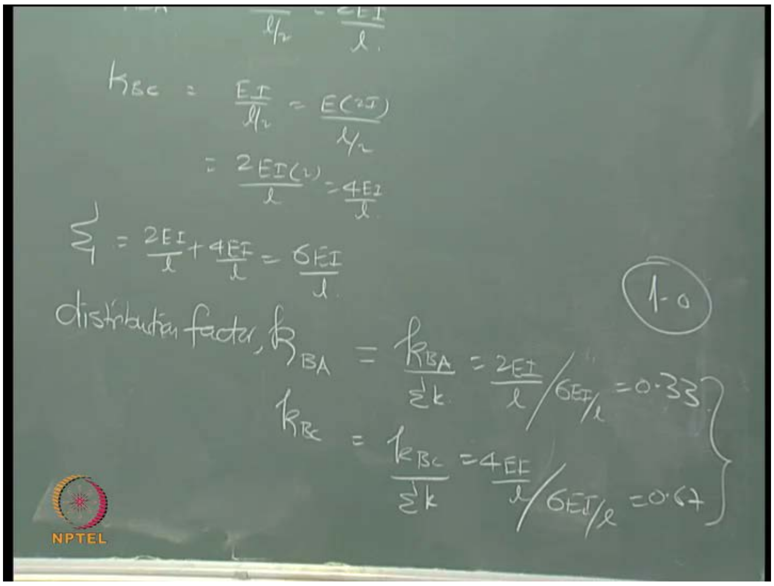


So, let us say, I want to find the stiffness of the member BA and BC at joint B. It is a hypothetical joint because it is a section where the moment of inertia is changing, so k BA and k BC. So, it is nothing but EI by L, whereas k BC is again EI by L, where this is going to be 4EI by L. Let me find the sum of the stiffness of these at the joint B, which will be 2EI by L plus 4EI by L, which is 6EI.

Let me find out, what I call, the distribution factor, which I call k for segments BA and segments BC separately; BA and BC separately. The equation is simply k BC by sum k. This is k BC by sum of k, so I have both the values here. I can substitute that, so which is 2 EI by L by 6 EI by L, which will give me 0.33 and this is going to be 4 EI by L divided by 6 EI by L, which will give me 0.67. At any junction, the sum of this should be unity. So, in the first step I would like to know, because the moment of inertia of the cross-section are varying distinctly at a defined section, how the load will be distributed. So, I am talking about the distribution factor. I would like to know this before I want to estimate my stiffness.

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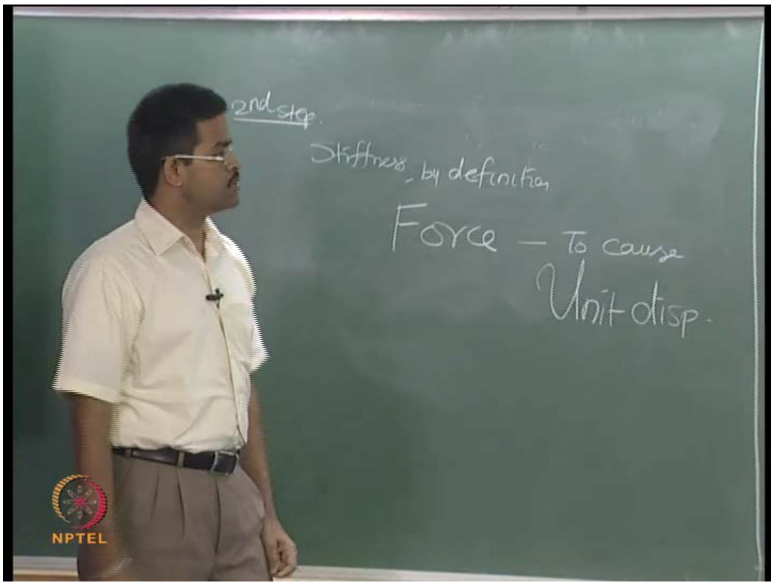


In the second step, I want to know, what will be the forces at this point, which can cause unit displacement at this point because stiffness, by definition, is force, which is responsible to cause unit displacement. So, where you want to cause unit displacement, I want to cause unit displacement where the force is applied, so at this point I want unit displacement. So, what would be the force responsible to create that displacement is what I am looking at stiffness of this whole beam.

Why I am looking at the stiffness of the whole beam? Because mass of the whole beam is known to me, if I know the stiffness being an idealized single degree freedom system model, I can easily find omega n, which is root of k by m. So, I am interested in knowing stiffness here. So, I will use the classical mechanics again to use or to derive this. Why I am following the same steps? Because this is the evident method by which people know how to find stiffness. So, we will use the same technique here while I apply it on dynamics problems for a simple example here.

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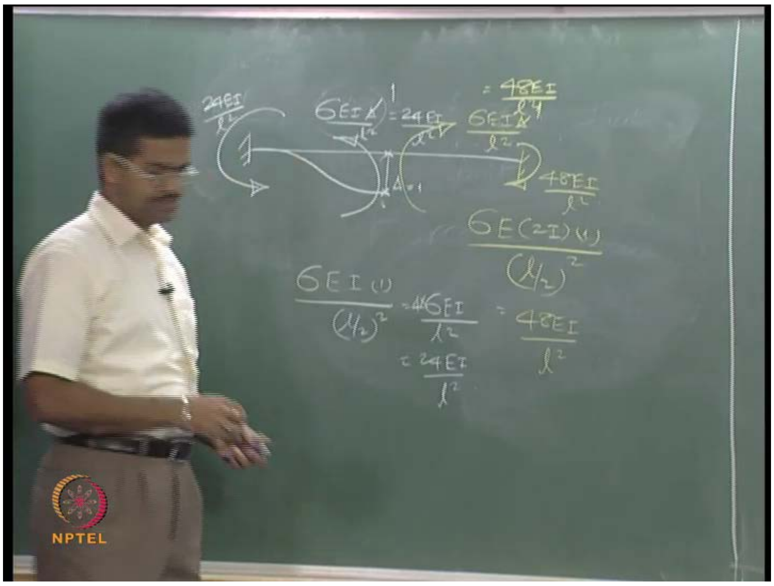


So, I have a beam, I have a point of interest here, I displace this point by unity without creating any disturbance to any other point. So, if I do that, obviously this will invoke a moment, which is 6 EI delta by L square delta in my case is unity, that is, by definition. And I in the left half is I, whereas l in the left half is l by 2, so I substitute that here or this, which will be 6 EI unity by l by 2 the whole square, which is 6 EI. So, let me write this value as 24 EI by l square. The same will get transferred here as 24 EI by l square.

Look at the second half of the beam. Now, this is displaced, so there will be a force applied here and this will also be equal to 6 EI delta by l square. Delta by definition is again unity, I by definition in the problem is 2I and l by definition in the problem is l by 2 for my segment. I substitute back again here, so 6E 2I unity l by 2 the whole square, so it becomes 48 EI by l square. So, I write that value here, 48 EI by l square will have the same effect here, which is also fixed, which is 48 EI by l square.

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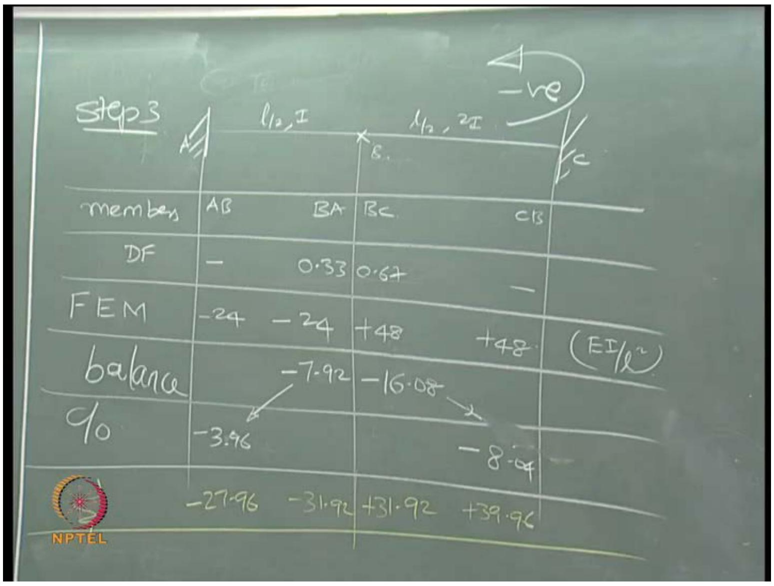


After doing this I will do a simple moment distribution to find out the stiffness. So, I will again draw the beam, fixed point of interest is fixed, so let me enter the members, this is point B, so BA, BC, AB, CD, these are my members. Let me write the distribution factor. We already saw BA is 0.33 and this is 0.67 of course, there are no factors done for the connections, which has got only one member. This is only one point, so we do not need at A and C.

Now, I want to know the unbalanced moments, I call them as fixed end moments. It is not finite element method; it is fixed end moments. So, there are somewhere anticlockwise moments, somewhere clockwise movements. I have to convert that with a numerical sign, so I will take anticlockwise as negative, that convention can be followed. Look at the member BA, look at the piece BA, it is anticlockwise of 24, I enter minus 24. Of course, the multiplier of EI by l square will remain common for the entire row, I take it out, it is not required to be written inside. Similarly, for the member AB, again anticlockwise, so minus 24, whereas for the member BC it is clockwise plus 48 and plus 48. So, at the joint B, I have got unbalanced moment, which is caused because of unit displacement created at this point where I am interested to find the stiffness for the entire beam. So, the unbalanced moment is positive, so the balance will be negative. The unbalanced moment is positive, so the balance will be negative.

If this is x, the value of x is nothing but the unbalanced value, which is 48 minus 24. In my case, it is 24 multiplied by the distribution factor, so in my case it is going to be 24 multiplied by 0.33, so 7.92. So, I will remove this, I will enter 7.92. And similarly, if you want to know this value, which I call as y, this value will be simply again that balance, which is 24 multiplied by 0.67, which is 16.08. After you do this balance I do a carryover, so simply carry over this to the next joint, carry over this to the next joint with the same sign, quality will be same, quantity will be 50 percent. So, minus 7.92 by 2, which is 3.96. Then in this case, my 8.0; try to sum them up. So, let us sum this up, 24, this is minus 27.96 minus 31.92 plus 31.92 plus 39.92.

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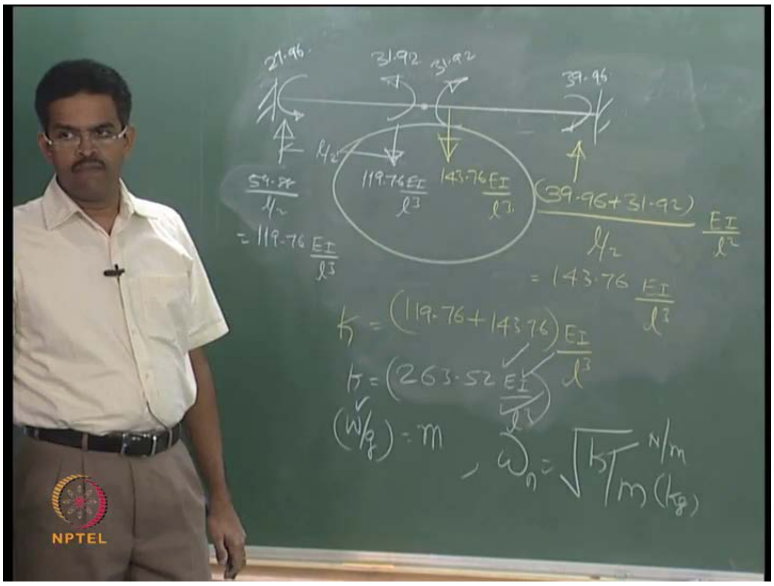


Once I get this, let me get back to step number 4. So, draw the beam again here, so at the joint B, BA had minus 31.92, which means, it is anticlockwise. So, I enter it here and of course, this is clockwise of the same magnitude. Since the joint is balanced we have stopped the iteration here, otherwise it will be keep on continuing. And you look at AB, it is again 27.96 anticlockwise and for CB it is 39.96 clockwise.

Now, let us look at the segment AB, I have got a couple or I have got a total moment of 31.92 plus 27.96, which is anticlockwise. I want to balance this by a couple; that value is nothing but 27.96 plus 31.92 divided by this span, which is l by 2. So, this value is now going to be 59.88, so I will remove this. I should say it is 59.88 by l by 2, which is nothing but 119.76. I already had a multiplier EI by l square here, so I will get EI by l cube. There is l again here and this value is again 119.76 EI by lq. Similarly, let us do for the segment BC, I have got a clockwise moment of 31 plus 39, I want to again balance this by an anticlockwise couple and if you, so look at this example. Let us say, here this value will be 39.96 plus 31.92 divided by l by 2 of EI by l square, so which amounts to 143.76 EI by l cube 143.76 EI by l cube. It is a classical mechanics.

Now, let us focus the values here. Now, the question comes, why do we have to focus the values here? Because it is the point where these force are responsible for unit displacement at this point. So, I am looking at the stiffness of this problem, so both of them are downward, which is acting in the same direction as that of the force applied. So, I should say, 119.76 plus 143.76 of EI by l cube will be equal to my k. So, this is going to be 263.52 EI by…

So, if I know my mass or if I know my W by g, which is my mass, because W is known to me I can find omega plus root of…. Because I know k, I know already Young's modulus of the member, I know the cross sectional dimensions of the member, I know length of the span or span of the member and I know W now or m now, can easily find omega and (( )) is provided, m is in kg and this is Newton per meter. You can even check the units here, EI by l cube E Newton per mm square. So, you will automatically get Newton per millimeter, convert that into a specific unit, what you want you will get is this as so many radians per second.

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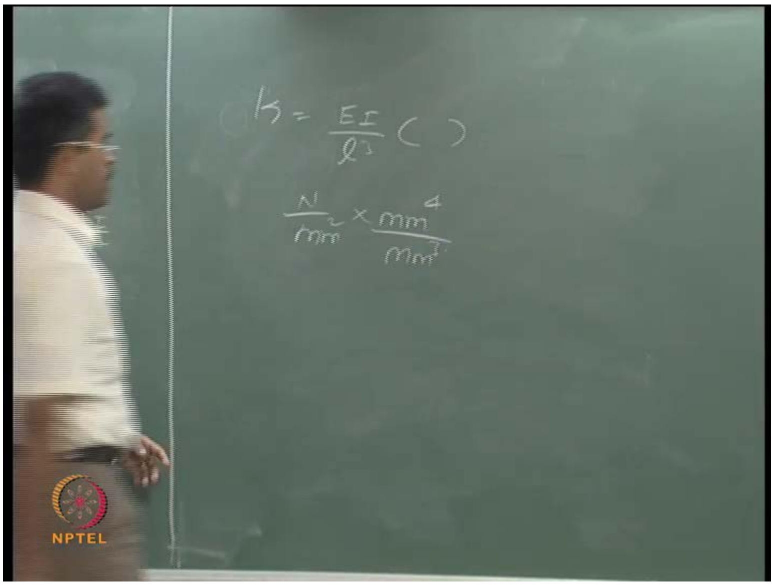

Now, if the problem says, add weight of the beam also, this m will be m plus self weight of the beam. Add the beam and do it in all these examples. There is one commonness; we are measuring the stiffness assuming the mass lump and degree of freedom at a single point. Suppose if we shift it in two degrees of freedom, then the problem starts in single degree. It is not there because single degree is only an idealized point where mass is concentrated. Though the beam is a continuous mass system, I am assuming the masses of the beam are concentrated here or neglect the self weight of the beam.

Do the vibration analysis only for the external force acting at a specific point. Wherever it is acting, do the same argument there, get omega. It is a very simple classical application of standard structural mechanics for dynamics problem of single degree. Even in the last example, where we saw for a truss problem, again unit load method is a classical example of applying it for single degree freedom system for dynamics application.

So, for single degree freedom systems, existing classical theories can be easily applied to find out vibration characteristics or dynamic characteristics of the member. Now, there is a confusion starts. Once when you have got a member with single degree of freedom, but force is of two in nature. Now, here the force is only one in nature. I am applying the force only in the downward direction. Suppose, if we have a problem where I have got degrees of freedom in a different format, but I want to condense it to an easy format, so let us derive what we call lateral stiffness of any given member.

So, we are focusing on derivation of stiffness and mass matrices, because for single degree, if you understand how they are done for two degrees and multi-degree, it will be easy for us to then work further, because we will have no confusion on arriving at the mass and stiffness matrices. We can work further, which are more complicated than getting, m, m and k and the real dynamic analysis problems where you have selected an offshore platform by an innovative form.

The problem starts in arriving at the k and m itself because these are non-classical structures or non-simplified structures. The difficulty will start once when you start working at the stiffness matrix and the mass matrix by given innovative form, which you wanted to derive for importance to put it in deep waters or ultra deep waters. So, main hitch will start only at the initial stage of the problem where you have got to arrive or sometimes derive the stiffness and mass matrices. So, the examples, what you see here, are very simple because they are all single degree, two degree, multi-degree where the defined points are already existing. We will also talk about deriving stiffness and mass matrices for new generations of, I mean, offshore platforms where how they can be derived from first principles.

So, you must understand how stiffness and mass matrices are derived from first principles. This will be a base for doing this derivation in the next module for new generation platforms where the form is innovatively derived to suit a specific requirement in water depth. So, at, at that point of time, k and m itself will become a nightmare. You will not be able to derive them. It is because of this reason, most of the people who work on dynamic analysis on, let us say, innovative form of offshore structures, depend on numerical analysis using software.

So, they model the structural system in the software, let the software arrive at the k and m for the given problem and (( )), you do not know whether it is right or wrong. You simply keep on doing the analysis and get the result, which is wrong because a form is innovative; you have no basis to derive k and m. Therefore, you depend on numerical analysis where you model this in a software using a finite element mesh and let the software arrive at k and m, whatever value it gets, do the analysis and get the response values, which may be absurd, which may be even wrong also because you have no clue how to derive that.

Though in this course we will not teach you how to derive stiffness and mass matrix for buildings and structures, I am taking only very few examples. Because many of you may not even know,, you have qualified structural engineering in B. Tech program where you have studied this concepts in B. Tech in classical structural mechanics. In my experience of these 18 years, I have seen that not even a single student was able to derive this; single student, Indian and foreigner not able to derive and they all waste their interest only on software development where they say, they make a black box model using software. Software gives them k and m, they even do not know what are the values, they do the analysis and they call them as dynamic response analysis, which is absurd, which is not correct.

Now, let us look at one more extended derivation of this where I have got two degrees of freedom, but I want to condense it in single degree of freedom. This is exactly many software do for simplifying the dynamic analysis. Let us see quickly, for a simple example, how this condensation can be done by a fundamental mechanics. We will see this, and then we move forward to two degree freedom system problems.

Any questions, any doubt here? Because the class may address some components on classical structural mechanics like movement distribution, distribution factor, unit load method. So, you may think we are deviating from dynamics to classical structural mechanics or advanced structural analysis. As I said, dynamic analysis has a foreword of analysis into it, so we must know what is the analysis we are looking at.

Now, if you all know this, I can skip this coolly and get back to two degrees of freedom. I have no problem at all because it will remain as a dark area for you forever because many literatures do not explain this in dynamic analysis text books. Many literatures, which talk about structural analysis, do not touch on dynamic prospective. So, you will have no bridging between these two and you will totally depend on or surrender yourself to a numerical analysis done by software. I am not commenting on any software, they are all excellent written by intelligent people, coding has been verified, tested, etcetera, but above all you should know, whether the answers derived by the software are right or wrong. You should have a checking mechanism to say closely it is right or wrong. You cannot simply surrender the whole exercise to a software developer and say this is my dynamic analysis, what x and y is giving me, that is wrong. So, that is why we are exposing it to you in this format.

So, we did a simple truss example in the last class. We did a very simple example of a beam in this class, which are all covering the whole structural classical mechanics theory, which are studied already. The one, which I am going to derive now, may be new for many of you. Let us see what is that newness you have, I mean, innovativeness in that because this is again not available in a standard literature. These are all available from research paper, which I am going to do now. It is very simple exercise we will do that because I want you to have a strong basis on deriving k and m, because I am going to demonstrate it for different platforms in the next module, talking from gravity platforms, (( )) towers, TLPs and (( )) tops, I will derive the k and m here.

At that time if you have any difficulty of how this coefficients have been derived, that is the end of your class. You will never follow further unless otherwise you get back to this class and try to understand this once again. You will stop there; your knowledge run will get frozen there. This happens to many, many students in the first class onwards, but still for sure it will happen from that time onwards, then I will show only pictures and you will keep on yawning and writing, that is not dynamic analysis. Understanding is more important. You need not have to understand 100 percent, but you have to understand 1 percent on your own and say confidently, yes, I have understood 1 percent, my job is done. That 1 percent is going to be definitely basics, which we are focusing in this class.

So, our, our objective is very clear. I will show you advanced analysis data, results, research papers; I will discuss all of them in this course. But to start with I do not want to take you or travel you to that end and then bring back you and say yes, I have done a great work. If you know how I have done it, at least you will believe that I have done it. Secondly, you will appreciate, that I have done it because it is difficult. If you have these two in your mind, you will then motivate to do it as similar as I have done; my job is done at that research level.

So, some fundamentals are very important. I cannot skip them. It may look very silly, that we are discussing something on mechanics on dynamics class. Many people have asked this to me when I was teaching, but I feel it is very, very vital for a student to understand what I am talking in the fundamentals before I extend it to my higher level of mechanics. I must teach this; therefore, it is important for us to understand.

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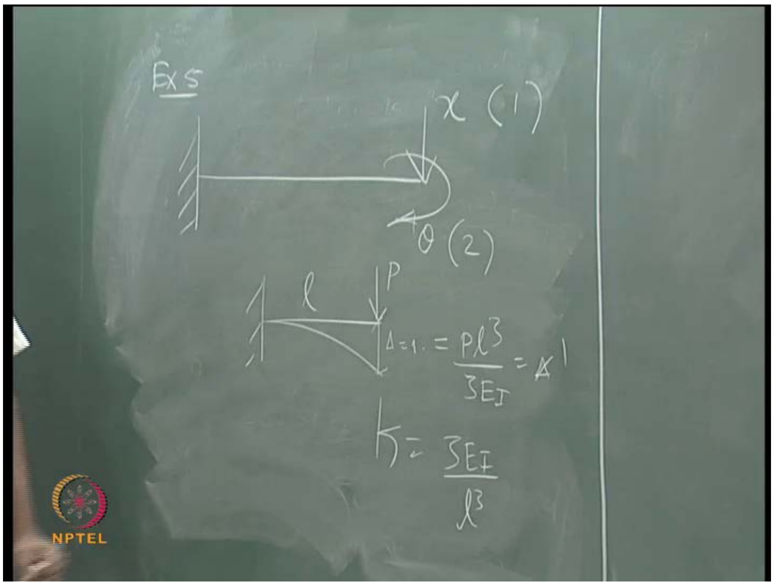


Now, I have another example here where there are two degrees of freedom, but I want to condense it. I am putting this example 5, because 4 we have already done. So, I have a cantilever beam. I have got two degrees of freedom for this problem, let us say, this is my degree of freedom x and this is my degree of freedom theta. I can call this, instead of x as 1 and instead of theta as 2.

So far we have been thinking that we want to know the stiffness matrix of a cantilever beam. What will you do for a loading, given vertical is this is P, this is l? If I say this is my unit deflection, this deflection is classically given by P l cube by 3 EI. If I say this is unity, the force responsible for unit deflections is my stiffness, I would simply say, it is 3 EI by l cube. You will become jittery only when I introduce theta to it, will it change? No guess, no clue, nothing, simply you have no clue at all whether it will change or not, because as long as theta is not introduced in the problem, you have absolutely no trouble when theta is there. You do not know how to handle it for such problems.

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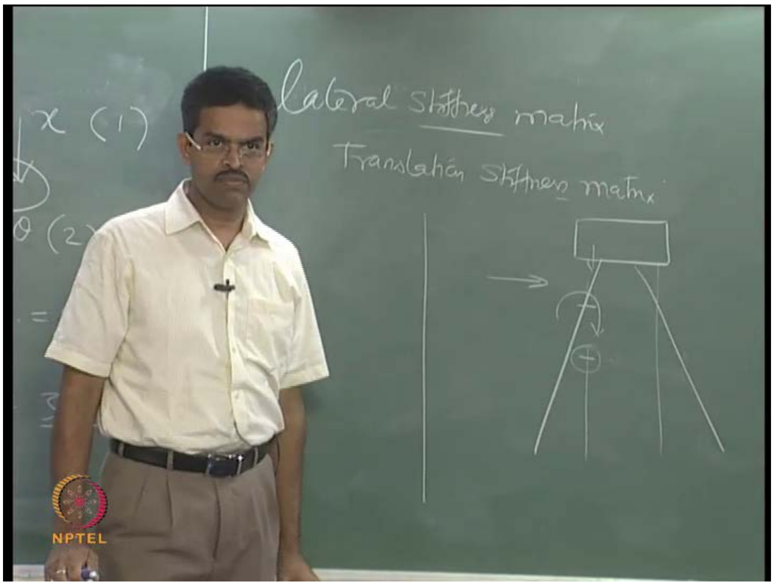


We have called the derivation of lateral stiffness matrix or translational stiffness matrix, why I am putting them as matrix? There is a reason, I will tell you this, I will also tell you what the significance of this problem is? Practically, first let us understand this problem. I think you are getting the gravity of the question. As long as theta is not a degree of freedom indicated in the problem, your stiffness would have been simply 3Ei by l cube, is that right? If theta is given, how do you handle to get stiffness confusion? We have no idea, so we are going to do that. So, I have taken deliberately two degrees, creating a confusion get k and I call that k as translational stiffness matrix or lateral stiffness matrix. There is a reason for this.

Look at this as a problem of a column of an offshore jacket structure. I have a column, let us say, this is my jacket platform, I am idealizing it or taking the simple example as a vertical member for understanding, I have a lateral load pickup. Any segmental member, assuming this boundary condition as fixed, I have a lateral load. On the other hand, because of the weight or because of inclination, there is moment also acting here. So, it is a very classical example of a member like this, which has got lateral load from wave and a moment because of weight or extensity of the load. So, it is a, you cannot idealize this problem as purely a lateral load problem and get stiffness, it is not correct. So, in such situation what do we do? So, let us remove this.

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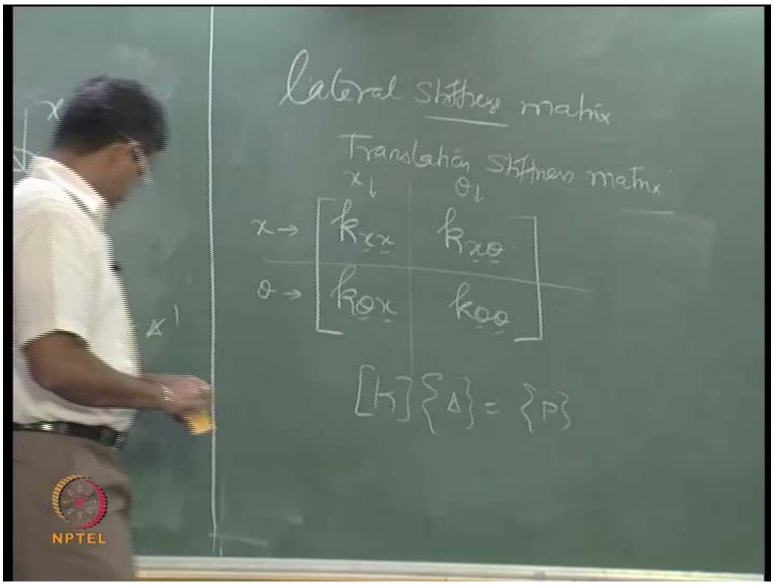


Let us say, since you have got two degrees of freedom here, I will now get what I call as a matrix. I call this coefficient as k xx, k x theta, k theta x, k theta theta. It is very easy to remember this; I will give you a clue. This row is x row, this row, I mean, this column is x column, this column is theta column, this row is x row, this row is theta row. So, row column, row column, row column, row column, row x, column x, row theta, column x. You can read it like this, so no confusion here.

Now, my job is how to get this, k xx, k x theta, theta x and theta theta, from first principles, then how to get ultimately the lateral stiffness matrix. We all know already, that multiply by displacement, will give me the force whatever may be the displacement, it may be rotational, it may be translational. So, I will extend this equation for my problem.

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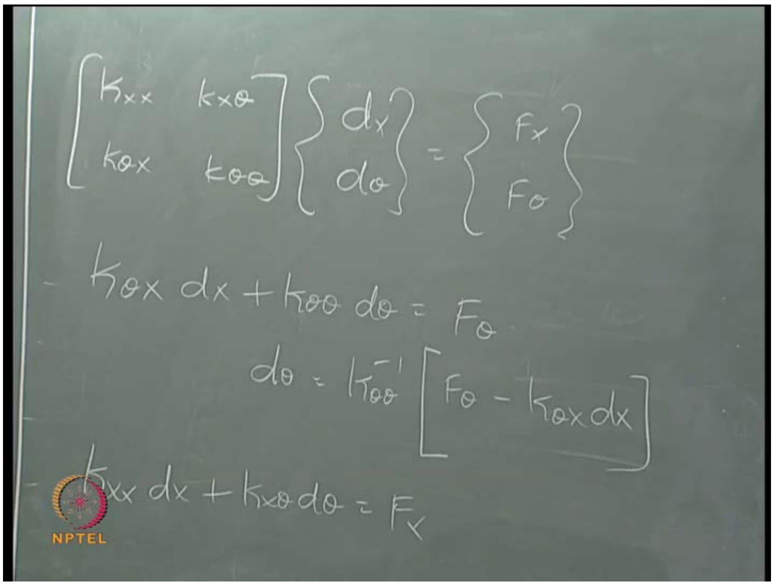


I should say, k xx, k x theta, k theta x, k theta theta of displacement x and displacement theta, should give me force x and force theta. So, let us multiply the second row. So, k theta x of d x plus k theta theta of d theta is F theta. So, I can get d theta from here, which will be k theta theta inverse of F theta minus and so on. Look at the first row, I say k xx d x plus k x theta d theta is F x.

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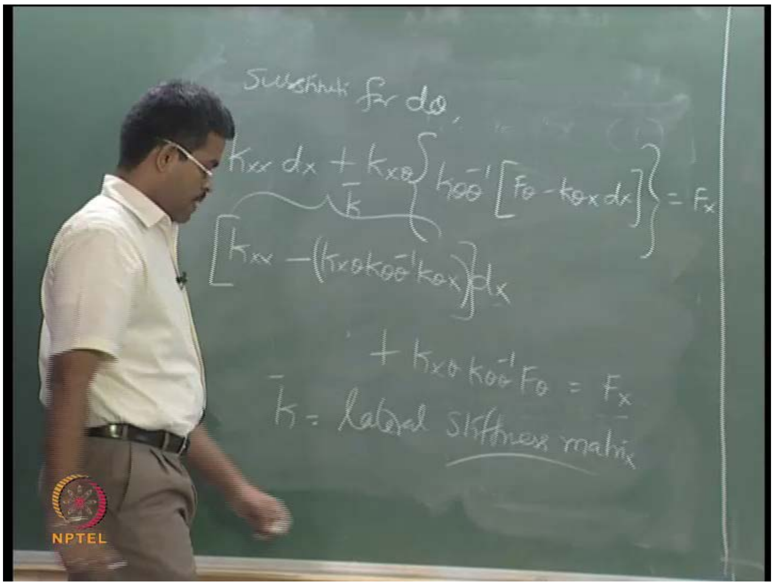


I substitute for d theta from this equation here. So, substituting for d theta, so let us say k xx d x plus k x theta of k theta theta inverse F theta minus k theta x dx will be F x. So, I have got two terms here, one is on dx, other is on d theta. Let us separate this. So, if you look at k xx minus k x theta k theta theta inverse k theta x of dx plus k x theta k theta theta inverse of f theta should be F x, ok.

Now, compare this equation with this equation. If my delta would have been only displacement degree, I will get k in that degree and force in that degree, that is a general expression. Look at this equation. If I am looking for the force in x degree and displacement in x degree, this should be a refined stiffness only in x degree. So, I call this as k bar, where k bar is my lateral stiffness matrix. So, if you want to derive this stiffness matrix once again, you may not remember it, there is a very easy shortcut to remember this. I will tell you, that shortcut quickly to remember this. So, I want to remember the lateral stiffness matrix of k xx. I can write it the same equation very simple here.

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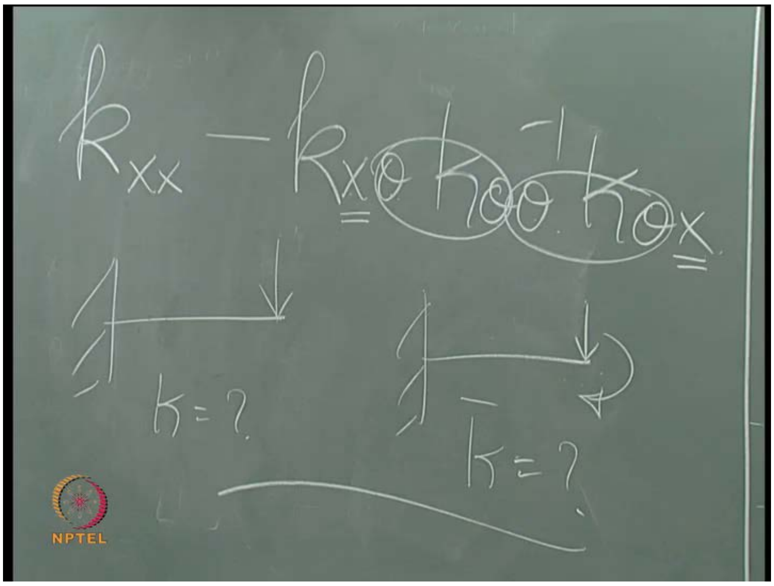


k xx minus k x theta k theta theta inverse k theta x, it means, I must get xx back. I am getting xx back, theta theta theta theta xx back; it is very easy to remember. Now, the question is not answered still. I have a cantilever beam, which has unit load here or any load here only in this direction. The k was very easy to remember, whereas I had a cantilever beam where I got two degrees of freedom, I am going to use k bar, will I get the same as this? If I am able to bridge these two, then derivation of k bar will be helpful for me to understand or to avoid the confusion of theta on to this problem. Is it clear? That was the confusion, right.

So, I am not going to derive k from the first principles here. I will derive k for this problem using k bar and see what happens to my k bar here. Now, there is a reason why I am calling this lateral stiffness matrix because it is very easy to understand here. From a general expression, any displacement connecting the force in a direction should give me stiffness only in the direction that is a general algorithm. It may have components of theta, that is why, I am calling this as condensed stiffness matrix or translational stiffness matrix or lateral stiffness matrix equivalent matrix. It has got components of theta also into it, remember that. So, when you have confusion of this order I can resolve it in this order using this. And this equation may not be derived, can be remembered; it, it is very simple. So, I will take up this problem quickly now in few minutes, find out x theta theta theta theta x and x x combine them in this equation and get k bar, see what happens, that is our objective. So, I will rub this. Any doubts here?

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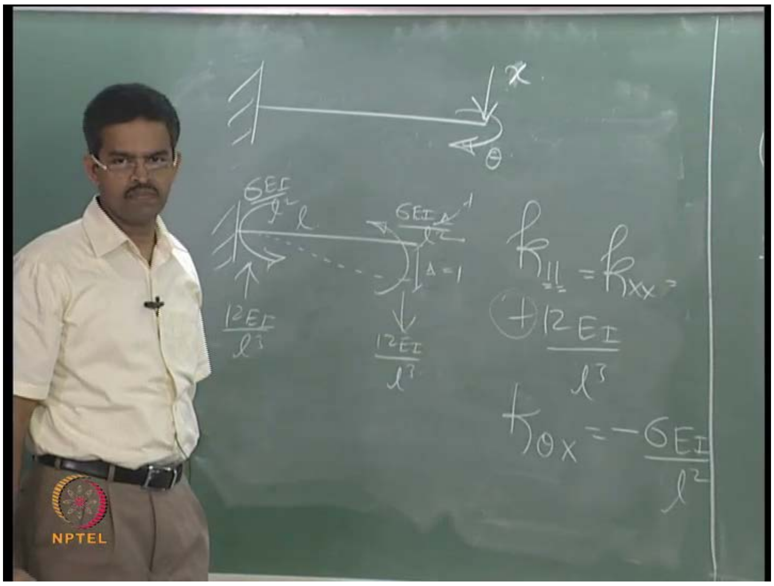


I will put this as theta, this as x instead of 1 and 2. So, what is stiffness? Stiffness is force, which can cause unit displacement. I will pickup unit displacement along x and get the force, right. So, I draw this pickup unit displacement along x and get the force, so this is going to be displaced position of my beam. So, if this is delta, this will be 6 EI delta by l square where this is my l. This is also 6 EI delta by l square, delta is unity, so there is a total movement of 12.

So, I get a couple, which is 12 EI by l cube; 12 EI by l cube. This is the force, which is responsible to cause unit displacement in x-axis or x-direction, so I should say k 11. Why 11, force in the first degree. Because of displacement in the first degree I can also call this as k xx, which is plus 12 EI by l cube. Why plus the displacement and the force are on the same direction. There is one more component, which is arising because of the displacement, that is, the moment. There is a moment here, that moment is again going to be k moment caused because of displacement k theta x; moment caused because of displacement, right. So, this is going to be equal to this value, this is opposite to this, so minus 6 EI.

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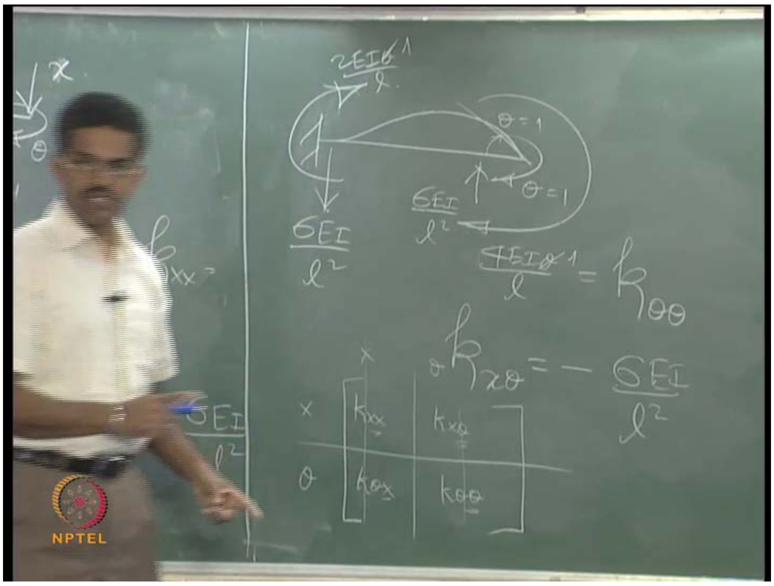

So, again I have got to give unit rotation. So, let me draw a unit rotation to give a unit rotation. You will have to apply a moment, which will be 4 EI theta by l, which will be transferred as 2 EI theta by l theta, is unity. Now, there is a moment total of 6 EI by l, there is a couple, which is 6 EI by l square and 6 EI by l square. Now, this is a force in theta direction because of unit displacement given in theta direction, so I should say this as k theta theta, which is nothing but plus 4 EI by l and this value is a force in x direction because of unit rotation given in theta direction, but this is opposite to the force applied here.

So, I should say, k x theta force in x because of displacement given in theta, which is nothing but this value is minus 6 EI by l square minus because this force is opposite to direction of x. So, now I have got a matrix, so this is x and this is theta and this is x and this is theta, this is k xx, k x theta, k theta x, k theta theta. Remember, stiffness matrix is always derived column-wise. First, I got xx and theta x I got this second, I got theta theta and x theta, I got this. It is always derived column-wise, it is never derived row-wise. Though there is symmetry, does not make a difference, but you must understand how they are derived. It is always derived column-wise, substitute back this. Quickly I will remove this here.

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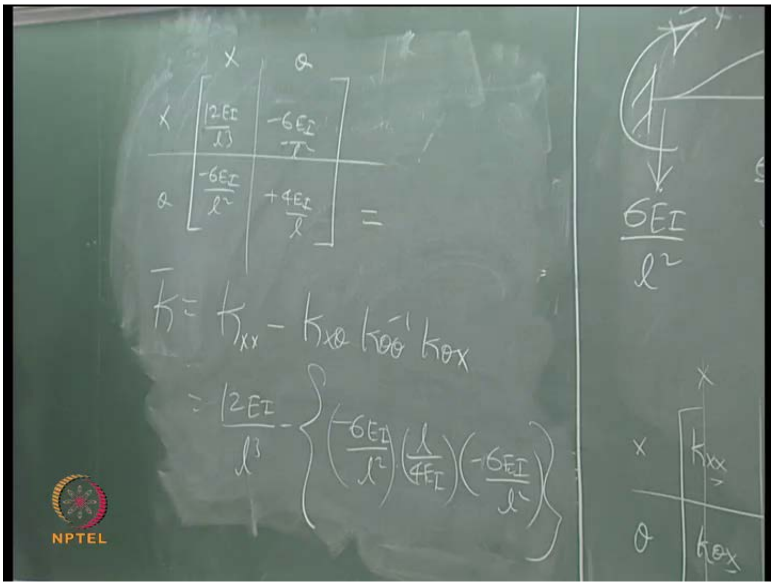

So, mine is a matrix now, so substitute for this value, it is going to 12 EI by l cube minus 6 EI by l square minus 6 EI by l square plus 4 EI by l. So, I want to write k bar; k bar is k xx minus k x theta k theta theta inverse k theta x. This is my x, my theta x theta, so 12 EI by l cube minus x theta minus 6 EI by l square k theta theta inverse l by 4 EI and k theta, k theta x minus 6 EI by l square. Simplify and tell me what do you get?

How much, how much?

It is exactly same, but I have used a different technique, therefore creating an additional degree of freedom should not cause embarrassment to work out k. Therefore, people use still this number as an equivalent lateral stiffness and solve the problem. I have taken theta into consideration, still I got k bar as same as k. I must understand this, why it is so.

It is, I demonstrated it because of this reason. So, introducing an additional degree should not cause panicity in working out k. In that situation I can use equivalence stiffness or I call lateral stiffness matrix and derive it back again with a simple expression like this.

We stop here, we have a next class where we will talk about two degrees of freedom, may be I have to finish the two degree and multi-degree in the next class or next two classes. I will extend by one more class because there are many examples I would like to show you in two and multi-degrees before we finish off or take quiz 1. So, till quiz 1, I will continue. The quiz 1 will be next Saturday, not day after tomorrow, so we will complete for the next week classes, as well as, may be another five classes. I will try to complete the two degree and multi degree, all methods including procedure till quiz 1. So, we can start from quiz-two session onwards, the fresh application on dynamic analysis of ocean structures directly, different examples we will pick up and do dynamic analysis for them.