Dynamics of Ocean Structures
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Module - 1
Lecture - 15
Comparison of Methods

We saw already in the previous lecture how to estimate zeta value experimentally. Analytically, we will see using half band power point method how to estimate the beta value.

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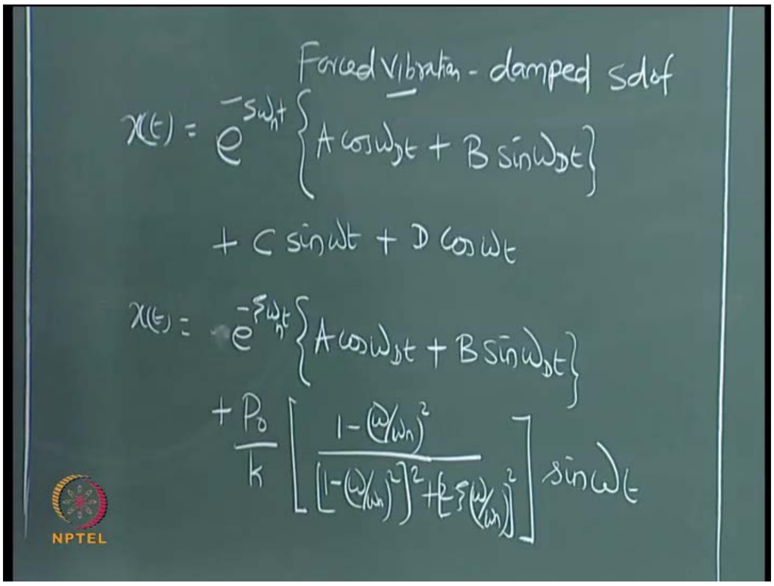

So, if you look at the forced vibration damped single degree, we already gave this expression, which is general case. We said x of t is given by… The general expression was omega t. Now, I will expand C and D, which we already wrote earlier, which will be given by… I think there is a natural frequency here.

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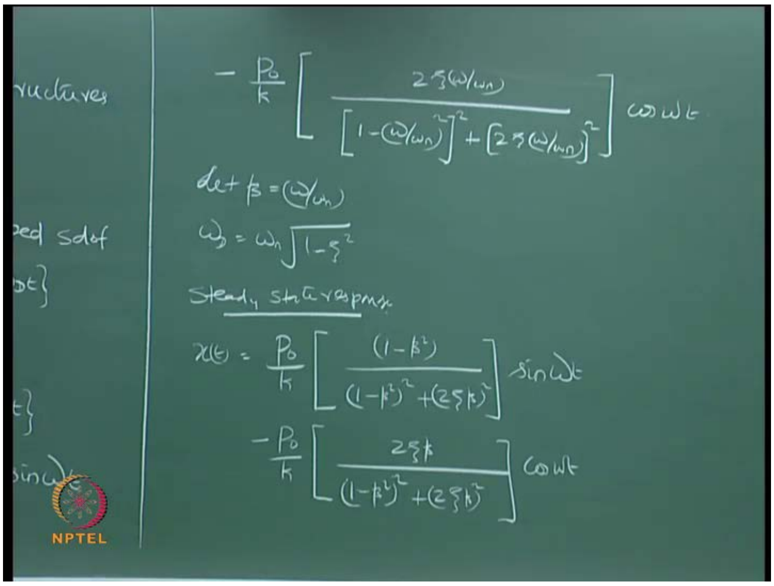

So, this was the expression, which we have. Is this ok? This is the expression what we expanded for c and d and I am just rewriting c and d. Whereas, we already said, this is my transient response and this is my steady state response, because this depends on the initial values or the initial conditions what you give to obtain a and b. They also depend on the fundamental frequency of the system, whereas there is no dependence of the initial condition in this two terms of sine omega t and cos omega t. Therefore, transient response, as we already saw is exponentially decaying. So, after a long interval of time, after the force vibration is applied to the system this response may decay, whereas this response will presently continue. Therefore, we can call this as my steady state response and we will open a window and see how I will focus only on the steady state response.

So, let I will explain a ratio. Let beta be omega by omega n and of course, omega d we know is omega n root of 1 minus zeta square. Therefore, my steady state response x of t becomes P naught by k 1 minus beta square by 1 minus beta square the whole square plus 2 zeta beta the whole square of sine omega t minus P naught by k 2 zeta beta by 1 minus beta square the whole square plus 2 zeta beta the whole square of cos omega t. I can express this vectorially can obtain the resultant of this value as shown here.

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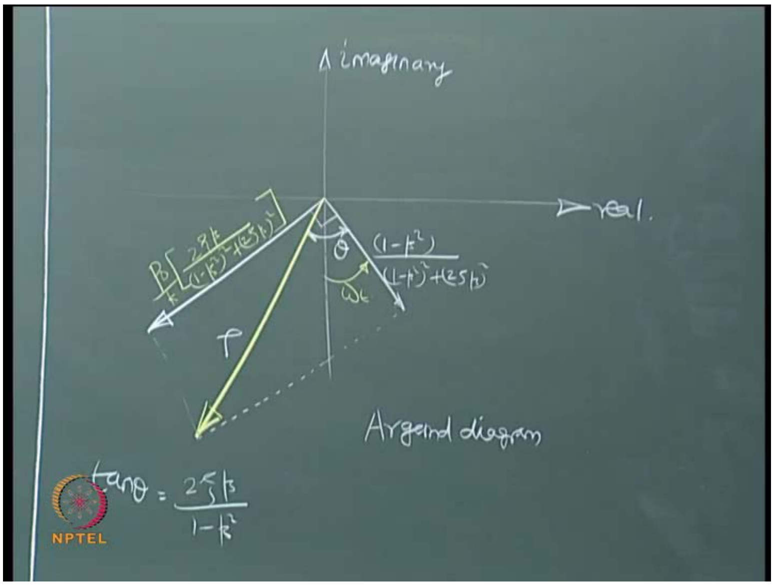

So, this component on the real arm, which is the cos component thatbecomes P naught by k. There is a negative sign here. I am plotting on the negative side, so P 0 by k 2 zeta beta by… Whereas, on the imaginary arm, this can be 1 minus beta square of 1 minus beta square the whole square plus 2 zeta beta the whole square. So, I call this resultant as rho. This is what I call as an Argand diagram. It is a vectorial representation of my resultant of the steady state response, which I am having here. So, from this figure I can easily find tan theta as, it is 90, so tan theta will be 2 zeta beta by 1 minus beta square. So, resultant will be the sum of squares of this, take a root of that so I write it here.

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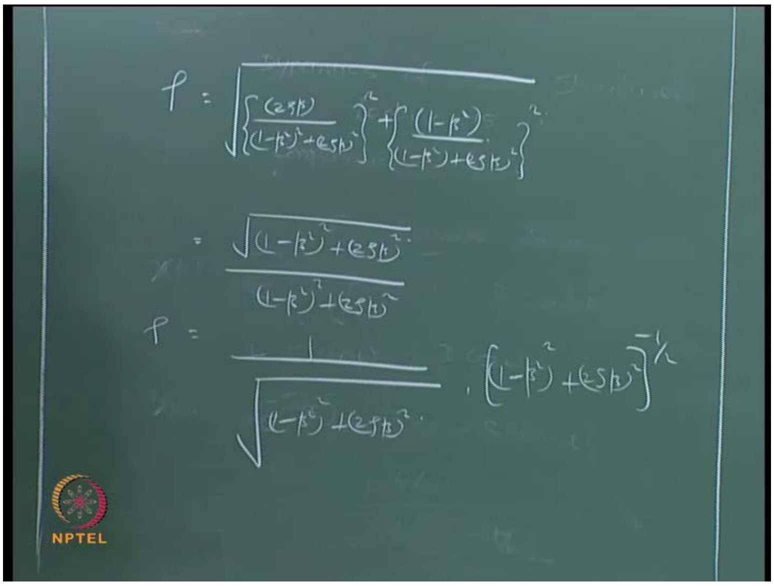

So, a resultant rho be root of squares of these two arms, which will be 2 zeta beta by… of the whole square, root. We simplify, I will get, 1 minus beta square the whole square plus 2 zeta beta the whole square root divided by, there is a square here in the root, it will go away and simply get 1 minus beta square square plus 2 zeta beta square. This is again a square of the numerator, so resultant is nothing but 1 by root of 1 minus beta square square plus 2 zeta beta square, or some literature write this as… It is one and the same. Now, I can express the steady state response except t instead of a function like this using this Argand diagram and the resultant rho and express steady state in a most comprehensive and closed form as seen here.

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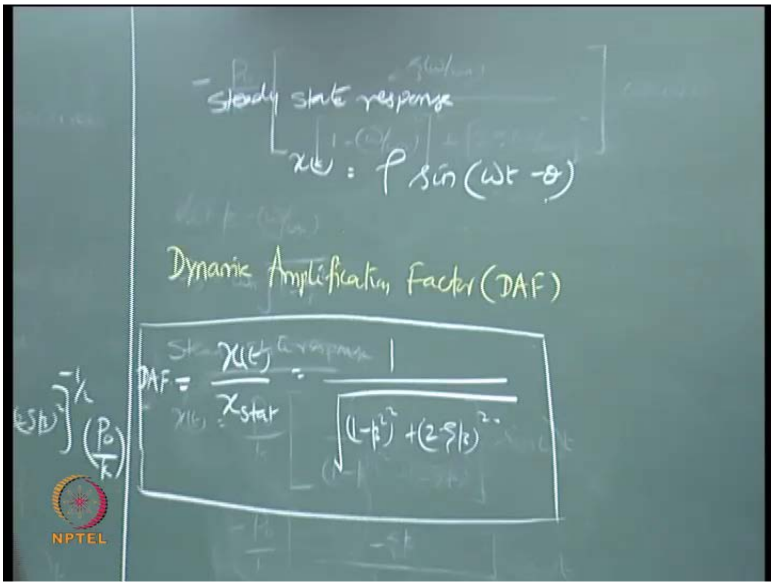

Therefore, steady state response x of t can be… Of course, the resultant of, maybe we can take any theta. Why I am saying minus omega? Why I am saying omega? Because in this function, both, I have got only the forcing frequency, I do not have any omega n. In my original result if you see c and d, you see the c and d value in your paper, I have multiplier of omegas, omega n does not appear, so sine omega t rho now becomes a resultant. As now we see here, that is also equal, let us put it like this. It is one and the same. And theta, of course, I have relationship from here.

Now, I want to find very important aspect of dynamic analysis what we call dynamic amplification factor. Literature addresses this as DAF, somebody calls this as dynamic magnification factor, it is one and the same. So, this is nothing but x of t by x static, a resultant, I am sorry, resultant has a multiple of P naught by k in both cases.

Here also it is there, please make the correction here, it is also there in both arms, it is there in both multiplier of omega and cos omega, I have P naught by k, therefore this will also be there. Please make this change; please make this change. So, P naught by k is my x static, so that goes away. I will simply get this value as simply 1 by root of 1 minus beta square the whole square plus 2 zeta beta the whole square; that is my dynamic amplification factor. I would like to plot this and see how does it look like? Any questions here, till here?

Few minutes, please turn back your literature and see where we are passing through. We started with free vibration analysis, undamped and then damped, then force vibration, undamped and damped both, resonance case we have studied, omega equal omega n, now we are landing up in a situation where a general solution for a forced damped vibration is being understood here. We have also discussed this in the last lecture for omega equal omega n, so are we getting here, all of us? This is very important. This is one of the important landmarks of understanding dynamic fundamentals on single degree. You must know what is the relationship for a dynamic amplification factor? I will tell you the significance of this in our study, because this is very important for us in ocean structures, we will talk about this.

In ocean structures you have studied about response amplitude operator, RAO. I will talk about that also later. I will connect these two, how they are important. Any doubt here, till any point here for anybody? Any, difficulty? Any confusion? I hope you have made this change, that is, a P naught by k multiplier here. If you look back, the derivation P naught by k was multiplied with sine omega, as well as, cos omega component. I missed out that in the Argand diagram here, that is why, I did not get, the spelling is A, Argand, it may look like e, I do not know, it is A; A, R, G, A, N, D, Argand diagram. So, it is a close form solution.

Now, for x of t, where theta is known, omega is, of course, known to me, it is a forcing function frequency and the resultant rho is obtained from here, which is a combination of ratio of the frequencies and zeta. So, dynamic amplification factor has got two variables. One is of course, beta, which is the ratio of the frequencies, that is, forcing frequency versus natural frequency of the system, it is a ratio. The other variable is zeta, which is C by CC.

Now, I have only one equation, I have got two variables, I cannot plot this, I have got to operate one as a constant value and keep on seeing how the other one varies for every value of this, that is what I am going do. So, preferably, which can be kept constant? The international standard say, keep zeta constant, keep on varying beta and plot this variation for every zeta; that is how it is done. There is a reason for this, why? If a plot can give me for every beta independently for every zeta, from the plot I can read all responses varying from undamped till damped for all frequency ratios.

So, DAF is a very comprehensive graph, which indicates the dynamic response of any system. Of course, in our case, it is single degree freedom system. So, it is a very important understanding of how the system will behave for different frequencies. Of course, here also you will see, at beta equals 1, what is the most interesting inference we derive from DAF, we will see that.

Now, let me plot this. Anyway, I am going to plot it not to scale, but I want you to plot it in Excel and try to see, are we resembling the same curve here? I am not showing the curve in Excel here, you can pick up every different value of zeta and beta and try to plot, you will get the same thing, which I am qualitatively showing you, which I will draw here. Is there any doubt? Any questions for, anybody here, because next three classes we will devote on solving problems on single degree, then we will introduce to multi degree and solution procedures.

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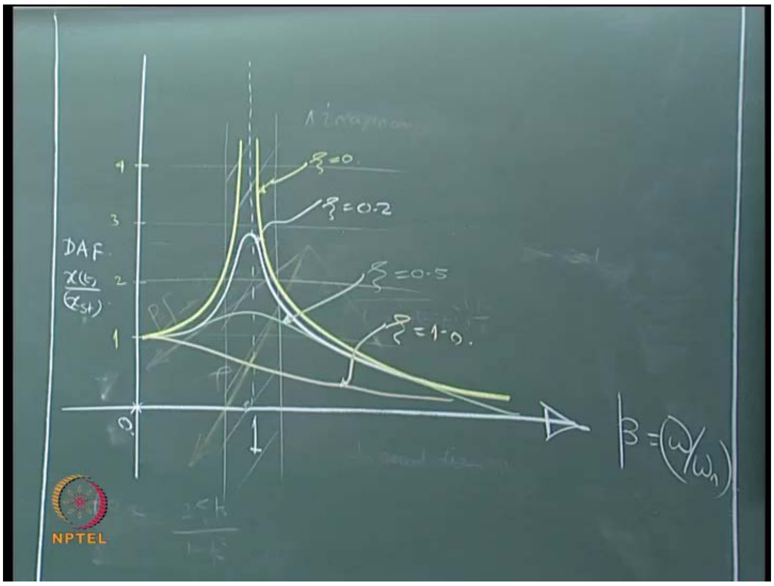

So if you want to plot this, of course, this is going to be my beta value, which is nothing but omega by omega. So, critically let us try to plot or draw a line where, let us say, this is 1, so near resonance frequency. And also, we are interested to know what happens at 0? So, it gets unbounded for zeta equals 0, undamped systems. The moment you introduce damping… Zeta 0.2, 100 percent damping. So, we infer more important things from this curve; let us see what are they? And of course, the vertical axis in this curve is dynamic amplification factor is nothing but x of t by x static. So, we infer some important information from this, can you tell me what are they?

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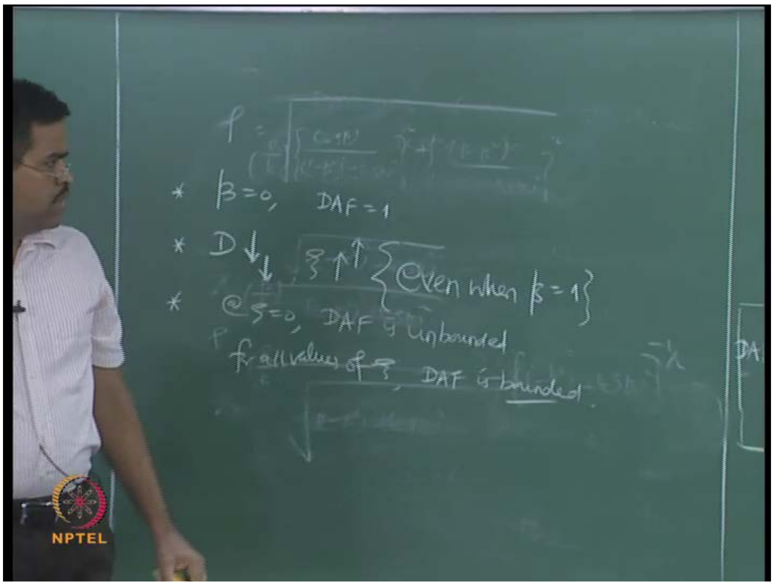

For beta equal 0 dynamic amplification factor will remain 1, it is not 0. Now, the dynamic amplification factor keeps on decreasing; the dynamic amplification factor keep on decreasing for increase in zeta, right. And this interestingly happens even when beta is 1, that is most important; that is very important. Thirdly, at zeta equals 0, what we call undamped system, DAF is unbounded. It is just shooting up, we do not know. For all values of zeta, except 0, DAF is bounded, what does it mean? You can predict the response.

You know what is the maximum response, how do you know that? You know x of t when x s of s t static, this is simply P 0 by k where P 0 is my amplitude of my excitation force and k is my stiffness of the system, which are known characteristics to me, right. Simply, doing any, without doing any dynamic analysis, as a thumb rule, if I know x static I can easily find x of t as a peak point for any specific zeta except 0. I can design the system for that. Now, interestingly, we extend this discussion further. Let us quickly look at this window, what is happening here on the resonance back because that is where we are very interested to know at this band what is happening. We would like to see, so obviously, I cannot see from this expression here, so I will take away this.

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So, we already know the dynamic amplification factor D is given by 1 by root of 1 minus beta square the whole square plus 2 zeta beta the whole square. So, D at beta equals 1, it is 1 by 2 zeta, is that right. Now, unfortunately at zeta equals 0 this equation cannot be quantified; cannot be quantified because it gets unbounded. If you look at experimental values or studies, experiments show that D is not simply 1 by 2 zeta. D is having some more correction to it. They say, it is 1 by 2 zeta of root of 1 minus zeta square. Some correction is there, we have neglected this. If you see the higher order powers, the derivation we have neglected this. Now, one may wonder why I am getting discrepancy between my analytical result and my experimental observation. Very simple, the difference between the equation a and b will not be much for higher values of zeta. For higher values of zeta the difference between these two will not be much that is why it is acceptable.

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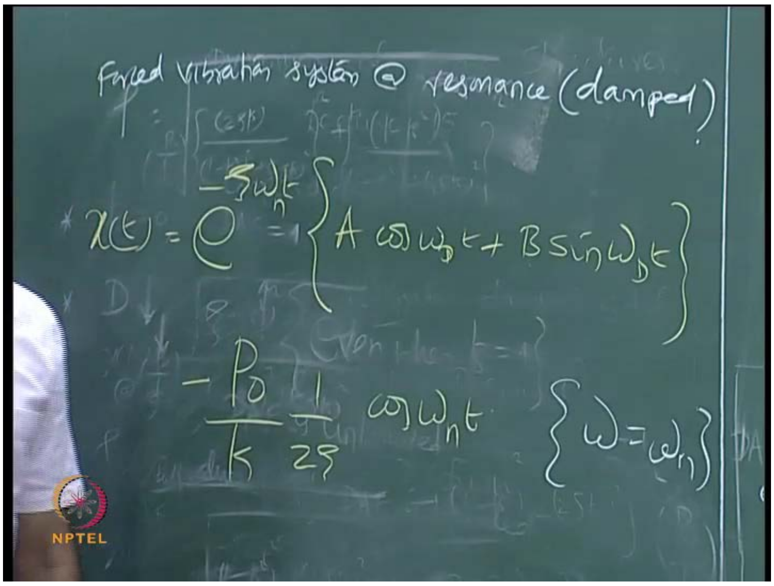

Now, I want you to write this equation, which already I know, which I have given you already, that what happens for a forced vibration damped system at resonance. What was the x of t given? What was the x of t derived that is I am looking? Now, at a forced vibration system at resonance, which is damped, just turn back and tell me what is your x of t or locate it, I will write it here. So, are we getting this equation? I am writing it here, we already derived it. Please see, is it there or not? We have already derived it. x of t was given by e to the power of minus zeta omega t omega n t A cos omega D t plus B sine omega D t minus P naught by k 1 by 2 zeta of cos omega n. Are we getting this expression? This was for omega equals omega n, a specific case we derived it. We have this expression with us right now. I have got a and b, I want to evaluate it, so let us substitute at t equals 0 x of 0 and x dot of 0 or 0.

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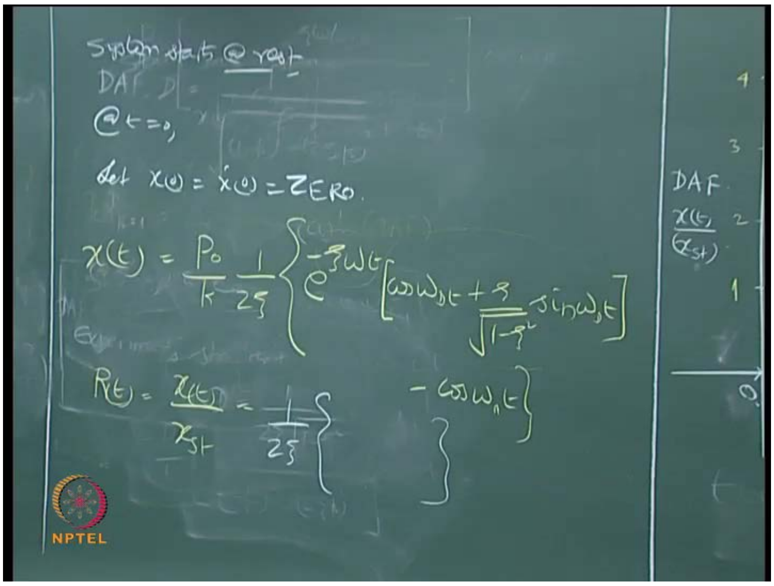

So, the system starts at rest. So, at t equals 0, let x of 0 x dot of 0 be 0. So, can I eliminate x and a and b and give me the full x of t? Hurry up, hurry up, quick. Say, x of t is given by P 0 by k 1 by 2 zeta e to the power of minus zeta omega t. I substitute omega or omega n, one and the same, because I am looking at the resonance condition. I may not write omega n here, I can even say omega, can you tell me why I am violating of writing omega n to omega here? Exact answer, why I am doing this? Resonance, that is fine, but I have defined resonance in a very different format. Resonating frequency is that frequency at which the forcing frequency maximizes the response; it is not the natural frequency.

Student: (( ))

Sorry, omega n, no, at resonance they will not again lie in your control. At resonance, again, the k and m characteristic will change. We are talking about resonance. Why I am writing omega here? because we are looking at the response at resonance. Resonance frequency is that forcing frequency at which their responses maximized. Therefore, I am writing it as omega, but hypothetically and mathematically, whether I write omega n or omega, it is one and the same. So, cos omega d t sine omega d t minus cos omega n t or omega t. Are you getting this equation?

So, this components are exponentially decaying the multiplies only with those two components of omega d terms where omega n t or omega t component is not exponentially decaying. It is away from this zeta omega t, it is out. This multiplies only for these two, not for these. Of course, P 0 by k 1 by 2 zeta will remain common once we eliminate a and b for this condition. Are we getting this or not? Yes or no? Now, let me work out response ratio r of t, which is x of t by x static. So, P 0 by k will go away, I will get this as 1 by 2 zeta of the whole story back again.

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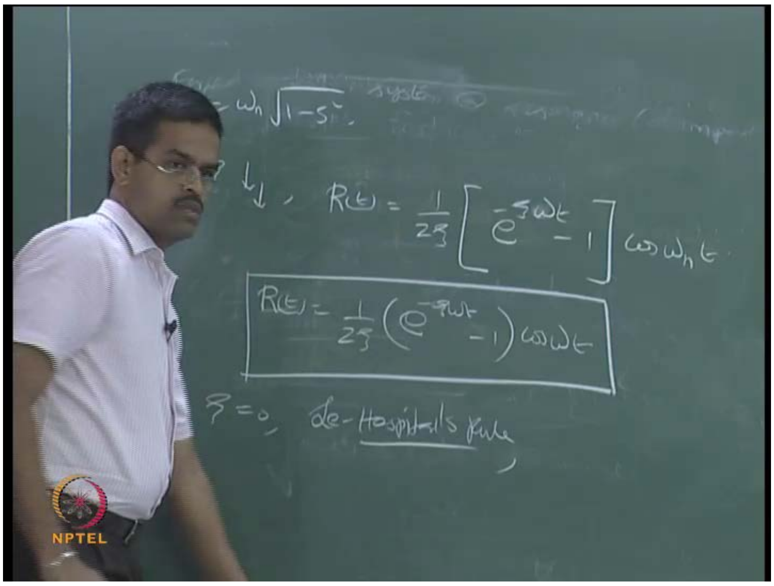

Now, I can rewrite this expression slightly in a different form saying, we know already omega d is omega n of 1 minus zeta square for zeta to be very small, small values of damping. Why I am saying that? I am trying to touch not the peak at zeta equal 0. Any value other than zeta equal 0, I will get a bounded value, right. I am trying to catch that particular point because at 0, I cannot catch any value other than 0. We already saw, that the response will get bounded.

So, for very, very small value of zeta, omega d and omega n will remain equal. So, in that case, my r of t will become 1 by 2 zeta, just think about it. What will happen to my r of t for very small value of zeta? This term will go away and omega d and omega n will all be same. So, I can write e zeta omega t minus 1 of cos omega n t or r of t can also be written as 1 by 2 zeta e zeta omega t minus 1 cos omega t. This is the standard form of writing response ratio at resonance for a forced damped system. Now, in this expression when I put zeta equal 0, r of t will become unbounded. So, I should apply Le-hospitals rule to solve this. Let us do that and see what happens.

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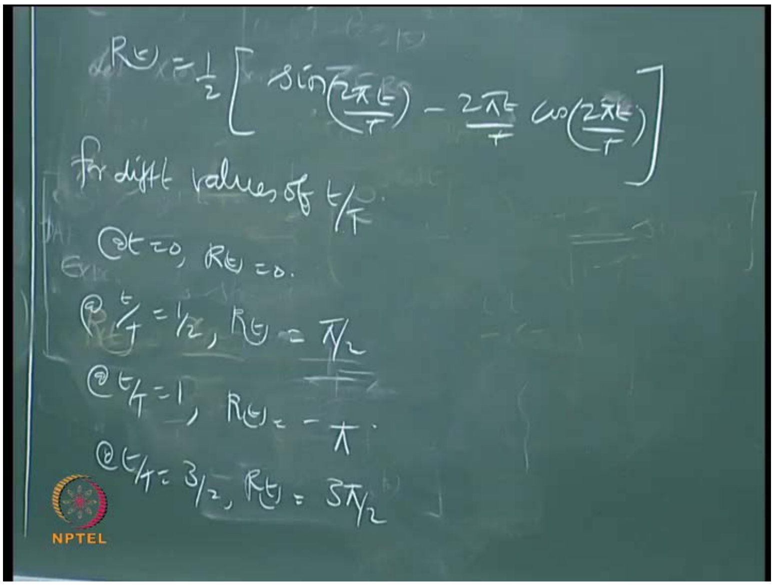

So, r of t now becomes ratios of sine and cos half sine omega t. I will differentiate sine omega t minus omega t cos omega t. I want you to plot this, for as we know omega is 2 pi by t. Since omega is 2 pi by t, plot this expression for different values of, let me write this, r of t as half of sine 2 pi t by T minus 2 pi t by T of cos of 2 pi t by T 2 pi t by T. Now, plot this for different values of t by T. So, we all know, that at t is equal to 0, r of t will be always 0 because there is a, multi, sine component will go away, there is a multiplier here, r of t will become 0. So, it starts at rest, which is satisfying the initial condition, which we assumed in the calculation. So, for t by T is equal to half what happens to my r of t? Quick, quick, pi by 2 at t by T is 1, what happens to my response ratio, minus pi, at t by T, half, 1, 1 and a half, 3 by 2? We already did this, 3 pi by 2, so let me try to plot this, we already plotted this also, but still.

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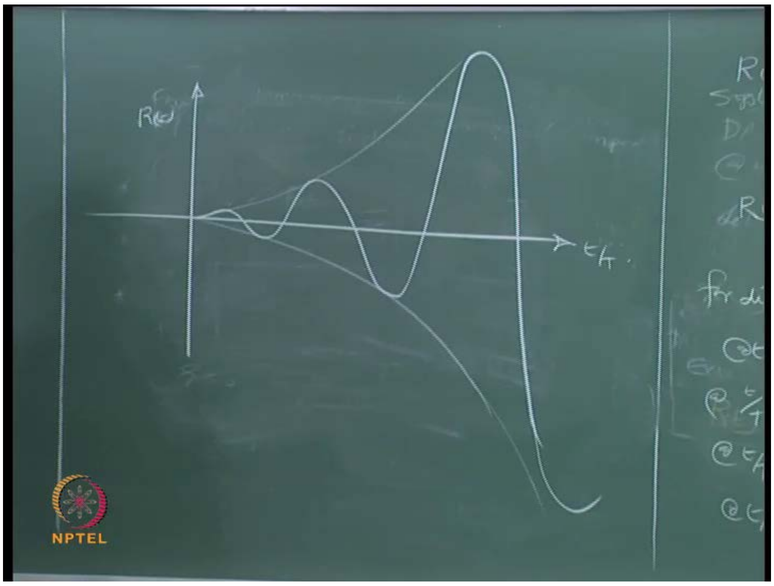

So, we have already seen this plot saying, that this will go, it is a bell widening starting from here and so on. On the other hand, remove this where r of t, I will write this expression again.

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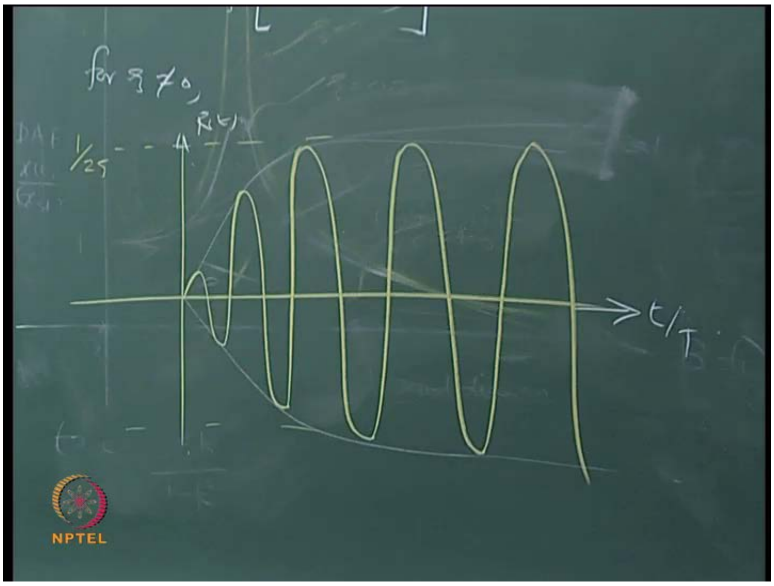

Is this r of t, yeah, where zeta not equal 0, so the plot will look like this. There will be a buildup of the response after that the response will get bounded at a value, which will be equal to 1 by 2 zeta. I want you to plot this mathematically, qualitatively I am showing you. This is t by T and this is r of t, please plot these expressions all and see what we are discussing is, I want to now write the inference between these two responses and compare them.

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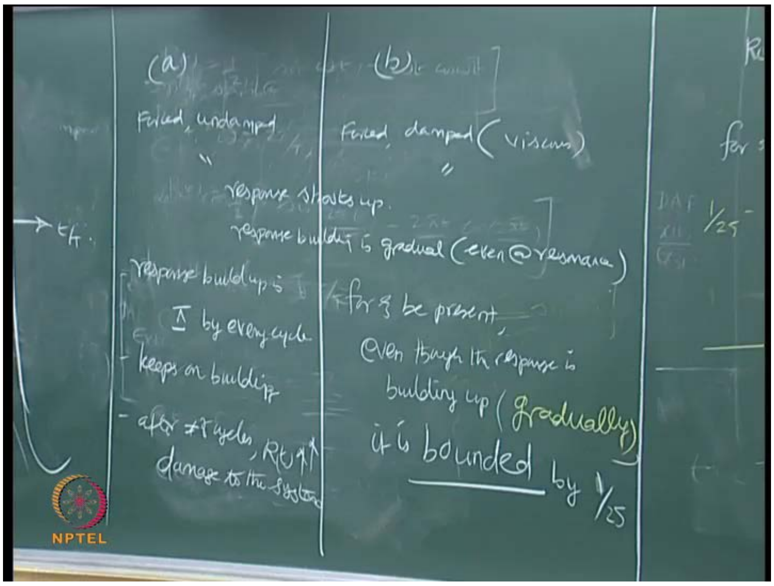

So, there are two response, I have one on my left, I have one on my right. Let us first identify these responses. The one on my left is forced undamped model with zeta 0. The one on my right is again forced, but damped model. I make it very specific; it is viscous damped in both cases. The commonness is response shoots up. It is shooting up, here also it is shooting up, and there also it is shooting up, shooting up. The response building is gradual even at resonance. In an undamped system, the response buildup is pi by every cycle, is it not? I think we have seen this value, this value and this value. We have seen, the difference of these two is pi and keeps on building.

So, there is a possibility, after specific number of cycles the response may shoot up very high, which can cause damage to the system. These are all at resonance, at resonance we are talking about. Whereas, in this case, once you introduce damping for zeta be present, even though the response is building, let me put it here, even though it is building up gradually, it is bounded by 1 by 2 zeta. So, there is a binding beyond which the response will not shoot up.

So, one may ask a question, at what number of cycles this 1 by 2 zeta bound will be reached after how many number of cycles? This 1 by 2 zeta bind or the upper bound of the response will be reached by the system. The answer is it depends on the value of zeta.

So, the lower the value of zeta you give, it takes large number of cycles to reach, is it not? The binding will be higher, but large number of cycles, slowly it will build up, that is why we prefer under damped systems in any structure engineering systems, but in specifically ocean engineering systems.

Now, all these discussions are for under damped systems. Zeta less than, I mean c less than cc, is it not? That is the argument of the whole discussion and by any chance if zeta is present in a system, the upper bound of the response even at resonance, even in resonance, even at resonance will not shoot up infinitely like in this case. So, in ocean engineering structures or ocean structural systems zeta will be inherently present, what is the inherent damping present in the system. Two, what are those inherent damping present in the system in ocean engineering structures.

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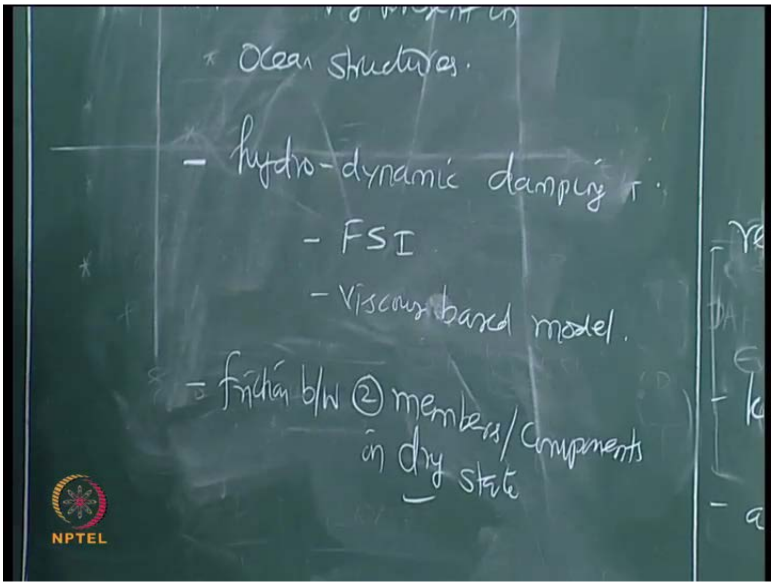

The inherent damping present in ocean structures is the following. One is, because of wave structure or fluid structure interaction, there is a component of hydrodynamic damping. This mainly arises from fluid structure interaction. I am just saying FSI and we already know, this is viscous, viscous based model because it depends on the velocity. The second kind of damping is also present in the system because of friction between two members' components in dry state.

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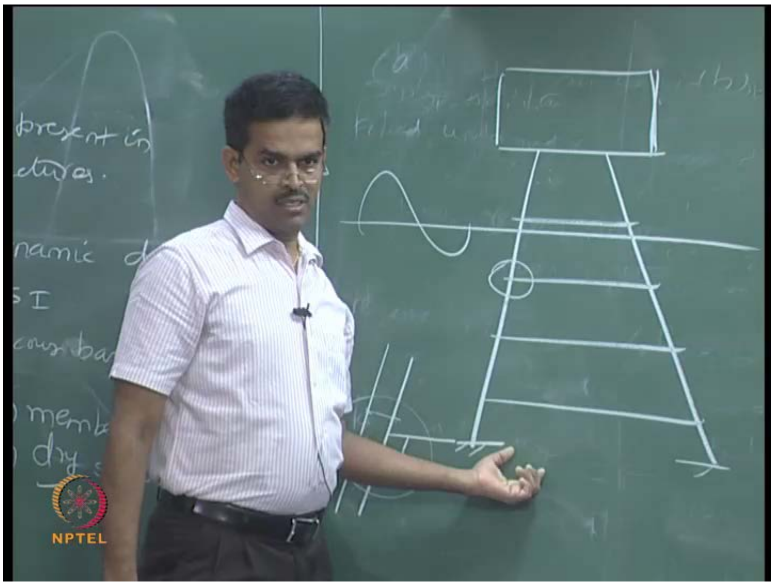

It is Coulomb damping where this, there, for example, we have an offshore platform, I will remove this. Let us say, I have a jacket structure in a top side bounded. Look at this intersection, there are two members. So, this junction will have your relative movement because of motion of the structural system or displacement of the structural system, especially even in fixed structures like these. In case of complained structures, like articular towers, guide towers, TLPs, this will be even more. So, they will develop coulomb damping.

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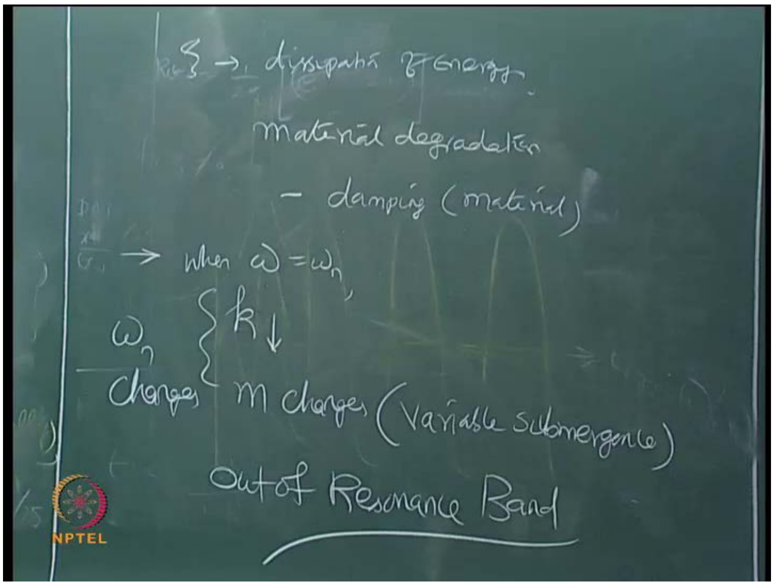

Thirdly and importantly, damping, if identify this as dissipation of energy or energy loss or strength loss indirectly because strength loss is directly a state of the stiffness energy, the loss or dissipation of energy of the system, there is a material degradation in ocean structures because of age. It can be due to corrosion, it can be due to sulfate attack, it can be due to environmental loads acting on the members. So, material degradation will also impose some kind of damping, which comes from the material. So, damping in anyway is inherently present in ocean structures. So, we understood just now from the literature, that if damping is present any even single degree freedom system, this is not a single degree, it is a multi-degree, and we will model this in the second module later. Even a single degree, if zeta is present even at resonance, the total response ratio, that is, r of t will be bounded by 1 by 2 zeta.

And most importantly, as we discussed in the last lecture, when resonance sets in, when resonance sets in, material degrades stiffness, decreases mass changes. I am saying changes this is because of variable submergence, we call put together omega n changes. I am not saying decrease or increase. Therefore the structure will be out of resonance band, so there is no problem.

Now, it is because of this reason deeper water offshore structures are always made flexible. It is because of this reason, because in deeper waters omega n band is of a different operational frequency. You want to fix omega band is of a different operational frequency. If you want to fix my structural system out of band of this, I must have a flexible system. So, that is because of the reason why all deeper water and ultra deep waters have become or designed flexible by form.

It is also because of this reason of material damping and Coulomb damping activation, articular structure survive because there is an excellent Coulomb damping effect given by the universal joint at the bottom. It is also because of this reason; because of the frictional damping arise between the guides towers survive, because there is a lot of fiction happening between the lead point, fair lead point. I think you remember what is a fair lead point? At the point, between the guide wire or the guidelines with that of the structural member. It is also because of this reason, perforated structures in coastal structures survive because in case of perforated members for coastal protection systems, which have been a recent development in breakwaters when you allow hydrodynamic, damping increase, the response decreases.

So, all these have been derived advantages in different forms of structural systems, which has been meant for coastal or ocean structures like this. So, in the next class, we will talk about some applied examples on single degree, then we will understand single degree and then we will move on to two degrees and multi degrees in the successive classes.

Any question? Any doubt? Now, you want to read completely because you will be asked to reproduce and understand and derive or write down, list down the inferences from different problems as explained in the blackboard for your examination. So, the questions may not be straight forward, so try to understand the whole literature, derive them also once and understand the limitations of every segment of derivation and it is relevant application in the ocean structural systems.