Dynamics of Ocean Structures
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Module - 1
Lecture - 13
Undamped and Damped Systems II

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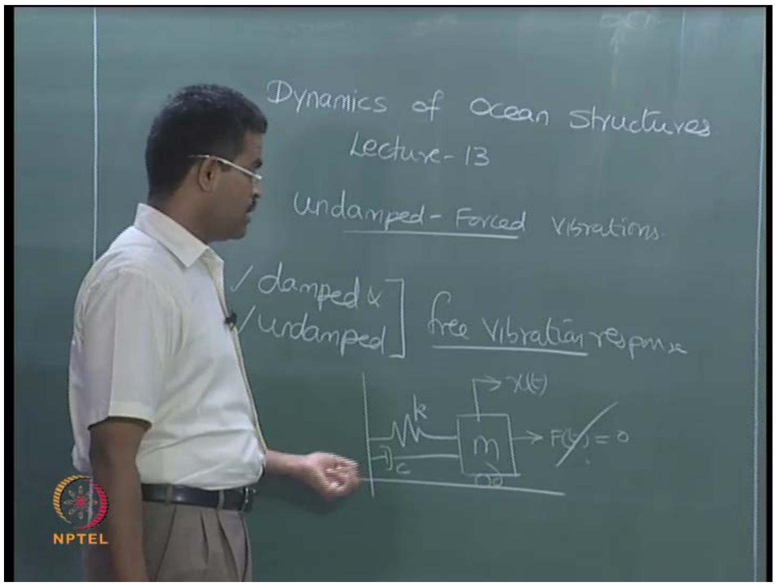

So, we have discussed so far the damped and undamped free vibration response. This we have discussed so far. So, we have already said when there is no excitation force happening to the given system, let us say you have a simple single degree freedom system model, you have the elastic restoring force always present in the system, and you may or may not have C present in the system. And of course, F of t is set to 0, that is why we say it is free vibration analysis or free vibration response.

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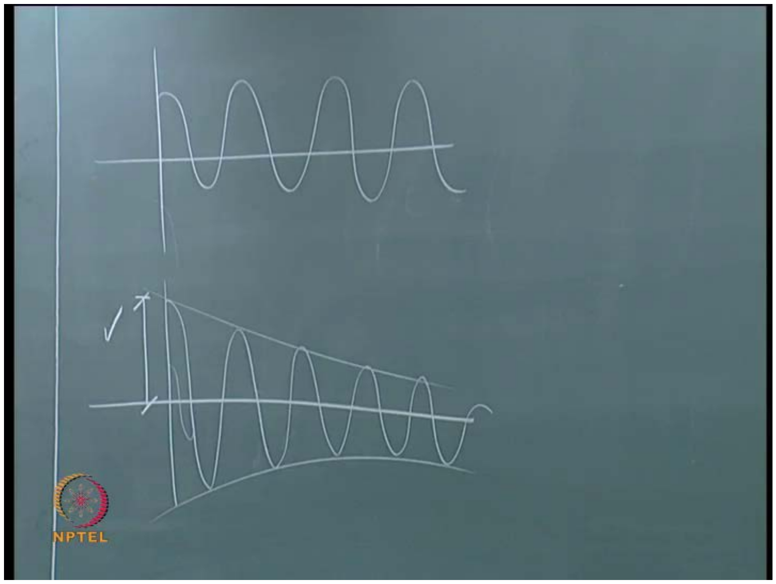

So, when the damping element is not present in the model, we already saw that the response for any specific initial value keeps on repeating, it is nature for an infinite period of time, hypothetically. The moment you introduce a damping component in the model, we have already seen, that there is an exponential decay of the response irrespective of whatever initial displacement you give to the model, how high it is? For example, you give initial displacement very low, still it will decay; if you give very high, still it will decay.

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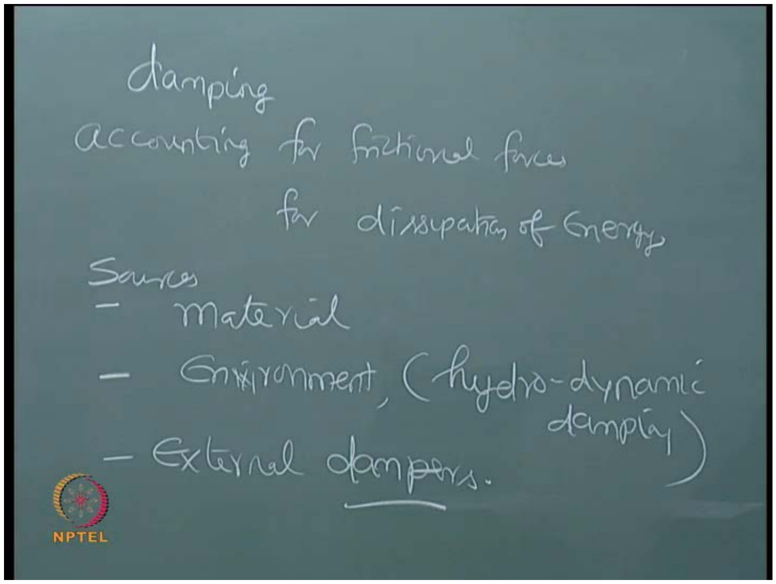

So, the decay is ensured if you have damping in this. The moment you say damping, damping is a model, which is accounting for any frictional forces or any dissipation of energy. So, damping accounts for this. Damping can come from the sources, it can come from the material degradation and material loss, it can also come from the environment. For example, in our case, hydrodynamic damping and so it can also have external dampers, which was introduced in the system to response or to control the response of the structure that we will see in the advanced level of dynamic class in this course later. How you can use external dampers to control the response, so you can have dampers in many sources. So, we said and we agreed that when the damping element is present, the response is anyway, for sure, is decay.

The moment you say damping, you have got two concepts here, one is what we call damping proportional to velocity, damping independent of velocity. So, we said, that I can have Coulombs damping or I can have the frictional damping based on the frictional forces and we already said, in case of Coulomb damping model there is an exponential decay, whereas in the frictional base model the cycle of dissipation or the loss of energy or the response reduction in every cycle is fixed. Therefore, you have got to undergo large number of cycles to attain a specific, desired response control from x 0 to x N and therefore, the cycle has to be large enough to attain because the step-by-step decrease is linear; it is not exponential, it is linear. So, these were the discussions in summary what we had, as far as damped and un-damped free vibration responses are concerned.

In the current lecture we will talk about un-damped again, but forced vibration. The moment I say forced vibration, F of t will be included in the system, it will be present. One may ask a question very violently, that F of t, which is arising from the sources of let us say, wave forces, wind forces, etcetera, then why we should exclude the presence of F of t in study at all. I mean, what is the relevance of the study? The relevance of the study is to understand what are the free vibration characteristic of the model itself.

Ultimately, we are interested in estimating a natural frequency at which this model is vibrating, which is otherwise not influenced by the excitation frequency, which can be present in the external agency or a force because the moment I have an external force present in the system, which is also a function of time, because we are talking about the dynamic loading, then this will have also a frequency of vibration. If that frequency at which this is exciting matches with the natural characteristic of the system, then the response of the system will be different from what we have so far seen. So, I would not isolate this discussion initially, that let me not have any forcing frequency in the system. I will take only the characteristic of the model itself and estimate the natural frequency, which we study.

Now, we will take up an external force also acting on the system. So, the source of external loading already we saw in the previous lectures. It can be from the wind loading, it can be from the wave loading, can be current, it can be impact load from ice, it can be hydro dynamic loading, it can be also sea floor movement, it can be from the earthquakes, it can be from the large displacement because of the erection forces, etcetera, caused to the system.

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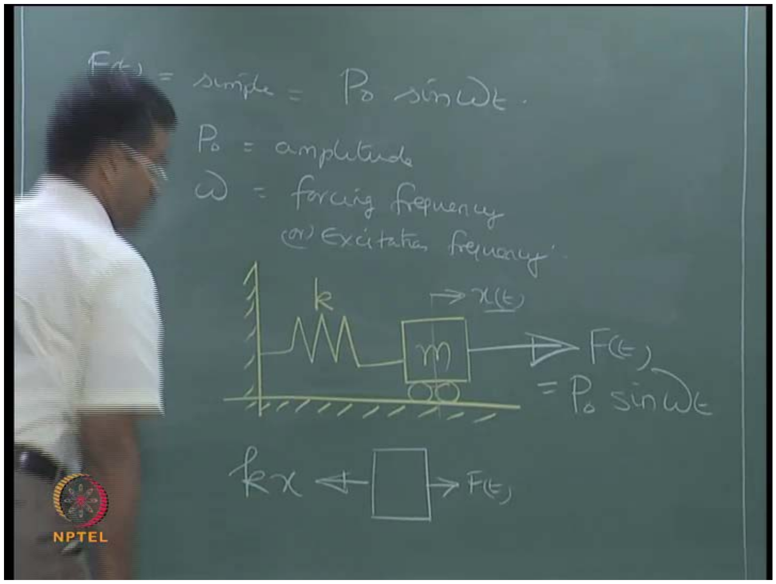

So, F of t, for example, to be as simple as in my model, I will take it as P naught sine omega t. So, P naught be the amplitude of the sinusoidal function, which I am assuming and omega is what I am going to call as the forcing frequency or excitation frequency. So, I will take up this. Now, if I introduce F of t presence in the model, then the model will have a different look in terms of its equation of motion. I am talking about the single degree freedom system.

I am imposing a resistance of motion only in one direction, this is my mass element, so this is my restoring force element, which I am going to call as k and I am measuring always, response from the CG of the mass center, which is x of t, which is the function of time. And of course, I am going to introduce the F of t present in my model, which is in my case P naught sine omega t. If you draw a free body diagram for this, from the Newton's law, you can straightaway say that a mass is subjected to F of t and this is opposed by a restoring force, which is k of x. So, I can write the force balance equation from Newton's law.

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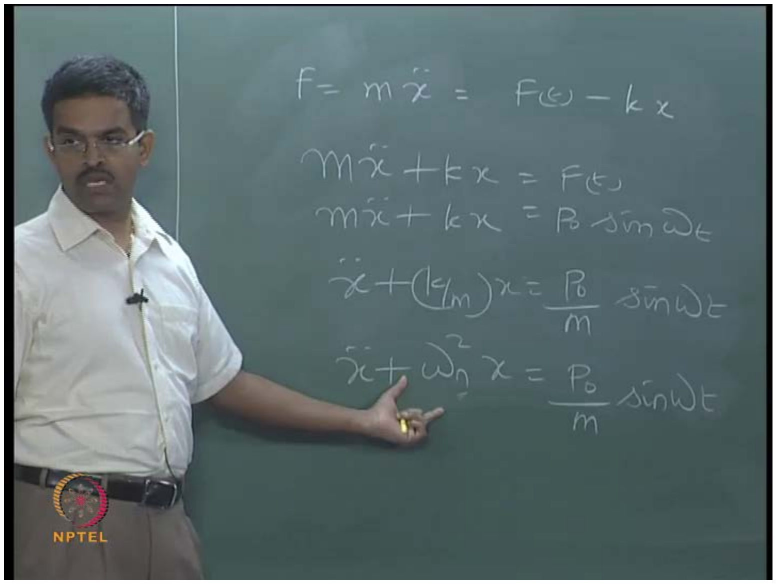

And Newton's law says, the force, which is mass into acceleration will be equal to F of t minus of k x because kx is an opposite sense of F of t. I replace it here, so m x double dot plus kx is my F of t, which is otherwise P naught sine omega t. So, m x double dot plus k x is P naught sine omega t divided by m as usual. So, x double dot plus k by m of x is P naught by m sine omega t. So, x double dot plus omega square n of x is P naught by m sine omega t.

Please note, that I am using suffix n here indicating it is natural frequency of the system. Earlier it was not having any significant difference because omega was not present in the discussion. Now, since omega is also present in the discussion, which is the frequency of the forcing function, therefore now we have got a distinctly right omega N and omega, this is a system characteristic, this may force in function. They are two different things. Now, this is again a second order ordinary differential equation, which we call as equation of motion.

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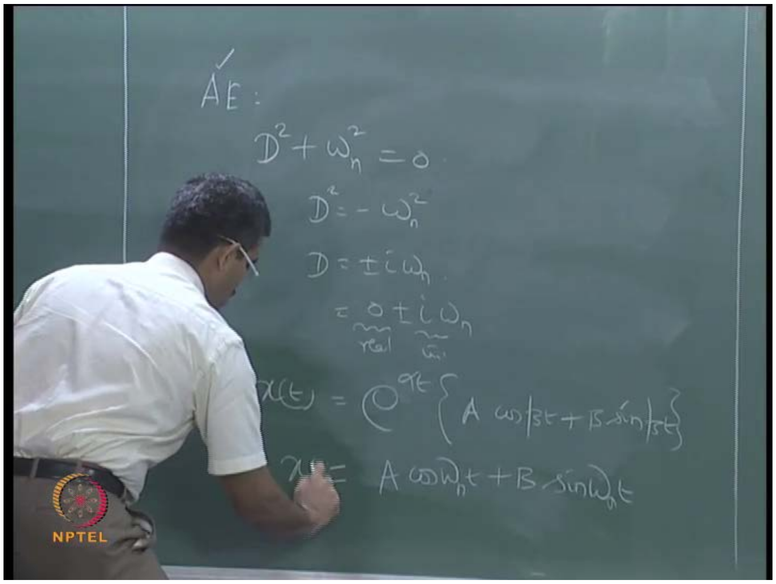

I want to solve this equation of motion using standard principle I will write down back, I will write what is called auxiliary equation for this problem. So, I should say D square plus omega n square is set to 0. We can remember the standard procedure for writing auxiliary equation. The right hand side is always set to 0 even though you have a right hand side function present here. If you want to solve this, then the second component of the solution will be a particular integral, which we will do later, so the right hand side is always set to 0, because there is an auxiliary equation. So, I write the complementary function for this.

Now, the roots will be, for example, D square now becomes minus omega n square or D is plus or minus I omega n. You can rewrite this as 0 plus or minus i omega N. This is my real part, this is my imaginary part, therefore the complementary function, which will be A x of t, which will be 0 e, 0 alpha t, alpha t A cos beta t plus B sine beta t. That is a general equation for my problem, this become 0, so A cos omega n t plus B sine omega n, that is my x of t, which is only my complementary function. It is only one part of the answer.

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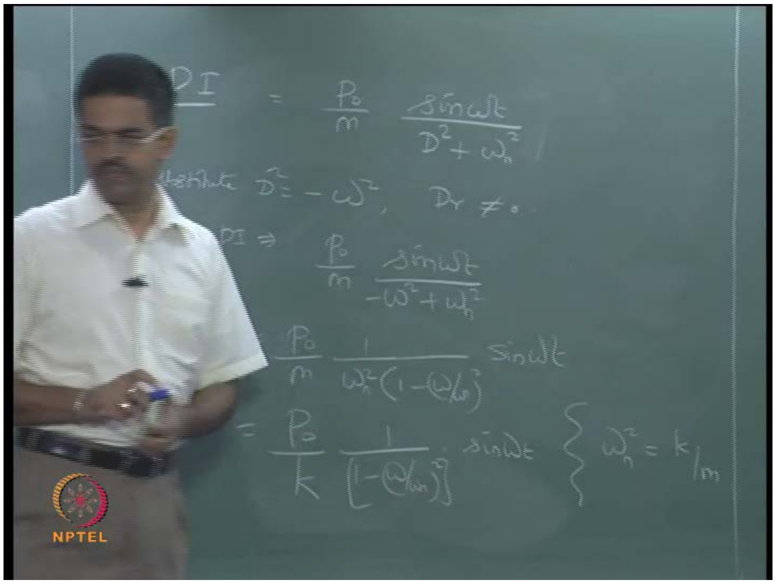

The second part of the answer starts from particular integral. It is nothing but P 0 by m sine omega t by D square plus omega n square. So, the procedure is, substitute D square as minus omega square in this equation provided your denominator should not become 0. If it becomes 0, then the procedure is slightly different. So, now, anyway, as long as omega and omega n are not equal in the discussion numerically, so it will not become 0. So, straightaway you can say, my P I is now going to be P 0 by m sine omega t by minus omega square plus omega n square. I can take out omega n out, so P 0 by m 1 by omega n square 1 minus omega by omega n the whole square of sine omega t.

There is a reason why I am taking omega n out instead of omega because omega n is represented as k by m, m will get cancelled, I will ultimately get this as P 0 by K 1 1 minus omega by omega n the whole square of sine omega t where omega n square is k by m . So, this is second part of the answer. The first part of the answer is here, the second part of the answer is here, the total solution is sum of these two answers. Let us try to sum them up and write the whole equation for x of t.

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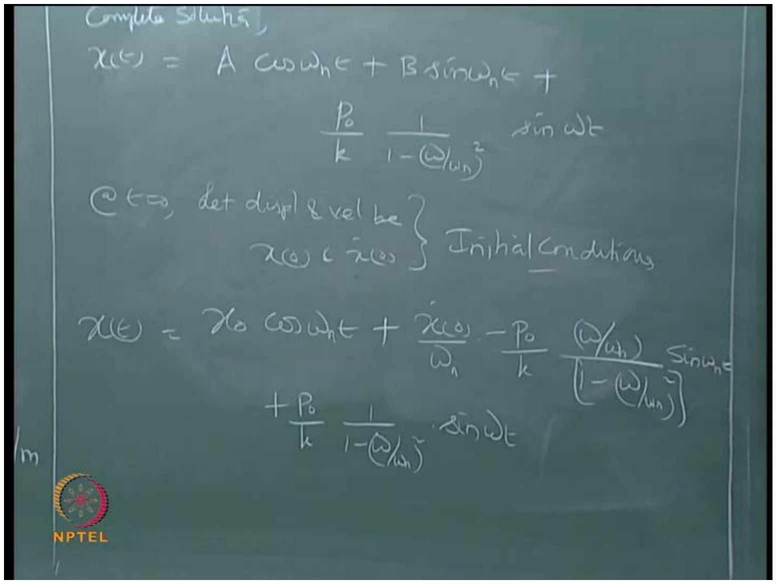

So, now x of t, the complete solution of the ordinary second order differential equation is given as A cos omega n t plus B sine omega n t plus P 0 by k 1 by 1 minus omega by omega n the whole square of sine omega t. Now, there are constants A and B, I have to eliminate them, so I will use, at t is equal to 0, let displacement and velocity be x 0 and x dot 0, which are initial conditions for the given problem. So, I will apply this and try to eliminate A and B and tell me what is your x of t.

So, you have to differentiate this equation once with respect to time, then use this relationship and find out B to find out A. Of course, you can directly use this, so do that and see what we are going to get as x of t. So, x of t will be x 0 cos omega n t plus B value, which will be x dot 0 B omega n minus P 0 by k omega by omega n by 1 minus omega by omega n the whole square of sine, omega t, omega n t of let us say, sine omega n t plus P 0 by k 1 by 1 minus omega by omega n the whole square of sine omega t. I think we are all getting this equation.

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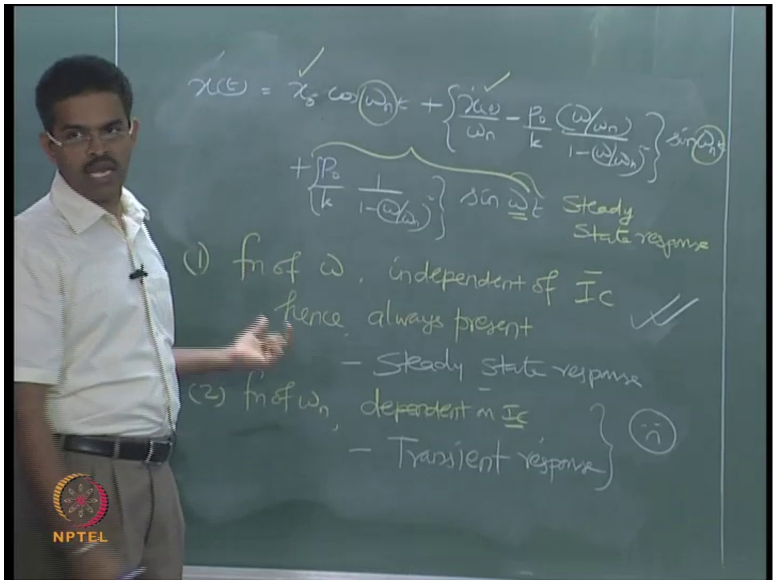

Let me rewrite this equation here to have more discussion once we got the complete solution for x of t, which I am rewriting here again, say x of t is x 0 cos omega n t plus x dot 0 by omega n minus P naught by k omega by omega n sine omega t. Now, there are two exclusive components, which we can distinctly see from this result. What are those two different components? The first component, what we see here, is a function of omega. So, it has got a function of omega, which is independent of any initial condition and hence always present because it is independent of initial condition. This is neither x naught nor x dot naught in this result, what we call as steady state response; this is what we call as steady state response.

Now, the second component, what you see here, are functions of omega n; so they are functions of omega n. It is dependent on initial conditions because x naught and x dot naught are reflected in the result. So, depending upon the value of x and x dot naught, it may even die down or diminish also, so may not be always present. And further, when you introduce damping to this existing system, now it is undamped, there is no damping here, further if you introduce damping to the system, you will see, that any response with functional natural frequency and dependant on initial conditions will decay. We have already seen that in the last. Therefore, this response is what we call transient response.

So, two categories of response are present, one is transient, other is steady state. So, therefore, the transient response may not be of not any interest to us because it may not continuously present, it depends on initial conditions, whereas the steady state response will always be present, therefore it is important for us. Let us zoom this window and see how I can further simplify this steady state response and what inference I can draw from the steady state alone, which will be interest for us.

So, on the other hand, when you talk about any real dynamic analysis problem where F of t is not set to 0 despite whether the function or the model is damped or undamped, the dynamic analysis generally focuses only on the steady state response of the model, it will not focus on this. Then the question alternatively comes is, what is it, then the significance of a transient response? Then what is the significance of this response? Why do we see this then? That can be a question for you to the exam.

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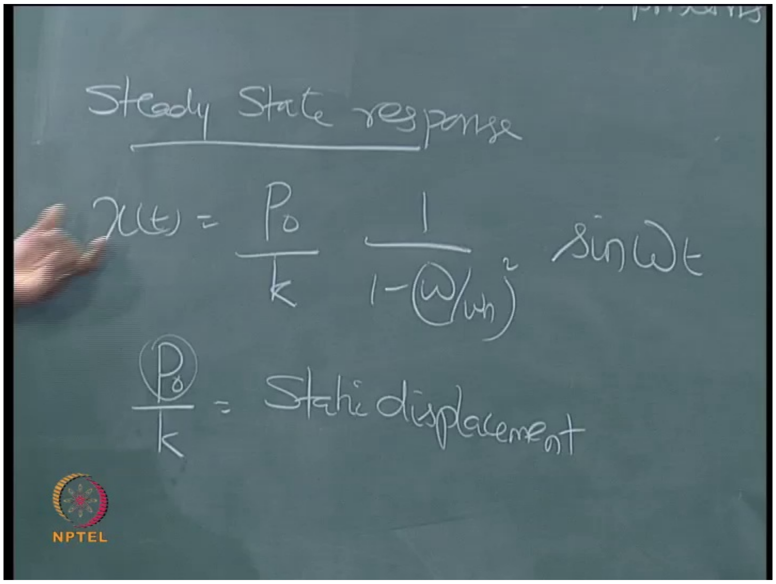

Transient response in x of t… That is very important. Steady state will, anyway, open the window now and explain. So, I will pick up the steady state response alone, which we say x of t is P 0 by k 1 by 1 minus omega by omega n the whole square of sine omega t. Interestingly, P 0 by k is, what is called, static displacement while P 0 is the amplitude or magnitude of the forcing function and k is the restoring coefficient or stiffness coefficient of the spring model given. Therefore, this is can be simply called as static displacement.

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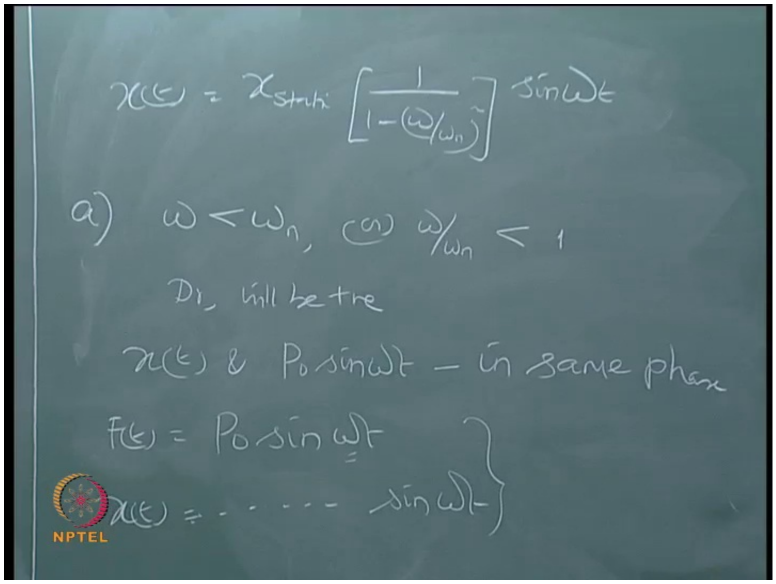

So, I can rewrite this equation as x of t is x static 1 by 1 minus sine omega t. Now, depending upon omega less than omega n or omega by omega n is less than 1, the denominator will be positive, is it not? If it is less than 1, the denominator will become positive. It means x of t and P 0 sine omega t will be in same phase. How can you say that? Look at F of t. F of t is P naught sine omega t, it is a sine function or a specific frequency, whereas x of t is also sine omega t, is it not?

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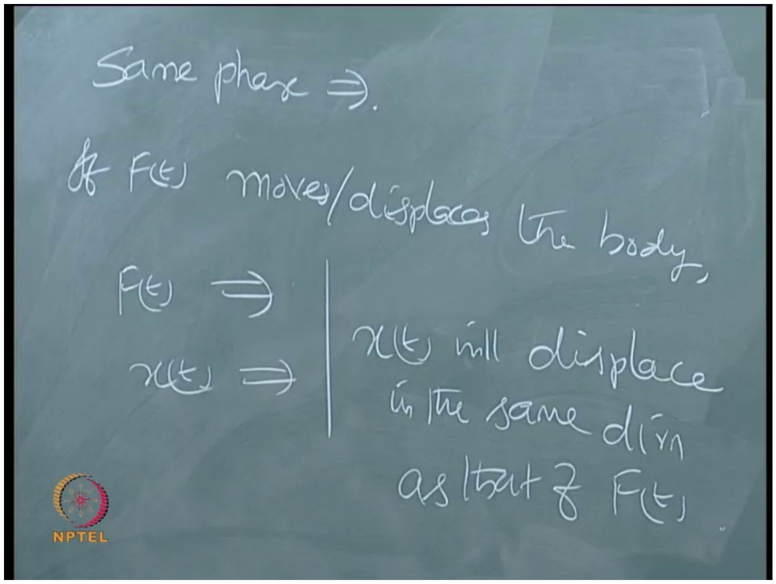

Say, on the same phase, what does it mean physically? What do you mean by same phase physically? Same phase means, same phase implies, if F of t moves or displaces the body and F of t is acting towards right, x of t will also be towards right. It means, x of t will move or will displace in the same direction as that of F of t. That is what same phase means.

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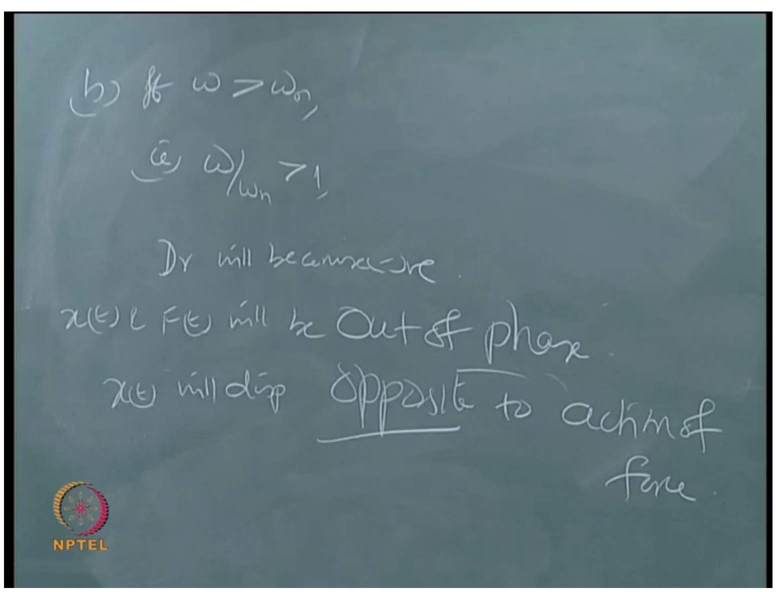

Let us take another case, case b. If omega is greater than omega n, that is, omega by omega n is greater than 1, denominator will become negative. So, sign will change, is it not? So, x of t and F of t will be out of phase. So, what does it mean out of phase? If F of t try to act from left to right, x of t will have displacement towards left, so x of t will move or will displace opposite to action of force.

Now, this is very interesting and very difficult to imagine. It is very difficult to imagine. A body is there, you are trying to push the body towards right, but the body is responding towards left. That depends upon the match between at what frequency you are pushing the body, on what frequency the body is developing itself in natural phenomenon. Then depending upon this value the body can even oppose an action opposite to the direction of application of force also. Mathematically, it is very clear from this equation.

Now, instead of looking both these cases in isolation, dynamics looks both of them together in a single equation because I want to incorporate this variation in a single equation, plot it and see what happens. Now, remember very carefully here, both these condition do not tell you what happens in omega equal to omega n; that is very, very important here. Please do not get confused these are two distinct domains;, omega less than omega n, omega greater than omega n. It is silent about omega equals omega n, what happens we are going to see that, that is very important.

Now, we all agree, that what is omega and what is omega n? Omega is the frequency at which the force is acting on the system. Where do you get this frequency from? Wave loading has a specific frequency; we know, it is function of time. Wind load has specific frequency; we have already seen equations, etcetera. So, these forcing function frequencies are known to us and they operate on a specific band because waves do not encounter offload structures on a specific frequency, is it not? There is a band, so both the possibilities are there.

The structure can move in line with the force. The structure can oppose the force because omega is a range. It can be equal to omega n, which we are not discussing now. It can be lesser than omega n and can be more than omega n also in a given service period, a given day of a structure also, 8 o’ clock omega can be lower than omega n, suddenly 9 am high wave comes and omega is exceeding omega n structure. It means, dynamic loading imposed on offload structures automatically triggers fatigue response to the system. It is inbuilt; you cannot avoid this because direction of reverse, it is possible with the omega. There is also a possibility omega will exactly lie equal to omega n, that we will discuss. Anyway, this discussion does not cover that, still pending with our discussion. So, A and B, instead of looking at distinctly, I want to unify them in a single expression and then plot and see what happens

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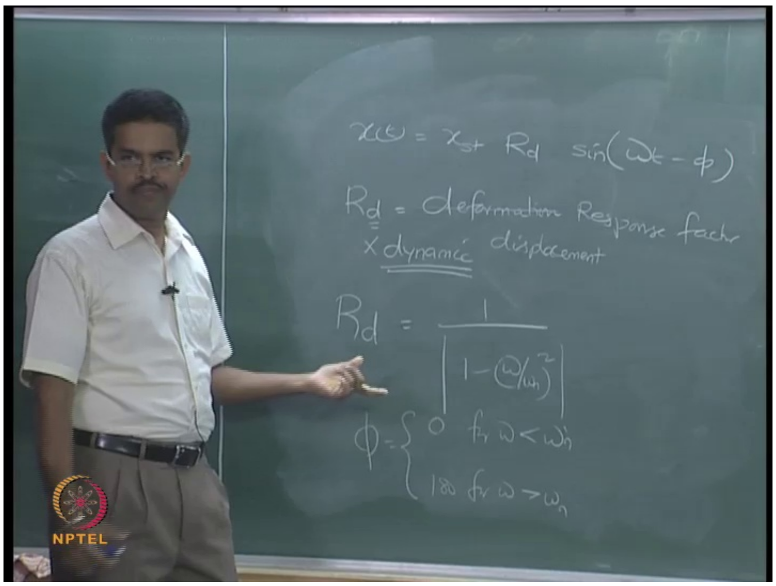

So, I can write an expression like this, x of t as x static R d sine omega t minus phi. I am introducing a phase lag here, I am retaining the equation of expression as omega itself, I am retaining the value also, sine value, sinusoidal function, still I have introduced a new term called R d. R d is called deformation response. Of course, it is a factor, so this d does not stand for dynamic, please remember this.

There is a common confusion, people use this, it is not dynamic, and it is deformation. If you are uncomfortable with deformation, you can even replace this as displacement also, it is one and the same. Then why people have used deformation? There is a specific reason for this, displacement is not the only, deformation can also have velocity, acceleration, etcetera. So, people say, there will be deformation response factor.

Now, I have to speak about the qualitative value of phi. What is about R d? So, I can now say, R d is simply given by 1 by absolute of 1 minus omega by omega n square. I am taking away the sign from this and giving that sign to my phi, so phi is equal to 0 or 180, when omega less than omega n for omega greater than omega n. So, I have a single expression now, which is x static. I mean, x of t is x static R d sine omega t minus phi. Now, I am interested to see what is the variation of R d with respect to omega by omega n, this equation? So, I will try to plot this. Since the sign convention is blocked off I will always get a positive response for this, then the plot will look like what I am drawing here.

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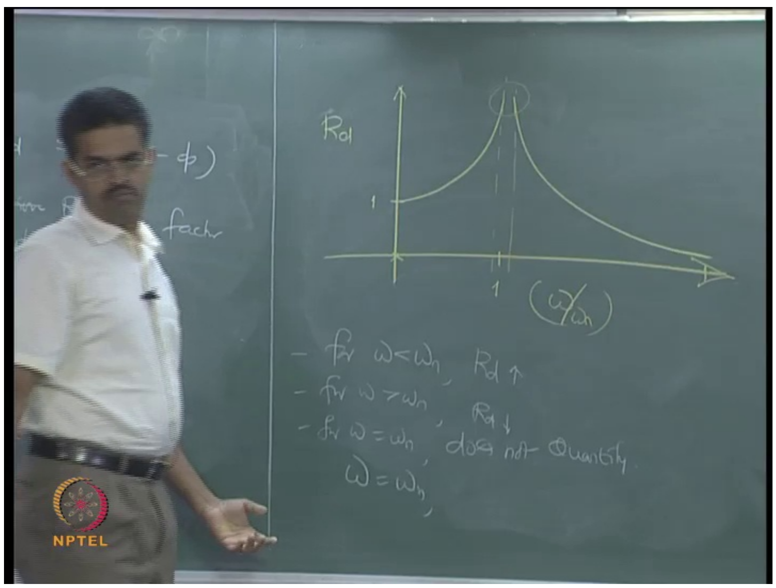

I am trying to plot omega by omega n, the ratio of the frequencies with respect to R d. For this ratio remaining 0 here, R d still be 1, start from here. For a lower value of omega compared to omega n, the denominator for any increase in omega, the denominator will keep on decreasing. Therefore, R d will keep on increasing. For any value of omega in a band more than omega n, for further increase in omega beyond omega n, denominator will be keep on increasing because there is a more sign and R d will be keep on decreasing.

There is a band, which is silent here. This is the point where omega is equal to omega n. This band is not discussed or it is not qualified by this expression, will not give you this band, can you tell me why? What is the problem? Sorry, that is a mathematical answer. It becomes infinity, I do not want that, I want a very clear mathematical answer for this. It is not a mathematical answer, it is a physical answer. I have not defined resonance so far, sorry, discontinuity, no, no, no; there is a very explicit answer for this, very explicit. I can give a hint, think of solving ordinary differential equation at denominator becoming 0.

The procedure what we used to derive, this expression at condition omega equal omega n is not correct, it is wrong. Look at the solution procedure what we just now discussed when omega equals omega n, the way in which I derived this expression is not applicable. It is wrong; therefore, I cannot apply a value in the domain where the equation is not justifiable at all. So, mathematically it is wrong to understand omega equal omega n from this equation, not from this figure. So, we must look into again the solution procedure starting from the beginning and substitute omega equals omega n and see what happens. So, that will give me a focus on further window, what is happening here. Secondly, as correctly told, the function becomes discontinuous at this location, so we do not know.

Let us have some information about this figure, some inferences here, very simple. For omega less than omega n, it is this band R d is increasing; for omega greater than omega n, R d is decreasing. The figure does not quantify the range for omega equals omega n, does not quantify. Now, when omega equals omega n, there is unbounded response seen in this figure. The response gets unbounded. We do not know what is the peak value of this response, is it not? That is, unbounded; R d becomes unbounded.

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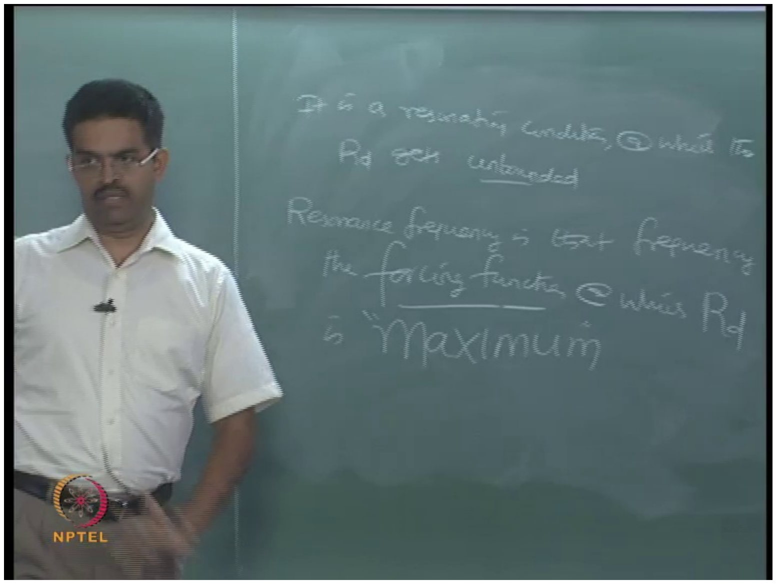

Now, let us define this state in the analysis saying, it is a resonating condition at which the dynamic response factor are, sorry, the deformation response factor gets or becomes unbounded. I am not saying infinity and all, it is unbounded, and therefore resonating frequency is that frequency of the forcing function. Remember, very important, this statement is very important, at which R d is maximum resonance is not addressed related to natural frequency; resonance is addressed to forcing function actually. Resonance is that frequency at which the forcing function makes R d maximum. So, in a given plot if you are able to find out a specific value at which the response factor gets its maximum amplitude, it is that frequency of the forcing function, which is called resonance frequency. So, it is very simple. Mathematically, there are two variables, I can fix one and vary the other with respect to the fixed value. What is fixed in the discussion is omega n, it is not omega. That is what I am meaning by this statement; I am fixing omega N and not omega.

Now, why, why not omega? What is the problem? There are two reasons, for this omega n is known to me in advance because it is a system characteristic. This k by m root, I know k and I know m of the given structural system because my platform is form driven. Form means, I know the geometry, I know the mass value, I know the stiffness value, so I can easily find omega n, even though I have no idea about omega. Therefore, I can fix this value right; that is one reason. The second reason is omega is a broader range; you cannot fix it at all. Therefore, we fix omega n and vary omega. See, that in that variation at which omega you get R d maximum, call that omega as resonance frequency, that is how it is defined, the reason why we are doing it like this.

Now, the argument still gets confused, because at this band it is silent. I am not quantifying and most of engineers are really worried about the response of a function or a system or a model or any engineering design when a dynamic excitation frequency matches with the natural frequency of the system because the response is going to, definitely, become maximum.

So, generally, there is a fear, that whenever resonance happens, two things are coming in mind. One, always people think the response at resonance will be unbounded, it is infinite, we cannot define because it is resonating, it is one. Always, any pre assumption made by every engineer thinking, that if omega equals to omega n, the response is going to become infinite. Therefore, I should not allow this condition to happen in reality because the structure will fail. But neither I can control omega, nor I will show you I can control omega n.

Also, I will show you how it is, I can give a clue. Omega n is a function of k and m for a single degree. m has component from variable submergence because the platform is not completely in water, not completely in air, it is partially immersed in water and the wave height is not still water, it is not horizontal, it is having elevations, which is sinusoidal defined. At any given instant of time, there is a possibility, the submerged volume can alter, therefore m will alter. When m alters, omega n cannot be fixed, omega n is also vary, offshore structures specifically.

Therefore, there is a very less probability, fortunately, in offshore structures that resonance will set in. It is because of this reason, blessed by God; offshore structures are never failed by a resonance activity, whereas for example, people quote failure of bridges. Tacoma Bridge in Japan is one example, where it is failed by fluttering phenomena in wind forces, where it is a resonating activity. You have never read any failure of offshore platforms in the literature, which has failed by resonance activity, never, because it is God blessed system where omega n is automatically shifting with omega.

Therefore, as an engineering design prospective in offshore structural design, we need not have to bother about resonance condition in the design there is no worry, but mathematically I must know what happens in this domain, it is very important for me. But I cannot use this expression because this expression was derived from a fundamental observation where the condition is never becoming valid. So, I must go back to my original equation and derive it back. Again, substituting omega equals omega n and see what happens, then I will discuss this band at which it is happening. So, there are only five minutes left over, we will touch it now here, it will take at least half an hour for me, I will not do that now. If you have any questions we will answer.

So, we have learnt steady state response and transient response from a damp, I mean, undamped force vibration model. We have picked up a simple example. The forcing function has amplitude P naught and then wave only with one frequency. With this possibility, the forcing function can also have different frequency set. We will discuss that later in this stochastic dynamics in the last module. But here we have taken a simple case to identify clearly the window where we are interested in.

We have also had a question, that why steady state is always considered to be important dynamic analysis vis. a. vis, transient response because transient response is dependent on the initial conditions of the problem. It may decay, it may die down, it may become insignificant in comparison to the steady state response where steady state response is always a function of forcing frequency, which will otherwise present in the system because we are talking about forced vibration, it will be present. Therefore, steady state will always exist.

Transient response may or may not be significantly there compared to steady state, so we do not bother about that. That is what the theory says as far as important steady state is concerned, but we still had a question, that why, then we at all steady, steady transient response. Then we took up a window on steady state alone. We derived an expression for deformation response factor. We plotted it. We clearly understood that there is a band at which the equation of R d does not quantify this value because the solution procedure to obtain R d was wrong at this band. We also understood, that this band, since these two frequencies match, it is expected, it is seen, that it is unbounded, which gives an alarming signal to any engineering prospective person saying at resonance (( )) platform fail because response factor rises to or shoots up to infinite or unbounded value.

But as far as we understand in dynamics of ocean structures, omega n is also vary because of one great advantage, which is variable submergence effect we have. It also varies with k; I will talk about that later in the second module. That is why, I initially said, offshore structures are form driven systems. The stiffness is not unique, it changes with time, and stiffness is the function of time actually. It is a very, very interesting problem, which very few structural systems on earth you will have when you talk about form driven systems.

As far as offshore systems are concerned, we are deliberately introducing the variation stiffness respect to time because I am working on this problem here. So, I will never attempt to have resonant band at all for a longer duration, for one instance it may happen. But how to attack that, how to design it, that we are going to discuss later at a glimpse. But you must appreciate now, that dynamic analysis of ocean systems is closely coupled with the design and functionality both.

Why design, because I am working on this band in the beginning itself, because omega n is my property of design. Why functionality? Because I am form deriving the system, function is not important for me, function is drilling of production. I understand that, but I am putting this platform in different water depths, making the platform either rest on flow or we hold it down by mooring or tethers, etcetera, or introducing articulations. So, I am changing the form behavior, so that k is also becoming a function of response, as well as time.

Therefore, this band will not actually bother me much in dynamic analysis of ocean systems. It is because of this automatic and inbuilt reason that offshore structures generally do not fail at resonance, generally they will not. But even at any one instant of time if resonant occurs or sets in, how the system escapes in resonance I will discuss it in the next lecture. Then you will convincingly agree with me, that resonance need not be looked upon in offshore design at all. I will explain this very clearly in the next lecture because I do not have that equation, this band here. Now, we cannot talk about that. Any question? So, we will stop.