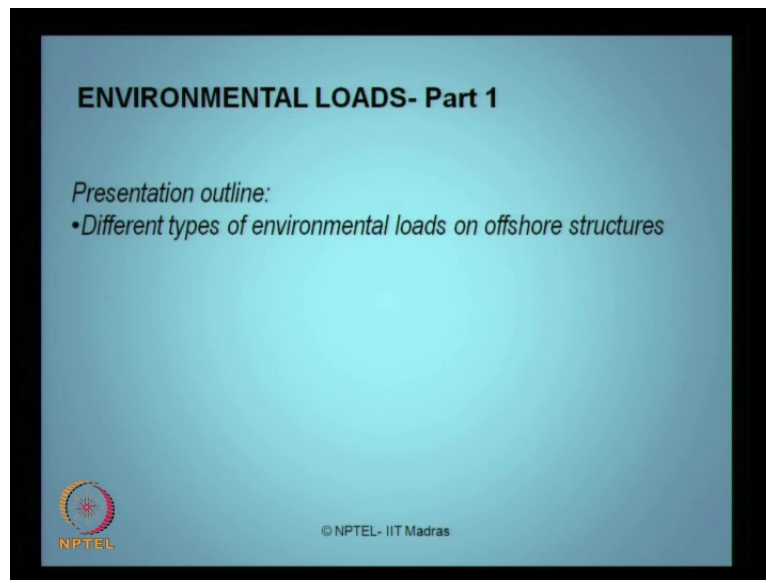


Ocean Structures and Materials
Prof. Dr. Srinivasan Chandrasekaran
Department of ocean Engineering
Indian Institute of Technology, Madras

Module - 1
Lecture - 7
Environmental Loads I

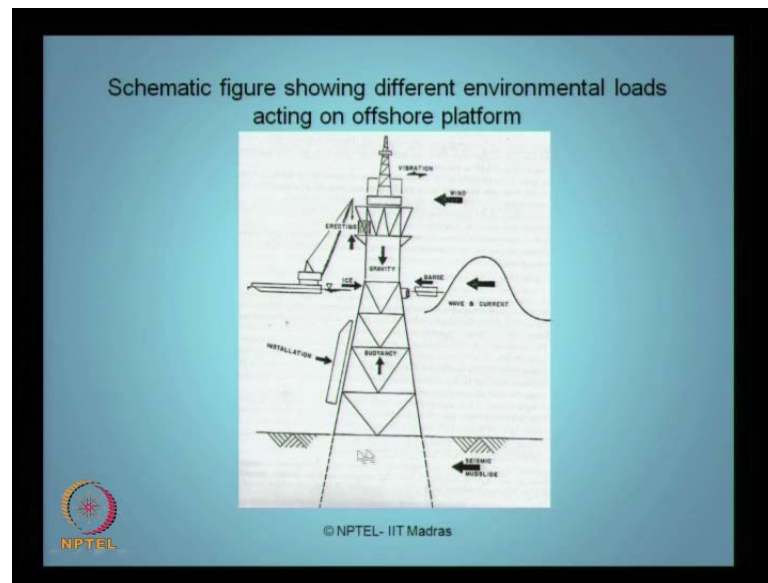
Ladies and gentlemen, welcome to the seventh lecture on module 1 of ocean structures and materials which is a virtual course under the brace of NPTEL, IIT Madras. Let us quickly see what we will be covering in this lecture as a presentation outline.

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As we have discussed different types of offshore structures in the previous lectures, in this present lecture and the next one followed by this, we will talk about different types of environmental loads that are acting on offshore structures.


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So, the focus of this lecture will be on different types of environmental loads on offshore structures. Quickly let us see a schematic figure which explains different environmental loads acting on offshore structures. As we all understand this is nothing but the drilling derrick which is being installed on the top side of the structure. We can see on the jacket braces of the structure, this is nothing but a launching truss where lot of erection loads can be happening. This is a wave action, this may be an action of the current, this may be action due to the lateral loads like wind, this may be the weight of the platform acting down, and this may be the buoyancy of the platform or the sum wisd volume of the platform acting up.

There can be other loads coming, because of anchoring of the ships or boards or any vessels which is used for offloading to that of the platform. So a schematic figure shows, what are the different ranges of forces acting on an offshore structure in addition to that, we also have a special forces from earthquakes as well on the sea floor. So ladies and gentlemen, you will appreciate that offload structure is one of the class of its variety which almost encounters all types of environmental loads that are available.

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Introduction

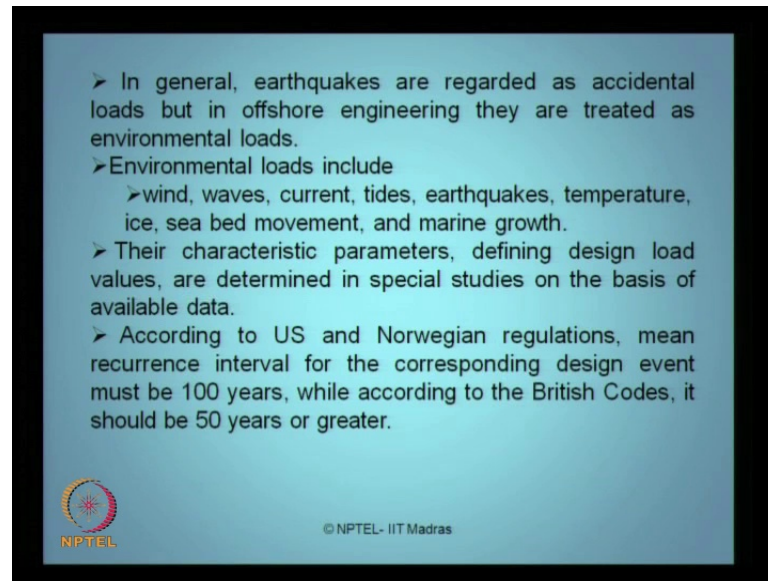
- Loads acting on offshore structures are classified as:
 - i) permanent loads or dead loads
 - ii) operating loads or live loads
 - iii) other environmental loads including earthquake loads
 - iv) construction and installation loads
 - v) accidental loads
- Design of buildings onshore is influenced mainly by the permanent and operating loads
- But design of offshore structures is dominated by environmental loads
 - especially from waves and loads during various stages of construction and installation.

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Now, the loads acting on offshore structures can be classified in the following manner. There can be permanent loads or dead loads acting on the system. There can be operational loads what we put them as live loads on the platform. There can be other environmental loads including earthquake forces which are coming on the platform. Certainly offshore structures are attracted by construction and installation forces; of course, there can be accidental loads which are caused by the weaseling pack on the structures. How to we actually know to compute all these forces.

In this lecture, we will give you a very brief summary about the calculation methodologies of this forces, the source of information where they are available to compute these focuses; however, this course will not elaborate more in detail about the computation aspects of the forces on offshore structures. Interestingly if you look at the design of buildings onshore, they actually influenced by the permanent and operating loads which we called them as dead loads or live loads; whereas if you look at the design offshore structures, they are essentially dominated by environmental loads. Especially the loads from waves and the loads during various stages of construction and installation can be the critical set of forces acting on offshore structures.

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➤ In general, earthquakes are regarded as accidental loads but in offshore engineering they are treated as environmental loads.

➤ Environmental loads include

- wind, waves, current, tides, earthquakes, temperature, ice, sea bed movement, and marine growth.

➤ Their characteristic parameters, defining design load values, are determined in special studies on the basis of available data.

➤ According to US and Norwegian regulations, mean recurrence interval for the corresponding design event must be 100 years, while according to the British Codes, it should be 50 years or greater.

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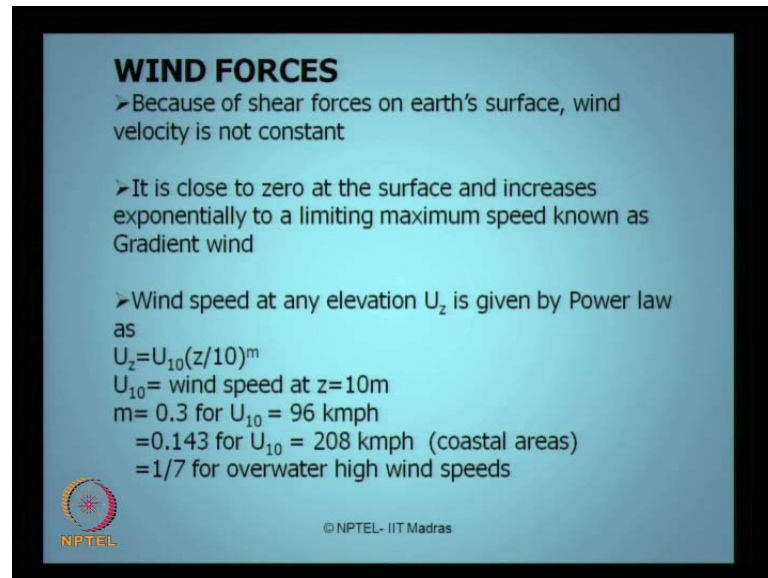
In general if you look at the very interesting point of view, earthquake loads or seismic loads are considered as accidental loads, but in offshore engineering they treated as one of the major source of environmental loads. Therefore, we can make a simple summary. Environmental forces include wind forces, wave forces, forces due to current acting on the sea, tides, earthquakes, temperature forces, ice forces because from the ice box; seed bed movement, and ultimately a very interesting follow-up of the force what the call them as force incurred due to marine growth.

Now their characteristic parameters, defining design load values are determined using special studies on the basis of available data. So all these forces are parametrically varying, there can be random variations in these forces; many of them are dynamic in nature and time-dependent. If you want to really look at the design characteristic parameters of these forces, they are actually available in international course based on special studies carried out on the basis of the existing data which has been recorded in the sea environment.

Now interestingly, if you look at different international codes quickly, according to the United States and Norwegian regulations, mean recurrence interval for the corresponding design event must be 100 years as per the design recommendation given in the code. If you look at the British standards, then this code says this mean recurrence interval can be either 50 years or greater. So there is a hint given to you here, that international codes


while recommending design forces for design of offshore structures, they also widely vary depending on their methodology how these forces have been estimated by these codes.

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WIND FORCES

- Because of shear forces on earth's surface, wind velocity is not constant
- It is close to zero at the surface and increases exponentially to a limiting maximum speed known as Gradient wind
- Wind speed at any elevation U_z is given by Power law as
$$U_z = U_{10}(z/10)^m$$
$$U_{10} = \text{wind speed at } z=10\text{m}$$
$$m = 0.3 \text{ for } U_{10} = 96 \text{ kmph}$$
$$= 0.143 \text{ for } U_{10} = 208 \text{ kmph (coastal areas)}$$
$$= 1/7 \text{ for overwater high wind speeds}$$

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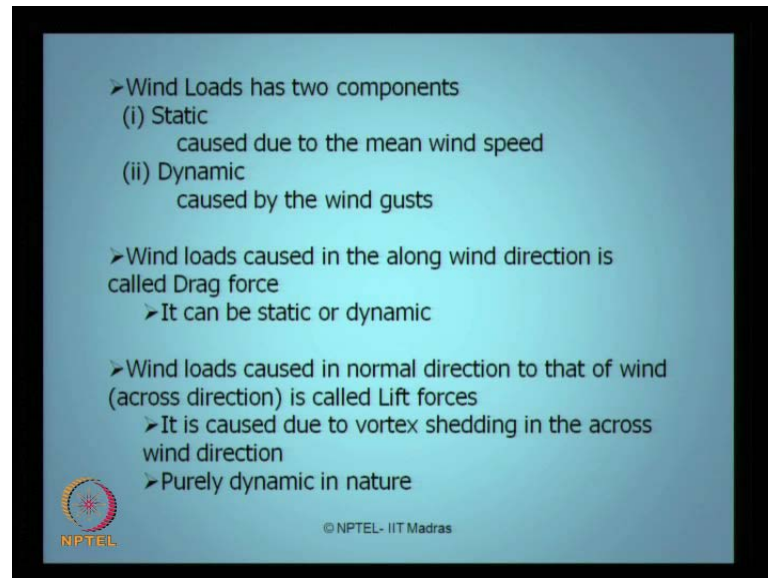
As a part of the calculation, let us look into detail how do we estimate wind forces coming on offshore platform? We are engineers; we have an idea how to estimate wind forces on land-based structures. The same phenomenon is applied here for offshore structure also. Essentially wind forces are created because of the sheer force acting on the earth surface. Interestingly wind velocity does not remain constant.

It is close to zero at the surface of the mean sea level and keeps on increasing exponentially, as you keep on increasing the height along the tower. On the other hand, the variation of wind velocity acting on any particular member of an offshore structure exponentially varies with zero at the mean sea level, and with the maximum at the top. Interestingly wind forces essentially depend on what we call wind velocity at any specific location.

If you put wind velocity as U_z , we have a power law which explains U_z calculation based on U_{10} . What is U_{10} ? U_{10} is actually the wind speed measured at a height of 10 meters from the mean sea level which we called as a reference datum for wind velocity measurements. So it is assumed and understood that, the wind velocity remains constant till 10 meter height from the mean sea levels above which it keeps on varying

exponentially, because there is a power m which is given to this equation and the values of these equations of m for different U_{10} velocity for coastal areas are available on the slide.

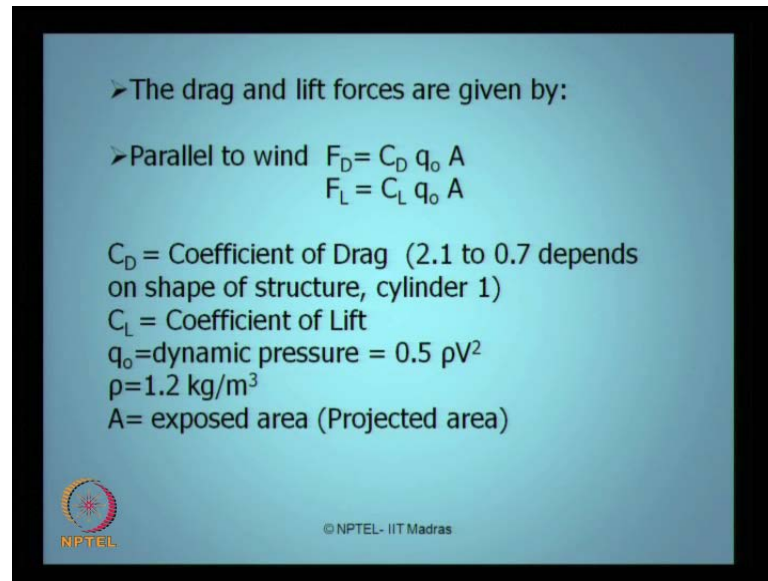
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- Wind Loads has two components
 - (i) Static
 - caused due to the mean wind speed
 - (ii) Dynamic
 - caused by the wind gusts
- Wind loads caused in the along wind direction is called Drag force
 - It can be static or dynamic
- Wind loads caused in normal direction to that of wind (across direction) is called Lift forces
 - It is caused due to vortex shedding in the across wind direction
 - Purely dynamic in nature

Wind loads have two vital components. One is what we call a static component; other is what we called as a dynamic component. Now the static component is caused due to what we call mean wind speed; whereas the dynamic component is caused by what we call gust wind. Now the wind loads caused in along the wind direction is given a specific name in the offshore literature; we call them as drag force. The drag force can be either static or it can be a time variant which we call as dynamic in nature. Wind loads which are caused in the direction normal to the direction of wind, what we call a literature as across direction; we call these kind of forces generated by across the wind as lift forces. It is essentially caused due to the vortex shedding in the across wind direction. This is purely dynamic in nature and strongly time-dependent.


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➤ The drag and lift forces are given by:

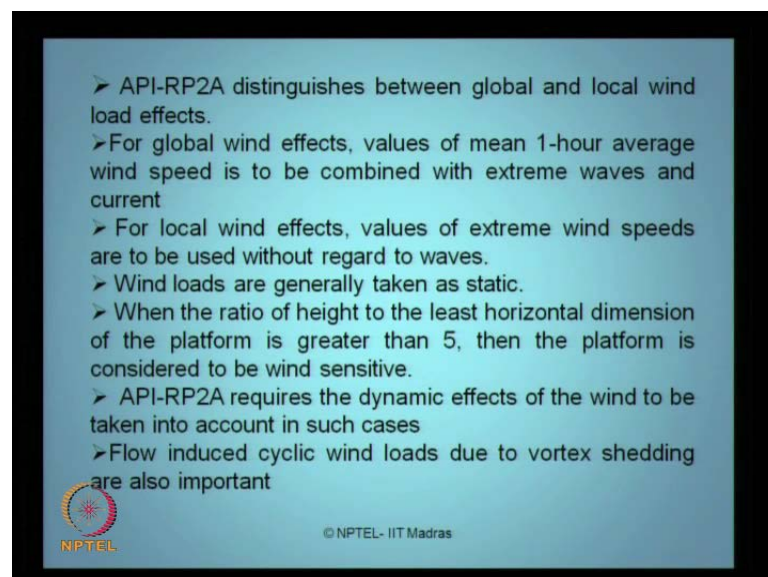
➤ Parallel to wind $F_D = C_D q_0 A$
 $F_L = C_L q_0 A$

C_D = Coefficient of Drag (2.1 to 0.7 depends on shape of structure, cylinder 1)
 C_L = Coefficient of Lift
 q_0 = dynamic pressure = $0.5 \rho V^2$
 $\rho = 1.2 \text{ kg/m}^3$
 A = exposed area (Projected area)

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Now the question comes as to how to estimate quickly the drag and lift forces on the members of offshore structures. Now parallel to wind is what we called as a drag force which is given by $C_D q_0 A$; whereas a lift force is given by $C_L q_0 A$, where A as we all understand is exposed area, what we call as projected area in the literature; q_0 is the dynamic pressure which is varying with the velocity V which is half ρV^2 , where ρ is the density of wind. C_L and C_D of course, are respectively the lift co-efficient and the drag coefficient and the drag coefficient. These values depend on the shape of the structure which is available in the literature in offshore design guidelines.

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➤ API-RP2A distinguishes between global and local wind load effects.

➤ For global wind effects, values of mean 1-hour average wind speed is to be combined with extreme waves and current


➤ For local wind effects, values of extreme wind speeds are to be used without regard to waves.

➤ Wind loads are generally taken as static.

➤ When the ratio of height to the least horizontal dimension of the platform is greater than 5, then the platform is considered to be wind sensitive.

➤ API-RP2A requires the dynamic effects of the wind to be taken into account in such cases

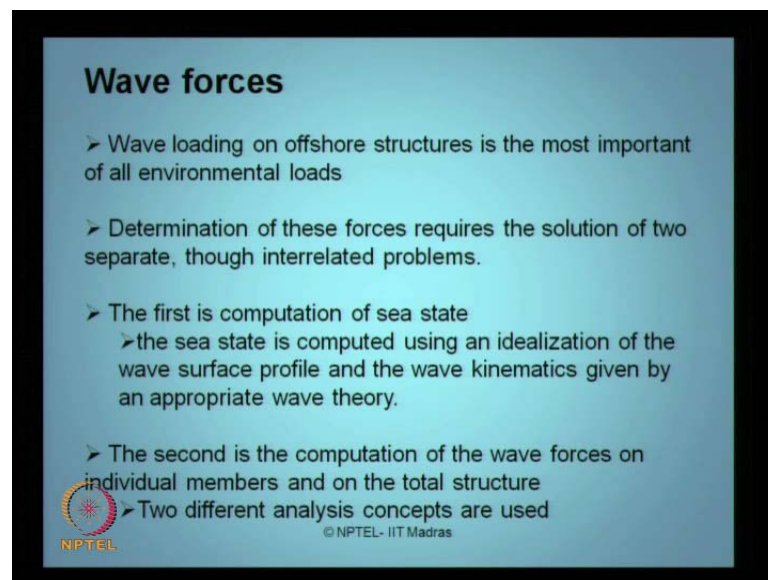
➤ Flow induced cyclic wind loads due to vortex shedding are also important

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If you look at American Petroleum Institute Recommended Practice which we call as API-RP2A; this distinguishes the wind forces between global and local wind effects. For the global wind effects, the value of one or average wind speed is to be combined with extreme waves and current; because in the environmental situation, an offshore platform is combined subjected to wind, wave, and current. You have to consider all of them acting simultaneously. In that situation, if you are talking about global wind effects then you must say the values of mean one or average wind speed can be combined with that of extreme waves and current.


For local wind effects, the value of extreme wind speeds are to be used without regards to the waves. Wind loads are generally taken as static in the analysis, but when the ratio of height to the least horizontal dimension of the platform exceeds 5, then the platform is consider to be what we call wind sensitive or aerodynamic sensitive. API-RP2A requires dynamic effects of wind to be taken in such cases. The flow induced cyclic wind loads which are essentially caused because of vortex shedding become very important in such cases. The second classified load what we have in offshore structure is arising from the waves. It is one of the predominant load which acts on members of offshore platforms.

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Wave forces

- Wave loading on offshore structures is the most important of all environmental loads
- Determination of these forces requires the solution of two separate, though interrelated problems.
- The first is computation of sea state
 - the sea state is computed using an idealization of the wave surface profile and the wave kinematics given by an appropriate wave theory.
- The second is the computation of the wave forces on individual members and on the total structure
 - Two different analysis concepts are used

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Wave loading is considered to be the most important of all the environmental loads acting of on an offshore platform. Determination of these foresees requires solution of two separate but interrelated problems. There are two issues related to this; one is how to

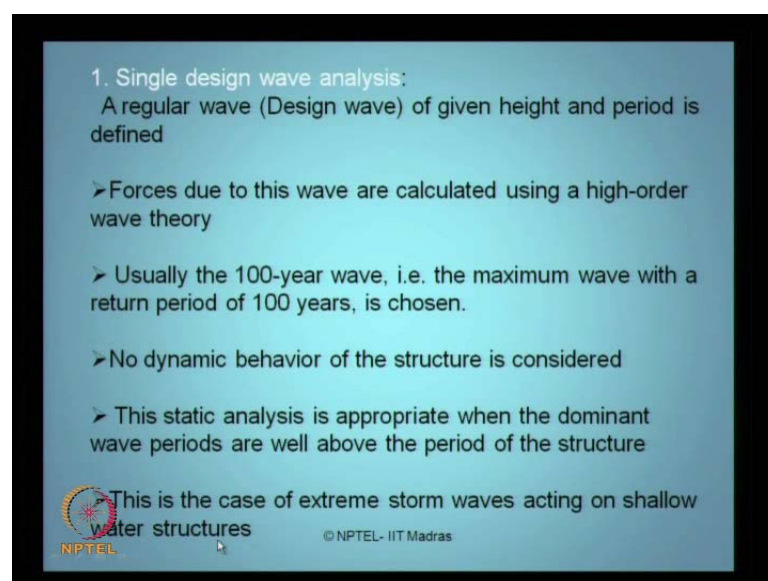
compute the sea state, the second is how to compute the wave forces on individual member and how to compute the sea forces on total of all these members.

If you look at the first problem where we say how to fix or how to estimate the sea state, the sea state is actually computed using an idealization of the wave surface profile and the wave kinematics. The wave kinematics include water particle velocity and variation, in terms of its velocity and acceleration along the depth at any special interval x and y at any specific time t . These are suggested in the literature by many classified theories.

So, which theory to use, what wave theory I will use, where are they applicable? These are in detail covered in many course and on many literature available wave hydrodynamics, but for the completion is sake in this lecture, we will discuss few theories and some equations for you which becomes handy to compute the wave sources on members.

So, if you want to locate the sea state which you are designing the system for, you must select and idealize the wave surface profile and then you must appropriately use any specific theory to compute the wave kinematics or water particle kinematics. The second problem associated with computation of wave forces is how to calculate these forces individually on each member and then on the total structure. Ladies and gentlemen to do this, we have got two different analyses available in the literature.

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1. Single design wave analysis:
A regular wave (Design wave) of given height and period is defined

- Forces due to this wave are calculated using a high-order wave theory
- Usually the 100-year wave, i.e. the maximum wave with a return period of 100 years, is chosen.
- No dynamic behavior of the structure is considered
- This static analysis is appropriate when the dominant wave periods are well above the period of the structure

This is the case of extreme storm waves acting on shallow water structures

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One is what we call single design wave analysis. A single design wave is nothing but a regular wave which we call in the literature as a design wave, whose wave height and period are defined. Now the forces due to this wave are calculated using a higher-order wave theory. Usually a 100 year wave; that is the maximum wave with the return period of 100 years is chosen. We have a record; let us say we have a record of past 50 years, 75 years, 100 years. Look at the wave height in the past 100 years record ascend them in ascending order, look at the maximum wave height for the return period of 100 years and take that specific wave as a single design wave for which you will compute the forces. In such cases, no dynamic behavior of the structure is considered.

Can you tell me why? Because we are looking for a hypothetical wave which occurred at maximum amplitude once in the past 100 years; it may not be sure that this wave will re-occur again and again. Therefore, no dynamic effect needs to be considered if you consider a single design wave analysis. So this static analysis is appropriate, when the dominant wave periods are well above the period of the structure. Ladies and gentlemen, you must now have an idea what are the time periods of range of offshore platforms starting from fixed jacket type platform, gravity based structures, which are stiff in nature.

Like compliant structures, which have got two hybrid range of frequencies which one is flexible and one set is highly fixed and so on. So, if you have the dominant wave period, so called selected for the design wave, is well above in the range of the period of the structure what you are designing, then this approach can be used. This is the case of extreme storm wave acting on shallow water structures. Essentially there is a limitation on this methodology; this can be successfully applied to shallow water structures up to the depth of 200 meters.

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2. Random wave analysis:

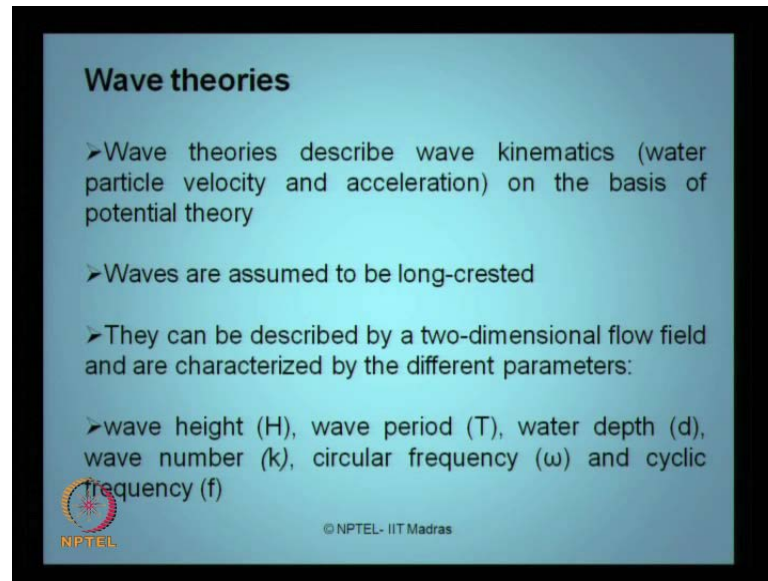
- Statistical analysis is done on the basis of a wave scatter diagram for the location of the offshore platform
- Appropriate wave spectra are defined to perform the analysis in frequency domain
- Appropriate wave spectra is used to generate random waves, if dynamic analyses for extreme wave loadings are required for deepwater structures
- With statistical methods, most probable maximum force during the lifetime of the structure is calculated using linear wave theory
- Statistical approach is chosen to analyze fatigue strength and the dynamic response of the offshore platform

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The alternate approach to compute the wave forces is what we call random wave analysis. As the name spells, it depends on statistical methodologies. Statistical analysis is done on the bases of wave scatter diagram. For every offshore location, the wave scatter diagram is available in the literature. Appropriate wave spectra are defined to perform the analysis in frequency domain. Appropriate wave spectra are used to generate a random wave, if dynamic analyses for extreme wave are required for deepwater structures.


As we all understand, when you go for offshore structures in deeper waters because of their compliancy, because of the flexibility introduced in the design by geometry and structural design, dynamic effects on these structural members becomes predominantly important. With statistical methods, most probable maximum forces during the lifetime of the structure are calculated using what we call linear wave theory. Statistical approach is finding its importance in analyzing the fatigue strength of members and of course, to estimate the dynamic response of these platforms in the literature.

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Wave theories

- Wave theories describe wave kinematics (water particle velocity and acceleration) on the basis of potential theory
- Waves are assumed to be long-crested
- They can be described by a two-dimensional flow field and are characterized by the different parameters:
- wave height (H), wave period (T), water depth (d), wave number (k), circular frequency (ω) and cyclic frequency (f)

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Now the question comes both of these require an appropriate wave theory. Let us see a very brief idea in few slides about wave theories. Now what do wave theories tell me? Wave theories describe wave kinematics. What do understand by wave kinematics or water particles kinematics? Water particles kinematics are nothing but the velocity and acceleration variation along the depth, along the spatial values of x and y with respect to time. It is derived on the basis of potential theory. There are some assumptions made in this. Waves are assumed to be long-crested waves. They can be described by two-dimensional flow field and they are characterized by different parameters as given below: First and foremost parameters are wave height and wave period; of course, it depends on water depth, wave number, circular frequency, and cyclic frequency.

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Common wave theories

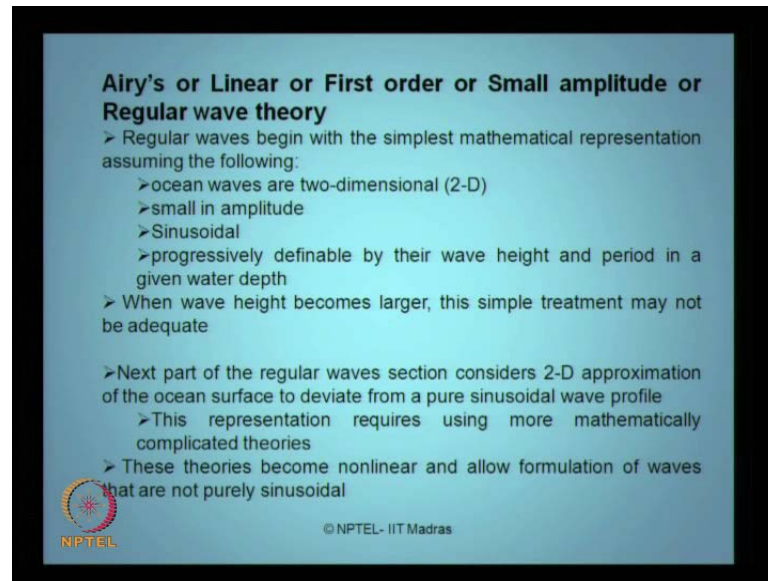
- i) linear Airy theory;
- ii) Stokes fifth-order theory;
- iii) Solitary wave theory;
- iv) Cnoidal theory;
- v) Dean's stream function theory; and
- vi) Numerical theory by Chappellear.

Definition of wave parameters

The diagram shows a sinusoidal wave profile. A horizontal dashed line represents the 'Still water level'. The vertical distance from the 'Mud line' (the bottom boundary) to the still water level is labeled 'd'. The vertical distance from the trough to the crest is labeled 'H'. The horizontal distance between two consecutive crests is labeled 'L = cT'. The wave celerity is labeled 'c = Wave celerity m/sec'. The NPTEL logo is visible in the bottom left corner, and the text '© NPTEL- IIT Madras' is at the bottom center.


There are many theories available in this literature to compute the wave forces. Let us see one of them immediately after this presentation now. Let us quickly list the list of theories available in literature. Linear wave theory, Stokes fifth-order wave theory, solitary wave theory, Cnoidal theory, Dean's stream function theory, and numerical theory given by Chappellear. All these theories have certain basic parameters which are predefined. So, the figure here shows you what are those parameters which are predefined, which is d as a water depth. I call this as may trough, this as may crest, this is my wavelength, and this is my wave celerity value, and this is my what I call still water level, and the distance between the vertical distance; between the trough and the crest is what I call as the wave height.

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Airy's or Linear or First order or Small amplitude or Regular wave theory

- Regular waves begin with the simplest mathematical representation assuming the following:
 - ocean waves are two-dimensional (2-D)
 - small in amplitude
 - Sinusoidal
 - progressively definable by their wave height and period in a given water depth
- When wave height becomes larger, this simple treatment may not be adequate
- Next part of the regular waves section considers 2-D approximation of the ocean surface to deviate from a pure sinusoidal wave profile
 - This representation requires using more mathematically complicated theories
 - These theories become nonlinear and allow formulation of waves that are not purely sinusoidal

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Let us quickly see very briefly the Airy's wave theory. Ladies and gentlemen Airy's wave theory has couple of analogous names given in the literature as linear wave theory or people address this as first order wave theory or some literature says this as small amplitude regular wave theory. Regular waves begin with the simplest mathematical representation assuming the following: Ocean waves or two-dimensional, the amplitude is very small, the profile is sinusoidal. It is progressively definable by their wave height and period in a given water depth. Now when the wave height becomes very large, the simple treatment may not be then adequate.

So, what is the next suggestion on this? The next part of the regular wave considers 2-D approximation of the ocean surface to deviate from the pure sinusoidal form which you have been assuming earlier. This representation requires more mathematically complicated theories; we will not look into that in this representation. These theories become nonlinear and allow formulation of waves as they are not purely sinusoidal. I will take up one example of Stokes fifth-order wave theory to explain you, how forces can be computed based on fifth order wave theory.

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Classification of water waves according to relative depth

Classification	d/L	kd	$\tanh(kd)$
Deep water	$\frac{1}{2}$ to ∞	π to ∞	~ 1
Transitional	$(1/20)$ to $\frac{1}{2}$	$(\pi/10)$ to π	$\tanh(kd)$
Shallow water	0 to $(1/20)$	0 to $(\pi/10)$	$\sim kd$

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If you look at this figure, this table gives you interesting information about classification of water waves according to what I call relative depth. You may wonder, what is relative depth? The relative depth is a relationship between water depth and wavelength what I called d by L ratio. So based on this, I can easily classify waves as waves deep water, transitional, and shallow water waves. I am not reading the numbers available on the table. You can read them and see; what are the lower and upper limits of d by L which I call relative depth value to classify water waves as deep water, transitional, or shallow waters. The most important part in this table which I want to focus you is that, the hyperbolic relationship of kd where k is the wave number and d is the water depth.

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Relative Depth	Shallow Water $\frac{d}{L} < \frac{1}{10}$ $kd < \frac{\pi}{10}$	Transitional Water $\frac{1}{10} < \frac{d}{L} < \frac{1}{3}$ $\frac{\pi}{10} < kd < \frac{\pi}{3}$	Deep Water $\frac{d}{L} > \frac{1}{3}$ $kd > \frac{\pi}{3}$
1. Wave profile	near A_0 -	$\eta = \frac{H}{2} \sin \left[\frac{2\pi x}{L} - \frac{2\pi t}{T} \right] + \frac{H}{2} \cos \theta$	- near A_0
2. Wave celerity	$C = \frac{L}{T} = \sqrt{gd}$	$C = \frac{L}{T} = \frac{gT}{2\pi} \tanh \left(\frac{2\pi d}{L} \right)$	$C = C_0 = \frac{L}{T} = \frac{gT}{2\pi}$
3. Wavelength	$L = T\sqrt{gd} = CT$	$L = \frac{gT^2}{2\pi} \tanh \left(\frac{2\pi d}{L} \right)$	$L = L_0 = \frac{gT^2}{2\pi} = C_0 T$
4. Group celerity	$C_g = C = \sqrt{gd}$	$C_g = cC = \frac{1}{2} \left[1 + \frac{4\pi d/L}{\sinh(4\pi d/L)} \right] C$	$C_g = \frac{1}{2} C = \frac{gT}{4\pi}$
5. Wave particle velocity			
(a) Horizontal	$u = \frac{H}{2} \sqrt{\frac{g}{d}} \cos \theta$	$u = \frac{H}{2} \frac{gT}{L} \frac{\cosh[2\pi(x+dy)/L]}{\cosh(2\pi d/L)} \cos \theta$	$u = \frac{H}{2} \frac{gT}{L} e^{k(y)} \cos \theta$
(b) Vertical	$w = \frac{H\pi}{T} \left(1 + \frac{z}{d} \right) \sin \theta$	$w = \frac{H}{2} \frac{gT}{L} \frac{\sinh[2\pi(x+dy)/L]}{\cosh(2\pi d/L)} \sin \theta$	$w = \frac{H}{2} \frac{gT}{L} e^{k(y)} \sin \theta$
6. Wave particle acceleration			
(a) Horizontal	$a_x = \frac{H\pi}{T} \sqrt{\frac{g}{d}} \sin \theta$	$a_x = \frac{gH}{L} \frac{\cosh[2\pi(x+dy)/L]}{\cosh(2\pi d/L)} \sin \theta$	$a_x = 2H \left(\frac{\pi}{T} \right)^2 e^{k(y)} \sin \theta$
(b) Vertical	$a_z = -2H \left(\frac{\pi}{T} \right)^2 \left(1 + \frac{z}{d} \right) \cos \theta$	$a_z = -\frac{gH}{L} \frac{\sinh[2\pi(x+dy)/L]}{\cosh(2\pi d/L)} \cos \theta$	$a_z = -2H \left(\frac{\pi}{T} \right)^2 e^{k(y)} \cos \theta$
7. Wave particle displacement			
(a) Horizontal	$\xi = -\frac{HT}{2\pi} \sqrt{\frac{g}{d}} \sin \theta$	$\xi = -\frac{H}{2} \frac{\cosh[2\pi(x+dy)/L]}{\sinh(2\pi d/L)} \sin \theta$	$\xi = -\frac{H}{2} \frac{gT}{L} e^{k(y)} \sin \theta$
(b) Vertical	$\zeta = \frac{HT}{2} \left(1 + \frac{z}{d} \right) \cos \theta$	$\zeta = \frac{H}{2} \frac{\sinh[2\pi(x+dy)/L]}{\sinh(2\pi d/L)} \cos \theta$	$\zeta = \frac{H}{2} \frac{gT}{L} e^{k(y)} \cos \theta$
8. Surface pressure	$p = \rho g \eta = 0$	$p = \rho g \eta \frac{\cosh[2\pi(x+dy)/L]}{\cosh(2\pi d/L)} - \rho g z$	$p = \rho g \eta e^{k(y)} - \rho g z$

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Summary of linear wave theory characteristics

These are summary of linear wave theory characteristics available in the literature. These are available in all references given to me in the beginning of this course. Just for the completion sake, I made a comprehensive table for you to easily calculate the desired parameters based on wave theories. I have no interest to read these values for you. You can always use this; these are standard literature available in the references given by me at the end of this lecture.

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Water particle kinematics

- Ocean surface waves are generated at any offshore site by the drag of wind on the water surface
- It is necessary to relate the surface wave data to water particle velocity, acceleration, and pressure beneath the waves, which can be achieved by using the appropriate wave theory (Dawson, 1983)
- Airy (1842) presented a relatively simple theory of wave motion
 - He assumed a sinusoidal wave form whose height 'H' is small in comparison with the wave length 'λ' and the water depth 'd'
 - Although not strictly applicable to typical design waves used in offshore structural engineering, the Airy's theory is valuable for preliminary calculations

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Let us focus now on what we call water particle kinematics. As we understand, the moment I say water particle kinematics, I must lead towards computation of velocity and acceleration variation along the depth, along the spatial interval x and y and to the respective time as well. So, ocean surfaces are generated at any offshore side by the drag of wind on the water surface. It is therefore necessary to relate the surface data to water particular velocity acceleration and pressure beneath the waves. This is given by Dawson, 1983, what we call appropriate wave theory.

Airy's theory was proposed in 1842, which presented relatively a simple theory of wave motion. Here we assume a sinusoidal waveform whose wave height is H and which is small in comparison to the wavelength λ and the water depth d . Although not strictly applicable to typical design waves used in offshore structural engineering, Airy's theory is very good for preliminary design calculations.

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Sea surface elevation

$$\eta(x, t) = \frac{H}{2} \cos(kx - \omega t) \quad \text{where} \quad k = \frac{2\pi}{\lambda} \quad \omega = \frac{2\pi}{T}$$

The horizontal and vertical particle velocities at any location (X, Y) and time t are given by:


$$\dot{u}(x, t) = \frac{\omega H \cosh ky}{2 \sinh kd} \cos(kx - \omega t)$$

$$\dot{v}(x, t) = \frac{\omega H \sinh ky}{2 \sinh kd} \sin(kx - \omega t)$$

The horizontal and vertical particle accelerations at any location (x, y) and time t are given by:

$$\ddot{u}(x, t) = -\frac{\omega^2 H \cosh ky}{2 \sinh kd} \sin(kx - \omega t)$$

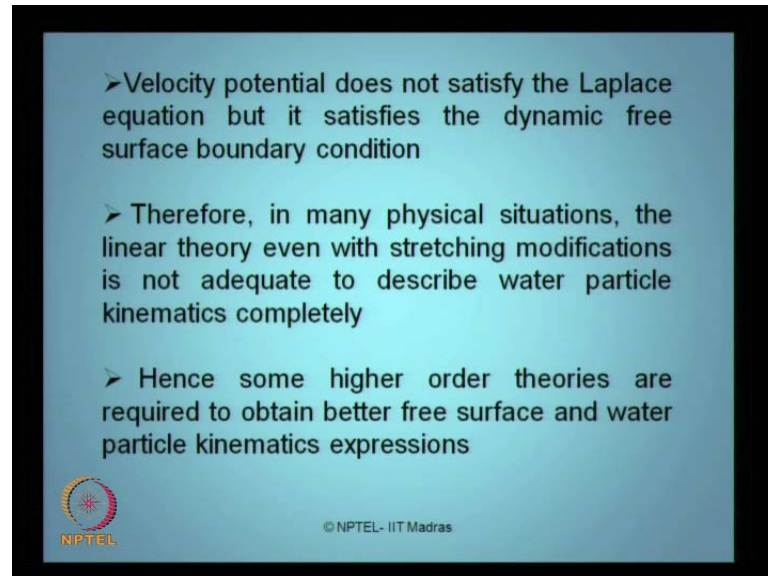
$$\ddot{v}(x, t) = \frac{\omega^2 H \sinh ky}{2 \sinh kd} \cos(kx - \omega t)$$


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I will quickly look at some of the equations given by Airy's theory for the handy calculations of the listeners. The sea surface elevation η is function of x and t where x is measured along the wave profile direction or the direction of wave propagation and t is of course any instantaneous time. The horizontal water particle velocity and the vertical water particle velocity at any special location X and Y which is also a function of time is available in these two equations. Similarly, the horizontal and vertical water particle

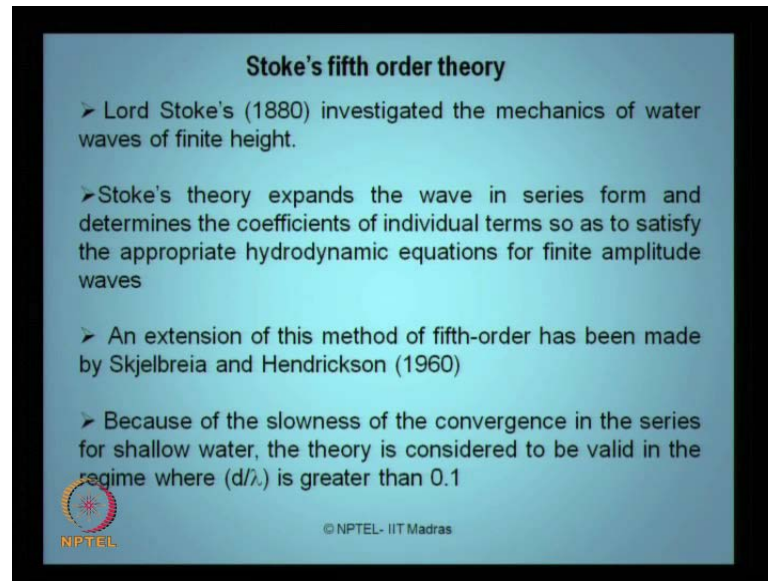
accelerations at any spatial location x and y and at any time t are given by these two equations.

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Once I know the water particle kinematics, then I can proceed further to compute the forces. The velocity potential does not satisfy the Laplace equation as per the earlier theory, but it satisfies the dynamic free surface boundary conditions. Therefore, in many physical situations, the linear theory, even the stretching modifications given by Hodgman, Wheeler, etc cannot be found to be adequate to describe the water particle kinematics completely. Hence some higher order theories are proposed by the researchers which have been used for such situations.

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Stoke's fifth order theory

- Lord Stoke's (1880) investigated the mechanics of water waves of finite height.
- Stoke's theory expands the wave in series form and determines the coefficients of individual terms so as to satisfy the appropriate hydrodynamic equations for finite amplitude waves
- An extension of this method of fifth-order has been made by Skjelbreia and Hendrickson (1960)
- Because of the slowness of the convergence in the series for shallow water, the theory is considered to be valid in the regime where (d/λ) is greater than 0.1

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Let us quickly see one of such higher order theory proposed in the literature which is called Stoke's fifth-order wave theory. Lord Stoke's in 1880 investigated the mechanics of water waves with finite height. Stoke's theory actually expands the wave in series form and determines the coefficients of individual terms, so as to satisfy the appropriate hydrodynamics equations for finite amplitude waves. An extension on this method of the fifth-order was been made by Skjelbreia and Hendrickson in 1960. Because of the slowness of convergence in the series for shallow water, the theory is considered to be valid in the regime of d by λ which is greater than 0.1, where d is water depth and λ is the wavelength.

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According to Stoke's fifth-order nonlinear wave theory, the sea surface elevation is given by:

$$\eta(x, t) = \frac{1}{k} \sum_{n=1}^5 F_n \cos[n(kx - \omega t)]$$
$$F_1 = a \quad F_2 = a^2 B_{22} + a^4 B_{24} \quad F_3 = a^3 B_{33} + a^5 B_{35}$$
$$F_4 = a^4 B_{44} \quad F_5 = a^5 B_{55}$$

Constants denoting wave profile parameter vis-à-vis B₂₂, B₂₄, etc. depend on the value (kd) and wave height parameter, 'a' which is obtained from the following equation:

$$\frac{kH}{2} = \left[a + a^3 B_{33} + a^5 (B_{35} + B_{55}) \right]$$


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Now according to Stoke's fifth-order nonlinear wave theory, the sea surface elevation is interestingly given by a sum of series. You can see here, the number n which is indicated here as well as here keeps on varying from n is equal to 1 to 5. It is because of this number upper bound of 5, this is called Stoke's fifth-order nonlinear wave theory. You may wonder why it is nonlinear; the summation results in a nonlinear plot of the sea surface profile. Now in this equation, if you want to compute the sea surface elevation x at a spatial time t, then I must have the coefficients of F₁, F₂, F₃, up to F₅ which are available here.

Now interestingly if you look at F₁, F₂ and so on, these are all functions of other constants as a, B₂₂, B₂₄ and so on; therefore, we must know what are these constants to compute these forces F₁, F₂, F₃, etc. The constants denoting wave profile parameter are given below; but however, in this equation a is what is called wave height parameter. Remember ladies and gentlemen, a is not the wave height, h is the wave height; anyway a is parameter related to wave height. So, H is the wave height, k is the wave number. If you know the constants B₃₃, B₃₅, and B₅₅, I can solve this equation to compute the so called wave height parameter a. Now the question comes, how do we estimate B₃₃ and so on because they are not only required here but also required here to compute the coefficients F₁ to F₅.

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Constants for Stoke's fifth-order wave theory as presented by Dawson (1983) and Patel (1989) are given below:

Where arithmetic constants are:

$$s = \sinh\left(\frac{2\pi d}{\lambda}\right) \quad c = \cosh\left(\frac{2\pi d}{\lambda}\right)$$

$$B_{22} = \frac{(2c^2 + 1)}{4s^3} c$$


$$B_{21} = \frac{c(272c^8 - 504c^6 - 192c^4 + 322c^2 + 21)}{384s^9}$$

$$B_{33} = \frac{3(8c^6 + 1)}{64s^6}$$

$$B_{35} = \frac{(88128c^{14} - 208244c^{12} + 70848c^{10} + 54000c^8 - 21816c^6) + (6264c^4 - 54c^2 - 81)}{12288s^{12}(6c^2 - 1)} + \frac{(6264c^4 - 54c^2 - 81)}{12288s^{12}(6c^2 - 1)}$$

$$B_{44} = \frac{c(768c^{10} - 448c^8 - 48c^6 + 48c^4 + 106c^2 - 21)}{384s^9(6c^2 - 1)}$$

$$\frac{(19200c^{16} - 262720c^{14} + 83680c^{12} + 20160c^{10} - 7280c^8)}{12288s^{10}(6c^2 - 1)(8c^4 - 11c^2 + 3)} + \frac{(7160c^6 - 1800c^4 - 1050c^2 + 225)}{12288s^{10}(6c^2 - 1)(8c^4 - 11c^2 + 3)}$$

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Now the coefficients of B 22 and so on are available in Dawson, 1983 and Patel, 1989, which are reproduced here for your ready reference. Now we look at these constant B 22 and so on varying till B 55, you will find there are some values of c and s available in this computations; c stands for cos hyperbolic 2 pi d by lambda, and s stands for sine hyperbolic relationship 2 pi d by lambda, which I call them as arithmetic constants in this coefficients of B II which is given by Dawson and Patel in these references.

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Horizontal and vertical particle velocities are given by:


$$\hat{u}(x, t) = \frac{\omega}{k} \sum_{n=1}^5 G_n \frac{\cosh(nky)}{\sinh(nkd)} \cos[n(kx - \omega t)]$$

$$\hat{v}(x, t) = \frac{\omega}{k} \sum_{n=1}^5 G_n \frac{\sinh(nky)}{\sinh(nkd)} \sin[n(kx - \omega t)]$$

$$G_1 = aG_{11} + a^3G_{13} + a^5G_{15} \quad G_2 = 2(a^2G_{22} + a^4G_{24})$$

$$G_3 = 3(a^3G_{33} + a^5G_{35}) \quad G_4 = 4(a^4G_{44})$$

$$G_5 = 5(a^5G_{55})$$

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Once I know these constants, I know the sea surface elevation η of t ; then I can now compute what I call the horizontal and vertical water particle velocities. Now to compute the horizontal and vertical water particle velocities which is interestingly required for the computing the forces on the members; again use a series which is varying from $n = 1$ to 5 that is why they call this as Stoke's fifth-order nonlinear wave theory, which depends on the wave number and the hyperbolic relationship of cos and sine and of course, there is a number n here and there are constants G_n which are coming in the play here. Now the G_n varying from 1 to 5 as shown here $G_1, G_2, G_3, G_4,$ and G_5 again depends on a , which you have computed in the earlier slide and also depends on further coefficient G_{11}, G_{13} and so on which we are now discuss the next slide

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Wave Velocity Parameters

$$G_{11} = A_{11} \sinh(kd)$$

$$G_{13} = A_{13} \sinh(kd)$$

$$G_{15} = A_{15} \sinh(kd)$$

$$G_{22} = A_{22} \sinh(2kd)$$

$$G_{24} = A_{24} \sinh(2kd)$$

$$G_{33} = A_{33} \sinh(3kd)$$

$$G_{35} = A_{35} \sinh(5kd)$$

$$G_{44} = A_{44} \sinh(4kd)$$

$$G_{55} = A_{55} \sinh(5kd)$$

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Now these are called wave velocity parameters. Again they depend on the sine hyperbolic relationship of kd which are available here. Depending upon the coefficients, they are also multiplied with $2, 3, 4,$ and 5 . They also depend on the further coefficients called A_{11}, A_{13} ; ladies and gentleman the earlier a what you saw which is wave height parameter is a small a , whereas here these are capital A 's which are coefficients.

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$$A_{11} = \frac{1}{s}$$

$$A_{13} = \frac{-c^2(5c^2 + 1)}{8s^5}$$

$$A_{15} = \frac{-(1184c^{10} - 1440c^8 - 1992c^6 + 2641c^4 - 249c^2 + 18)}{1536s^{11}}$$

$$A_{22} = \frac{3}{8s^4}$$

$$A_{24} = \frac{(192c^8 - 424c^6 - 312c^4 + 480c^2 - 17)}{768s^{10}}$$

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Now how to estimate this? These are again available in further constants as given in this slide which is the function of s in c which are called arithmetic constants explained in the previous slide.

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The expressions for accelerations are given below:

$$\ddot{u}(x, t) = \frac{kc_s^2}{2} \sum_{n=1}^5 R_n \sin n(kx - \omega t)$$

$$\ddot{v}(x, t) = \frac{-kc_s^2}{2} \sum_{n=1}^5 S_n \cos n(kx - \omega t)$$

Wave speed c_s is given by the following equation:

$$c_s = \left[\frac{g}{k} (1 + a^2 C_1 + a^4 C_2) \tanh kd \right]^{1/2}$$

$$C_1 = \frac{(8c^4 - 8c^2 + 9)}{8s^4}$$

$$C_2 = \frac{(3840c^{12} - 4096c^{10} + 2592c^8 - 1008c^6 + 5944c^4 - 1830c^2 + 147)}{512s^{10}(6c^2 - 1)}$$

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Now the expressions for acceleration are now available as u double dot and v double dot, which are respectively the horizontal and vertical water particle acceleration. Then the water particle acceleration again is the function of n; n varies from 1 to 5 and of course c s is a order what I called wave speed is given with this equation. Wave speed depends on

wave number as well as constants a , which is a wave height parameters which you computed in the last slide. Now the constant C_1 and C_2 which are required to compute the wave speed C_s is available below, which is also function of c and s which are nothing but the arithmetic constants explained in the previous slide. Now if I know the C_s value as computed from here, for any special variation x and t , you can compute u double dot and v double dot; provided I know the values of R suffix n and S suffix n .

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Acceleration Coefficients

$$R_1 = 2U_1 - U_1U_2 - V_1V_2 - U_2U_3 - V_2V_3$$

$$R_2 = 4U_2 - U_1^2 + V_1^2 - 2U_1U_3 - 2V_1V_3$$

$$R_3 = 6U_3 - 3U_1U_2 + 3V_1V_2 - 3U_1U_4 - 3V_1V_4$$

$$R_4 = 8U_4 - 2U_2^2 + 2V_2^2 - 4U_1U_3 + 4V_1V_3$$

$$R_5 = 10U_5 - 5U_1U_4 - 5U_2U_3 + 5V_1V_4 + 5V_2V_3$$

$$S_1 = 2V_1 - 3U_1V_2 - 3U_2V_1 - 5U_2V_3 - 5U_3V_2$$

$$S_2 = 4V_2 - 4U_1V_3 - 4U_3V_1$$

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Now R suffix n and S suffix n are available in this equation what we call them as acceleration coefficients. So ladies and gentlemen, as you see Stoke's fifth-order theory extends its complication in terms of arriving at the constants which are required to estimate the wave profile parameters.

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$$S_3 = 6V_3 - U_1V_2 - U_2V_1 - 5U_1V_4 - 5U_4V_1$$
$$S_4 = 8V_4 - 2U_1V_3 + 2U_3V_1 + 4U_2V_2$$
$$S_5 = 10V_5 - 3U_1V_4 - 3U_4V_1 - U_2V_3 + U_3V_2$$
$$U_n = G_n \frac{\cosh(nky)}{\sinh(nkd)}$$
$$V_n = G_n \frac{\sinh(nky)}{\sinh(nkd)}$$

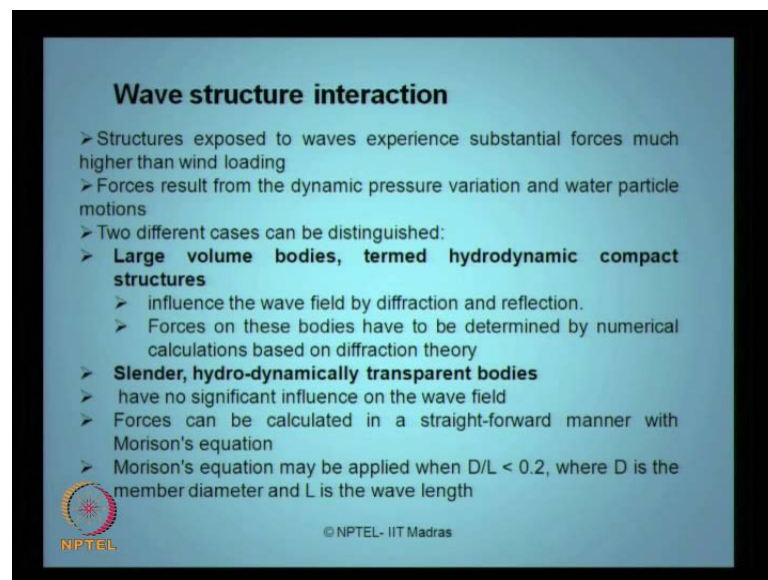
Once I know these constants, I compute; because these constants against depends on what we call V_n 's and U_n 's, where as U_n and V_n are functions of G_n 's which have been explained in this previous slide. And now if I know U_n and V_n , keep on substituting them I get these acceleration constants. Once I get these acceleration constants, now I am very clear to understand how to estimate my sea surface elevation $\eta(x, t)$ as a series of five waves which varying 1 to 5; the n number varying 1 to 5. I also know how to compute the water particle velocity in the horizontal and vertical direction. I also know how to compute the water particles acceleration in the horizontal and vertical direction.

Ladies and gentlemen, we all now strongly understand that it depends on 3 parameters. The spatial distance of x which is along the direction of propagation of wave, depends on the point y which is measured along the water depth and also a spatial time interval t . Of course Stoke's fifth-order theory is not as simple as you saw in Airy's wave theory, because there is a linear wave theory; whereas Stoke's fifth-order is highly nonlinear wave theory. Therefore, it depends on lot of constants which are to be used for computing the sea surface profile on the water particles kinematics.

Once I know from these two set of theories like Airy's wave theory or Stoke's fifth-order; for example, there are many other theories available in the literature. You can use appropriate wave theory to compute the forces on a platform. I have given two examples

of computing this; using one, what we call linear wave theory which is Airy's wave theory. The other one is Stoke's fifth-order. Now the question comes after you know how to compute this water particles kinematics based on these theories, how do re-proceed further to compute the forces.

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Wave structure interaction

- Structures exposed to waves experience substantial forces much higher than wind loading
- Forces result from the dynamic pressure variation and water particle motions
- Two different cases can be distinguished:
 - **Large volume bodies, termed hydrodynamic compact structures**
 - influence the wave field by diffraction and reflection.
 - Forces on these bodies have to be determined by numerical calculations based on diffraction theory
 - **Slender, hydro-dynamically transparent bodies**
 - have no significant influence on the wave field
 - Forces can be calculated in a straight-forward manner with Morison's equation
 - Morison's equation may be applied when $D/L < 0.2$, where D is the member diameter and L is the wave length

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Now we will speak about what we call wave structure interaction. The structures exposed to waves experience substantial forces which are much higher in magnitude compared to the wind forces. These forces result from dynamic pressure variations and water particles motions. Two different cases can be distinguished; one is what we call a large volume bodies, other is what we call slender, hydro-dynamically transparent bodies. How are they different in terms of wave structure interaction? It is very interesting to know, large volume bodies they are termed as hydrodynamic compact structures. The influence of the wave field by diffraction and reflection will be accounted in such cases.

Forces on these bodies have to be determined by numerical calculation based on diffraction theory. Whereas if you have a slender body which is hydro-dynamically transparent, then they have no significant influence on the wave field because the wave calmly passes through this body. Forces can be computed in a straight-forward manner using a very classical equation is given by Morison which is Morison's equations, but unfortunately Morison's equation has a very serious limitation. Morison's equation can

be applied only when you have a D by L ratio less than 0.2, where capital D stands for the member diameter and L stands for the wavelength. Now interestingly if you have an offshore structural member whose D by L is less than 0.2 can, you can use the compact equation given by Morison's equation to compute the forces on slender, hydro-dynamically transparent bodies.

On the other hand from the slide, you can easily conclude that if the member whose D by L exceeds 0.2; if a member whose D by L exceeds points two, where D is the diameter of the member and L is the wavelength. If this ratio exceeds 0.2, then they are called large volume bodies; you must use diffraction theory to compute the forces. If fortunately, I should say fortunately, because Morison's equation gives you a close form solution to compute the forces on these members, which is less complicated and less numerically tiresome compared to that of diffraction theories. So, if you have a D by L value of any given member on offshore platform which is less than 0.2, I identify this as hydro-dynamically transparent body; in term it is called as slender bodies.

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> Steel jackets structures are regarded as hydro-dynamically transparent
 > Wave forces on the submerged members can therefore be calculated by Morison's equation

$$F = \frac{1}{2} \rho C_d D \dot{u} |\dot{u}| + \frac{\pi D^2}{4} \rho C_m \ddot{u}$$

> where, F is the wave force per unit length on a circular cylinder
 > (\dot{u}, \ddot{u}) are water particle velocity and acceleration normal to the cylinder
 > water particle kinematics are computed with the selected wave theory at the cylinder axis
 > ρ is the density of sea water
 > D is the member diameter including marine growth
 > C_d, C_m are drag and inertia coefficients respectively

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For example, steel jackets structures are regarded as hydro-dynamically transparent members because they have D by L value much lower than 0.2. Wave forces on these members when they are submerged can therefore be estimated calculated using Morison's equation; using very closed form simple equation as shown here. It depends

on C_d and C_m ; C_d is what we call a drag coefficient; C_m is what we called inertia coefficient.

It depends on \dot{u} and \ddot{u} , which are horizontal water particles velocity and horizontal water particles acceleration. Ladies and gentleman now will appreciate the necessity for computing the water particle kinematics, if you want to really estimate the forces on the members; because force on the member depends on water particle kinematics. As you see here, D of course the diameter of the member, you can easily compute force per unit length on a circular cylinder whose diameter is D using this classical equation.

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>For compliant structures like TLPs, forces due to relative motion is given by:

$$F = \frac{1}{2} \rho C_d D (\dot{u} - \dot{x}) |\dot{u} - \dot{x}| + \frac{\pi D^2}{4} \rho C_m \ddot{u}$$

>Values of C_d and C_m depend on the wave theory used, surface roughness and the flow parameters
 >According to API-RP2A, $C_d \sim 0.6$ to 1.2 and $C_m \sim 1.3$ to 2.0 . Additional information can be found in the DNV rules.

>Total wave force on each member is obtained by numerical integration over the length of the member.
 >Fluid velocities and accelerations at the integration points are found by direct application of the selected wave theory.

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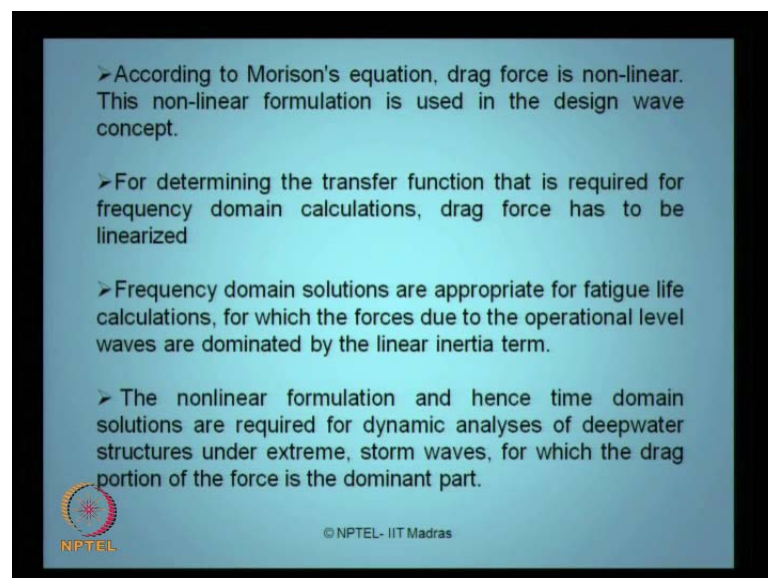
Now in case of compliant structures, since the structure is also going to have a relative motion with respect to the wave action. Then in that case, you cannot use the existing equation which is to be slightly modified, incorporating the relative velocity term in the equation. So, it is nothing but half $\rho C_d D$; whereas it is not \dot{u} , it is $\dot{u} - \dot{x}$, where \dot{u} is the horizontal water particle velocity; whereas \dot{x} is horizontal structural velocity given by the member.

So, I am looking for relative a velocity term now here and this term become nonlinear. So that is why we call, since it is associated with the drag force, we called this as nonlinear drag term. Now the values of C_d and C_m in both the above equations depend on the wave theory what you are using. It also depends on the surface roughness and the

flow parameters. There are international code guidelines given; C_d can be varying anywhere from 0.6 to 1.2, whereas C_m can vary anywhere from 1.3 to 2.0.

If you really want to know more addition information on how to compute the drag and the inertia coefficients for a given sea state, you can refer to standard DNV rules available in the literature. Now the total wave force on each member is thus obtained with the numerical integration over the length of the entire member. The fluid velocity and acceleration at the integration points are found by direct application of the selected wave theory.

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A slide with a light blue background and a black border. It contains four bullet points in black text. At the bottom left is the NPTEL logo, and at the bottom center is the copyright notice '© NPTEL- IIT Madras'.

- According to Morison's equation, drag force is non-linear. This non-linear formulation is used in the design wave concept.
- For determining the transfer function that is required for frequency domain calculations, drag force has to be linearized
- Frequency domain solutions are appropriate for fatigue life calculations, for which the forces due to the operational level waves are dominated by the linear inertia term.
- The nonlinear formulation and hence time domain solutions are required for dynamic analyses of deepwater structures under extreme, storm waves, for which the drag portion of the force is the dominant part.

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According to Morison's equation, the drag force is nonlinear. This nonlinear formulation is used in the design concept. For determining the transfer function that is required for frequency domain calculations, the drag force has got to be essentially linearized. Therefore, frequency domain solutions are appropriate for fatigue life calculations, for which the forces due to the operational level waves are determined by linear inertia term. The nonlinear formation and hence time domain solutions are required for dynamic analysis of deep-water structures under extreme, storm waves, for which the drag portion of the forces is the dominant part in the calculation.

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➤ In addition to Morison's forces, lift forces and slamming forces can be important for local member design.

$$F_L = (1/2) \rho C_L Dv^2$$
$$F_S = (1/2) \rho C_S Dv^2$$

➤ where C_L , C_S are the lift and slamming coefficients respectively

➤ Lift forces are perpendicular to the member axis and the fluid velocity

➤ related to the vortex shedding frequency

➤ Slamming forces acting on the underside of horizontal members near the mean water level are impulsive and nearly vertical.

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Now in addition to the Morison's force what you saw, you also have forces which are arising from the lift and slamming constant which is F_L and F_S as given by these equations. The lift force essentially is perpendicular to the member axis and the fluid velocity is related to the vortex shedding frequency; where the slamming forces acts on the underside of the horizontal members near the mean water level and which are impulsive and nearly vertical.

Thanks.