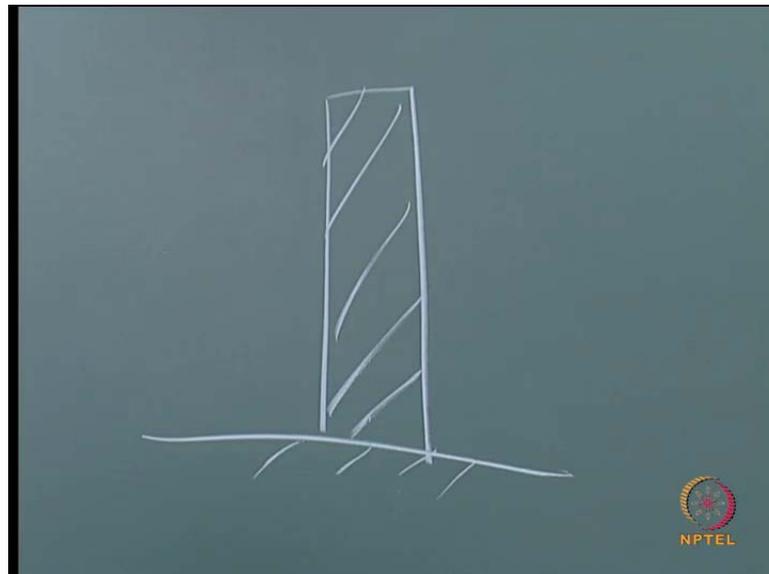


Wave Hydro Dynamics
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Module No. # 02
Wave Motion and Linear Wave Theory
Lecture No. # 06
Standing Wave Theory

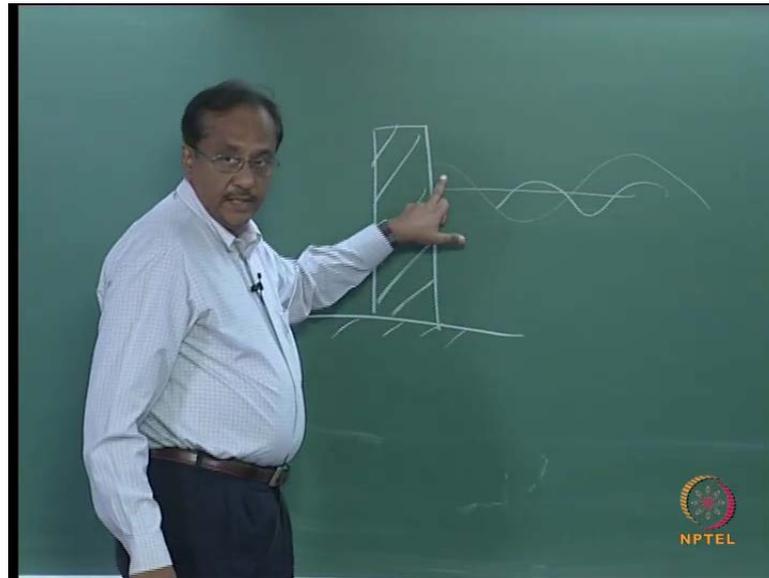
We have already seen the characteristics of progressive waves. So, progressive waves are the most important type of waves, which we normally deal with, for example, in order to calculate the forces acting on structures or behaviour of the ocean waves as they propagate from the deep ocean to the coastal area. So, what exactly is a standing wave? Standing wave is a kind of waves, which are formed due to pure reflection; do we have these kinds of waves in the ocean? **Yes**, we do have these kinds of waves in the ocean.

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Particularly, think of structures **which are** which are **impermeable** and I mean impermeable and vertical.

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So, in which case, you see that when the waves are propagating, hits the structure and the entire wave gets reflected back. It is to be said that, the forces exerted by reflected waves are much more than non reflected waves, reflection is it good? **No**, it is not good particularly **in the...** As you, if you consider reflection, later you will see reflection is not good for the simple reason the forces, the velocities, the pressures, if everything increases in the vicinity of the structure.

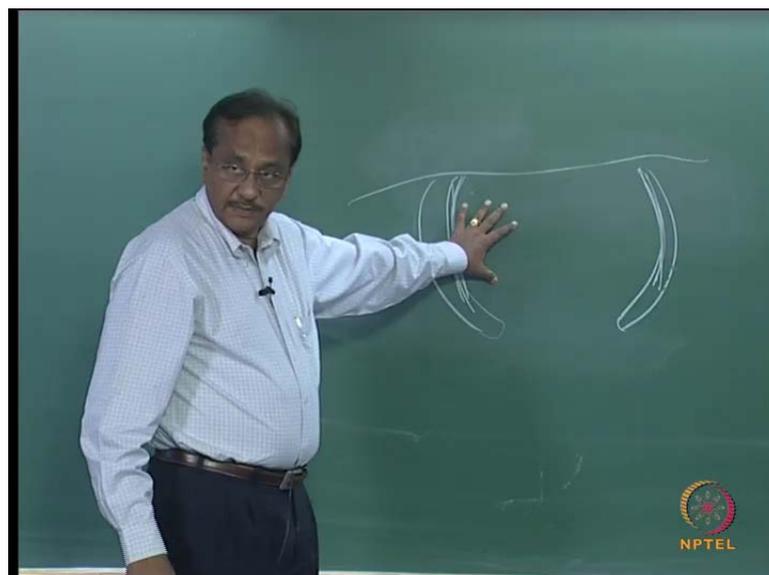
The pressures exerted on a vertical wall will be much more compared to if it is a sloping wall or if it is a permeable wall, the force acting on the structure is expected to be much less compared to an impermeable wall. So, from the force or pressure's point of view reflected waves are not good, but there are situations, where you cannot avoid reflected waves.

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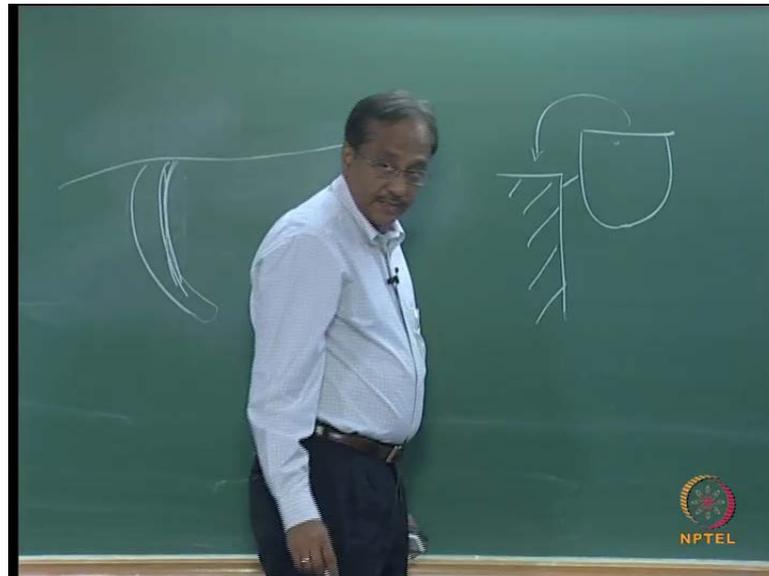
The other aspect is when you consider harbors, etcetera. If you have a reflected wave inside the harbor, the amplification of the wave climate inside the harbor due to reflection increases. So, what happens, if the amplification of the waves inside the harbor takes place, then that is going to be creating lot of confusion, lot of problems. So, actually you have break waters or any kind of structures in order to serve as a sheltered area for vessels.

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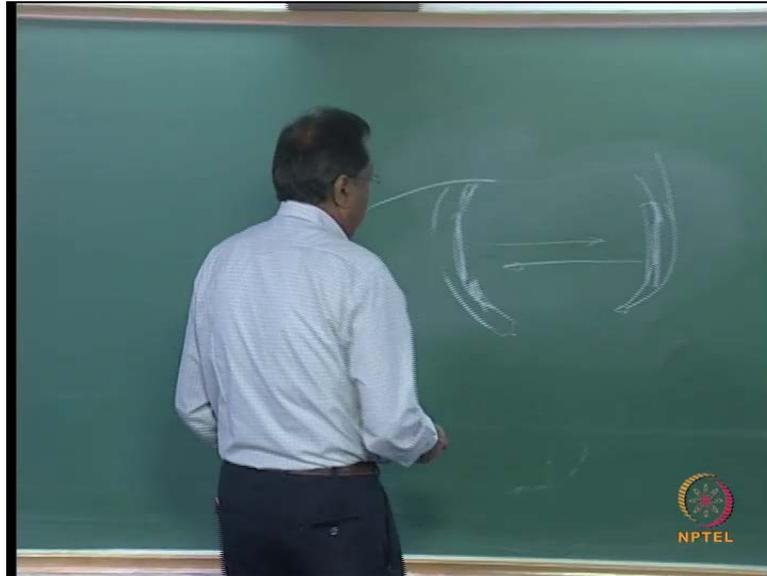
But, if you design structures or the obstructions in such a way they have vertical walls and impermeable walls, then you will have problems, you have to be very careful while designing such structures.

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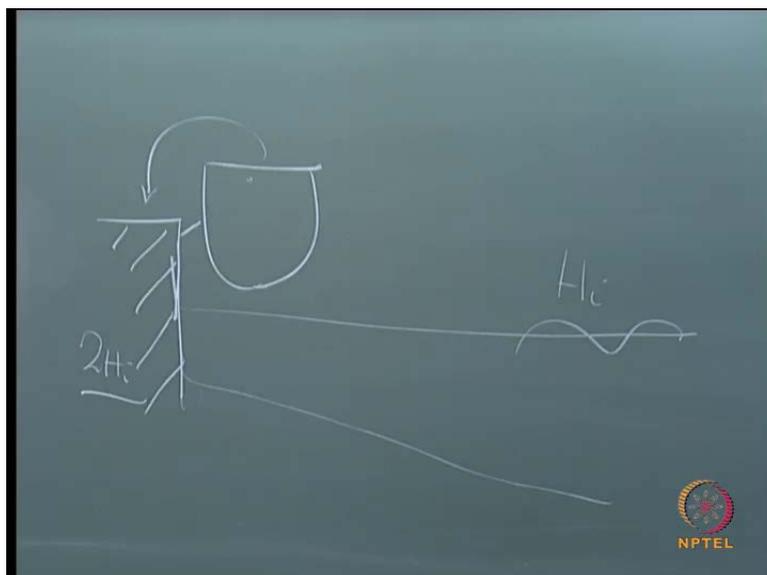
The one typical structure is when you want, **you can have** you should have something like this inside the harbor for berthing of vessels. You berth the vessel, so you can off load or on load passengers or goods. So, in that way you need to, you will be forced to have vertical structures; whether it is desirable or not from **from** first point of view, it is not desirable. And also from reflection point of view it is not desirable, but still you will be forced to go in for vertical type of structures, which is going to lead to reflection.

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So, whatever waves coming from one end of the structure, if this there is a vertical structure here, that wave will travel and if we have this side another vertical structure, this wave will travel like this. So, for example, in an enclosed basin for typical example is like this, you have a berth here, and also you have a berth here; what can happen, could happen is a wave going and hitting this, because **this is a perpendicular, I mean** this is a vertical structure **(O)** going to get reflected. So, this oscillation is going to generate standing waves.

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Later you will see that, when you have a vertical structure like this, and when the waves are propagating from the deep water, if this is H the wave height, the wave height **in the** on the wall will be $2H$ which we will try to prove. So, from basic propagating waves, we have **have** already seen progressive waves, we have already seen that the pressures, etcetera will be pressures or the particle velocities are going to be a function of H and it is going to increase. Now, you see the effect of vertical wall. So, hence it is extremely important to understand the basic physics and hydrodynamics of standing waves.

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Different forms of ϕ

We have earlier seen the different forms of ϕ

$$\phi_1 = \frac{-ag}{\sigma} \cdot \frac{\cosh k(d+z)}{\cosh kd} \cdot \cos kx \cdot \cos \sigma t$$

$$\phi_2 = \frac{ag}{\sigma} \cdot \frac{\cosh k(d+z)}{\cosh kd} \cdot \sin kx \cdot \sin \sigma t$$

$$\phi_3 = \frac{-ag}{\sigma} \cdot \frac{\cosh k(d+z)}{\cosh kd} \cdot \sin kx \cdot \cos \sigma t$$

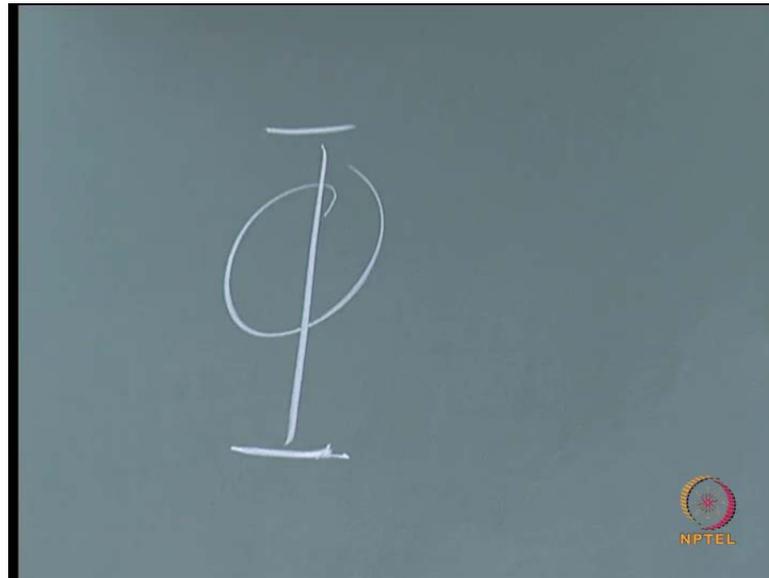
$$\phi_4 = \frac{ag}{\sigma} \cdot \frac{\cosh k(d+z)}{\cosh kd} \cdot \cos kx \cdot \sin \sigma t$$

For complete details: Dean, R.G and Dalrymple, R.A. "Water Wave: Mechanics for Engineers and Scientists, Prentice Hall Inc, 1984.

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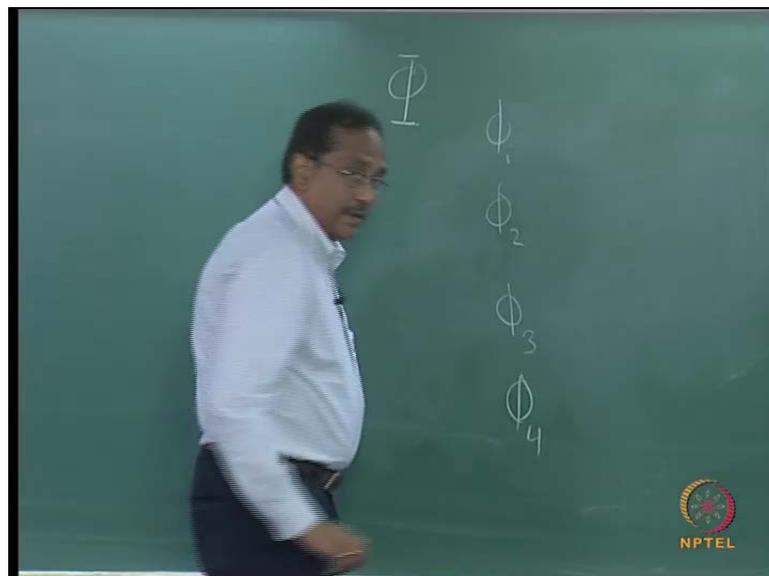
You should recollect that when we **when we** try to obtain the velocity potential.

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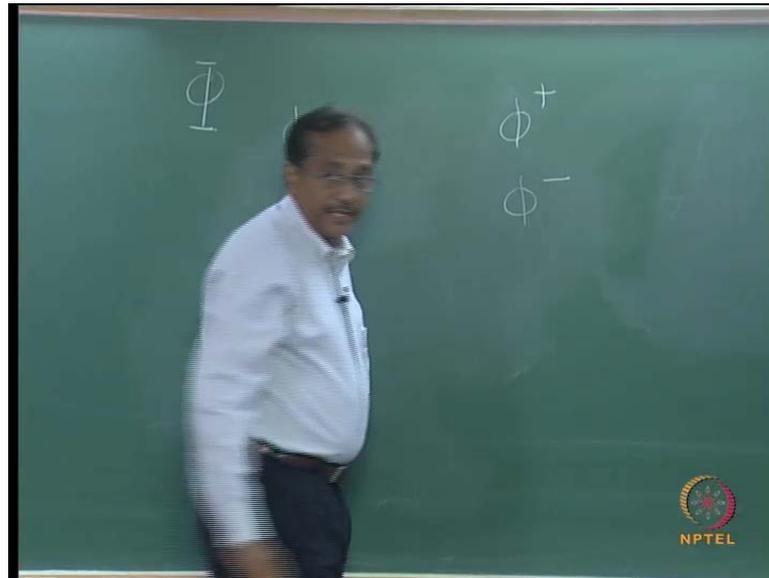
When, we try to obtain the velocity potential phi.

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Then we dealt with the progressive waves, we landed up with four forms of equations, phi 1, phi 2, phi 3 and phi 4. You should check the previous lecture or the lecture material **what we** what you have in hand, for all this **five** four forms of solutions for the velocity.

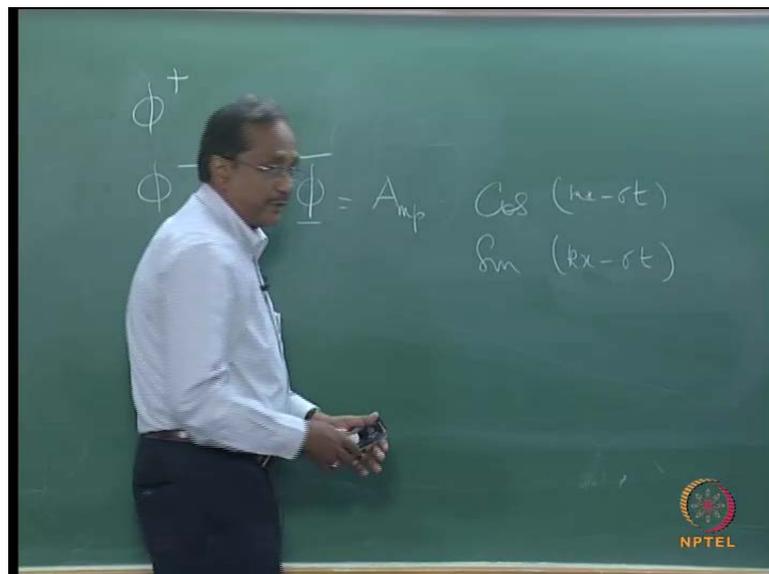
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Then what did we do, we took any of the two forms of the velocity potential, we added them or subtracted them in order to get the positive velocity potential or the negative velocity potential **ok.**

So, we will retain the four forms of the velocity potential, but we will not get into the photo potential initially **(0)**, in order to understand the basics of standing waves. So, I am just taking the form of phi 3 and phi 4, and phi 4 is marked with yellow color that is the one which we are **we are** going to do, we are going to take up.

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So, in the case of photo velocity potential, remember we had a kind of an amplitude into $\cos kx - \sigma t$ or maybe $\sin kx - \sigma t$ and this was called as the phase **this was called as a phase**. In order to express the expression in this form, we used to take **we** took two forms of any two forms of the velocity potential. Now, we are not doing that, instead we are just taking the form of ϕ and we take that and examine the physics behind it.

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Standing Waves

Consider one of the solutions among the four

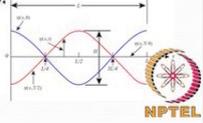
$$\phi = \frac{ag}{\sigma} \frac{\cosh k(d+z)}{\cosh kd} \cdot \cos kx \cdot \sin \sigma t \quad (1)$$

$$\eta(x,t) = \left. \frac{1}{g} \frac{\partial \phi}{\partial t} \right|_{z=0} = a \cos kx \cdot \cos \sigma t$$

where $\sigma^2 = gk \tanh kd$.

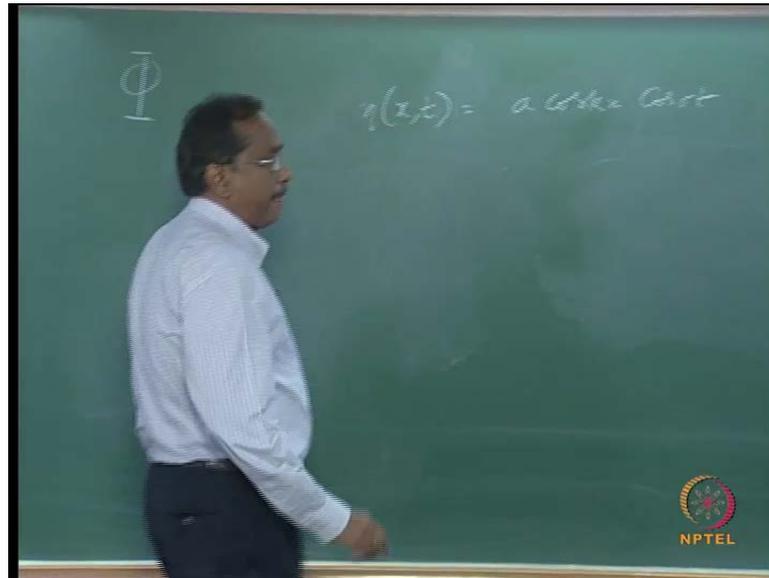
- The wave form is shown in Figure (next slide). At $\sigma t = \pi/2$, the wave form is zero for all x .
- At $\sigma t = 0$, it has a cosine shape and at other times, the same cosine shape with different magnitudes.

Assume walls on both ends.



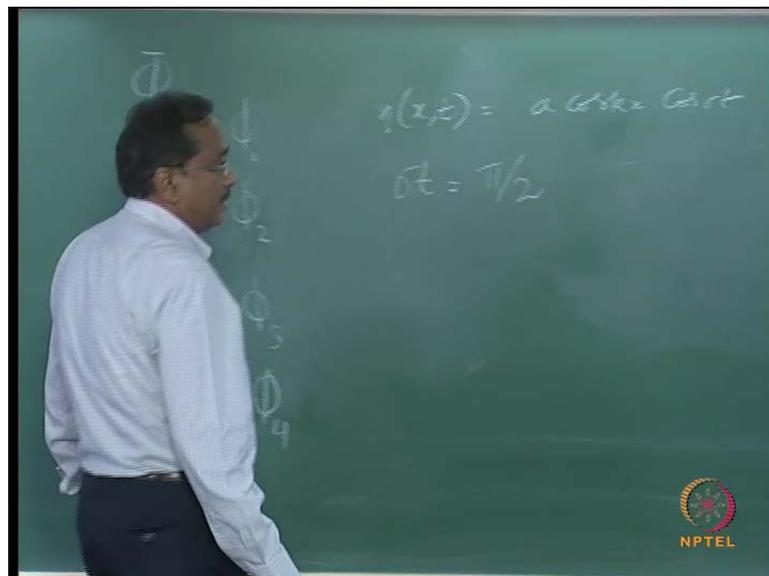
So, consider one of the solutions among the four that is what I have **I have** taken considered here that is your ϕ .

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Under this you see that, eta of x comma t using the what condition, the dynamic free surface boundary condition, we get a into cos of k into x into cos of sigma t. Of course, sigma square is **g k by** g k into tan h k d, the illustration or the variation of the eta will be explained in the next slide with much more details.

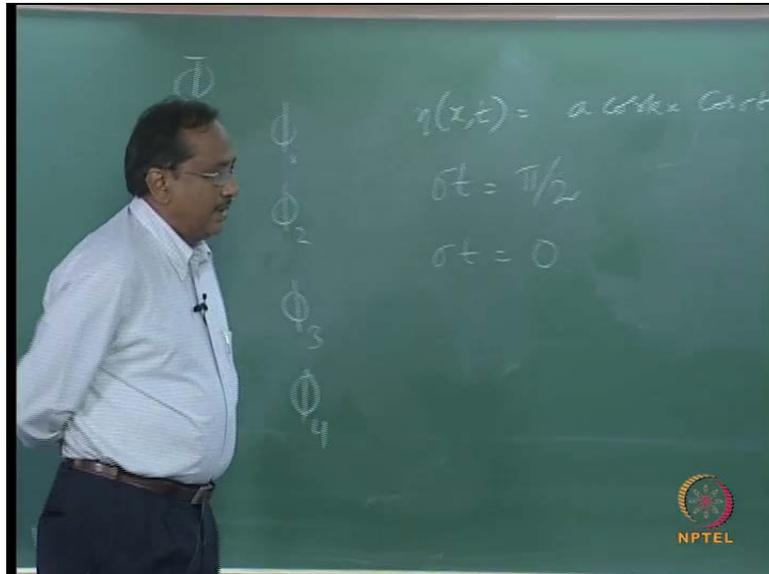
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But in this slide, now the right hand side bottom you see that at sigma t, then sigma t equal to how much, when it is equal to pi by 2 what will happen, then pi by 2 that is cos 90 **cos 90**, it is going to be 0. So, that means the wave formed will be 0 for all values of

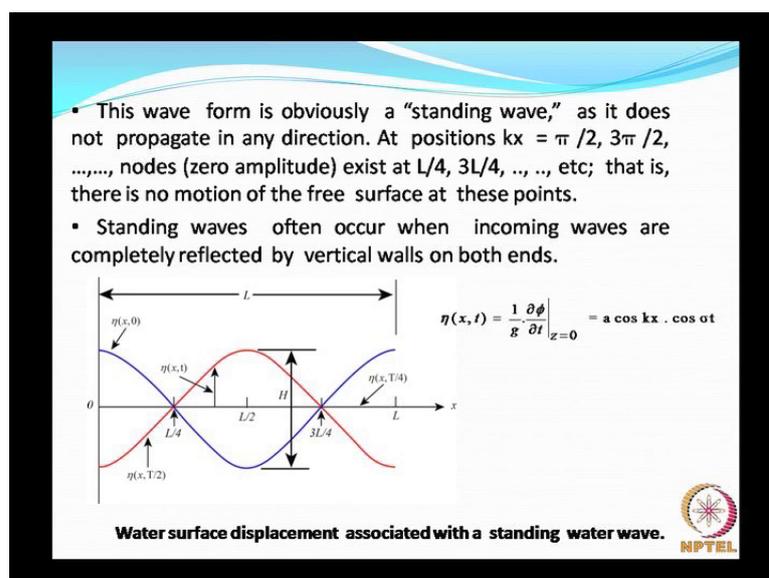
x, when sigma t is going to be pi by 2. So, this in fact facilitates the position of your zero amplitude **is that clear.**

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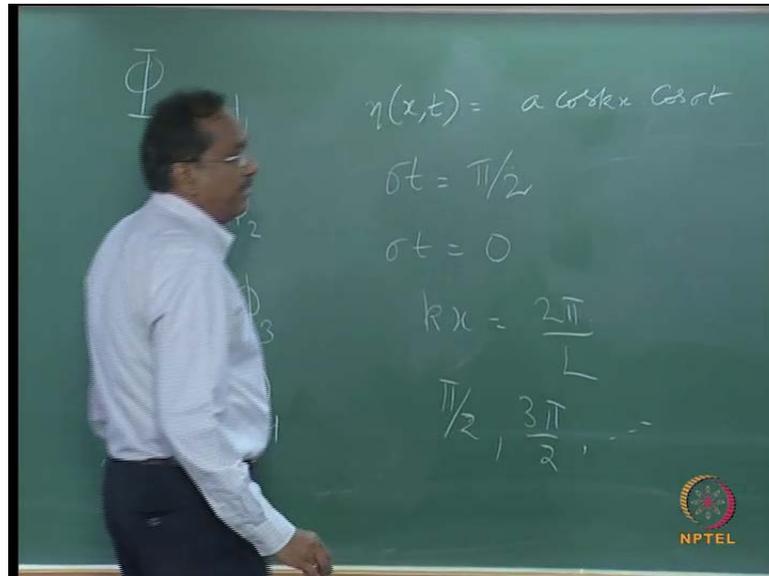
Now, our sigma t equal to 0, what will happen, this will become as a cosine shape factor. Now, this cosine shape, the shape of the cosine function will **will** be will follow and the amplitudes will be varying, the variations in amplitude will take the shape of a cosine curve.

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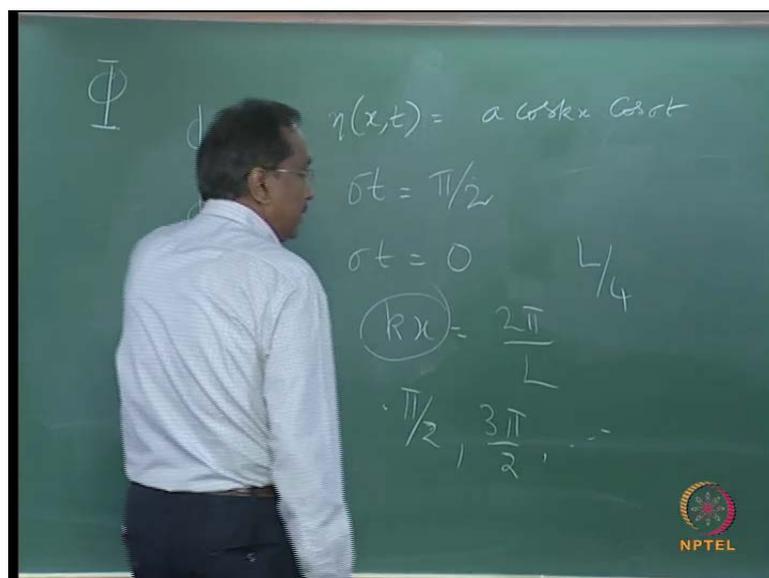
And that is what it is illustrated in the right hand, in the figure in the right hand corner below. Now, this form is obviously a standing wave, because it does not propagate, it is not propagating.

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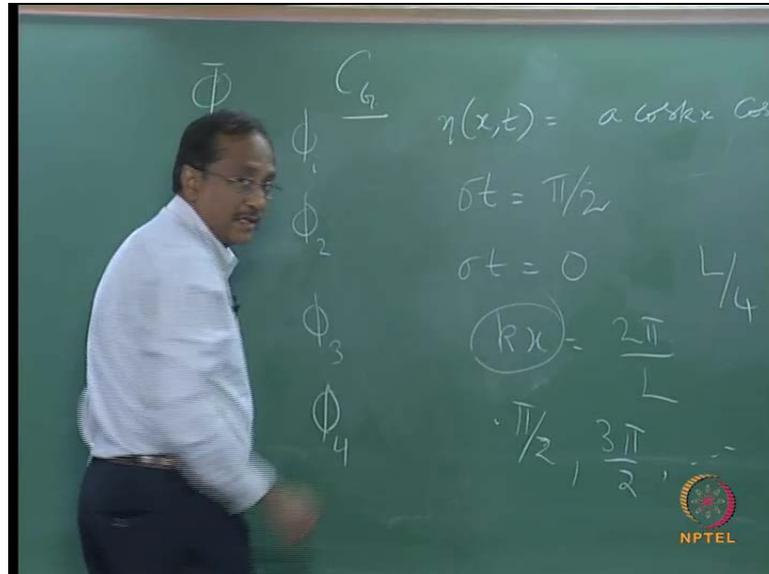
Why, but at any **any** position, let us talk about the position of the x axis, we are talking about the spatial variation, kx is 2π by L . So, if kx equal to π by 2 **pi by 2** or 3π by 2 , etcetera, what will happen, you will have nodes of zero amplitudes at what locations?

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At locations L by 4 , because look at this at L by 4 , $3L$ by 4 , etcetera, that is there is no motion of the free surface at these points **is that clear**.

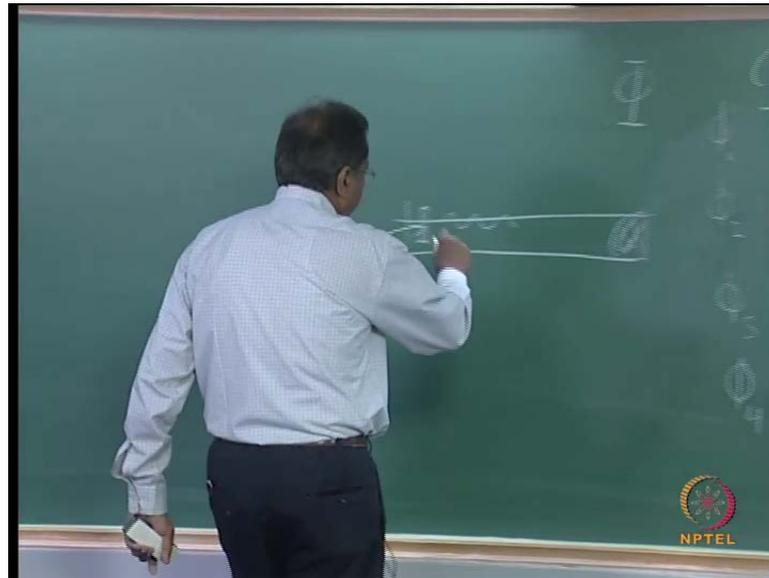
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So, this is similar to what we had already seen when we were looking at the variation or the explanation for group **(0)**, recollect we found out the nodes, position of the nodes and try to get the speed of the nodes in order to get the velocity.

Standing waves often occur when incoming waves are completely reflected, if the structure is only vertical wall, and is impermeable, you can expect standing waves. And standing waves **standing waves** means, if you have a container that is if you have a wall on either ends, then you will see that the waves from one end moves, hits the wall of the other side and the entire energy gets back. So, this is **this is** not to be in fact strictly generated in the lab, and that is why we have seen the necessity of having an absorber.

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If you look at any wave tank you see that, you will have one wave maker in order to generate the waves in the tank and on the other end, you will see that you have a absorber in order to absorb the waves, so has to have only progressive waves enough for testing of structures. So, this kind of a standing wave is now represented by the expression given on the right hand side, that is eta of x comma t, etcetera, that is a equal to a into cos k x into cos sigma t which is valid for the form of phi 4, how did you get that, by using the dynamics free surface boundary condition **is that clear.**

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Consider another standing wave (another form of ϕ)

$$\phi(x,z,t) = \frac{ag}{\sigma} \frac{\cosh k(d+z)}{\cosh kd} \cdot \sin kx \cdot \cos \sigma t \quad (2)$$

The difference between the former and the present solution is that the 'x' and 't' terms are 90° out of phase.

The associated water surface displacement is $\phi_4 = \frac{ag}{\sigma} \frac{\cosh k(d+z)}{\cosh kd} \cdot \cos kx \cdot \sin \sigma t$

$$\eta(x,t) = \frac{1}{g} \frac{\partial \phi}{\partial t} \Big|_{z=0} = -a \sin kx \cdot \sin \sigma t \quad (3)$$

as determined from the linearized DFSBC

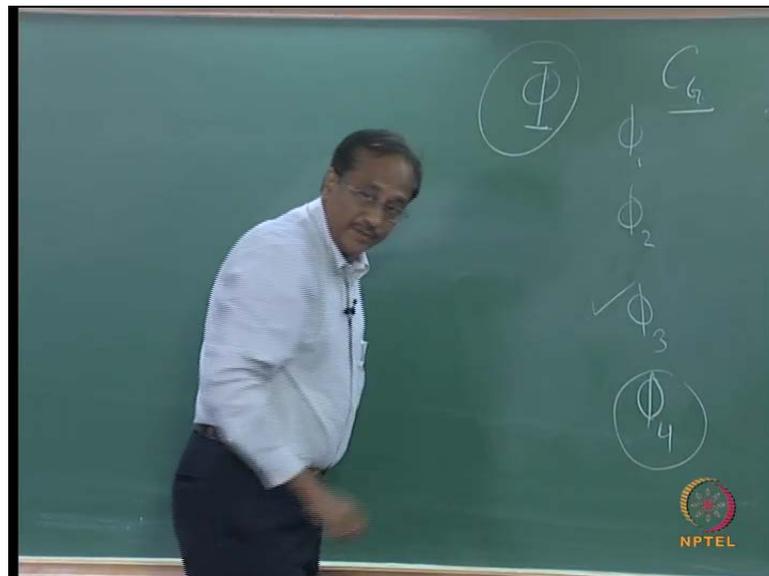


Now, consider another **another** standing wave, may be we consider phi 3. So, already we considered phi 4, now we are considering phi 3. So, the difference between the former and the present will be the out of phase by 90 degrees, you look at this, you see that you have a **a** sin sigma t, other one has cos sigma t

So, it is totally out of phase by 90 degrees, now you see that the associated water surface elevation for a phi 3 is going to be minus a into sin k x into sigma **is that clear**. Of course, this is, they are from dynamics free surface boundary condition.

So, now you see that, what we are dealing is with the Laplace equation **and it is a** and it is linear.

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So, this permits us to add or subtract solutions that could be obtained based on this example only, based on this kind of facility only we could, in fact add subtract one of the forms of two forms of, any two forms of the velocity potential in order to obtain the velocity potential for a progressive wave. Now, we subtract the present velocity potential in equation 2 from the velocity potential which has been represented as phi 4.

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Addition or subtraction of the solutions are valid as the Laplace equation is linear

If we subtract the present velocity potential in Eq. (2) from the velocity potential, ϕ_4 ,

we obtain

$$\phi = \frac{ag}{\sigma} \frac{\cosh k(d+z)}{\cosh kd} (\cos kx \sin \sigma t - \sin kx \cos \sigma t) \quad (4)$$
$$= -\frac{ag}{\sigma} \frac{\cosh k(d+z)}{\cosh kd} \sin(kx - \sigma t) \quad (5)$$


Strictly speaking, we are subtracting phi 3 from phi 4 **phi 3 from phi 4**. When we do that, the resulting equation will be as shown in equation 5, this is nothing but our classical velocity potential which we have already seen.

I am just repeating whatever I have told under basic physics, basic wave mechanics.

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This new velocity potential has a water surface elevation, given as

$$\eta(x,t) = \frac{1}{g} \frac{\partial \phi}{\partial t} \Big|_{z=0} = a \cos(kx - \sigma t) \quad (6)$$

If the two $\eta(x, t)$ corresponding to the two velocity potentials are subtracted,

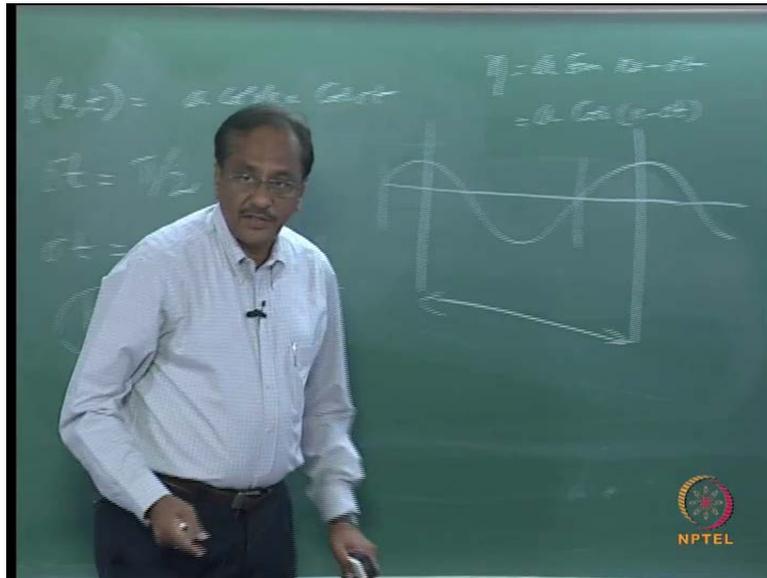
$$\eta(x,t) = a \cos kx \cdot \cos \sigma t + a \cdot \sin kx \cdot \sin \sigma t = a \cos(kx - \sigma t) \quad (7)$$

which is the same result. This proves that the total boundary value problem has been linearized and hence superposition is valid for all variables in the problem as stated earlier.



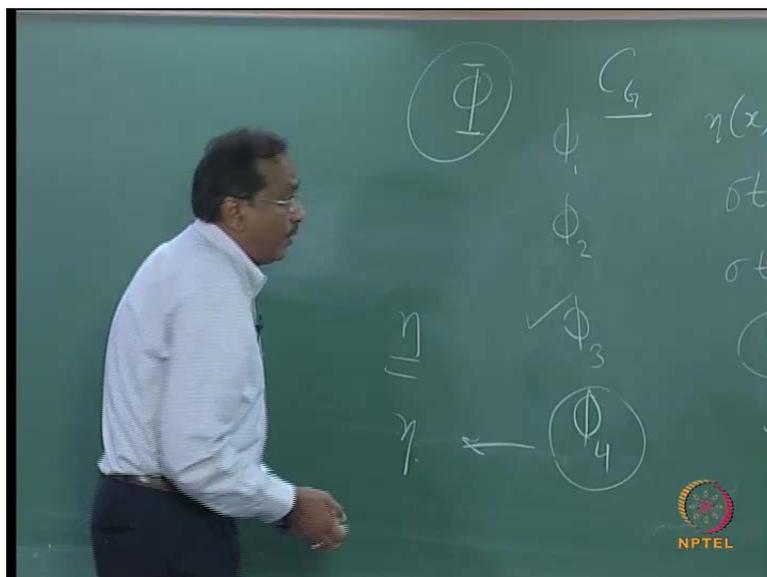
Now, this velocity potential is bound to have the eta variation as shown in here, that is it is going to be for a cosine curve. At this point of time, I would like to reiterate what I have said earlier.

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So, you can have a into sin of kx minus σt or a into cos of kx minus σt . So, certain books use this form of the expression, certain books use this form of expression, Both are fine, because what it means is when you have a surface wave like this, the first expression (Refer Slide Time: 18:30), consider this portion of the wave in order to examine the characteristics, whereas the second form considers this portion of the wave. As long as you consider one full cycle, the characteristics are taken care whether you are considering a sin curve or a cosine curve.

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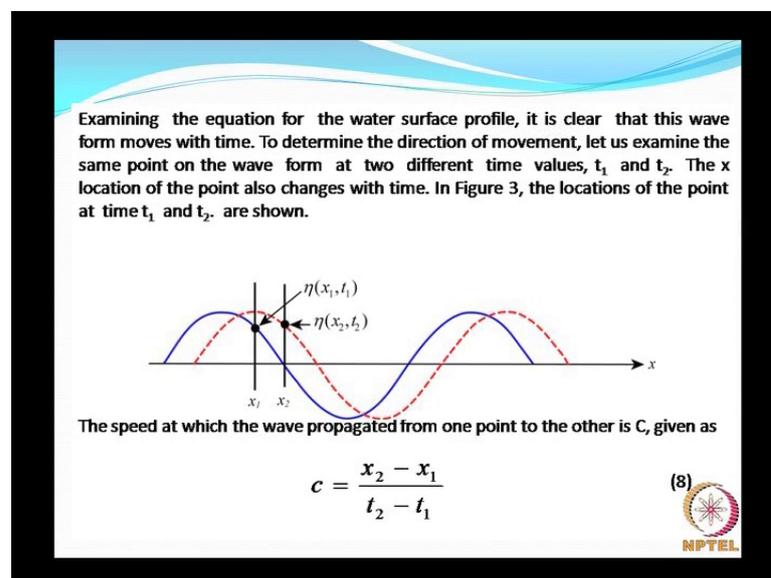


Now, you see that we had a two forms of velocity potential that is, this form of velocity potential, and this form of a velocity potential which will have the corresponding eta, what you see here is, for the total velocity potential that is summing up two velocity potentials that is having a space and a time separately earlier.

But now, we have brought space and time within the phase as you can see, this is a total velocity potential. Now, if you take the eta, you know the expression for phi 3 and phi 4 given in probably the first slide of this lecture, their corresponding eta is also given. So, look at the, where there is for one solution, and then you have here the other one.

So, use if your superpose boat, corresponding to these potentials, then you will see that this eta is nothing but a into cos of k x minus sigma t, which is nothing but which is the same as what we have got here, by adding two separate velocity potentials, you understood? So, what why we have done this is to prove that the superposition is valid is that clear.

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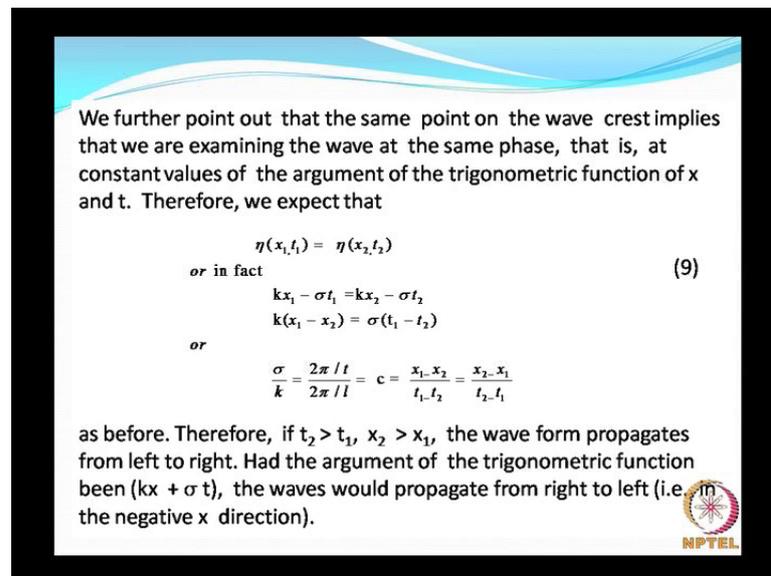
So, now having said about this, now we will move on to examine the water surface profile. Now, I consider, so let us to determine the direction of moment, let us examine some point far away from a two different time values that is, you have the yellow sorry blue and the red, and a position is fixed, the x location of the point also changes in time although we fix up a point. So, this position of this point can be the crest of a wave or a

position dou somewhere here in a red color wave, almost the same point as we are talking in the blue color wave.

Both are travelling in the same direction, now the x location of the point also will change with respect to time **that is clear**. So, in this figure you see that, the locations of point **points point** at t 1 and t 2 are clearly seen. Now, if you consider them, the speed at which the wave propagated from one point to the other that is naturally what is it, celerity. So, and that has to take place in that interval of time.

So, x 2 the distance, the difference in the distance x 2 minus x 1 divided by t 2 minus t 1 is going to be your speed **is that clear**.

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We further point out that the same point on the wave crest implies that we are examining the wave at the same phase, that is, at constant values of the argument of the trigonometric function of x and t. Therefore, we expect that

$$\eta(x_1, t_1) = \eta(x_2, t_2) \tag{9}$$

or in fact

$$kx_1 - \sigma t_1 = kx_2 - \sigma t_2$$
$$k(x_1 - x_2) = \sigma(t_1 - t_2)$$

or

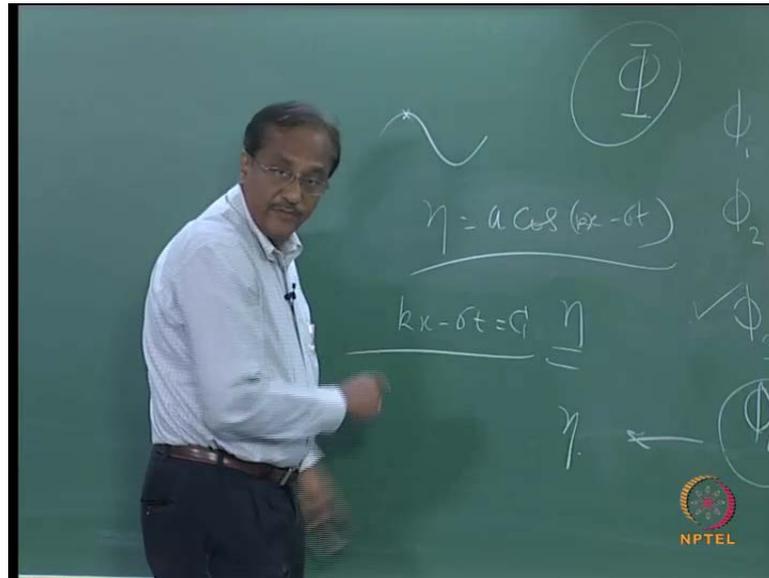
$$\frac{\sigma}{k} = \frac{2\pi / t}{2\pi / l} = c = \frac{x_1 - x_2}{t_1 - t_2} = \frac{x_2 - x_1}{t_2 - t_1}$$

as before. Therefore, if $t_2 > t_1$, $x_2 > x_1$, the wave form propagates from left to right. Had the argument of the trigonometric function been $(kx + \sigma t)$, the waves would propagate from right to left (i.e. the negative x direction).



Or we further point out that the same point on the wave crest implies that, we are examining the waves with the same phase.

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So, when we remember I would like you to recollect again, this is what we did, we identified a point or a wave while moving so that we try to move with the same speed of that of the wave at this point. Then in that case the phase, there is no phase difference, then we said we said that $kx - \sigma t$ is equal to a constant from which we derived C is equal to L by t for a propagation, I mean for a propagating wave, now the same thing holds good here.

Since, we are examining the wave at a same point, that is constant values of argument of the trigonometric functions will be same, equal. So, in this **in this** case $kx - \sigma t$ must be equal to $kx_1 - \sigma t_1$ because we are talking about the same phase. So, then when we go about deriving, then finally you see that the velocity **comes out** comes down to the difference between the spaces that is $x_2 - x_1$ divided by $t_2 - t_1$ **ok is that clear**.

So, if you look back, so this you have here x_2 , x_1 , $x_2 - x_1$ is marked there, x_1 is also marked there, if you use that picture, if t_2 is greater than t_1 , x_2 is greater than x_1 , and the wave form propagates from left to right **from left to right**. Suppose, if the argument of the trigonometric function that is, if you had use a positive sign instead of a negative sign inside that, then the waves will be moving in the right side, this we have already seen under a progressive wave **understood**.

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WATER PARTICLE KINEMATICS FOR STANDING WAVES

The original velocity potential we derived represented a pure standing wave, ϕ_4 (Note that $a=H_s/2$)

$$\phi = \frac{H_s g}{2\sigma} \cdot \frac{\cosh k(d+z)}{\cosh kd} \cdot \cos kx \cdot \sin \sigma t \quad (10)$$

with

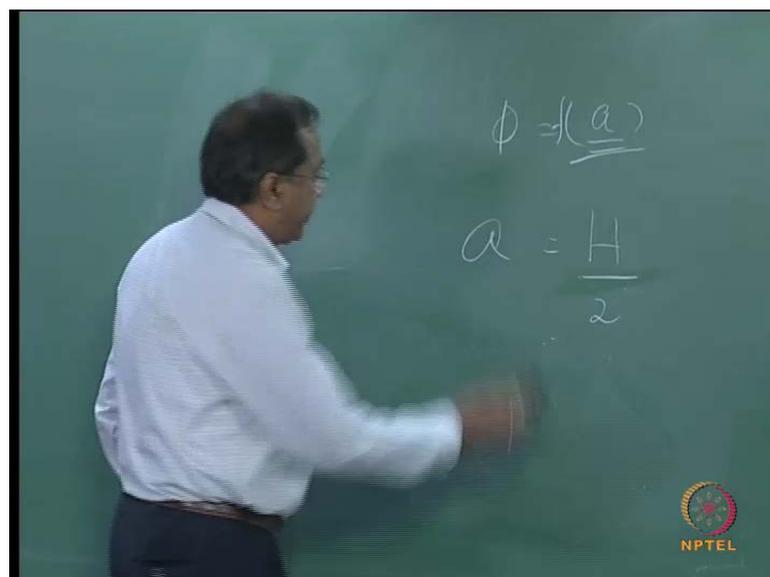
$$\eta = \frac{H_s}{2} \cos kx \cos \sigma t$$
$$\sigma^2 = gk \cdot \tanh kd$$

where H_s denotes the height of the standing wave and is twice the height of each of the two progressive waves forming the standing wave.



Now, let us look at the water particle kinematics for standing waves (No audio from 26:08 to 26:29). The original velocity potential may be ϕ_3 or ϕ_4 , it does not matter, you can take anything which ever you want. So, the original form of a velocity potential that in this case should be ϕ_4 , what we have derived is for a pure standing wave.

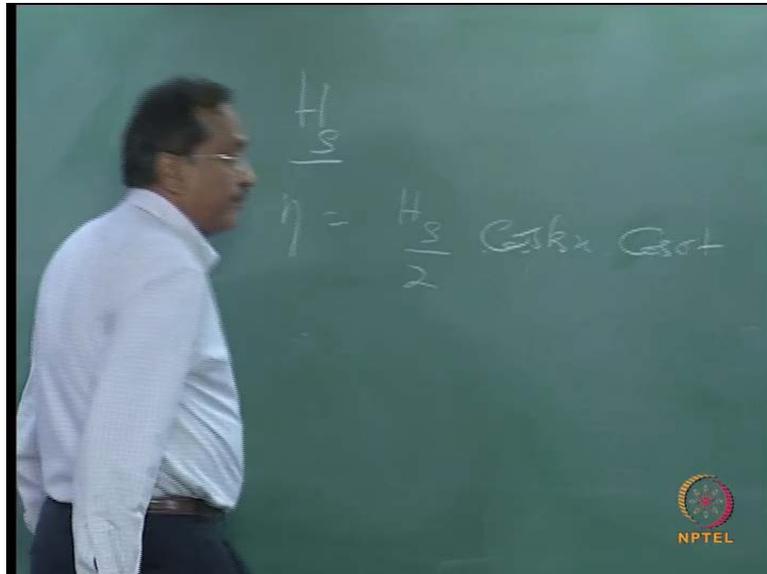
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Earlier we retained function as, I mean the velocity potential as in the function of amplitude. Now, since we are going to talk about the standing wave, let us revert, I mean

let us start talking in terms of wave height, wave height is nothing but amplitude is equal to wave height divided by 2.

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So, the velocity potential now since we have considered the standing wave, I am denoting H as H suffix s, then I have a H suffix s, that means it is a standing wave. For this velocity potential, now my eta is going to be as a function of H by cos of k x into cos sigma t. So, H s is the height of the standing wave and as I said earlier, it should be twice the height of the each of the two propagating waves that should be merged together to form as a standing wave.

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WATER PARTICLE KINEMATICS FOR STANDING WAVES

The velocity potential for a standing wave can be re-derived by subtracting the velocity potential for two progressive waves of the same period with heights H_p propagating in opposite directions.

$$\phi = -\frac{H_p}{2} \cdot \frac{g}{\sigma} \cdot \frac{\cosh k(d+z)}{\cosh kd} \cdot \sin(kx - \sigma t) + \frac{H_p}{2} \cdot \frac{g}{\sigma} \cdot \frac{\cosh k(d+z)}{\cosh kd} \cdot \sin(kx + \sigma t) \quad (11)$$
$$\phi = \frac{H_p g}{\sigma} \cdot \frac{\cosh k(d+z)}{\cosh kd} \cdot \cos kx \cdot \sin \sigma t \quad (12)$$

$\phi = \frac{H_p g}{2\sigma} \cdot \frac{\cosh k(d+z)}{\cosh kd} \cdot \cos kx \cdot \sin \sigma t$

NPTEL

So, particle kinematics in the case for the standing waves, the velocity potential for a standing wave can be re derived **re derived** by subtracting the velocity potential of two progressive waves, herein we are considering two progressive waves, but when will the standing wave take place, when they are moving in the opposite direction **when they are moving in the opposite directions.**

So, I consider here one wave, so I am using here as a H_p means a single propagating wave, progressive wave. I have a progressive wave here and this is nothing but the form of the expression, what we have obtained by superposing any of the two forms of velocity potential while deriving the expression for progressive waves. So, any two forms can be superposed, but only thing is what we are trying to do here is, we have to take care of this signature and in this the sign, because the wave is going to move in the opposite direction. Only then you can generate a, you can have a standing wave **is that clear** or can proceed.

Remember that, so this is your standing wave as we have seen earlier, and now this is the progressive wave, single progressive wave which is now of this form that is out of the phase kx and the σt is out of bracket. So, **oh sorry** now this is the form of velocity potential for standing wave, the orange color is for progressive waves, now both are same.

So, by examining these two expressions what you see, you see the cycle of 2 appearing you see the factor 2 appearing here, H_p equal to H_s divided by 2 which indicates clearly that the height of the standing wave is twice the progressive wave height the wave height or twice the wave height of a progressive height of the progressive wave is that clear.

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WATER PARTICLE KINEMATICS FOR STANDING WAVES

Comparing the two velocity potentials, (eqs. 10 & 12), it is clear that $H_p = H_s/2$. Therefore, a standing wave of height H_s , is composed of two progressive waves propagating in opposite directions, each with height equal to one-half that of the standing wave.

$$\phi = \frac{H_s g}{2\sigma} \cdot \frac{\cosh k(d+z)}{\cosh kd} \cdot \cos kx \cdot \sin \sigma t$$

$$\phi = \frac{H_p g}{\sigma} \cdot \frac{\cosh k(d+z)}{\cosh kd} \cdot \cos kx \cdot \sin \sigma t$$



And then is what is explained here, comparing the two velocity potentials, so we get the factor 2 which is again illustrated in this slide, what I have explained in the earlier slide is again repeated here, shall I proceed. Now, as we did now that we have the velocity potential, once you know the velocity potential you know what to do with the velocity potential in order to get the velocity components, orbital velocity components as you have done as you have done earlier.

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WATER PARTICLE KINEMATICS FOR STANDING WAVES
Velocity Components

$$\phi = \frac{H_0 g}{2\sigma} \cdot \frac{\cosh k(d+z)}{\cosh kd} \cdot \cos kx \cdot \sin \sigma t$$

The velocities under a standing wave are readily found to be

$$u = -\frac{\partial \phi}{\partial x} = \frac{ag}{\sigma} \cdot k \cdot \frac{\cosh k(d+z)}{\cosh kd} \cdot \sin kx \cdot \sin \sigma t \quad (13)$$

$$w = -\frac{\partial \phi}{\partial z} = \frac{-ag}{\sigma} \cdot k \cdot \frac{\sinh k(d+z)}{\cosh kd} \cdot \cos kx \cdot \sin \sigma t$$

where for convenience the subscripts has been dropped. Using the dispersion relationship,

$$u = \frac{\pi H}{T} \cdot \frac{\cosh k(d+z)}{\sinh kd} \cdot \sin kx \cdot \sin \sigma t$$

$$w = \frac{-\pi H}{T} \cdot \frac{\sinh k(d+z)}{\sinh kd} \cdot \cos kx \cdot \sin \sigma t \quad (14)$$


So, the **the** velocity potential for **for** this purpose is again introduced here so that you need not have to go back, this is what we have already derived for which use the usual expressions of a u and w for and also using the dispersion relationship, you can get in terms of H and t as we have done for progressive waves earlier.

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WATER PARTICLE KINEMATICS FOR STANDING WAVES
Velocity Components

$$\phi = \frac{H_0 g}{2\sigma} \cdot \frac{\cosh k(d+z)}{\cosh kd} \cdot \cos kx \cdot \sin \sigma t$$

The velocities under a standing wave are readily found to be

$$u = -\frac{\partial \phi}{\partial x} = \frac{ag}{\sigma} \cdot k \cdot \frac{\cosh k(d+z)}{\cosh kd} \cdot \sin kx \cdot \sin \sigma t \quad (20)$$

$$w = -\frac{\partial \phi}{\partial z} = \frac{-ag}{\sigma} \cdot k \cdot \frac{\sinh k(d+z)}{\cosh kd} \cdot \cos kx \cdot \sin \sigma t$$

where for convenience the subscripts has been dropped. Using the dispersion relationship,

$$u = \frac{\pi H}{T} \cdot \frac{\cosh k(d+z)}{\sinh kd} \cdot \sin kx \cdot \sin \sigma t$$

$$w = \frac{-\pi H}{T} \cdot \frac{\sinh k(d+z)}{\sinh kd} \cdot \cos kx \cdot \sin \sigma t \quad (21)$$


Up to this absolutely no problem, because it is similar to what you have already seen earlier.

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WATER PARTICLE KINEMATICS FOR STANDING WAVES
Velocity Components

As with the velocities under a progressive wave, these velocities increase with elevation above the bottom.

The extreme values of u and w in space occur under the nodes and antinodes of the water surface profile as shown in Figure, where u and w are zero under the antinodes and nodes, respectively.

Distribution of water particle velocities in a standing water wave.

Now, as the velocity under a progressive waves as with the velocities increase with the elevation, the particle velocities are kinematics is going to with respect to the variation along the depth, it is similar what is going to happen when a progressive wave passes.

It is basically a surface waves, so you need, you will have the dominance of the particle velocities, accelerations near the free surface and as you go down towards the seabed, it is going to decrease that implies from the expressions also what we have got so far. Now, the extreme values of u and w that is horizontal and the vertical velocities in space **in space** under the nodes and antinodes of the water surface will be is shown in the figure given below **in this** on this slide.

You see that u and w both, for example, if you take this is node, node is nothing but points of zero amplitudes, antinodes are locations of maximum amplitude which we have already seen. So, you see that, here the node the horizontal water particle velocity is going **to dominant** to be dominant and the vertical velocity is going to be 0 and vice-versa in the anti-node under the anti-node your vertical velocity is going to be the is going to be the maximum and of course, it is going to vary along the depth **is that clear**. So, now when we are discussing about this, we are considering to be assume that there are two vertical balls on either side, only then you can really have a pure standing wave taking place **is that clear**.

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WATER PARTICLE KINEMATICS FOR STANDING WAVES
Velocity Components

Note that the horizontal and vertical components of velocity under a standing wave are in phase; that is, the time-varying term “sin σt ” modifies both velocity components and, at certain times, the velocity is zero everywhere in the standing wave system.

It is therefore evident that at some times all the energy is potential (Eq.14), at other times all the energy is kinetic.

$$u = \frac{\pi H}{T} \cdot \frac{\cosh k(d+z)}{\sinh kd} \cdot \sin kx \cdot \sin \sigma t$$
$$w = \frac{-\pi H}{T} \cdot \frac{\sinh k(d+z)}{\sinh kd} \cdot \cos kx \cdot \sin \sigma t$$

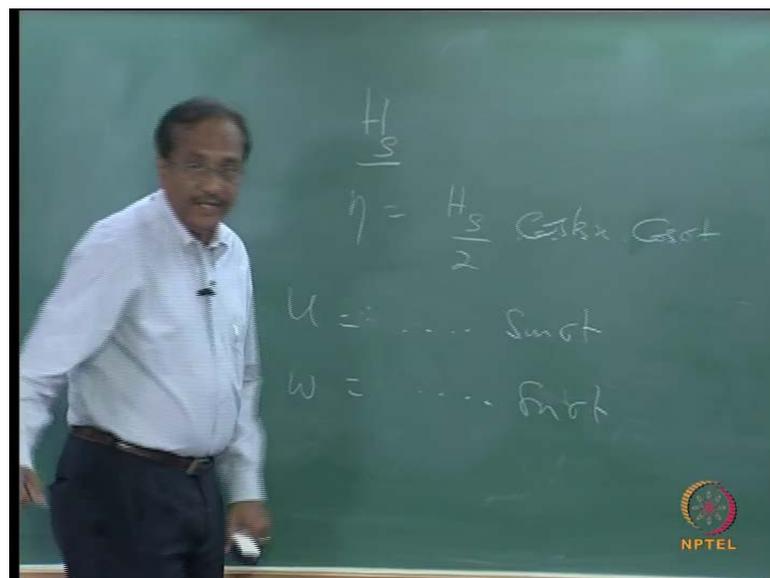
If $\sigma t = 0, \pi, 2\pi, \dots$ no kinetic energy will exist and for other values of σt , kinetic energy will exist.



See, note that both the horizontal and vertical **vertical** components of the velocity are in phase, that is quite surprising, because in the case of **in the case of** progressive waves, it will be the other way.

So, now you see that, it is going to be or they both are in same phase, both are having sin of sigma t.

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So, u will be sin of sigma t as in the, as we are seeing, whereas this is **also sorry** also is sin of t, both are in the same phase that is time bearing sigma t modifies both velocity

components and at sometimes, because they **they** dictate the velocity components, but at sometimes the whole thing will become 0 also **ok**.

So, velocity is 0 everywhere at certain value of the phase σt , the whole thing will become 0. So, there would not be any magnitude of the orbital velocity. Now, it is therefore evident that at times all the energy is potential and at some other times, all the energy will be kinetic **kinetic**, because this depends on the value of σt , if you have a value of σt , you will have certain values for your u and w . But it is all going to be both are going to be in the same phase **is that clear**.

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WATER PARTICLE KINEMATICS FOR STANDING WAVES
Velocity Components

If a progressive wave were normally incident on a vertical wall, which is reflected back from a vertical wall on the other end generates a standing wave in front of the wall.

The lateral boundary condition at the vertical wall would be one of no flow through the wall, or $u = -\partial\phi/\partial x = 0$ at $x = x_{\text{wall}}$, where, x_{wall} is the Location of the wall.

Inspection of the equation for the **horizontal velocity**, Eq. (13), shows that at locations $kx = n\pi$ (where n is an integer), the no-flow boundary condition is satisfied.

$$u = \frac{\partial\phi}{\partial x} = \frac{ag}{\sigma} \cdot k \cdot \frac{\cosh k(d+z)}{\cosh kd} \cdot \sin kx \cdot \sin \sigma t$$

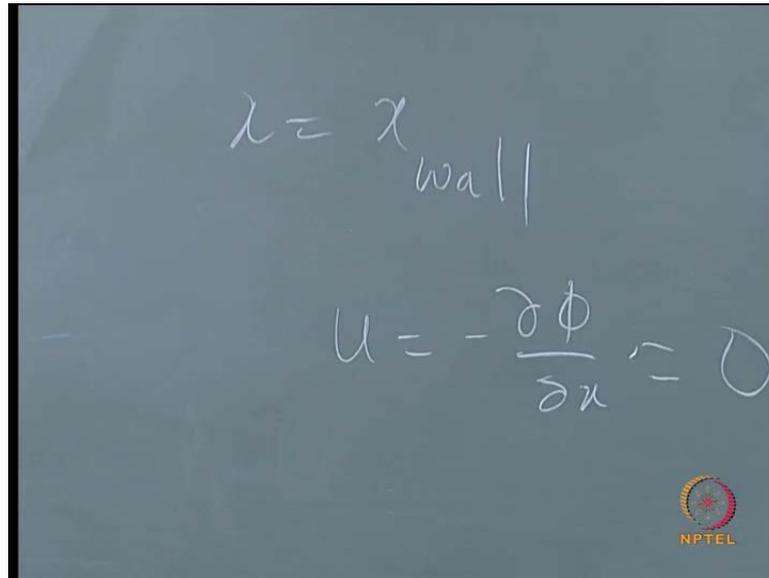
$$w = \frac{\partial\phi}{\partial z} = \frac{-ag}{\sigma} \cdot k \cdot \frac{\sinh k(d+z)}{\cosh kd} \cdot \cos kx \cdot \sin \sigma t$$



Now, again we continue the particle velocities, particularly when we talk about a vertical wall, when we have a vertical wall, what is the first, what is that kinematic bottom boundary condition, vertical velocity normal to the sea bed is 0, now you are having a vertical wall and you have a wave coming and hitting the wall.

So, naturally the velocity normal to the vertical wall is going to be 0. So, using that at any x **x** equal to let me say at the **at the** wall is going to be 0.

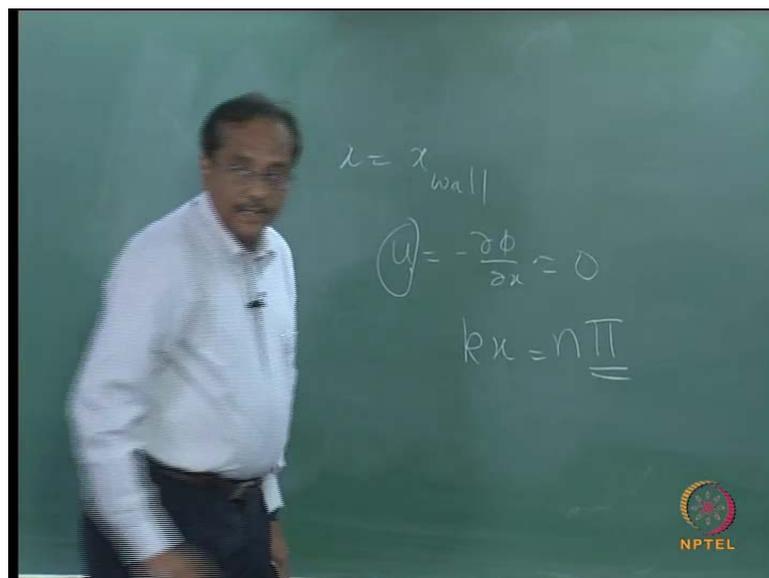
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$$\lambda = x_{\text{wall}}$$
$$u = -\frac{\partial \phi}{\partial x} = 0$$

That is equal to u equal to **is that clear**. Now, if you look at the expression for the horizontal velocity as you can see from equation 2, this is the horizontal velocity and the bottom one is the vertical velocity.

So, if you look for the horizontal water particle, velocity given in the right hand side corner.

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$$\lambda = x_{\text{wall}}$$
$$u = -\frac{\partial \phi}{\partial x} = 0$$
$$kx = n\pi$$

What does that show at locations kx equal to $n\pi$, n is an integer value, when n is an integer value what will happen, there would not be any flow that is **no** flow boundary

condition will be satisfied **is that clear**. So, no there would not be any flow into the boundary, that is what it implies.

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WATER PARTICLE KINEMATICS FOR STANDING WAVES

Velocity Components

Therefore, a standing wave could exist within a basin with two walls situated at two antinodes of a standing wave.

This is, in fact, the simplest model of uniform depth lakes, estuaries, and harbors where standing waves, called seiches, can be generated by winds, earthquakes, or other phenomena.

The local accelerations under a standing wave are

$$\frac{\partial u}{\partial t} = a\sigma^2 \cdot \frac{\cosh k(d+z)}{\sinh kd} \cdot \sin kx \cdot \cos \sigma t$$

$$\frac{\partial w}{\partial t} = -a\sigma^2 \cdot \frac{\sinh k(d+z)}{\sinh kd} \cdot \cos kx \cdot \cos \sigma t \quad (15)$$

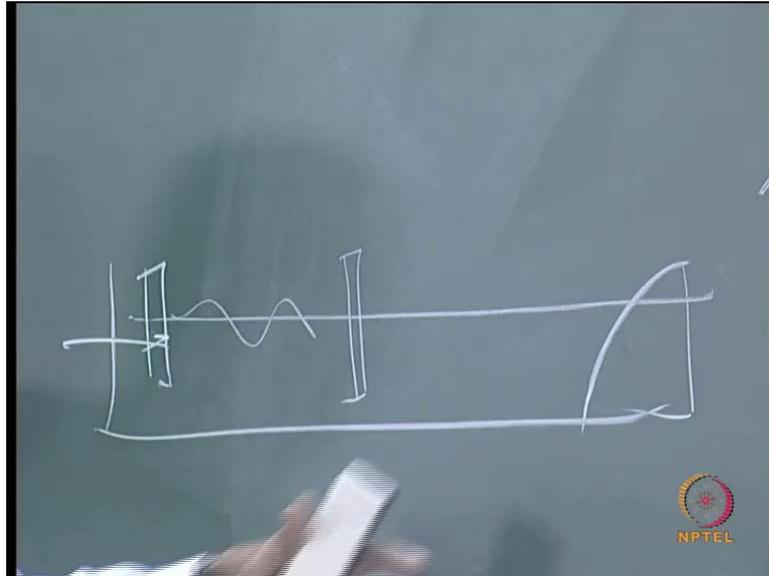
Under the wave antinodes, the vertical accelerations are maxima, while the horizontal accelerations are zero, and under the nodes, the opposite is true.



Therefore, a standing wave could exist within a basin, for example we have a wave basin or we have a vertical wall I mean a flume, now you remove the wave absorber, you generate a wave, allow the wave to, allow the wave maker to run for some time, what will happen, the wave will travel, go hit the wall and come back, and what will happen you will initially have some standing wave for certain location, certain stretch.

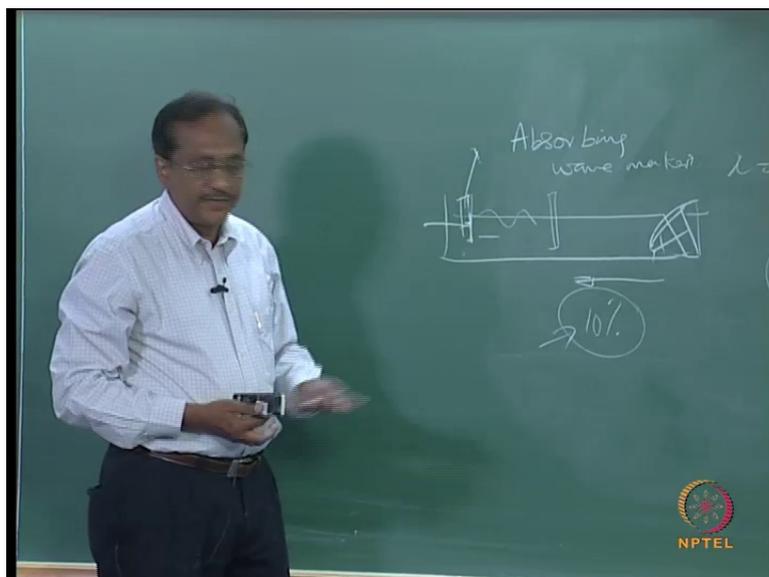
But then, again the waves which are coming back from the wall and hitting the wave maker again will get re reflected. So, these are all called as re reflected waves.

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So, for example, if you have a flume facility, strictly speaking what you are supposed to have, you are supposed to have progressive waves, in order to test the structure for example, ability measuring of forces some structure, etcetera, may be measuring forces on a tubular number which is quite common. When you want to do such **such such** studies, you have to have the wave maker which would be moving up and it will be generating, you remove this, then you are not generating what is going to happen in the field, in the field you are going to have a progressive waves.

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So, even in case you have this wave absorber, this wave absorber cannot absorb 100 percent of the energy of the propagating waves, there will be a certain degree of a reflection taking place, may be 10 percent and this energy, if it reflects back and if it is hitting the structure, hitting this wave maker. So, the next waves, next series of waves is not going to be of the same quality of waves which you want to generate you are, because you are able to generate a wave with a additional 10 percent of reflection.

In order to avoid this re reflection, we also have what is called as absorbing wave makers **absorbing wave makers** that it what it does is, it digitally it can be altered, the moment of the wave maker can be altered through the signals which we get, this it can be done electronically through a software and the control signal files for driving the, while driving the wave **is that clear**.

So, coming back that is another part of the story what we are already discussing about the standing waves. Now, this there are some areas where you will have this kind of a **a** problem, for example uniform depth, lakes, estuaries and harbors where standing waves they are referred to as **(O)**, this could be generated even by winds, waves, etcetera.

Standing waves in lakes or harbor base and etcetera are not good. So, if a tsunami enters a harbor basin, what will happen, it will keep on oscillating, one thing is if it is a **a** long period wave and enters into a harbor and inside the harbor you have most of them not that much not that absorbing type, then you are running into a trouble. So, what will happen, the disturbance will be continuous, because there is no kind of absorbing.

So, what will happen, you are more in lines, etcetera can be snapped after some time. So, the having seen the velocities, or orbital velocities, you can easily obtain the expressions for the accelerations, under the antinodes the vertical accelerations are maximum and while the horizontal accelerations are 0 and under nodes, the opposite the reverse takes place. So, that is your story about the particle velocities and accelerations under the standing waves.

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PRESSURE FIELD UNDER A STANDING WAVE

To find the pressure at any depth under a standing wave, the unsteady Bernoulli equation is used as in the case for progressive waves.

$$\frac{p}{\rho} + \frac{u^2 + w^2}{2} - \frac{\partial \phi}{\partial t} + gz = c(t) \quad (22)$$

Linearizing and evaluating as before between the free surface and at some depth (z) in the fluid, the gage pressure is

$$p = -\rho gz + \rho \frac{\partial \phi}{\partial t}$$

or

$$p = -\rho gz + \rho g a \frac{\cosh k(d+z)}{\cosh kd} \cos kx \cos \sigma t$$
$$= -\rho gz + \rho g K_p(z) \eta \quad (23)$$


Now, let us move to the pressures under a standing wave, but to find the classical Bernoulli equation is going to be used, same as what we have used for progressive waves and only thing is the velocity potential will be for a standing wave.

So, velocity potential for a standing wave we have already seen, just evaluate them and go through the same procedure as I have already told you for the progressive waves, then you will get finally, a summation of this static head and the dynamic part and there you see that, the pressure response factor is also coming into picture that is \cos into k into d plus z divided by $\cos k d$. Please refer to my lecture material on progressive waves under basic wave mechanics.

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PRESSURE FIELD UNDER A STANDING WAVE

$$p = -\rho g z + \rho g a \frac{\cosh k(d+z)}{\cosh kd} \cos kx \cos \sigma t$$
$$= -\rho g z + \rho g k_p(z) \eta$$

where the pressure response factor $K_p(z)$ is the same as determined for progressive waves.

Note that under the **nodes**, ($kx = \pi/2, 3\pi/2, \dots$) the pressure is solely hydrostatic.

Again, the dynamic pressure is in phase with the water surface elevation, and as before it is a combined result of the local water surface displacement and the vertical accelerations of the overlying water particles.



So, this is continuous and there is what you would have and note that, under nodes **under nodes** that is equal to kx equal to $\pi/2$ or $3\pi/2$, etcetera, you see that the pressure is going to be solely hydro static **is that clear**. So, after the lecture, you please have a look at the slides, go through carefully, slowly, I am sure that you are in a position to understand.

And all this part of the lecture are available in the book on, the book on wave mechanics for basic wave mechanics written by Dean and Dalrymple which is a classical text book for people who are interested in knowing about the mechanics of ocean waves. Again, the dynamic pressure is in phase with the water surface elevation, as you can see from the phase elevation and as before, it is a combined result of local water surface displacement and vertical accelerations of the overlying water particles.

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FORCE DUE TO A STANDING WAVE

The force exerted on a wall at an anti-node can be calculated by integrating the pressure over depth per unit width of wall .

$$F = \int_{-d}^{\eta_w} p(z) dz = \int_{-d}^0 \left[-\rho g z + \rho g \eta_w \frac{\cosh k(d+z)}{\cosh kd} \right] dz + \int_0^{\eta_w} \rho g (\eta_w - z) dz$$

Where the surface elevation at the wall , $\eta_w = a \cos \sigma t$,



Now, having seen the pressures, you we move on to the force, the force exerted on a vertical wall at an anti-node that is where you have the maximum amplitude which is expected to be twice the wave height, that can be calculated as minus d the integration has to take place from the sea bed up to the wave elevation, that is the first integration which can be split into two halves, you integrate up to the still water and from the still water to the eta.

As we have seen the how to derive the energy, etcetera **you know**, potential energy, kinetic energy, we used to we have taken that integral, the same way you do this, and here eta w you see that is equal to the wave elevation at the wall **at the wall**.

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FORCE DUE TO A STANDING WAVE

$$F = g\rho \left(\frac{d^2 + \eta_w^2}{2} \right) + \rho g d \cdot \frac{\tanh kd}{kd} \eta_w \quad (24)$$

to first order,

$$F = \rho \frac{gd^2}{2} + \rho g d \frac{\tanh kd}{kd} \eta_w \quad (25)$$


So, once you do this, the first order wave force due to a standing wave can be directly obtained and as you can see here. So, this can usually be used for calculating the wave forces on a vertical wave. However, you can cap with the pressures at the different elevations along the wall and integrate it.

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FORCE DUE TO A STANDING WAVE

The force on the wall consists of the hydrostatic contribution, plus an oscillatory term due to the dynamic pressure. The maximum force occurs when $\eta_w = H/2$,

$$F_{\max} = \rho g \frac{(4d^2 + (H)^2)}{8} + \rho g d \cdot a \cdot \frac{\tanh kd}{kd} \quad (26)$$


The force are consists of hydrostatic components as well as the dynamic component. So, what I will do is, anyway this completes the basics of the standing waves up, and when we are talking about the wave forces acting on vertical walls that **that** is coastal structures

under the coastal engineering module, then there I will mention more about how to calculate the forces at the exerted on the vertical walls, and there I will also consider both due to non breaking waves as well as the breaking waves. So, that will give a better, much more clear picture apart from whatever we have seen today. Are there any... So with this I am concluding the standing wave, information on the standing waves. And the next aspect is partially standing waves; partially standing waves can be due to some amount of permeability or some slight sloping structures. So, again if you want to have details about this read the go through book of Dean and Dalrymple.