

Wave Hydro Dynamics
Prof. V. Sundar
Department of Ocean Engineering
Indian Institute of Technology, Madras

Module No. # 02
Wave Motion and Linear Wave Theory
Lecture No. # 05
Wave Motion Problems

Having seen the basics of wave mechanics, we will now try to understand the subject with the help of a few worked out examples.

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Problem-1

A wave flume is filled with fresh water to a depth of 2 m. A wave of height 0.3 m and period 2.2 sec is generated. Calculate the wave celerity, group celerity, energy and power.

Solution

$$\frac{d}{L_o} = \frac{d}{L} \tanh kd$$
$$L_o = 1.56T^2 = 1.56 \times 2.2^2 = 7.5504 \text{ m}$$
$$\frac{d}{L_o} = \frac{2}{7.5504} = 0.2649$$

From wave tables

$$\frac{d}{L} = 0.2810 \text{ corresponding to } \frac{d}{L_o} = 0.2649$$
$$L = 7.1174 \text{ m}$$



As I said earlier, the **problem** problems related to the determination of or the behavior of a structures or the determination of the forces on structures etcetera; or even to understand the physics of the behavior of ocean waves itself. We have laboratory test facility, which is referred to as wave flume. So, before commencing any type of testing of structures, we need to know about the characteristics of the waves that can be simulated in the wave flume.

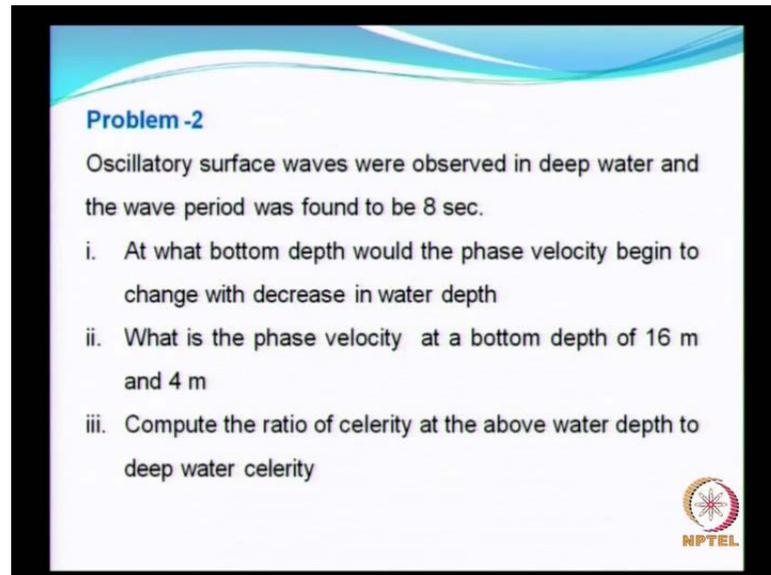
So, I start the lecture from that, with the example given here. So, if wave flumes here is filled with fresh water to a depth of about 2 meters and assume that, a wave of height 0.3 meters and a period of 2.2 seconds is generated. We are required to calculate the wave celerity, group celerity, energy and power. For all of which, we have already seen the formulas, so it is just substitution of formulas and also it gives you a feeling for the variables. So, later we will be dealing with the **characteristics** wave characteristics in the open ocean, so this is to start with only with the wave flume.

So, the first step would be to determine the wave length. So, we have already seen this relationship that is d by L naught and d by L , which is going to be an implicit equation, so we need to solve that. So, the first step would be, determine L naught, which we know is a 1.56 into T square, so doing that we will get 7.55 meters as the deep water length.

Then, we calculate d by L naught, so for the present problem it comes to 0.26 . We have already seen the wave tables, where in we had the different columns; one column giving you the d by L naught then, d by L then, $k d$, $2 k d$ then, $\tan h k d$, $2 h \sin h k d$, $\cos h k d$ etcetera. So, that table permits us to get the value of d by L corresponding to this value of d by L naught.

I suggest you also refer to these tables, which is available in many of the standard books, which have been given under the list of references. So, the corresponding d by L is 0.281 and hence with this you can because, the water depth is known to you as 2 meters, you can easily calculate your wave length, which is going to be 7.1 meters. So, this we have already done in one of our earlier explanation concerning the wave length and deep water wave length.

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Problem -2

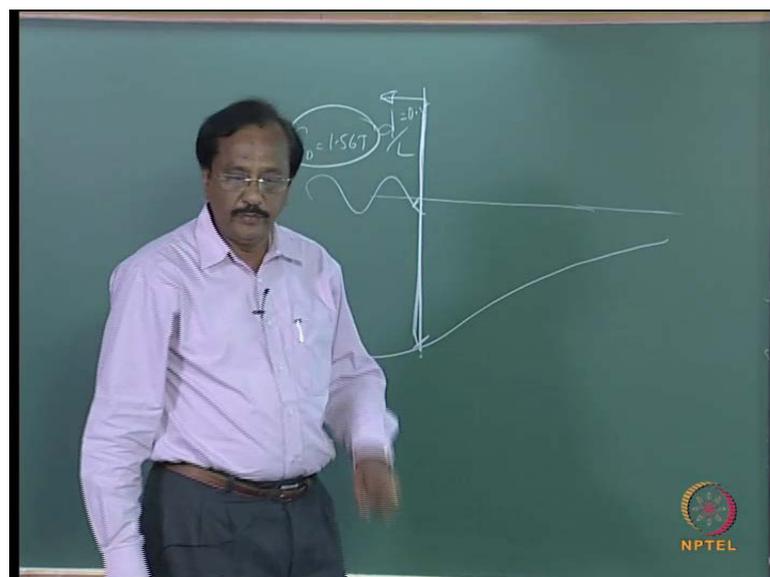
Oscillatory surface waves were observed in deep water and the wave period was found to be 8 sec.

- At what bottom depth would the phase velocity begin to change with decrease in water depth
- What is the phase velocity at a bottom depth of 16 m and 4 m
- Compute the ratio of celerity at the above water depth to deep water celerity

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So, we will go into the second problem. Second problem says, oscillatory surface waves that observed in deep water and the wave period were found to be 8 seconds. The question is at what water depth would the phase velocity begin to change with the decrease in depth, what does that mean? We have already seen that, when the waves propagate from deep to shallow water.

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The lecturer is standing in front of a chalkboard. On the board, there is a diagram of a wave with a vertical line representing the water depth d . A horizontal line is drawn at the top of the wave, and a vertical line is drawn from the water surface to the bottom. The equation $d = 1.56T$ is written on the board. The NPTEL logo is visible in the bottom right corner.

So, when d by L , d equals to 0.5 this is the area. So, beyond this you know that C naught is going to be a function of only wave period. So, when only when this condition reaches

your water depth I mean the waves will start sealing the seabed and that is the point at which your celerity or the speed of the wave will start changing, that is what is being asked. So, that particular point is nothing but, L naught by 2 which we will see later. What is the phase velocity? The next question is, what is the phase velocity at a bottom depth of 16 meters and 4 meters. So, you are given two water depths for which you need to calculate the celerity and probably, make a comparison with the deep water celerity.

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Solution

(i) $L_o = 1.56T^2 = 1.56 \times 8^2 = 99.84 \text{ m}$

The celerity will change, when water depth is less than

$$\frac{L_o}{2} = \frac{99.84}{2} = 49.92 \text{ m}$$

(ii) **At $d = 16 \text{ m}$**

$$\frac{d}{L_o} = \frac{16}{99.84} = 0.1603$$

From wave tables

$$\frac{d}{L} = 0.1923 \text{ corresponding to } \frac{d}{L_o} = 0.1603$$



Solution as usual you start of with your L naught and then, calculate your now in this problem you are supposed to calculate this L naught by 2. What is this L naught by 2, that is what I had just now explained, so that gives the water depth at which the speed of the wave is supposed to get change. And what is that water depth? That is going to be around 50 close to 50 meters.

So, water depth less than 50 meters, the wave length will keep on changing as the depth decreases. So, let us start with the other problem. The next subdivision is at water depth of 16 meters, I calculate my d by L naught and from the tables I calculate my d by L .

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$L = 83.203 \text{ m}$
Celerity = $\frac{L}{T} = \frac{83.203}{8} = 10.4 \text{ m/s}$
At $d = 4 \text{ m}$
 $\frac{d}{L_0} = \frac{4}{99.84} = 0.0400$
From wave tables
 $\frac{d}{L} = 0.0833$ corresponding to $\frac{d}{L_0} = 0.0400$
 $L = 48.025 \text{ m}$
Celerity = $\frac{L}{T} = \frac{48.025}{8} = 6.003 \text{ m/s}$



Calculate my wave wavelength and then, your celerity is calculated as 10.4 meters per second. So, you repeat the same calculation for water depth equal to 4 meters. Then you see that, the celerity decreases the speed of the wave decreases **as your wave** as the water depth decreases, that is what it implies. Now, you look at the variation now. So, here it is 10 meters per second and which is reduced to 6 meters per second in 4 meters water depth.

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(iii) **Ratio of Celerity**
 $C_0 = 1.56 \times T$
 $= 1.56 \times 8 = 12.48 \text{ m/s}$

At $d = 16 \text{ m}$,
 $\frac{C}{C_0} = \frac{10.4}{12.48} = 0.833$

At $d = 4 \text{ m}$,
 $\frac{C}{C_0} = \frac{6.003}{12.48} = 0.481$



So, the deep water celerity as we have calculated earlier is 1.56 into T , so which will be around 13 meters per second. So, when you try to work out the ratio this gives around a 0.83 is the variation. Whereas, as you go shallower you see that the order of difference would be almost 50 percent. So, you need to be very careful in calculating the wave length for the corresponding water depth, for which you are interested in designing the structure.

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Problem 3

Consider a particle initially 8m below the SWL and 20m above the sea bed. After the wave motion is established, what is the size and character of the orbit of the particle? Repeat the calculation for a particle at the surface and at the sea bed. The wave period, $T=10\text{sec}$ and deep-water wave height $H_0=3\text{m}$

Solution

Water depth, $d = 8+20 = 28\text{m}$

$L_0 = 1.56T^2 = 1.56 \times 10^2 = 156\text{m}$

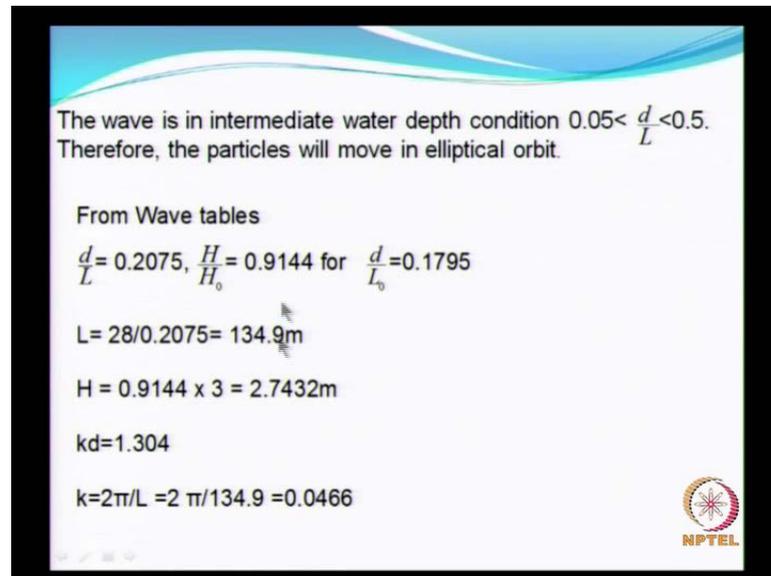
$\frac{d}{L_0} = 28/156 = 0.1795$

So, that is **those are** those are the two basic examples for calculating the most fundamental parameters, which you will be dealing with in wave mechanics. So, we move on to the third problem. Is there any doubts I think it is all quite straightforward. So, consider a particle I am talking about a fluid particle initially at 8 meters below the still water line, so this is the still water line and this distance is 8 meters and 20 meters this is not to the scale anyway and 20 meters above the seabed.

So, this is the elevation within the fluid medium, where you are required to find out some information. So, what are that information? You are supposed to find out the size and character of the orbit of the particle. Once you calculate your d by L you will know what will be character of the particle whether it will be a circular or an elliptical. So, let us find what are the parameters given to you, the parameters given to you are the wave period and the deep water wave height.

Remember you are given the deep water wave height. So, from this you get the water depth as 28 meters, the d by L naught as calculated as is calculated as shown here (Refer Slide Time: 12:11).

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The wave is in intermediate water depth condition $0.05 < \frac{d}{L} < 0.5$.
Therefore, the particles will move in elliptical orbit.

From Wave tables
 $\frac{d}{L} = 0.2075, \frac{H}{H_0} = 0.9144$ for $\frac{d}{L_0} = 0.1795$

$L = 28 / 0.2075 = 134.9\text{m}$

$H = 0.9144 \times 3 = 2.7432\text{m}$

$kd = 1.304$

$k = 2\pi/L = 2\pi/134.9 = 0.0466$

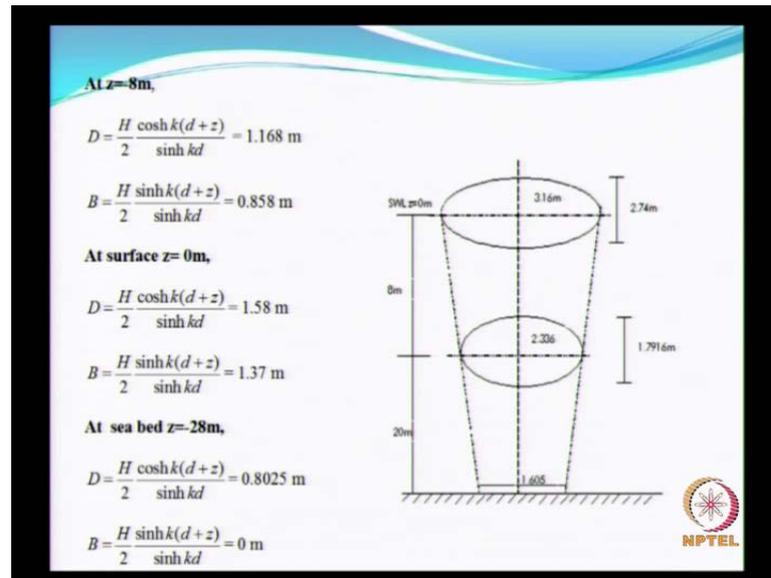


And from the wave table you also have a column that directly gives you H by H naught on the assumption that, the seabed contours are parallel based on that assumption you can use.

So, for this d by L you also now get here H by H naught. And so once you have calculated this you can calculate your wave length and also, the wave height. So, you see the difference the the common mistake of students usually normally make is try to use they use deep water wave height or water depth maybe, 10 meters or 5 meters or maybe the deep water wave length for 5 meters or 10 meters. This is the common mistake people make.

So, that is the reason why I am I am showing all the differences with the help of worked out examples. Now, you can appreciate the difference, so you have to be very careful. And all these things can also be either calculated or you can also try to get from the tables.

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So, let us take the 8 meters at z equal to 8 meters, z equal to minus 8 meters, this is minus (Refer Slide Time: 13:53). So, this is your still water line minus 8 meters is at this location. Recollect the formulas which we have seen the expressions, which we have seen for calculating the particle displacements. See, the particle displacements are provided here the formula for particle displacements are provided here.

This is the horizontal displacement whereas, this is the vertical displacement and this is nothing but, the semi major axis and the semi minor axis, which we have seen. So, substituting be sure that, z equal to minus 8 is substituted here, water depth is already known to you that is nothing but, 28 meters. Then, you calculate your D and that D only say two times the D is this, this is the mean axis. So, this is the semi major axis and this is going to be the semi minor semi major axis. So, the whole thing will be 2.3 something. And the value of D , which is the semi minor axis, is something like 0.86. So, the whole thing will be around 1.79.

So, before that itself you know that, because the **d by L value is the** d by L value is **0.24** 0.2 you know that it is going to be elliptical orbit. So, next at the surface, repeat the same calculations you will see that the value is 1.58 and 1.37 and B becomes 0 at the seabed, because of the formula. So, what and we also know that the vertical displacement will be almost negligible in intermediate water depth, near the seabed or it will be very negligible.

So, now you see how you are elliptical orbit, how the waves I mean fluid particle will be undergoing the motion here (Refer Slide Time: 16:39). So, it will be an elliptical orbit. So, if the direction of wave propagation is that like this, so **the orbit** the particle will be moving like this. And here its displacement will be as shown here and here it will be like this that clearly explains. And also, gives us a feeling for the magnitude. What could be the order of displacement for a given wave height and for given environmental conditions.

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Problem 4
 For a wave of height 1.5 m and period 6.5 sec, plot the variation of orbital velocity and acceleration in the vertical and horizontal directions of a particle at a position 2.8 m below SWL and 12 m above the sea bed. Estimate the maximum velocities at this position, at SWL and at the sea bed.

Solution:

$$d = 2.8 + 12 = 12.8 \text{ m}$$

$$\frac{d}{L_0} = 0.2245$$

From wave tables, $\frac{d}{L} = 0.2459$

From which $L = 60.19 \text{ m}$



So, that was a problem on the horizontal water particle displacement as well as the vertical displacements. Now, we going for the next problem which is fourth example, this is the fourth example. Now, we have a wave height of 1.5 meters and **6 meters** 6.5 second is the period. We are required to plot the variation of orbital velocity and acceleration, both in the vertical and the horizontal directions and the particle position is also given, which is 2.8 meters below the SWL I mean the Still Water Level and 12 meters above the sea bed.

You are now required to estimate the maximum velocities at this position some of the position that is, at this position means at that particular elevation of z equal to **2 point** minus 2.8 meters; **at the sea level** at the still water level and at the sea bed. Similar to what we have done for, the particle displacement in the last problem. So, you repeat the

calculations, you will you know now I think you **you** have enough knowledge in getting all the wave length and other usage of tables etcetera.

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At $z = -2.8$ m

$\cosh k(d+z) = 1.498$
 $\sinh k(d+z) = 1.115, \sinh kd = 1.607$

Substituting the values, we get

$$u_{\max} = \frac{\pi H \cosh k(d+z)}{T \sinh kd} = 0.6132 \text{ m/s}$$

$$w_{\max} = -\frac{\pi H \sinh k(d+z)}{T \sinh kd} = -0.5207 \text{ m/s}$$

$$\dot{u}_{\max} = -\frac{2\pi^2 H \cosh k(d+z)}{T^2 \sinh kd} = -0.5927 \text{ m/s}^2$$

$$\dot{w}_{\max} = -\frac{2\pi^2 H \sinh k(d+z)}{T^2 \sinh kd} = -0.5033 \text{ m/s}^2$$



So, you can also get all this information like your $\cosh kd$ etcetera, for z equal to minus **2 by** 2.8. Once you substitute all this values in these expressions that is u_{\max} is given by this expression. So, make sure that minus 2.8 meters is inserted there. Now, many of the students forget that **forget that** sine and they **(())** some value, which is totally incorrect. So, then also when you are answering to the questions of this nature you have to be very careful of the sign and this is another common mistake the students make.

So, u_{\max} is going to be positive in this case; it is around 0.6 meters per second whereas, w_{\max} will be a negative one with 0.52. I had already indicated how all these things vary the phase variations, etcetera I do not want to repeat again. And then, \dot{u}_{\max} what do you mean by \dot{u}_{\max} ? \dot{u}_{\max} etcetera that is the phase. See, you are supposed to have u equal to this much into $\sin \theta$, that $\sin \theta$ equal to 1. So, when $\sin \theta$ is equal to 1 you have the maximum value for velocity.

So, similarly, \dot{u}_{\max} will be as given here and w_{\max} is also given here. Although it looks like, it is just a simple substitution of the values for the variables and obtaining the result. I see many of the students making a number of mistakes in using this simple formula. So, when you are doing such important works you should also have the basic physics behind your mind.

So, for example, if you are having a sine curve and then, you are writing u_{\max} equals to a negative sign what does it imply for some guy some people can may get answer something they write u_{\max} equal to 0. So, these are all absurd. So, when you are working on not necessarily this area, any area when you are working with problems. You should also have the physics behind your mind, when you are trying to solve all this problems.

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At $z = 0m$

$\cosh k(d+z) = 2.4509, \sinh kd = 2.238$

Substituting the values, we get

$$u_{\max} = \frac{\pi H \cosh k(d+z)}{T \sinh kd} = 0.794 m/s$$

$$w_{\max} = -\frac{\pi H \sinh k(d+z)}{T \sinh kd} = -0.725 m/s$$

$$\dot{u}_{\max} = -\frac{2\pi^2 H \cosh k(d+z)}{T^2 \sinh kd} = -0.7675 m/s^2$$

$$\dot{w}_{\max} = -\frac{2\pi^2 H \sinh k(d+z)}{T^2 \sinh kd} = -0.7008 m/s^2$$



Now, at z equal to 0 that is at still water line. So, again you substitute all this values get all the values I mean u_{\max} . So, naturally u_{\max} is expected to be higher at the still water, so that is what you are getting here. And then w_{\max} , \dot{u}_{\max} .

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At $z = -14.8$ m

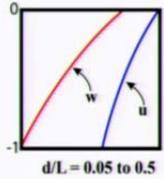
$\cosh k(d+z) = 1$, $\sinh kd = 2.238$, $\sinh k(d+z) = 0$

Substituting the values, we get

$$u_{\max} = \frac{\pi H \cosh k(d+z)}{T \sinh kd} = 0.3239 \text{ m/s}$$

$$w_{\max} = -\frac{\pi H \sinh k(d+z)}{T \sinh kd} = 0$$

$$\dot{u}_{\max} = -\frac{2\pi^2 H \cosh k(d+z)}{T^2 \sinh kd} = -0.3131 \text{ m/s}^2$$

$$\dot{w}_{\max} = -\frac{2\pi^2 H \sinh k(d+z)}{T^2 \sinh kd} = 0$$


$d/L = 0.05$ to 0.5



You repeat the calculations for z equal to minus 14.8. Now, if you plot if you just put all this values here along the y axis along horizontal axis for example. So, **I** we have worked out only for three different elevations. You can do it for the entire water depth **right** and then, at each elevation you will have velocity acting and if you join **all them** all of them; the curve is something like this and it is going to be a hyperbolic variation. And this is what is explained earlier, the variation of particle displacement etcetera. And now we are trying to understand with an example, is that ok. So, we will go into the next problem.

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Problem 5

A wave with a height 4.5 m and wave length 75m propagates in a water depth of 20m. Determine the local horizontal and vertical velocities at a depth 6m below the SWL at a position one sixth a head of the wave crest.

Solution:

$L = 75\text{m}$

$\frac{1}{6}L = 12.5\text{m}$

$$\theta = \frac{12.5}{75} \times 360 + \frac{1}{60} \times 360 = 120$$

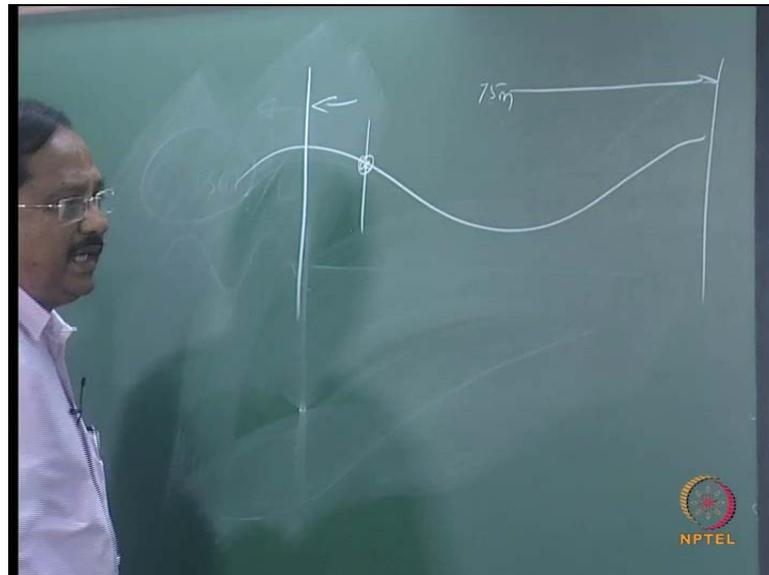
At $z = -6\text{m}$

$$u = \frac{\pi H \cosh k(d+z)}{T \sinh kd} \sin \theta = 0.782 \text{ m/s}$$

$$w = -\frac{\pi H \sinh k(d+z)}{T \sinh kd} \cos \theta = -0.907 \text{ m/s}$$


So, earlier we had looked into the maximum velocity and maximum acceleration. There may be at some point of time, there may be an interest of finding out all these values of these variables, for not at the maximum point, but at some other phase. So, for example, here a wave with a height of 4.5 meters and a wave length is also given here as, 75 meters propagates in a water depth of 20 meters. Determine the local **and** horizontal and vertical velocities in a depth 6 meters below the still water line, what does that mean? That means you are having a wave.

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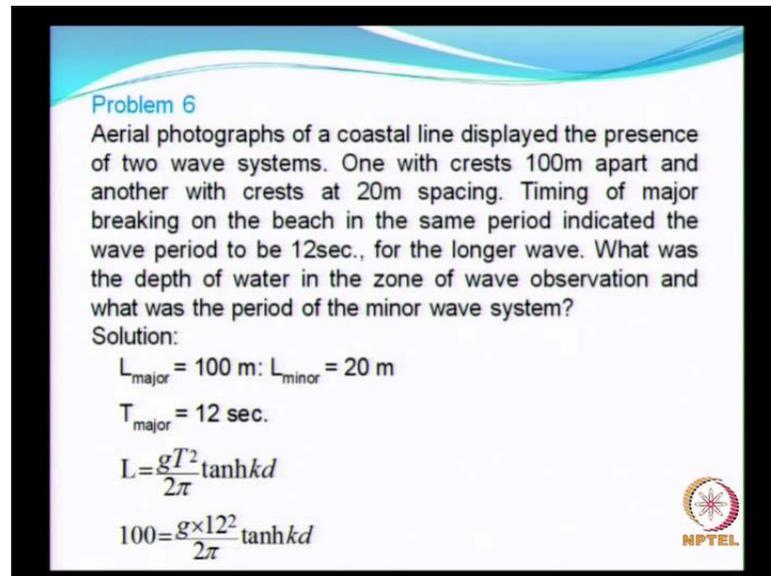


The problem says that, there is wave moving, the total wave length is 75 meters and it says that, it is one sixth a head of the wave crest, that is one sixth is somewhere here (Refer Slide Time: 25:51). What we are supposed to find out is, what are the values of your local horizontal and vertical velocities and accelerations? Or in this case, its just simply asked the for the velocities, at still water level at the position of one sixth of the a head of the wave crest; So, that means I calculate 1 by 6, that is what is indicated here.

So, this will be now 12.5 meters. Then, I can calculate the phase, the total wave length is for 360 degrees (()) 12.5 I can calculate. So, the angle will be something like 120 degrees, is that clear. So, then for this value this is the phase. And now you see **the** at different z's I mean the at different elevations.

So, in this case it is minus 6 meters. So, use this and then make sure that you are using the appropriate sine value I mean the sine or cosine and then, that will result in a value of. So, this will be definitely less than the u_{\max} or w_{\max} .

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Problem 6
Aerial photographs of a coastal line displayed the presence of two wave systems. One with crests 100m apart and another with crests at 20m spacing. Timing of major breaking on the beach in the same period indicated the wave period to be 12sec., for the longer wave. What was the depth of water in the zone of wave observation and what was the period of the minor wave system?

Solution:

$$L_{\text{major}} = 100 \text{ m}; L_{\text{minor}} = 20 \text{ m}$$

$$T_{\text{major}} = 12 \text{ sec.}$$

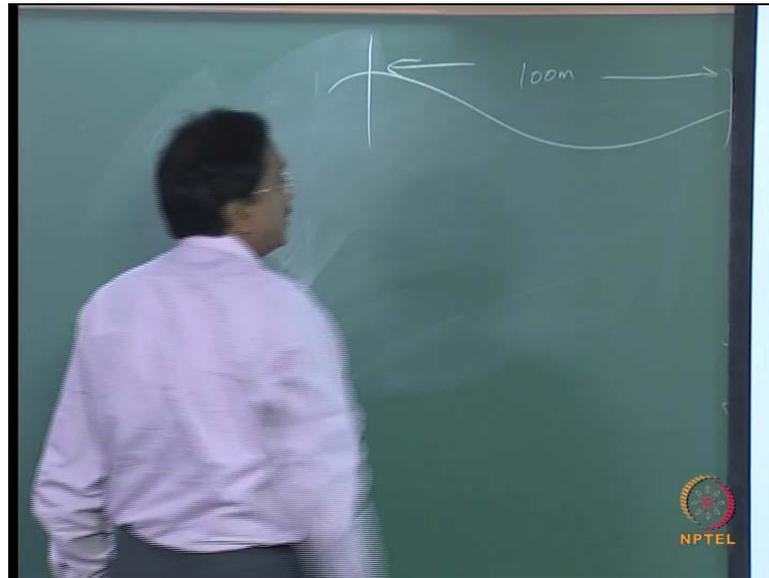
$$L = \frac{gT^2}{2\pi} \tanh kd$$

$$100 = \frac{g \times 12^2}{2\pi} \tanh kd$$



So, this **this** problem may be useful particularly, when you are using the **(())** etcetera to understand the or to derive the characteristics of waves. The problem states that, aerial photographs of a coastal line displayed the presence of two wave systems. The exercise is more of getting used to dealing with the different variables associated in this subject, that is the purpose of all this example, worked out examples **one of the one** two wave systems; one with crest apart 100 meters. So, one is I have a wave length of 100 meters.

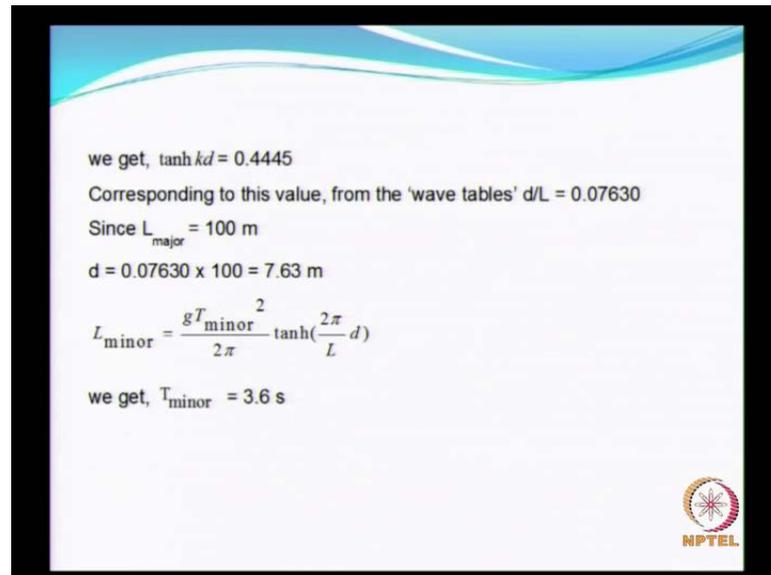
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And another has a wave of 20 meters, so there are two waves which is propagating. And at the time of major breaking, when the measurements are being taken it is observed that the wave period for this wave for the longer wave is 12 seconds. What is required by you is to find out, what is the depth at which the waves were observed that is one thing. The next one is what is the period of the **smaller the** wave with a smaller length? How do we use the same equation to arrive at all these values?

So, you see L major is I am calling it longer wave as L major and the smaller one as L minor. So, this is going to be 100 meters and this is 20 meters and the T major is 12 seconds. So, use your usual expression that is the expression for the wave length and for this. So, you should not get mixed up. Now, this two is a one pair. So, use that gravitation constant **you know**. So, I can determine the $\tan h k d$. Once I get the value of $\tan h k d$ then, I can just simply get my $k d$ value.

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we get, $\tanh kd = 0.4445$
Corresponding to this value, from the 'wave tables' $d/L = 0.07630$
Since $L_{\text{major}} = 100 \text{ m}$
 $d = 0.07630 \times 100 = 7.63 \text{ m}$
$$L_{\text{minor}} = \frac{gT_{\text{minor}}^2}{2\pi} \tanh\left(\frac{2\pi}{L} d\right)$$

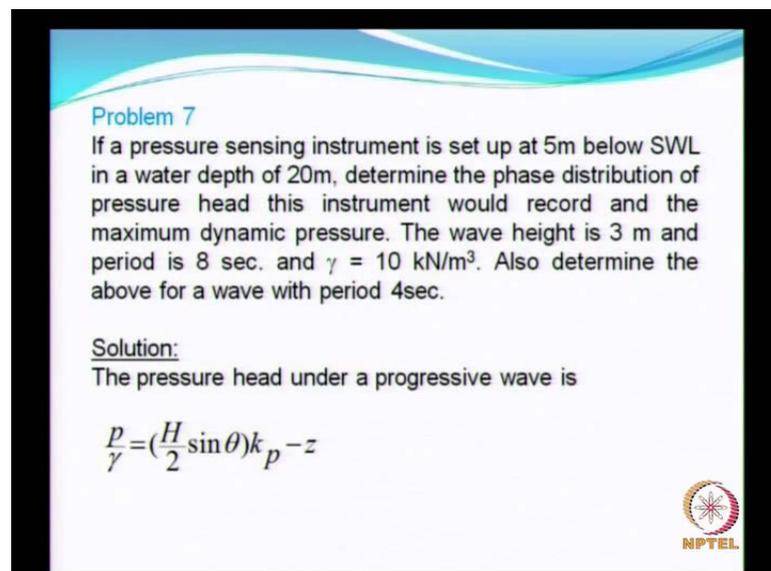
we get, $T_{\text{minor}} = 3.6 \text{ s}$



Or d by L value I can directly get from the wave tables. Once that is obtained since, the wave length is 100 meters for this wave you can get the water depth and water depth here in this case appears to be 7.63 meters.

Now, that **you know** the depth and it is clearly said in the problem that, the depth of observation is same. Now, use this depth and L minor is already given to you as 20 meters substitute that and use this equation to get the T of the I mean the wave period of the smaller wave. I hope you are not tired, shall we go a head.

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Problem 7
If a pressure sensing instrument is set up at 5m below SWL in a water depth of 20m, determine the phase distribution of pressure head this instrument would record and the maximum dynamic pressure. The wave height is 3 m and period is 8 sec. and $\gamma = 10 \text{ kN/m}^3$. Also determine the above for a wave with period 4sec.

Solution:
The pressure head under a progressive wave is

$$\frac{p}{\gamma} = \left(\frac{H}{2} \sin \theta\right) k_p^{-z}$$


So, having seen some of this, now we move on to dynamic pressures. I have already said, why are we interested in knowing about the dynamic pressures under propagating waves. One is the example for monitoring of I mean warning of tsunami etcetera. And also the other one is for getting the obtaining the wave climate **with the** with the measurement of pressure you can get the wave climate from the pressures.

So, the problem says, if a pressure sensing instrument is set up at 5 meters below the still water level in a water depth of 20 meters, determine the phase distribution of the pressure head **this would** this instrument would record and also the maximum dynamic pressures. Because, when you put a pressure sensor it is going to continuously record the pressure. So, one is how does this record look like the phase information. And all these things we are discussing about a regular wave only.

The wave height is 3 meters and the period is 8 seconds, γ is 10 kilo Newton per meter cube. Also determine the above that is the pressure and the distribution for a wave with a period of 4 seconds. So, we are basically having two waves of same wave height, but of two different periods; one is a long wave and another is a short wave.

So, we have a derived the pressure distribution from the basic Bernoulli equation that is linearized Bernoulli equation. So, the pressure distribution pressure head is given by like this, where **where** your k_p is **where your k_p is** pressure response factor. So, if you are not able to understand go back to the basic lecture material, so all where everything is described in detail.

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For $T = 8 \text{ sec}$, $d = 20 \text{ m}$

L is calculated as 88.85 m

$k = 0.0706$

$kd = \frac{2\pi}{L} \times d = 1.414$

$k_p = \frac{\cosh k(d+z)}{\cosh kd} = 0.741$

Dynamic pressure head, $\frac{p}{\gamma} = \frac{3}{2} \times 0.741 \times \sin \theta$

Maximum dynamic pressure, $p = 1.5 \times 0.741 \times 10006.2 \times \sin 90^\circ = 11121.9 \text{ N/m}^2$
 $= 1.11 \text{ m of water column}$

Total Pressure, $p = \left[\left(\frac{3}{2} \sin \theta \right) 0.741 + 4 \right] 10006.2 = 51146.7 \text{ N/m}^2$



So, let me take T equal to 8 seconds. So, usual procedure wave length is calculated I am skipping that you all know how the wave length is calculated. And once wave length is calculated, I calculate the pressure response factor that is what is needed all other things are known. So, pressure response factor is calculated as 0.741, what will be the pressure head? The dynamic pressure head will be only this part (Refer Slide Time: 35:42). I am not considering the static pressure.

So, when you take the dynamic pressure head alone, so $\frac{3}{2}$ this is 3 is the wave height and there is a pressure response factor and this is going to be your pressure head. The next question is, what is the maximum dynamic pressure? So, I just simply calculate the pressure this is the shifted here. And then, where is the maximum dynamic pressure going to occur this formula was derived for a sinusoidal wave.

So, at sine 90 the pressure is going to be maximum and that is what we get here. And this is something like so much 11000 Newton per meter square, which is approximately 1.11 meter of water column. So, this is the dynamic pressure head this is the maximum dynamic pressure (Refer Slide Time: 36:57). And what is the total pressure head? The total pressure **total pressure** is I just take this gamma into out and this into gamma will be the total pressure, which is given here (Refer Slide Time: 37:15).

But of course, this you **you** note that here, there is one sine theta, so the pressure will be varying as per your sine theta. If you want the variation the phase variation of the

pressure you use this phase variation. So, the phase variation of the dynamic pressure will be the same as that of u or your η .

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For $T = 4$ sec., $d = 20$ m, $\frac{d}{L} = 0.8$,

$L = L_0 = 25$

$k = \frac{2\pi}{L} = 0.2512$

$kd = 5.024$

$k_p = \frac{\cosh k(d+z)}{\cosh kd} = 0.2848$

Maximum dynamic pressure, $p = \left[\frac{3}{2} \times 0.2842 \times \sin 90^\circ \right] 10006.2 = 4265.7 \text{ N/m}^2$

Note:
For a wave period of 4 s the total pressure is less when compared to the wave period of 8 s for the other conditions not being changed. **So, for longer waves the pressure is more.**

Now, you go in and repeat the same calculations for T equal to 4 that is you are considering a smaller wave. And now you **you** see that, the maximum dynamic pressure is 4265. So, compared to what we got here for a 8 second wave you see that this is much less. So, what does this indicate? The pressures exerted by long waves are higher than the pressures exerted by smaller waves, so that is what is now demonstrated.

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Problem-8

A subsurface pressure type recorder is installed at a depth of 4m at the point where water depth is 12m. The average maximum pressure and the period registered by the recorder are 3bar and 9 sec respectively. Compute η , $\gamma = 10006.2 \text{ N/m}^3$.

Solution:

$k = 0.0714$

$k_p = \frac{\cosh k(d+z)}{\cosh kd} = 0.84$

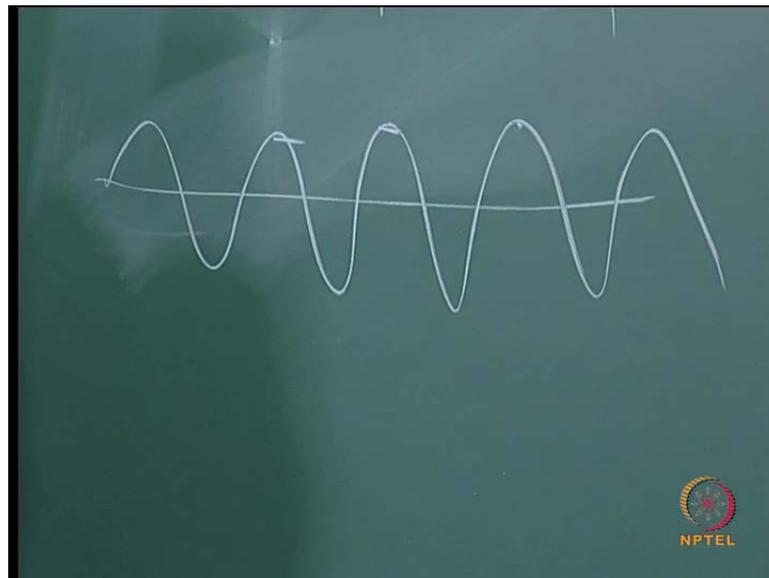
$\frac{p}{\gamma} = \eta k_p - z$

$\frac{3 \times 10^5}{10006.2} = \eta \times 0.84 - (-4)$ $1 \text{ bar} = 10^5 \text{ N/m}^2$

$\eta = -1.193 \text{ m}$

Next, we go on to again one more problem with the pressure measurement. A subsurface pressure type recorder is installed in a water depth of 4 meters at a point, where water depth is 12 meters. So, here z equal to minus 4 meters and water depth d equals to 12 meters, the average maximum pressure and the period registered is by the recorder is 3 bar and 9 seconds. What is this average maximum pressure? Average maximum pressure indicates.

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When you are measuring we are considering a **a** dynamic pressure, which is sinusoidal going to be its sinusoidal, but this crests there may be small difference, sometimes the difference can be slightly large also. So, we take the average of that and that is what it indicates. So, the average dynamic pressure that is measured is here.

So, you can try to get the solution as given here k is calculated then, k_p pressure response factor is calculated, you know the value of the pressure. Now, the idea is to find out at the point at which you have measured what is the point over the wave cycle it corresponds to. Since I have used a **maximum** average maximum pressure, it can be slightly different. So, this will be when a trough is there that is from this picture, we get the value of η as minus 1.19 meters. Now, we move on to the next problem.

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Problem 9
A maximum pressure of 1bar is measured by a sub-surface pressure recorder located at 0.5m above the sea bed in a water depth of 10m. The average wave frequency is 0.08 cycles/sec. and $\gamma = 10006.2 \text{ N/m}^3$. Determine the wave height.

Solution:

$$\frac{p}{\gamma} = \eta k_p^{-z}$$

The maximum pressure would occur at $\eta = H/2$ (Crest of wave)

$$p_{\max} = 100000 \text{ N/m}^2, \gamma = 10006.2 \text{ N/m}^3,$$
$$z = -(10-0.5) = -9.5$$
$$T = 1/0.08 = 12.5\text{s}, L = 118.4\text{m}$$


A maximum pressure of 1 bar is measured by a **surface** sub-surface pressure recorder that is located 0.5 meters above the sea bed in a water depth of 10 meters. So, the average wave frequency is given as 0.08 cycles per second and gamma **you know** now here, since it is the maximum pressure that has been recorded. So, you can directly get your wave height also, how do we get this? So, p_{\max} is given here and z can be calculated. So, your T can be calculated as **12.5 meters** 12.5 seconds from which you can calculate your wavelength.

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$$k_p = \frac{\cosh k(d+z)}{\cosh kd} = 0.8743$$

Substituting for the variables in the above formula

$$\frac{100000}{10006.2} = 0.8743 \times \frac{H}{2} - (-9.5)$$

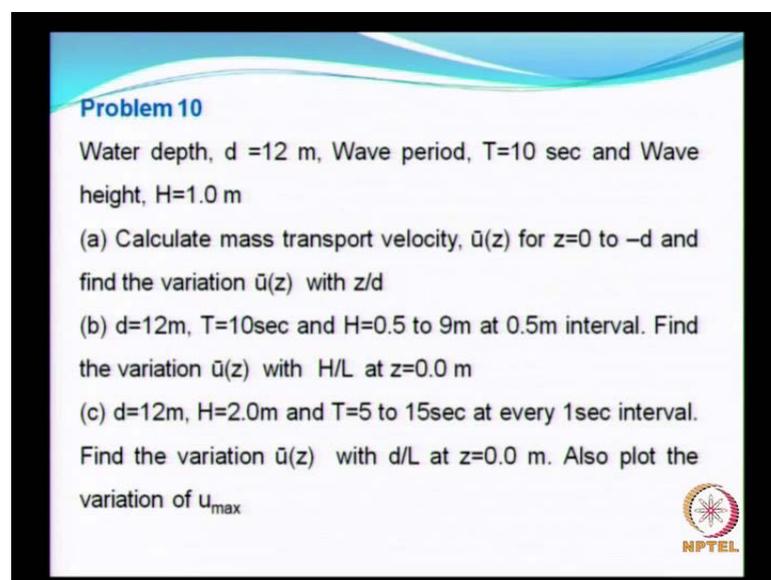
We get,

$$H = 1.13\text{m}$$


$k p$ also can be calculated. Now, you substitute this in η , η is now the maximum pressure it can record that is when the crest. So, that is why I have put H by 2 here in γ in the original formula of γ into that equation, which we have seen earlier.

So, you can get you will get wave height is equal to so much. So, when the wave height is of this order you will get an average pressure as measured or indicated in the problem, is that clear, so now having seen few problems on pressure. We will now move on to a problem that relates to mass transport velocity.

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Problem 10

Water depth, $d = 12$ m, Wave period, $T = 10$ sec and Wave height, $H = 1.0$ m

(a) Calculate mass transport velocity, $\bar{u}(z)$ for $z = 0$ to $-d$ and find the variation $\bar{u}(z)$ with z/d

(b) $d = 12$ m, $T = 10$ sec and $H = 0.5$ to 9 m at 0.5 m interval. Find the variation $\bar{u}(z)$ with H/L at $z = 0.0$ m

(c) $d = 12$ m, $H = 2.0$ m and $T = 5$ to 15 sec at every 1 sec interval. Find the variation $\bar{u}(z)$ with d/L at $z = 0.0$ m. Also plot the variation of u_{\max}

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Here, in we will also first initially understand the variation of mass transport velocity as a function of different parameters. And finally, we will also try to relate, how it is related with the horizontal particle velocity. What is the kind of magnitude or what is the kind of difference we have within these two parameters that is the is idea of this problem.

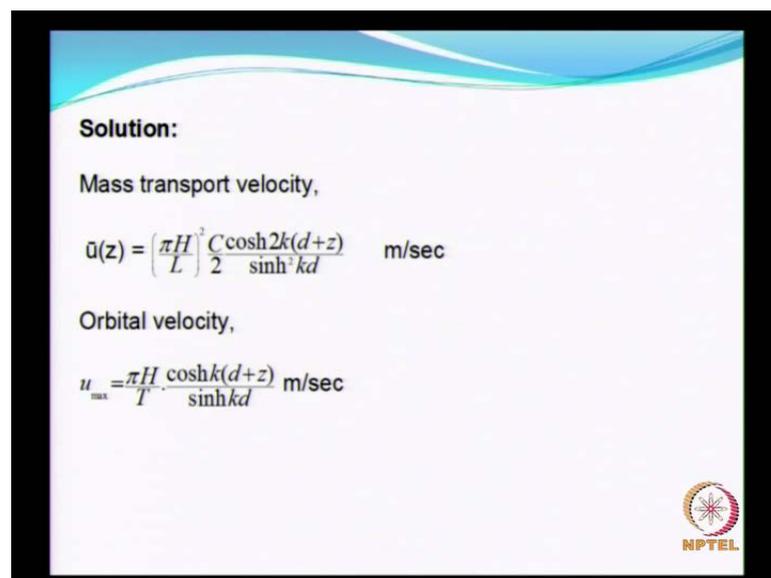
So, we are considering the water depth of 12 meters, wave period is 10 seconds, wave height is 1 meter. The first subdivision is calculate the mass transport velocity that is given as, \bar{u} of z for z equal to 0 to minus d . So, what we are trying to do is how the mass transport velocity is varying along with depth. And the variation of the mass transport velocity with z by d that is nothing but, the variation along the water depth.

The second subdivision is, we are interested to find out the variation of mass transport velocity as a function of γ . As we have already seen in the formula that, if the wave γ

)) H by L increases it increases. And I also indicated that, the practical example another practical example is that during a storm and that is the time when you have lot of debris from the ocean getting washed away towards the shore and that is the time when your wave steepness is expected to be more. So, the formula also clearly reflects this.

So, we will just check how it varies. And finally, for a given water depth of 12 meters and wave height of 2 meters we will vary the wave period from 5 to 15 seconds. And then, try to find out the variation of the mass transport velocity as a function of d by L. And this exercise we will try to do it only at the still water line, because at each elevation is you can easily get that is not a problem. But, we will just look at how it varies only at the still water line you understood now. **So, first.** So, finally, we will also try to **vary it** draw the variation of u max.

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Solution:

Mass transport velocity,

$$\bar{u}(z) = \left(\frac{\pi H}{L} \right)^2 \frac{C \cosh 2k(d+z)}{2 \sinh^3 kd} \quad \text{m/sec}$$

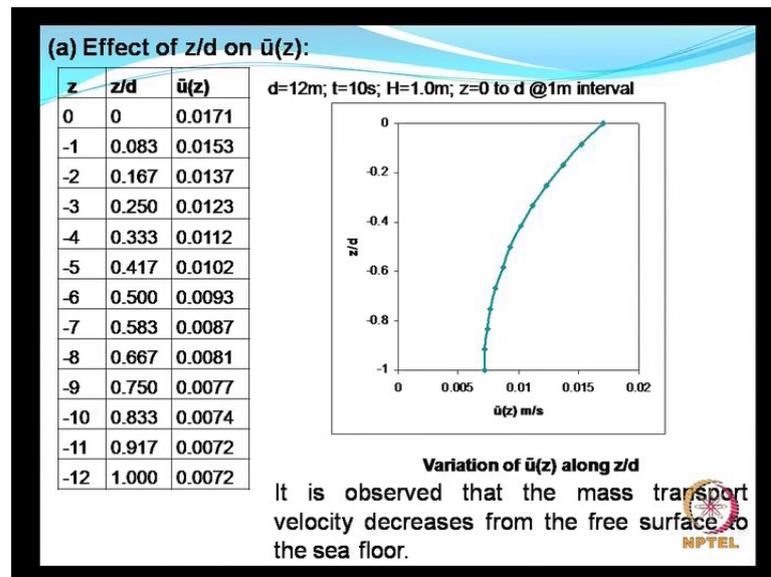
Orbital velocity,

$$u_{\max} = \frac{\pi H}{T} \frac{\cosh k(d+z)}{\sinh kd} \quad \text{m/sec}$$



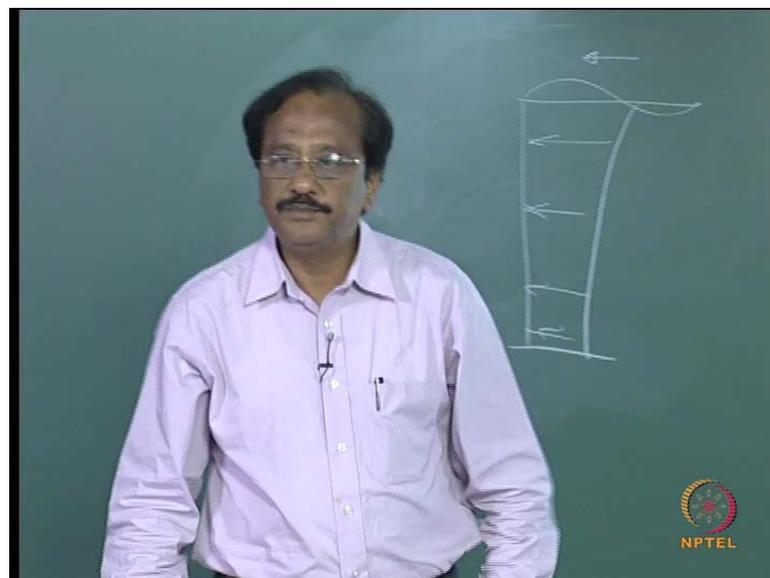
So, this is the formula for mass transport velocity top one, which is going to be a function of steepness H L square and then, this is the celerity and all other parameters are already known to you. Then, this is the orbital velocity and this is we are considering only the maximum orbital velocity (Refer Slide Time: 45:50). So, the phase variation etcetera will not come into the picture.

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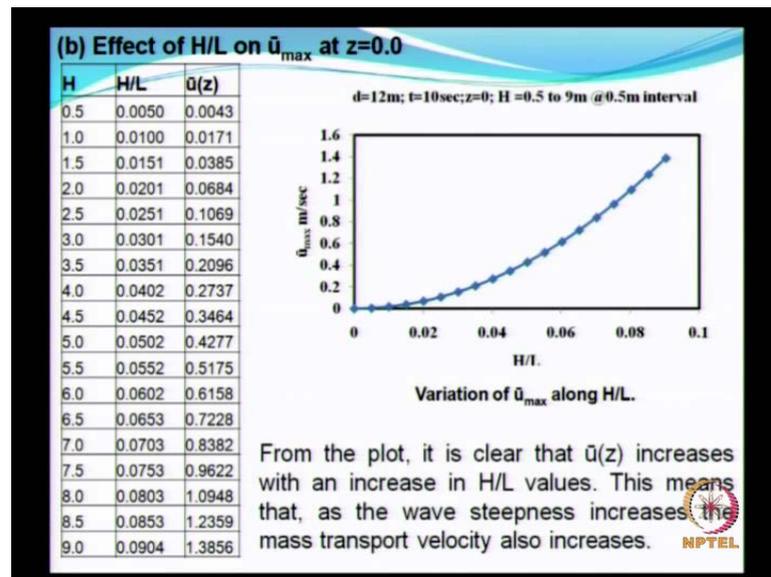
Now, you see that we have used the above formula to calculate the variation of the variation of \bar{u} as a function of z by d . So, this is your still water line and this is the this is the sea bed; and the variation is like this. In fact, if you look at this in elevation.

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This is sea, this is the wave direction this is what we have tried to show here. So, you see that it is of course, a hyperbolic variation and this gives the kind of variation you can anticipate with respect to the mass transport velocity (Refer Slide Time: 46:48).

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Now, using the same thing we have just varied the wave height, just to find out how all this variations can look like. So, you see that, there is a steep increase in the mass transport velocity with the increase in wave steepness. And a small increase in wave steepness is quite good enough to result in a substantial increase in the variation in the mass transport velocity. This has been done only at the still water line that is z equal to 0. Just to demonstrate how it varies with respect to the H by L .

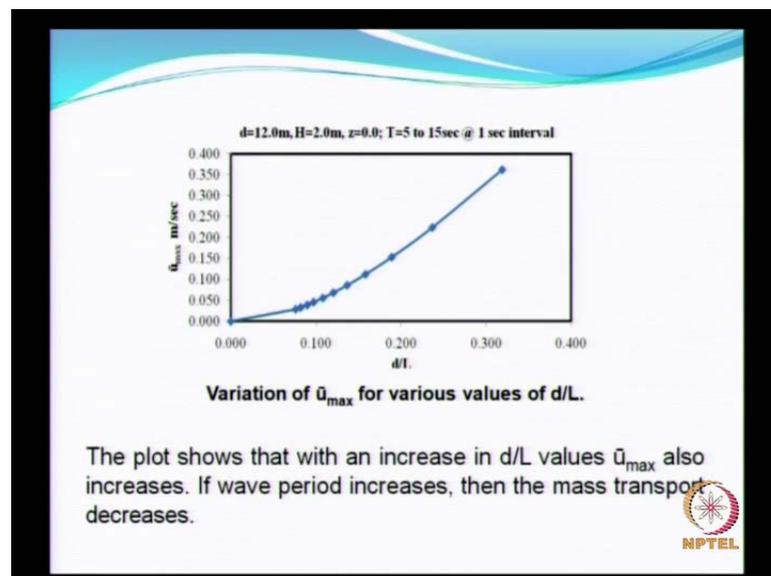
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(c) Effect of d/L on $\bar{u}(z)$:

T	L_0	d/L_0	d/L	L	C	\bar{u}_{max}	u_{max}	u_{max}/\bar{u}_{max}
5	39	0.308	0.319	37.6412	7.528	0.362	1.30267	3.59804
6	56.16	0.214	0.237	50.6971	8.450	0.224	1.15952	5.17618
7	76.44	0.157	0.189	63.4585	9.065	0.153	1.08113	7.04799
8	99.84	0.120	0.158	75.9013	9.488	0.112	1.03482	9.22155
9	126.36	0.095	0.137	87.8477	9.761	0.086	1.00378	11.64692
10	156	0.077	0.121	99.5851	9.959	0.068	0.98255	14.35976
11	188.76	0.064	0.108	110.9057	10.082	0.056	0.96569	17.28953
12	224.64	0.053	0.097	123.3806	10.282	0.046	0.96080	20.87657
13	263.64	0.046	0.090	133.4668	10.267	0.039	0.94460	24.05266
14	305.76	0.039	0.082	146.0743	10.434	0.033	0.94529	28.37048
15	351	0.034	0.076	157.2739	10.485	0.029	0.93961	32.53078

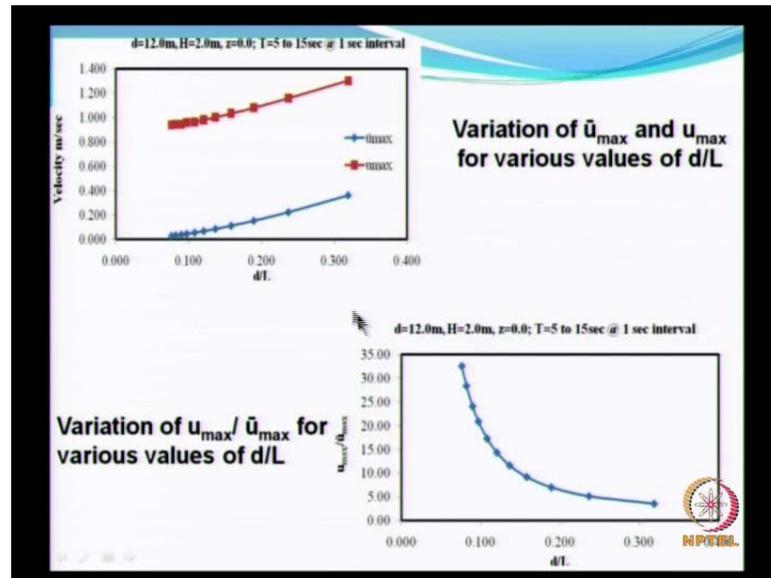
Then, the relative water depth, how does it vary with respect to relative water depth. This table gives you the variation of the relative water depth as you can see here. So, as d by L increases, this is the variation of the mass transport velocity (Refer Slide Time: 48:05). And this is the variation of the orbital velocity that is maximum horizontal water particle velocity. And this two are evaluate are the still water line, because these values will vary along the different elevation and this is the ratio. So, the ratio keeps on increasing as here d by L decreases, is that clear.

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So, this is the variation of the mass transport velocity for different values of d by L . So, you see that it increases.

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Now, this is the variation of your mass transport velocity and your horizontal water particle velocity given here. And this is the u_{max} orbital horizontal orbital velocity at the still water of course, and both are at the still water, and this is the mass transport velocity (Refer Slide Time: 49:04).

So, the ratio is plotted here and you see that, there is a steep decrease in the ratio and as d/L increases it reduces. So, we have started the basic of wave mechanics from what is meant by wave height, what is meant by wave period, then wave length and then wave moves what are the happenings that is the distribution of particle velocities, how these particles are moving in elliptical or circular orbit depending on the wave conditions, how the waves are classified according to apparent shape, water depth, then type of as per the origin etcetera.

And then we went on to understand about the pressures; dynamic pressures, static pressures etcetera; and how the pressures vary under a crest, under a trough. And then how all this variables are useful, and why they are needed for I mean, for the sake of design of maritime structures etcetera. So and then finally, we have also looked at the mass transport velocity. Another physical important parameter, which is related to I mean coastal or ocean engineering practice. I think, with this we have completed the basic wave mechanics, and then we will look into the other chapters later.