Wave Hydro Dynamics

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Wave Motion and Linear Wave Theory

Module No. # 02

Lecture No. # 04

Wave Motion III

So, today we will continue with the phenomena of group celerity, energy, power and mass transport velocity.

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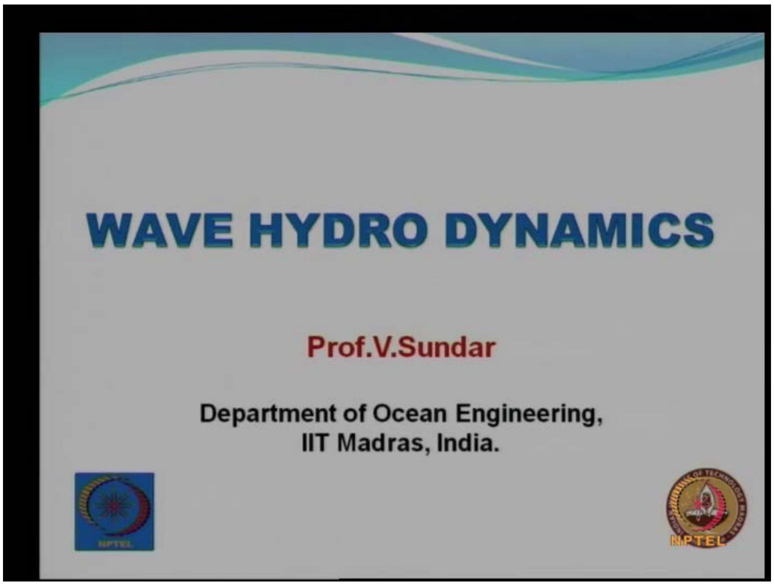

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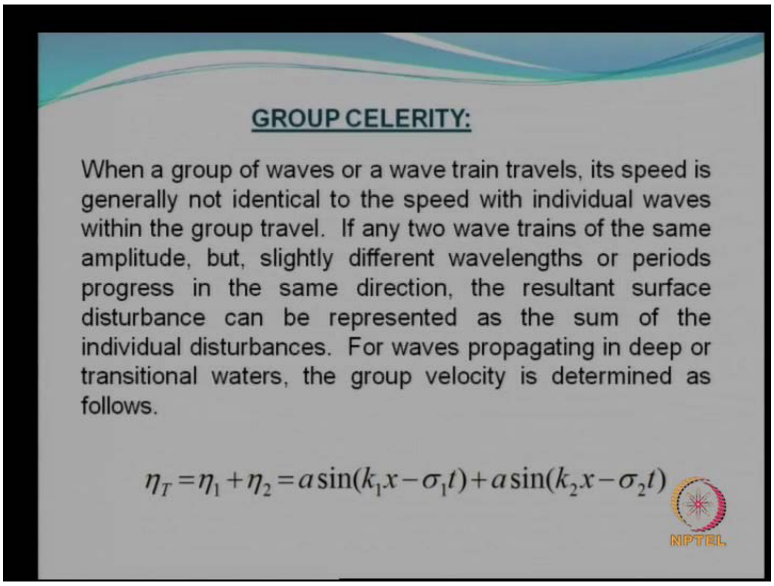

What is group celerity? The definition is very clear here. When a group of waves or a wave train travels, its speed is not identical to the speed with which the individual waves within the group move. So, if any two wave trains of same amplitude, let us assume that two wave trains are moving with the same amplitude, but has a slightly different wavelengths and we are assuming that they propagate in the same direction. The resultant surface disturbance can be represented as summation of these two components (()) disturbances.

So, let us say that eta 1 and eta 2 are now represented as shown here, so this is eta 1 and this is eta 2, both having same amplitude, but different frequencies indicated by k 1 or k sigma 1 and k 2 or sigma 2, so we linearly superpose.

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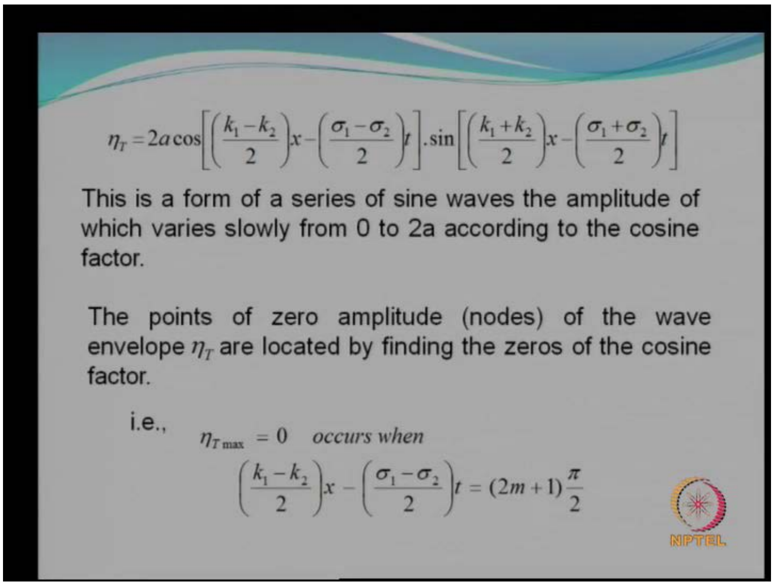

So, when you add these two, it can be represented as basic trigonometry, so 2 a into cos of this argument plus sine of this argument, what does this indicate? The surface due to the superposition of the two sinusoidal components will again be a sine form of the disturbance or the displacement.

So, which is given by the phase is now given by this quantity, sine of this quantity, where as this will the whole thing will indicate will be the amplitude, so this is a form of series of sine waves with the the amplitude varying between 0 to 2, and the amplitude of this is going to be dictated by this factor, that is the argument of the cosine factor. See, in one of the last previous classes we have also seen, how the disturbance can look like when you superpose two or more than two components, so try to recollect what you have seen earlier.

So, this disturbance will have some kinds of what is called as points, which are called as nodes, nodes are nothing but, points of 0 amplitude, so in a normal sinusoidal wave also you have point of 0 amplitude right, so when you superpose you expect at some points you there should be some 0 0, the amplitude should be 0. So, how do you get this since, the argument of the cosine factor that is the cosine function, is going to dictate the amplitude, so that is going to can be represented like, what I am trying to say here is, eta max will be 0, when this argument takes a value. What is that value, that can be represented as in this form, that is 2 m plus 1 into pi by 2, so depending on the value of m you will be having 0 points or 0 amplitudes, is that clear so we will go further.

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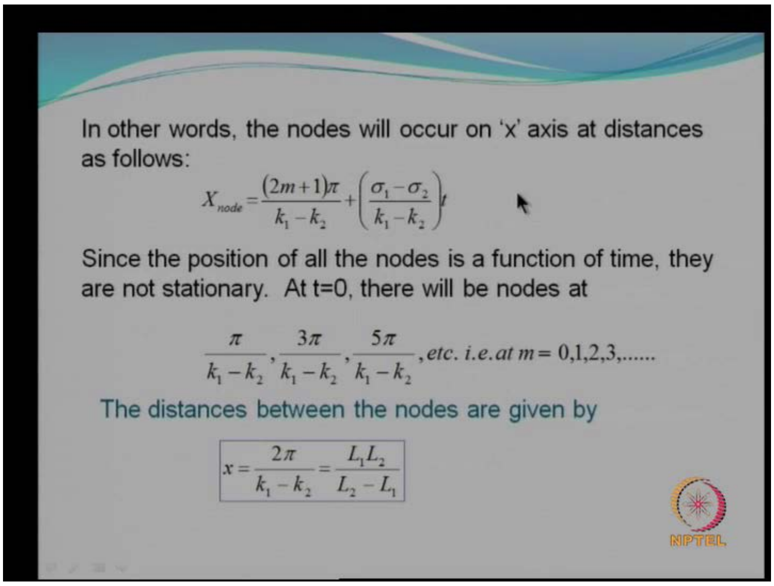

So, in other words, nodes will occur on the x axis, please remember that we are working with the two axis, one is time axis and another is the space axis. So, now we are talking about the space axis which is x axis, so at what locations you can determine this x, which is now represented from the earlier equation or expression as shown here.

So, along the propagation of the wave, you have a line which is represented as the space and then when you visualize this at a given point of time you will have a different locations points of 0 amplitude, so the since, the position of all the nodes is a function of time, so they are not stationary that is obvious right.

So, let us say that at any given point of time, if you want to take a photograph at a given point of time in a channel where you have generated some such kinds of waves, then you will see different locations along the your testing facility at different locations you will have the 0 amplitude; and that is going to be at time t equal to there will be nodes at this (()), so you have to just put t equal to 0.

So, this value x nor will take different values depending on the value of m, now you have nodes at distances as given here, what is the distance between any two nodes, so any two nodes will be the difference between these two as given here. But, what is k 1 and k 2, k

1 and k 2 can be represented as 2 pi divided by L 1, 2 pi divided by L 2, which when substituted you will get as shown here. So, the distance between the nodes is now represented in terms of wavelengths of the two waves which are superposed.

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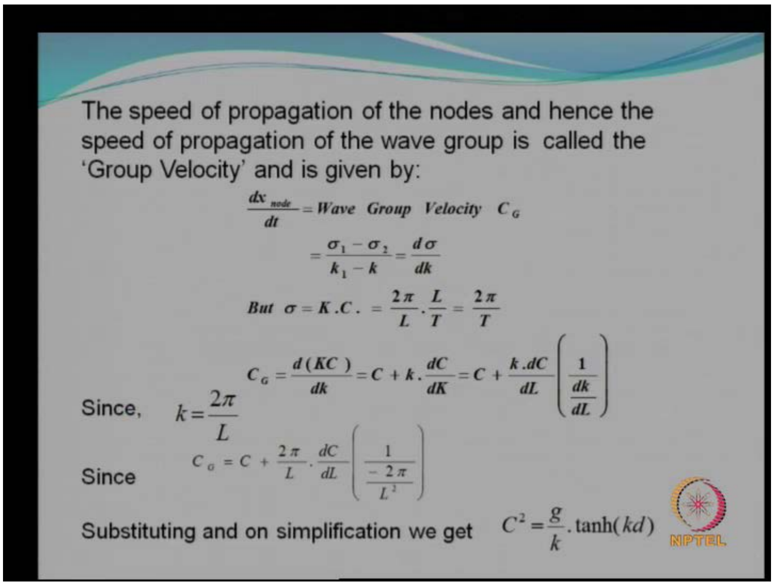

Now, once you have found out this x, you just differentiate with respect to time and when you differentiate it with respect to time obviously it is going to give you the speed, and what is that speed, that speed is nothing but, the speed with which the entire waves is going to move as a group; which obviously will be different from the speed with which the individual waves will be moving, so this is called as the group celerity.

So, when we continue this, so this can easily be given as d sigma by d k, but sigma can be sigma is what, 2 pi by T which is nothing but, a product of k which is the 2 pi by L and celerity. So, now I proceed now, I differentiate sigma which is represented now by K C and when you proceed with the differentiation you will finally land up with this kind of an expression, but we have already seen that the celerity is given by g by k tan h k d, what is this relationship? This is nothing but, dispersion relationship.

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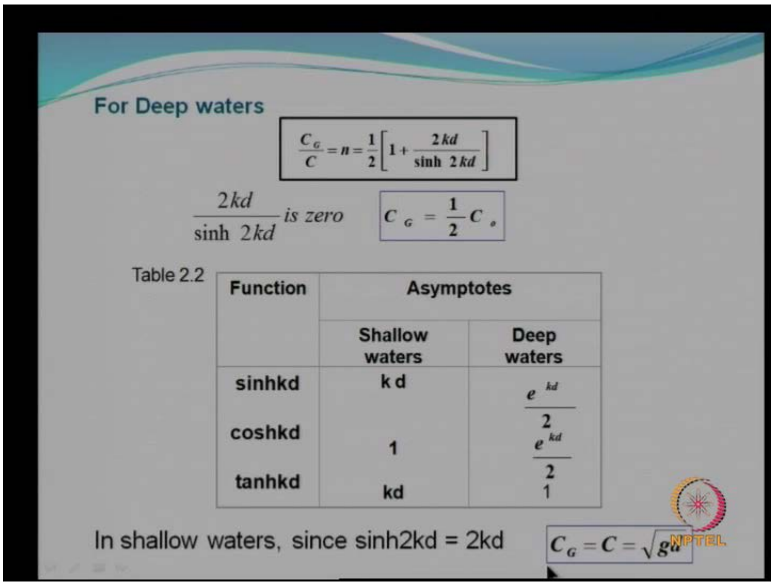

So, now substituting and when you do that in this equation, you can represent C G that is group celerity divided by the celerity of the wave equal to it is normally refer to as small n and this is, and this can be expressed in terms of k d as given in this equation. Now, what happens to this group celerity, again we go back to the behavior of the hyperbolic functions, so in deep waters as shown here, and shallow waters as shown here, under deep waters this factor can be will be set to zero, and hence C G is nothing but, half into deep water celerity, because we are talking about deep waters, so you have a suffix deep water for deep waters.

Now, if you repeat the same thing in a similar way, we can for the shallow water conditions, C G that is a group celerity equal to the celerity itself, and we already know that in in shallow waters, the celerity is going to be function of only water depth as given here. So, thus we see that the group celerity is half of the celerity in deep waters, where as it is same as that in shallow waters, same as the celerity in shallow waters.

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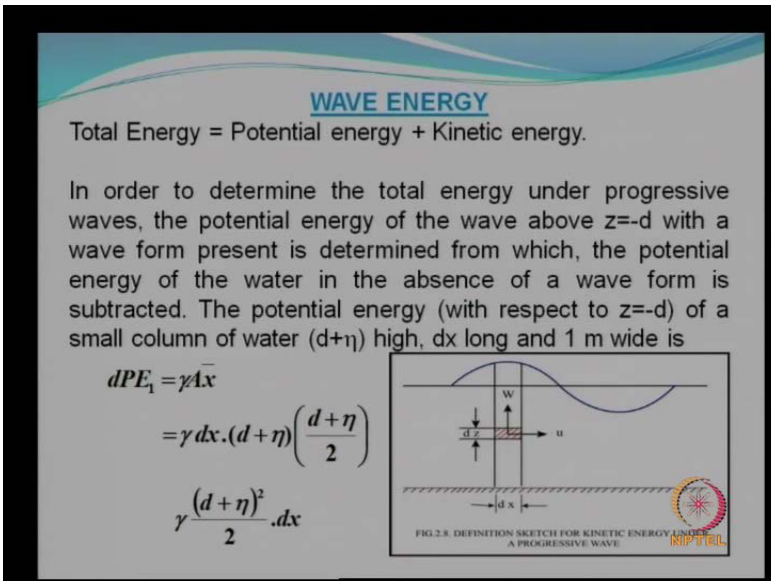

We will try to understand all this parameters later with the help of some few worked out examples, that exactly all these parameters are going to be used. The next topic would be the wave energy, when a wave is propagating, the total energy will be a sum of the potential energy and the kinetic energy.

So, in order to determine the total energy, the potential energy above minus z equal to sorry, z equal to minus d with the wave form present is determined and then we try to find out the potential energy only due to the water depth; and then we subtract from the first one, that is with the wave form then we get the potential energy due to the wave form alone.

So, the potential energy with respect to minus z equal to minus d that is from this location, so for this we are considering a small column of water, so d plus eta, and the d x is a length over a cycle or over a wave which we are considering; so we are assuming 1 meter width of the wave crest, that is perpendicular to the board is the wave crest, so we consider 1 meter. So, from the basics you know that, potential energy due to this water column can be gamma into A into x bar, so now x bar will be d plus eta this water column divided by 2, hence you get d p 1 as as given here.

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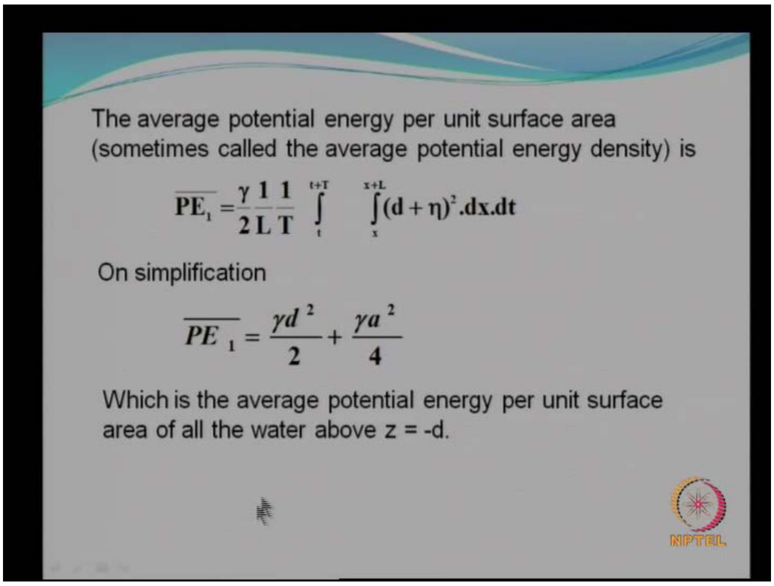

What we have considered there is the potential energy due to the wave under and the water depth at a given location over a distance d x, but the average potential energy their unit surface area, and this also called as average potential energy density, we have to in order to determine this, you have to consider one full cycle, so one full cycle is what, from crest to crest or from still water to still water, after it makes a (()) sorry 2 pi. So, now p 1 potential energy will be from any point t to t plus T or and this will also vary with respect to space that we are having it as x to x plus 1, so this is quite simple.

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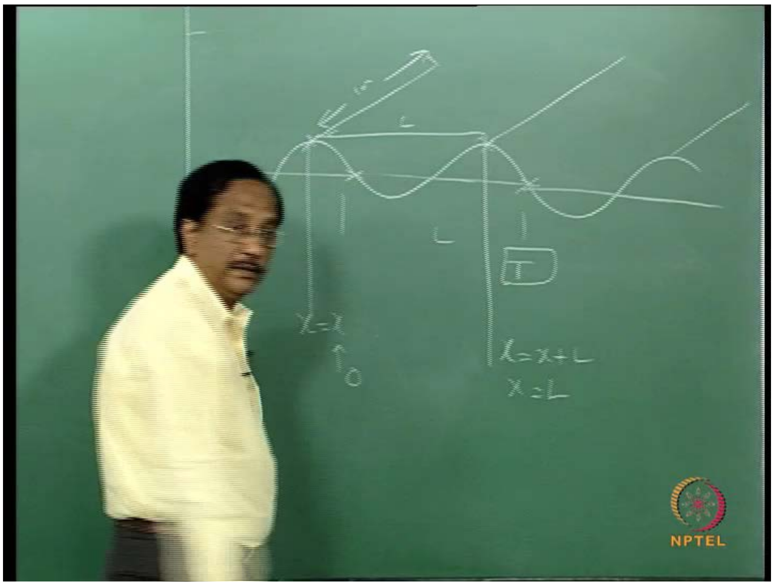

Now, you have a wave, now this is the crest we are considering, so we are taking 1 meter crest width, and now I can take any portion of the wave, but I have to consider one full cycle, so I can start this is a series of waves which is propagating, so I can start from here to here or here to here, so this is what, this is one wavelength or this is one wavelength and the time taken for covering moving or travelling one wavelength equal to a period.

So, I am defining, I can take this as x equal to x to this equal to x equal to x plus 1, so I can have this 0 and this equal to L, so basically what you are trying to do is you are considering one full wavelength, and the same thing holds good for the period time, so that is why we say t to t plus and since, we are taking the average over the one cycle, so you have 1 by L and 1 by T and half.

So, then this is the and when you carry out the integration and on simplification you will get that the potential energy due to water depth and the wave will be as shown here; and this is the average potential energy per unit surface area of all the water that is above z equal to minus d that is from the sea bed, having seen, the potential energy both the wave and the water depth.

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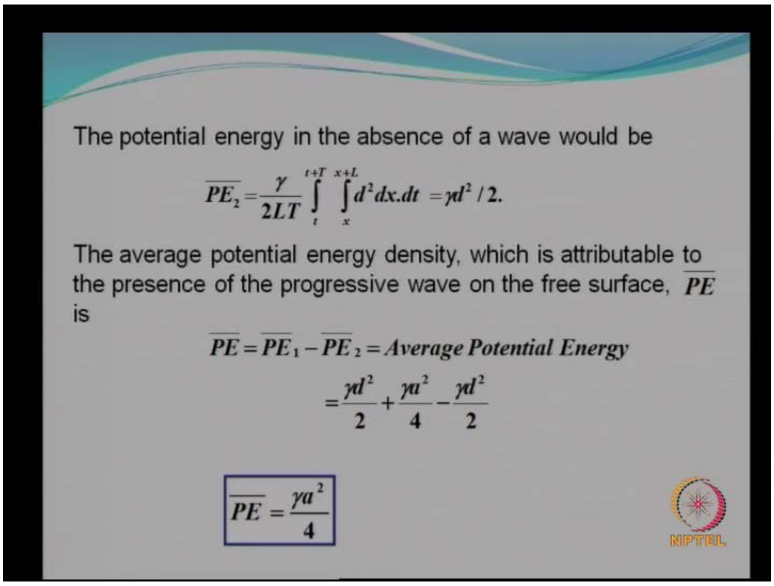

Let us consider only the water depth alone.

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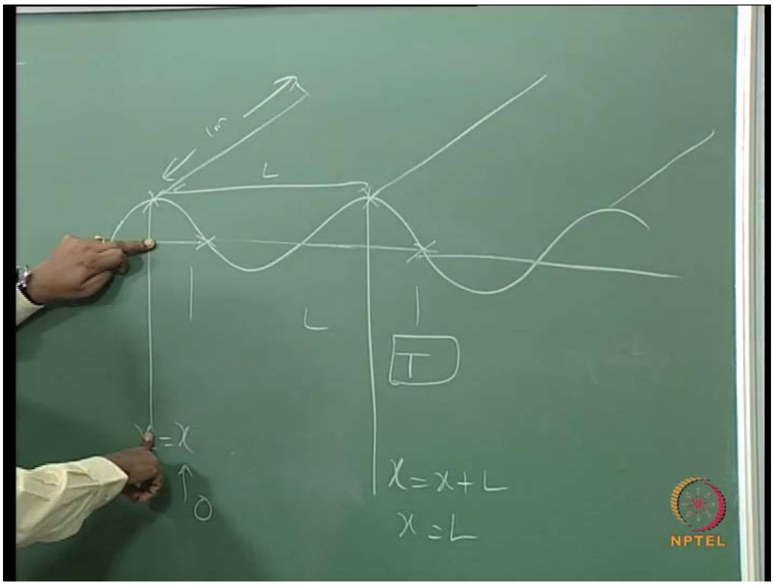

So, this can be written as double integral d into (()) and when you carry out the integration this what you would get, now what is the potential energy, average potential energy due to the wave form alone. That is nothing but, P E 1 which we have got here minus P E 2 which we have got here, and hence the total potential energy due to the wave form alone will be gamma into a square divided by 4, here you see, the amplitude of the wave.

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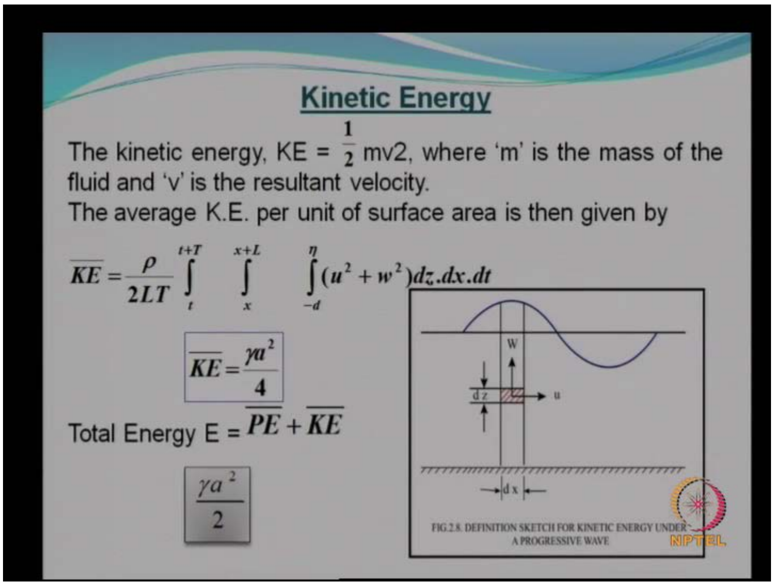

So, now having computed the evaluated the force, I mean the potential energy, we move on to the kinetic energy, kinetic energy by definition you all know that it is half into m v square, so this kinetic energy will be due to what is your velocity, we are considering two-dimensional flow, hence you will have u in the direction of wave propagation and then W in the vertical direction. And we are considering on a small element of this d z compared to the potential energy, there is one additional integral which would be coming here, because the the kinetic energy is going to be varying along the depth.

We have already seen how the horizontal water particle velocity and vertical particle velocity vary along the depth, so that integration also will be coming into picture, so you have to integrate some minus d 2 eta; we have seen the expressions for u and w substitute in this and then carry out the integration, you will get K E equal to gamma a square by 4, now add potential energy and the kinetic energy that will result in gamma a square by 2. So, what is this gamma a square by 2 that is the total energy and in terms of wave height it will be gamma h square by 8, because a is equal to h by 2.

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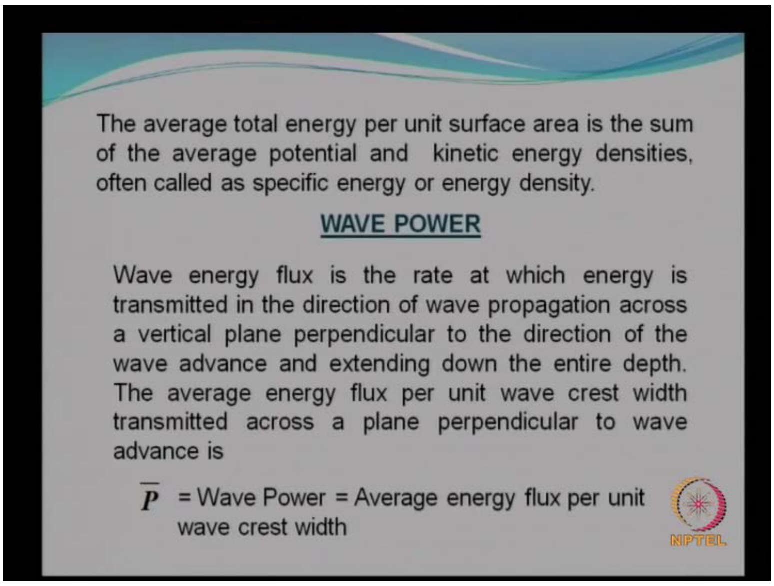

So, the average total energy per unit surface area is the sum of the average potential and kinetic energy densities often called as energy density or specific energy. Now, let us move onto wave power, what is wave power? Wave power is also refer to as wave energy flux, so when a wave is moving the rate at which the energy is transmitted in the direction of wave propagation extending from the top that is a free surface all along the entire water depth, you have to consider the entire water depth. So, the rate at which the energy is transmitted in the direction of wave propagation extending from the sea bed up to the free surface is called as your wave power, how do you get this?

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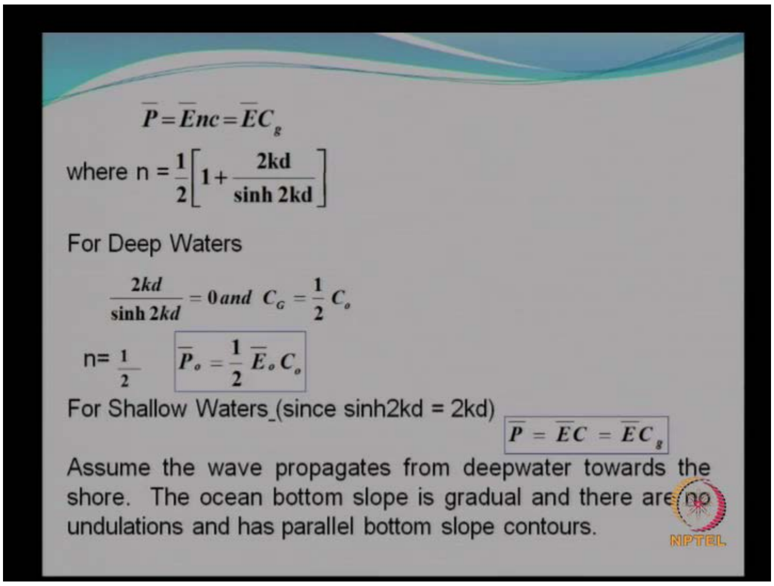

Wave power is going to be nothing but, the product of wave energy and the group celerity, and this is group celerity already we have seen that, it is a product of celerity into n, n is given by this; and this also we have earlier seen what happens to group celerity, it is going to be half into celerity in deep waters. So, hence what is going to be the power in deep waters powers in deep waters, because of this approximation, I we can say that it is half into E naught earlier class I said the meaning of suffix 0.

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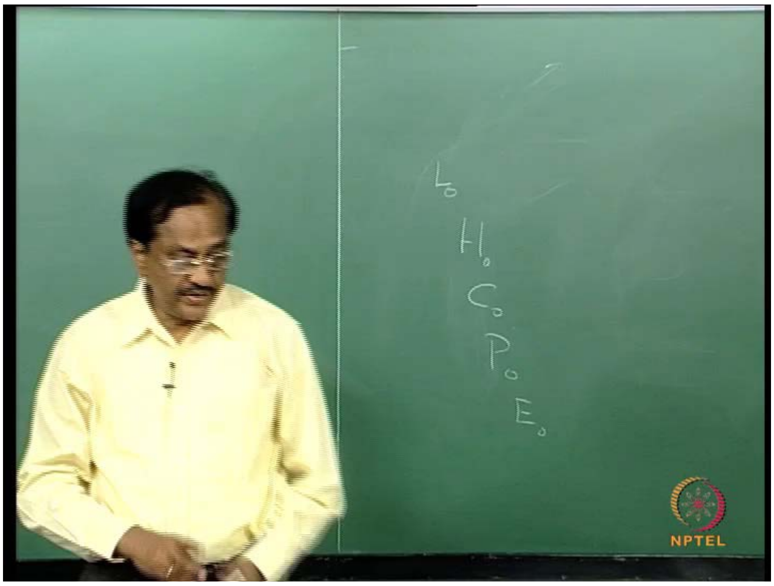

Always suffix 0, pertains to deep water conditions, so you have variable such as wavelength, wave height, celerity, power, energy all these things can be represented as suffix 0, and at the breaker zone when the waves are breaking at that particular point, we refer this instead of 0.

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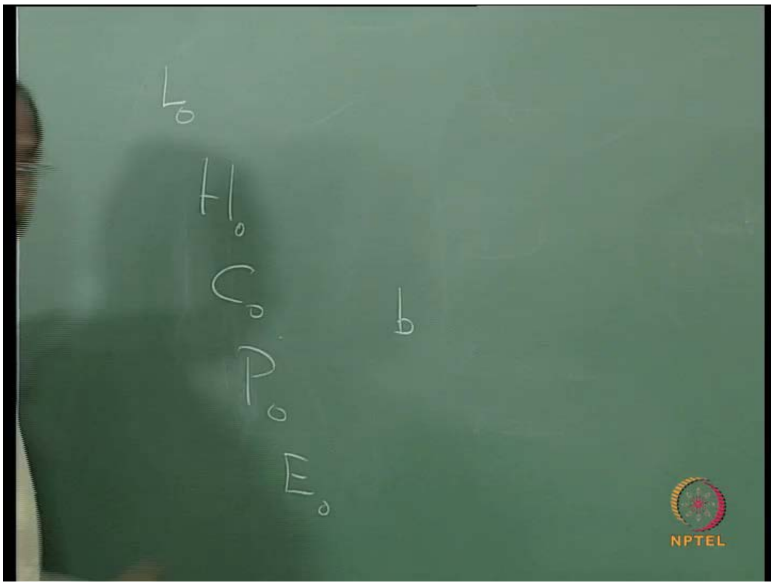

We use b, on now since this is pertaining to deep waters we have half into E naught into C naught, but in shallow waters P bar will be nothing but, E into C because, in shallow waters group celerity will be same as that of C (Refer Slide Time: 24.01). So, we have we have the power for the wave when it is in deep waters, and the power for the wave when it is in shallow waters; so now we know that waves propagate from deep waters to shallow waters.

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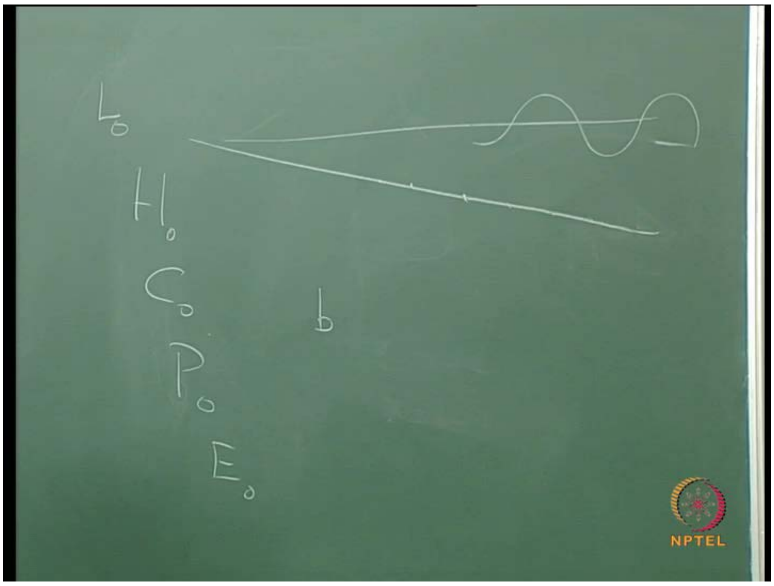

We assume that the sea bed ocean bottom is gradual with no undulation, when I say no undulation it is almost like a ramp, when you have a ramp and when the deep waters, when a waves are propagating from deep to shallow waters; we also say use the word parallel bottom depth contours that is, because of this assumption the bottom dense contours are suppose to be parallel to each other.

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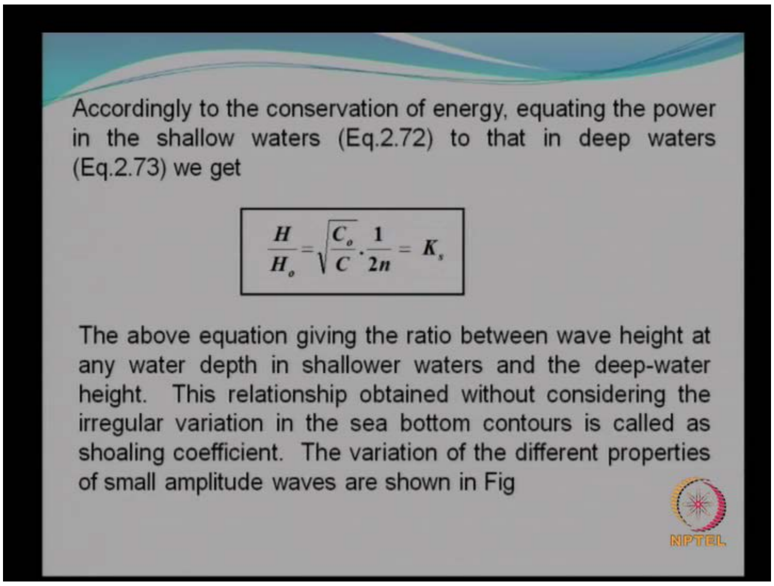

under that assumption, we can equate P naught e*+qual to P in order of as a part of the conservation.

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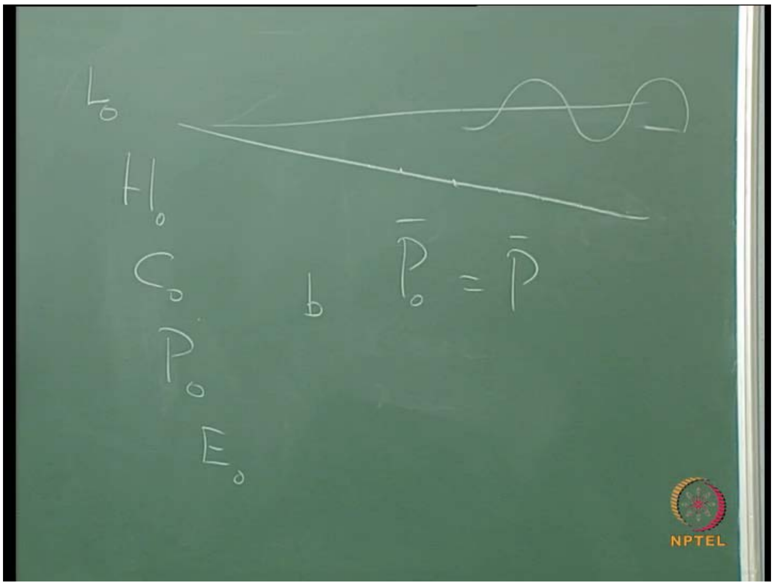

So, when you have this.

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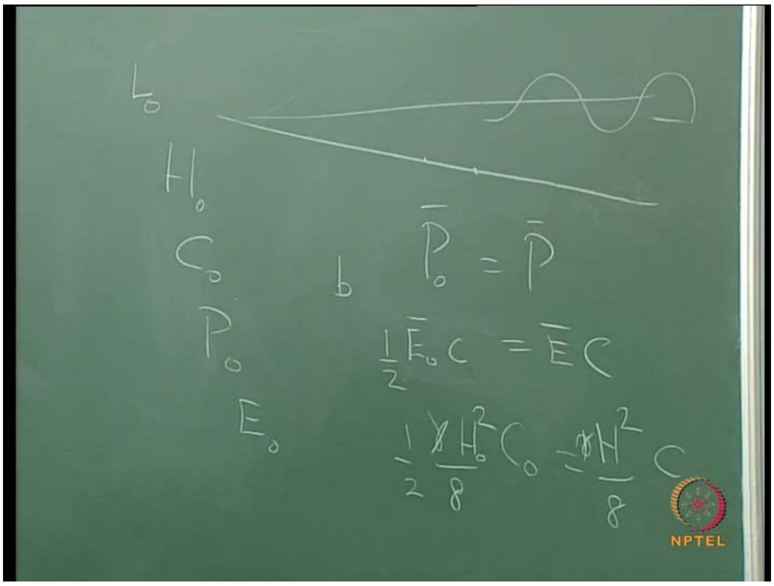

This is nothing but E naught into C half equal to E by C. What is e is gamma H square by 8 I use suffix 0 and I can use C naught and that is going to be equal to of course, I have this 1 by 2 and then H square by 8 and I will still retain my C. So, what are you getting we will get our relationship between the deep water wave height and wave height in any water depth as you can see here. What is this, this is nothing but, refer to as shoaling coefficient, so this gives the variation of wave height at any given water depth, once you know the deep water wave height (No audio from 27:13 to 27:20).

So, usually the deep water wave height can form as one of your input variable, normally it may be available to you and you want to come up with a structure in a water depths of 10 meters and the deep water wave height is say it is given as the 3.5 meters and in the event you assume that the water depths of contours are parallel to each other, which in fact is not the right way its only a first approximation. Because, when you want to design you are suppose to take care of the undulations of the sea bed also, if incase you do not take it and in case you assume that sea bed contours are parallel to each other then you can use this to get the wave height.

So, with the help of a problem later we will see what happens, when you do not include the undulations of the sea bed and when you consider only the shoaling coefficient. So, the above equation gives the ratio between the wave height at any water depth not necessarily in shallow waters in any water depth of a function of once you know the deep water wave height.

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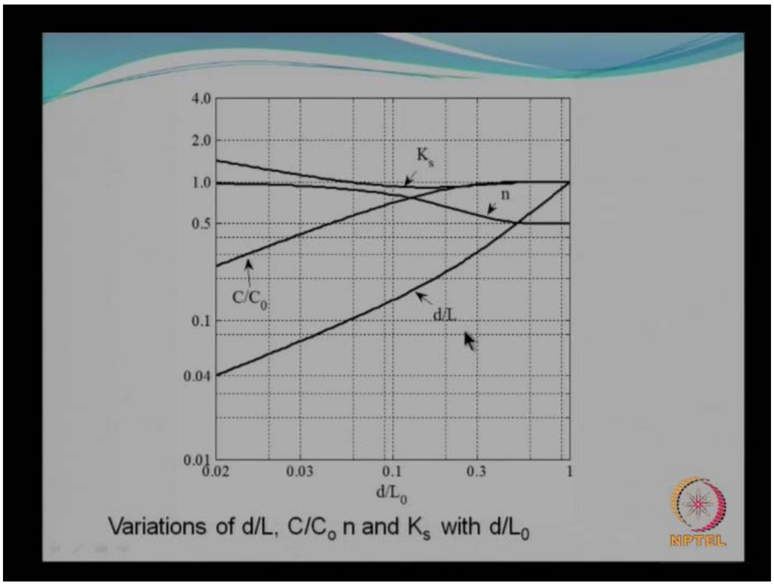

As I said we are not considering the irregularity of this event, we will look at this picture which provides us the variations of d by L that is water depth divided by the wavelength in any given water depth, the variation of the speed in any deep water depth divided by the deep water celerity. The variation of n which is the ratio of the group celerity and the celerity, but finally, the shoaling coefficient which we have already seen as a function of d by L naught, L naught is a deep water wave length. So, now let us consider a simple example to understand this.

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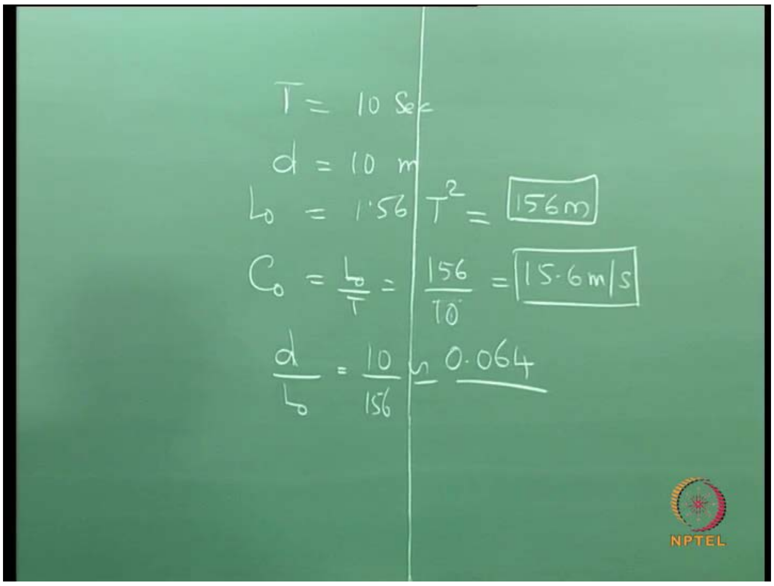

Usually, what you are given is the wave period and you will be asked to find out the parameters wave characteristics in a given water depth, so let me say that water depth is 10 meters and wave period is 10 seconds; now what is L naught, L naught is 1.56 T square which will be 156 meters. What is deep water celerity, deep water celerity is L naught divided by T which is 146 divided by time period that is equal to 10 seconds 15.6 meters per second. So, this is these are the values of the wavelength and the celerity in deep waters, can we use a use a same values for for a wave when it is 10 meters water depth, we will just check now, what is the kind of discrepancy we might have.

Now, let us using this picture d by L L naught is how much, d by L naught will be 0.64 that is 10 divided by 156 approximately, so from this picture you can easily get the value of d by L, the value of C by C naught, value of n, n variation is shown here or the shoaling coefficient all this the the values of all these parameters can be obtained with the help of simple graphical representation. So, now we going back to this, from this take the value of d by L naught and arrive at the value of d by L in which case it will be something here.

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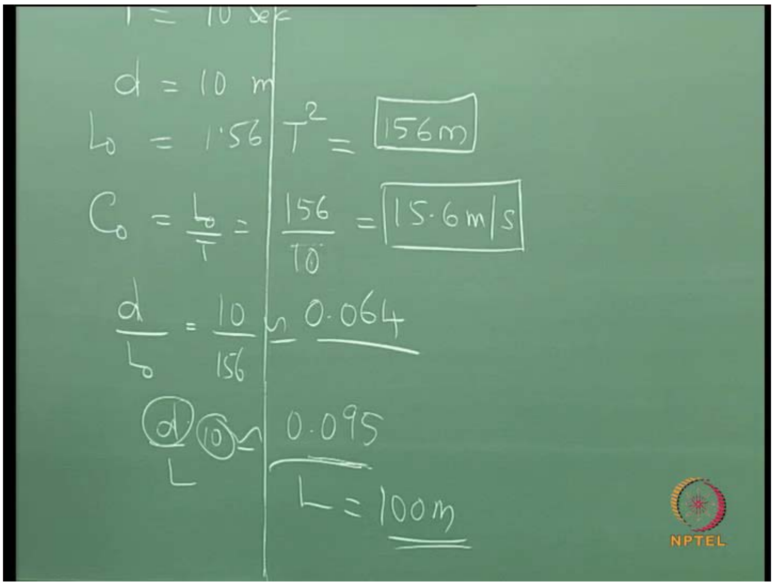

So, d by L will be approximately equal to 0.095 is that ok, so d is already you want the wavelength for water depths of 10 meters, so L will be, so now d will take the value of 10 sorry, L based on this relationship will work out how much? It will be approximately 100 meters.

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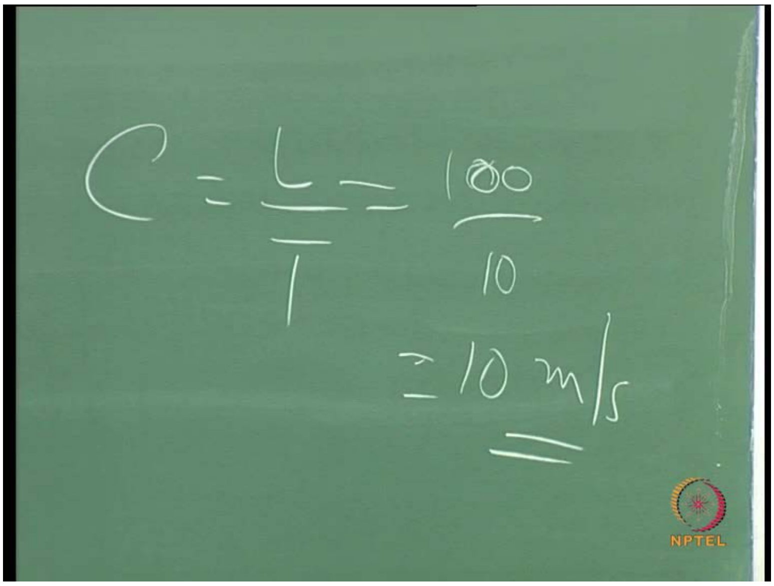

And what will be your celerity? 100 divided by T is 10 seconds, so this will be just 100 meters per second approximately. So, you look at the difference, my 156 meters was the deep water wavelength, but the wavelength in 10 meters water depth is going to be just

100 meters, so you see the difference and the celerity is going to be different it is going to be almost 10 meters, this will be around 10 meters compared to 15 meters per second.

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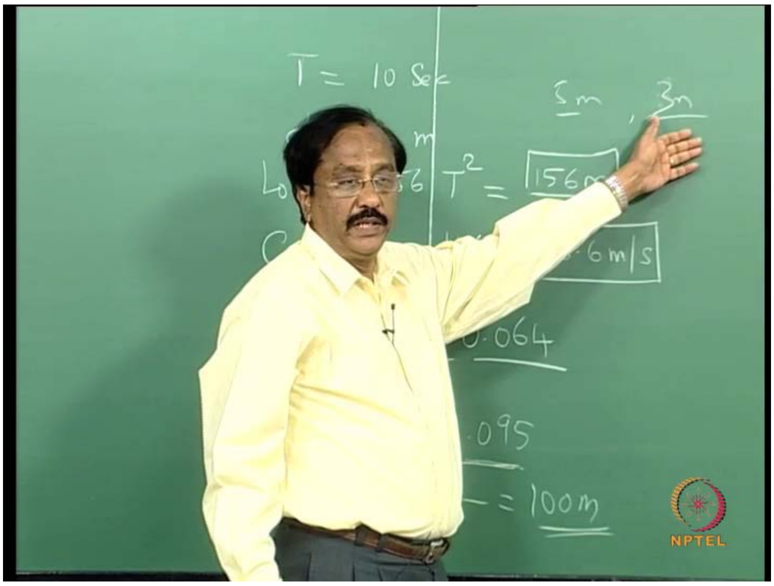

so suppose if you are, if your water depth where you would not have your structures is say 5 meters or even 3 meters, so in which case the difference between the deep water wavelength and the shallow water wavelength in much shallower water will be much larger, so you have to be very careful while calculating this wavelength, so this is one of the basic requirement before you start any kind of I mean any kind of information required for the ocean waves.

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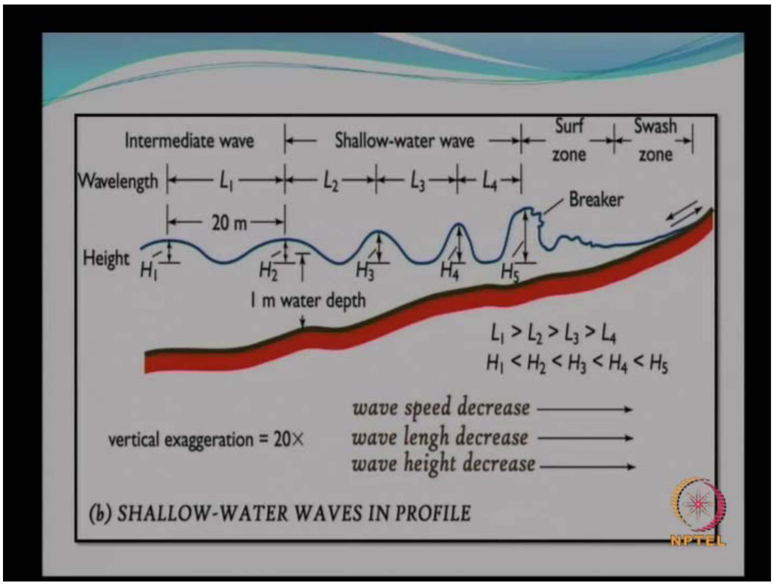

So, this pictures shows what happens to the wavelength and the wave height again this we will we will try to understand with the help of a problem, so as waves propagates from deep to the shallow waters the wavelength keeps on decreasing. So, and that is what is indicated here, L 1, L 2 etcetera, where as the wave height initially decreases and somewhere close to the breaker zone, it will again start increasing and then it becomes unstable and breaks, so this is what we have understood so far.

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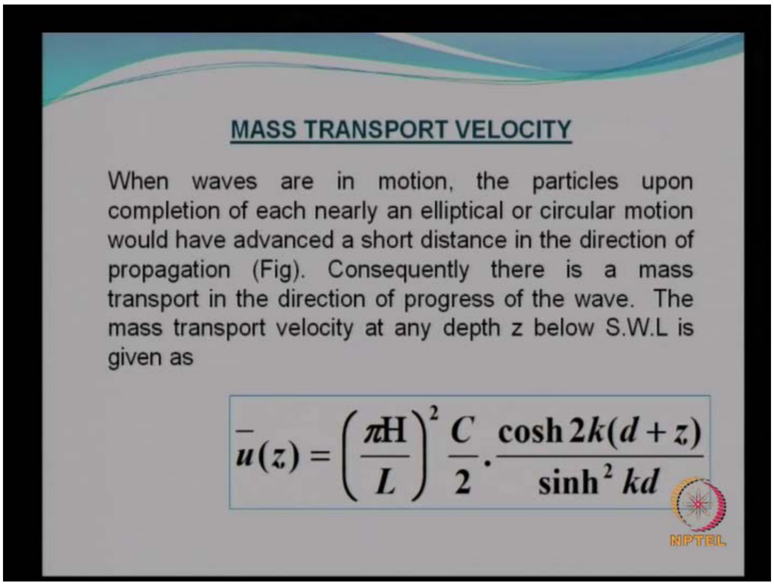

Now, before going into the mass transport velocity just brush up what we have seen so far, what are the velocities we have come across so far, we have seen celerity of the wave which is nothing but, the speed with which the wave is moving, that is given by L by T and then we also heard a discussion about orbital velocity. Orbital velocity is needed for the estimation of the forces and how all these variables are varying under different water depth conditions etcetera.

Now, we move on to what is called as mass transport velocity, what exactly is mass transport velocity, earlier we we said that wave is a oscillatory motion according to which, if you are able to sit on the wave on the crest of the wave, you are suppose to undergo a motion depending on the water depth conditions either elliptical orbit or circular orbit. If this is true and then you go to the beach, and say for example, you would have seen people near the beach, they throw a ball and what happens to ball, it comes back to you, so then what is happening to the oscillatory wave, oscillatory motion, according to the oscillatory motion that ball should be remaining there, but what happens it comes back to the shore.

Not only it comes back to the shore it does not mostly, it does not come straight to you, either it is deflected on on to your right side or to your left side, this is because of other phenomena which is called as near shore currents and these near shore currents are due to the breaking of waves, which we will not see in this our course. But, what you should understand is that the ball is coming back to you, then do you mean to say that the oscillatory motion, what we have seen is wrong, no and particularly when you have storms after cyclone or such things, what happens you will see so many debris from the ocean washed to the shore, this is because of the phenomena of mass transport velocity.

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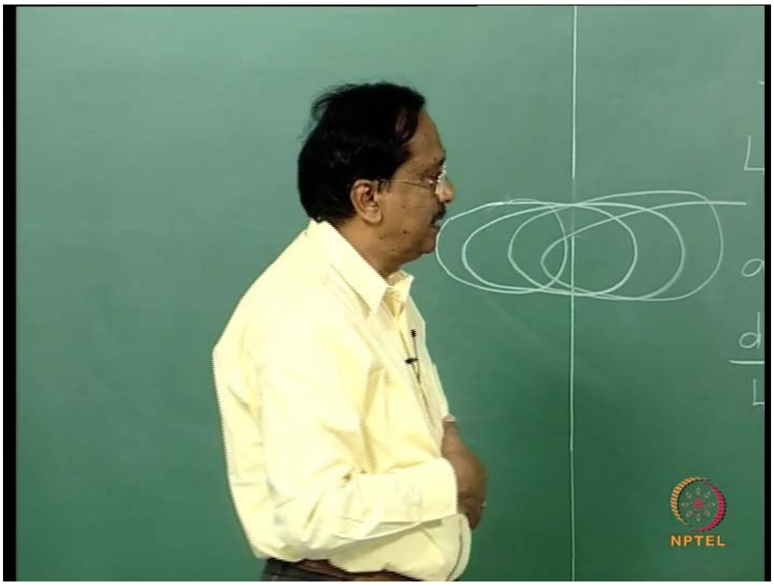

That is when the wave is propagating, it is suppose to complete one cycle just before completing one cycle it will not end exactly at a point where it is supposed to end, but instead there is a slight drift in the direction of wave propagation, so you see that the particle or the floating object which you are talking about is drifted in the direction of a propagation. And the velocity with which it drifts is called as the mass transport velocity, how does the mass transport velocity vary, mass transport velocity you can derive as shown here.

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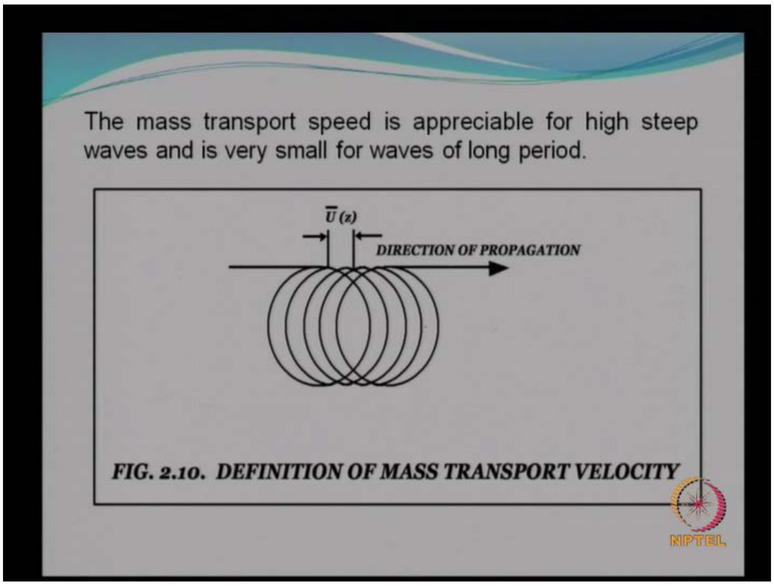

Can vary as a function of stiffness, usually when you have storms etcetera, you see that the stiffness of the waves are large and that is the time when you have lot of debris washing away to the shore, because of this kind of a phenomena, does it vary along the depth yes, mass transport velocity will be maximum near the surface and it will be varying along the depth as a cos hyperbolic variation (Refer Slide Time: 39:53).

So, now you see that mass transport velocity group celebrities, celerity, orbital velocity are all different and it has its own contribution to the mechanics of waves; and this is what I wanted to tell you mass transport velocity is appreciable for stiffness, and is very small for waves of long period long period waves.

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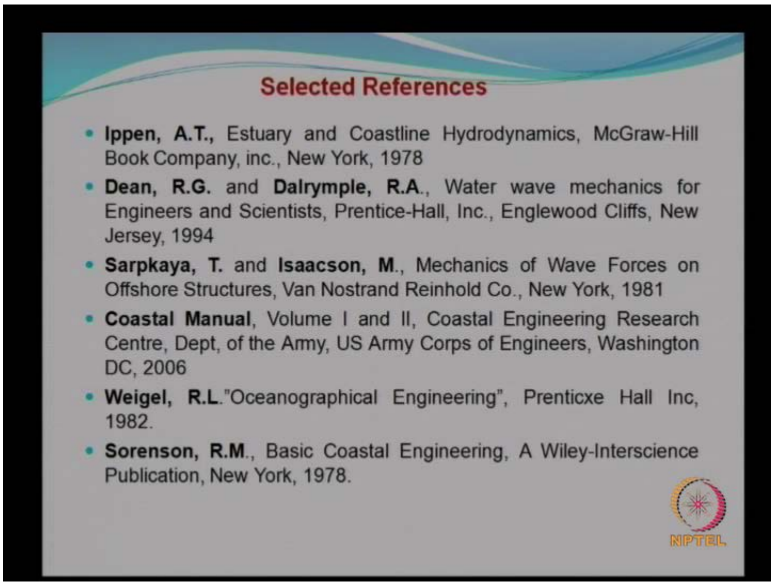

So, I have given lectures on the basics of phenomena of waves and there are several books in the subject, I have followed the book on Ippen, by Ippen who has brought out clearly the expressions, Dean and Dalrymple this book also is quite good and some of the animations we saw are, because of the contributions made by Dalrymple.

I have also referred to Sarpkaya and Isaacson, mechanics of wave forces we will be taking materials from his books, particularly when we are discussing about the wave forces and the other books are mentioned here, there are several other books like one more professor S. K. Chakraborty that is also hydrodynamics of offshore structures, that is unfortunately it is not listed here. So, they are several books, so I suggest if you are interested in having more information about wave mechanic, some of these books can be very very useful.

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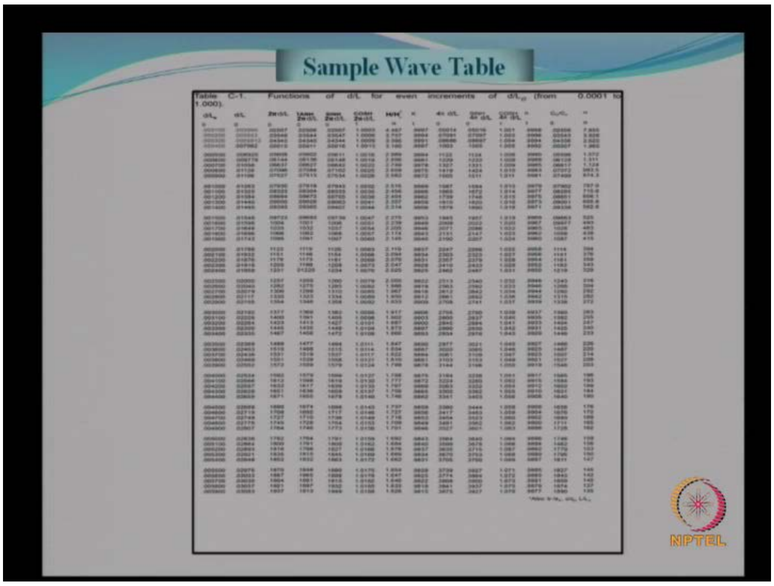

So, before I conclude, I like to say about the wave tables, I hope you you remember that we had discussed about the iteration equation that is an implicit equation to be solved by the equation process, that is.

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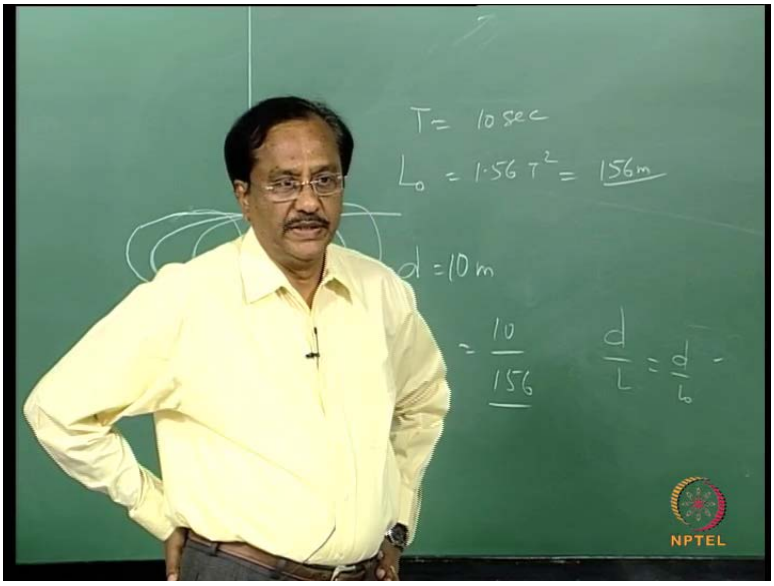

d by into tan hyperbolic k d, so you when you want to solve for the solve the equation for wavelength the task is made easier by using wave tables, the wave tables provide you it is quite convenient for us, because all mostly you will be having deep water wave height, the wave periods will be the input to you.

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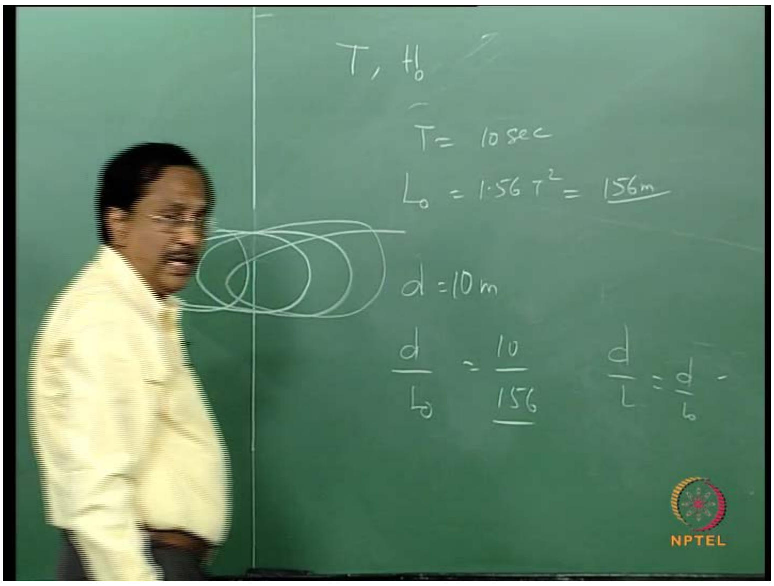

Sometimes, you will be given just waves height or wave height in deepwater waves, deepwater's and then based on this variables you parameters you are you may be asked to or it may be that require for you to get the wave height and other parameters in a given water depths may be 10 meters or 15 meters or whatever it is.

So, further the first step is you calculate your d by L naught as I have indicated, now once you calculate d by L naught you can arrive using this tables d by L to k d sorry this is k d, tan h k d, sin h k d, cos h k d then the variation that is nothing but, the shoaling coefficient, then this is the pressure response factor at the sea bed for the getting for obtaining the pressures, under progressive waves (Refer Slide Time: 45:41).

Then you have 2 k d, sin h 2 k d, cos h k d then the variable n which we saw under the group celerity, then the other the other variables are indicated here; so you can use this wave tables for obtaining all the variables, I mean parameters for solving your problems which will be using later.

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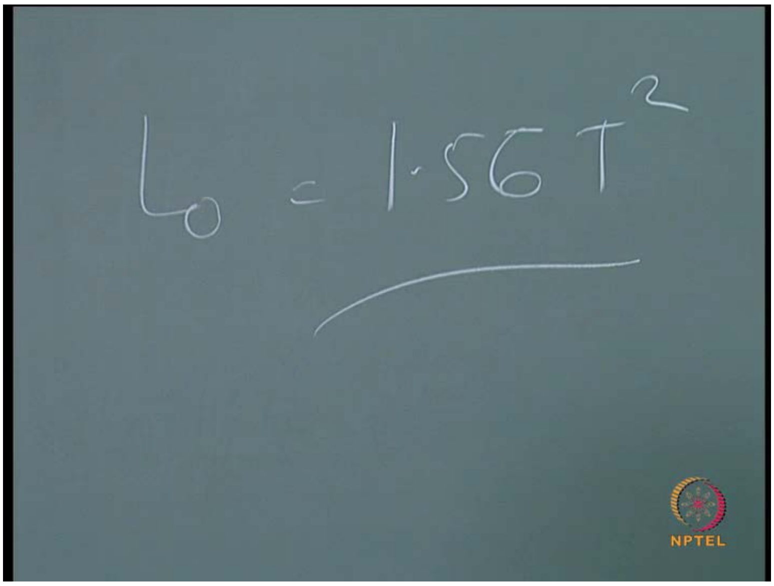

Looking at the wave length again, we have already seen how the deep water wavelength is defined it is just, 1.56 into T square all of us know.

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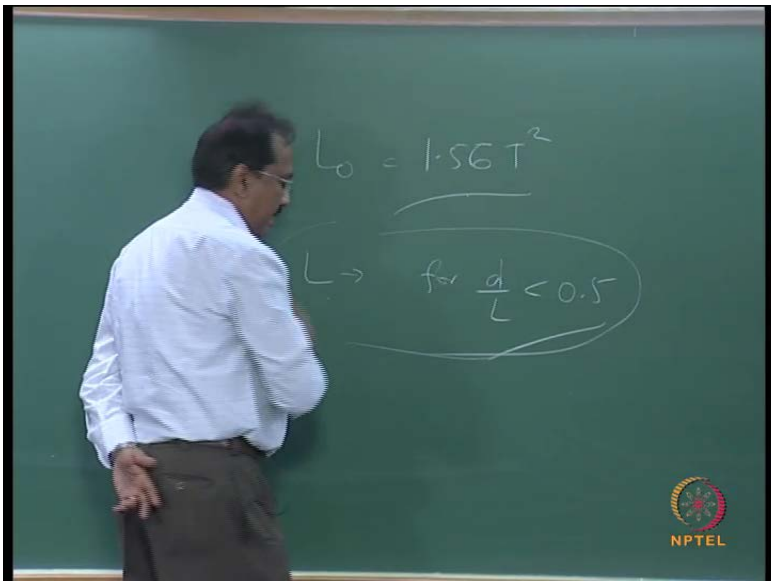

and then we use the iterative solution to obtain the wavelength for the case when d by L is less than 0.5, so if d by L is greater than by 0.5 that does not come into picture that is what we have already seen. So, although you have nitration equation, iterative equation for solving for the wavelength, there has been an attempt by several investigators starting as early as 79 by 1 hunt. And then all the way till 2008 by bob you, well they have come up with the (()) solutions for determining the wavelength.

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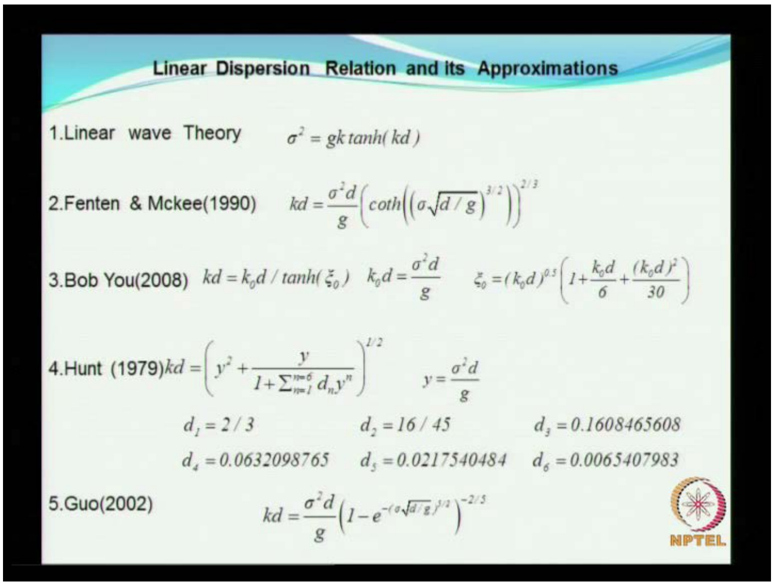

Let us have quick look at some of this formulas, they are just 5 in number, so starting with hunt he has defined as a summation of 6 summations, so d 1 all the way d 2 they are having this are defined as a co-efficience you as indicated there; and sigma is usually has the usual definitions. And then that is sigma is 2 pi by d and then d is the water depth, so hence you know the parameter why from this slide, so you can pluck in this parameter and then use this expression to obtain the value of wavelength.

Now, once you get k d, k d is nothing but, 2 pi by L, so from which you can straight away get your wavelength of course, I am not touching about the, I am not talking about the linear linear wave theory, because by now you all Know how to calculate the wavelength using the linear wave theory. Then moving on to Fenten and Mckee it is a quite popular method Fenten has done lot of work on the non-linear waves also, so here you see that the k d can be obtained by a simple expressions as defined here, sigma is known to you d, all the variables which are used there are known to us, so you can determine and this method, using this method you can determine the wavelength.

Similarly, you have by Guo, who has come up with this kind of an expression and again close form solution where in you can get the, and then of recently we have Bob you in 2008, coming up with this expression. And any of these expressions can be used to determine the wavelength (()) shallow waters, I mean shallow as well intermediate water depths.

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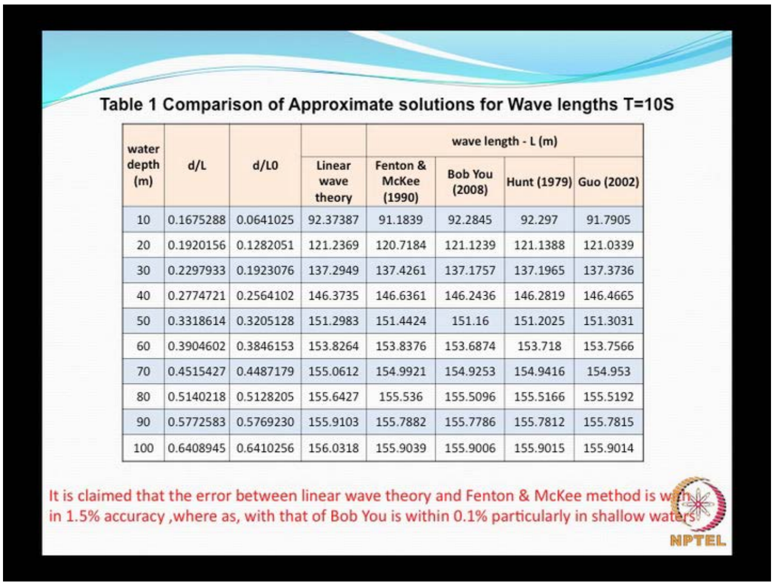
Table 1 Comparison of Approximate solutions for Wave lengths T=10S

water depth (m)	d/L	d/L0	wave length - L (m)				
			Linear wave theory	Fenton & McKee (1990)	Bob You (2008)	Hunt (1979)	Guo (2002)
10	0.1675288	0.0641025	92.37387	91.1839	92.2845	92.297	91.7905
20	0.1920156	0.1282051	121.2369	120.7184	121.1239	121.1388	121.0339
30	0.2297933	0.1923076	137.2949	137.4261	137.1757	137.1965	137.3736
40	0.2774721	0.2564102	146.3735	146.6361	146.2436	146.2819	146.4665
50	0.3318614	0.3205128	151.2983	151.4424	151.16	151.2025	151.3031
60	0.3904602	0.3846153	153.8264	153.8376	153.6874	153.718	153.7566
70	0.4515427	0.4487179	155.0612	154.9921	154.9253	154.9416	154.953
80	0.5140218	0.5128205	155.6427	155.536	155.5096	155.5166	155.5192
90	0.5772583	0.5769230	155.9103	155.7882	155.7786	155.7812	155.7815
100	0.6408945	0.6410256	156.0318	155.9039	155.9006	155.9015	155.9014

It is claimed that the error between linear wave theory and Fenton & McKee method is within 1.5% accuracy ,where as, with that of Bob You is within 0.1% particularly in shallow waters.

So let us see, how the values for these wavelengths using this different methodologies look like, comparison of approximate of formulas are solutions for wavelengths that is (()) solutions; so we have a taken here, the linear theory and we have varied the water depths from 100 to 10 to 100 in steps of 10 meters. And then go about calculating the d by L d by L naught then by using the linear theory, you are got the wavelength here and then the use the using the other four methods you get the wavelengths which is indicated in this rows.

So, if you compare for a particular water depth you see that the variation, the kind of variation you would have is something like this, you see that the based on this, it has been claimed that the error between the linear wave theory and that of the method of a Fenten and Mckee is within an accuracy of 1.5 percent where as with that of Bob you it is within 0.1 percent, particularly in the case of a shallow waters. So, this this few slides explain, how you can make use of (()) solution for determining the wavelengths, so in case you can just use a calculator to get all these values for the wavelengths in different water depths, that is apart from the deep water wave conditions, is that clear, so I will stop here.