Wave Hydro Dynamics
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Module No. # 02
Wave Motion and Linear Wave Theory
Lecture No. # 03
Wave Motion II

We will today continue with, what we had left in the last class.

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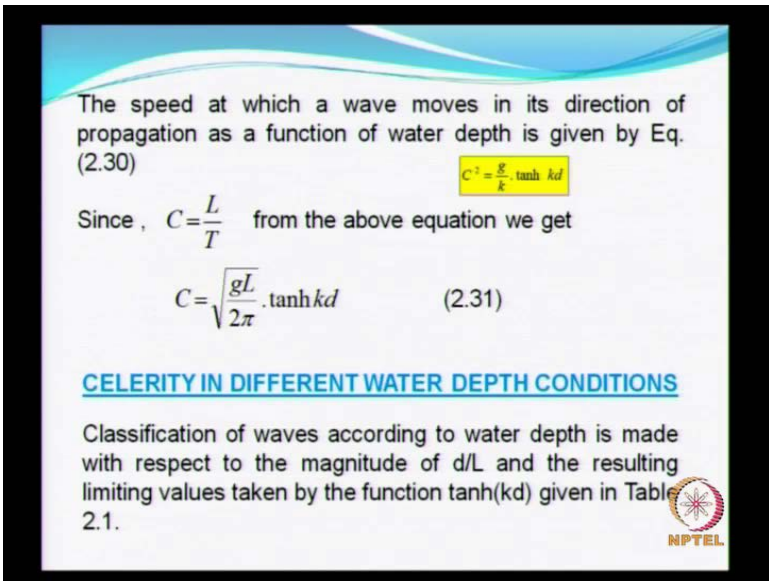

So last class, we had established a relationship for the dispersion relationship is called as a the dispersion relationship, where in C is the speed of the wave and g is the gravitational constant, then k is the wave number and d is the water depth; this establishes the relationship between the wavelength or the wave period and water depth. So, we have already seen that, celerity is going to be just L by T that is wavelength divided by the wave period. And if you substitute this in the above expression, because k is just 2 pi by L then you will land up with this expression for C in terms of wavelength and period wavelength and water depth.

So, now let us examine the variation of the celerity in different water depth conditions. So, I would we would be can consider this as a function of by examine the function tan h tan hyperbolic of k d.

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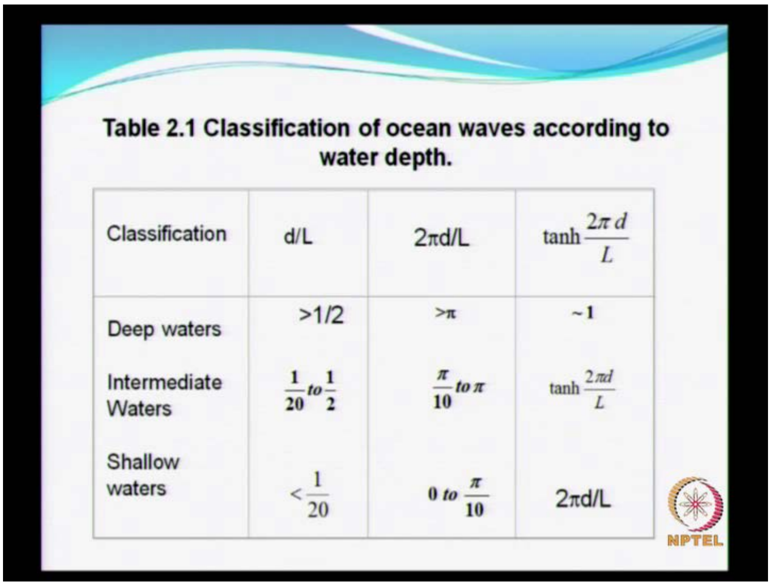
Table 2.1 Classification of ocean waves according to water depth.

Classification	d/L	2πd/L	$\tanh \frac{2\pi d}{L}$
Deep waters	>1/2	>π	~1
Intermediate Waters	$\frac{1}{20}$ to $\frac{1}{2}$	$\frac{\pi}{10}$ to π	$\tanh \frac{2\pi d}{L}$
Shallow waters	< $\frac{1}{20}$	0 to $\frac{\pi}{10}$	2πd/L

As you can see in this table. So, when d by L which is in the second column is greater than half, we call this as a deep water conditions. Under which condition under which situation your k d or 2 pi by d 2 pi d by L will be close greater than pi whereas, tan hyperbolic k d will be approximately equal to 1.

In the case of intermediate waters, this d by L will be ranging between 1 by 20 and half. On the corresponding variation for the k d and 2, I mean tan h k d are shown here; so tan h k d will remain as tan h k d in intermediate water depth, which will reduce it reduce to k d in the case of shallow waters, that pertains that is when we say shallow waters d by L should be less than 1 by 20 or 0.05. So, the speed of the wave will now, vary according to variation in variation of this parameter or tan hyperbolic k d, under different water depth conditions.

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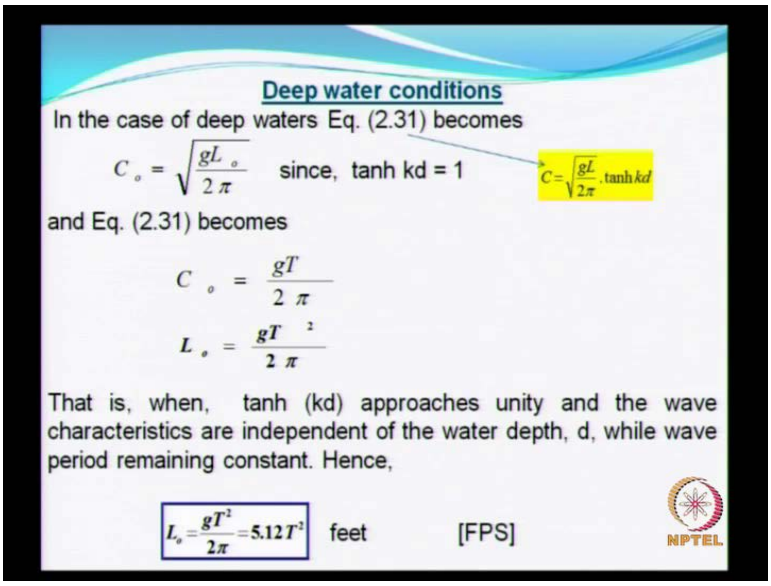

This is the equation we are examining now and using the, in the case of deep water conditions, that is d by L greater than half, you will have tan h k d equal to 1, that will lead us to on substituting you will have only C equal to root of g L by 2 pi. So, in wave mechanics any variable like celerity, wave height, wave length, of a wave which is in deep water, will be associated with the subscript 0 as you can see here. This for us to easily identify whether the wave of that particular magnitude of wavelength or celerity, etcetera whether it is in deep water, etcetera.

Later we will also see that, suffix b is used for the waves in breaker zone. Now this equation this equation that is 2.31 as we have seen earlier, in which we obtained the expression for celerity as the function wave period, will now reduce to as you can see here(Refer Slide Time: 04:31). From which again C naught is nothing but, L naught by t, so L naught will become g into T square by 2 pi which is; that means, the wave length of a wave, which is propagating in deep waters, will have a value g by 2 pi into T square which is just the function of wave period.

And similarly the similarly celerity or a speed is going to be just a function of… So, this in case of this can easily be mention as written as 5.12 into T square in a FPS units or in term wavelength in (()) or as 1.56 T square meters in MKS(No audio from 05:28 to 05:38).

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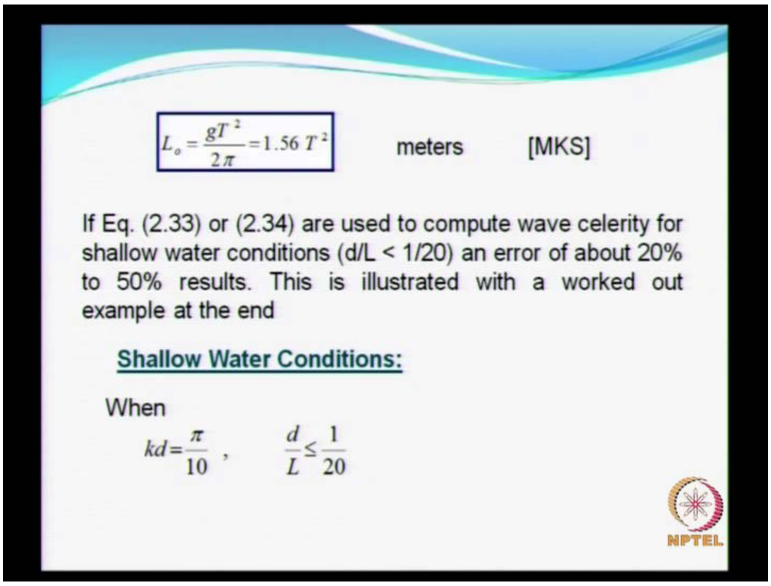

So, you see that, this quite straight forward to evaluate the wavelength in deep waters and it is also very easy, but remember that you are not suppose to use this expression for a wave propagating in intermediate water depth or shallow waters. The later we will see that, in intermediate waters the speed will be dependent on both wave period and water depths. In deep waters it the wave speed or the wave lengths is not a function of water depth at all.

So, instead of going the intermediate conditions, we will just move on to shallow water conditions, because in the intermediate water depth conditions the definition of celerity is nothing but, L by T, that is wave length divided by wave period; and wave length will definitely be a function of water depth. So, now we will move on to the other extreme condition, that is shallow water condition because we have just seen the deep water condition, in the shallow water conditions I have already mentioned you it is repeated here.

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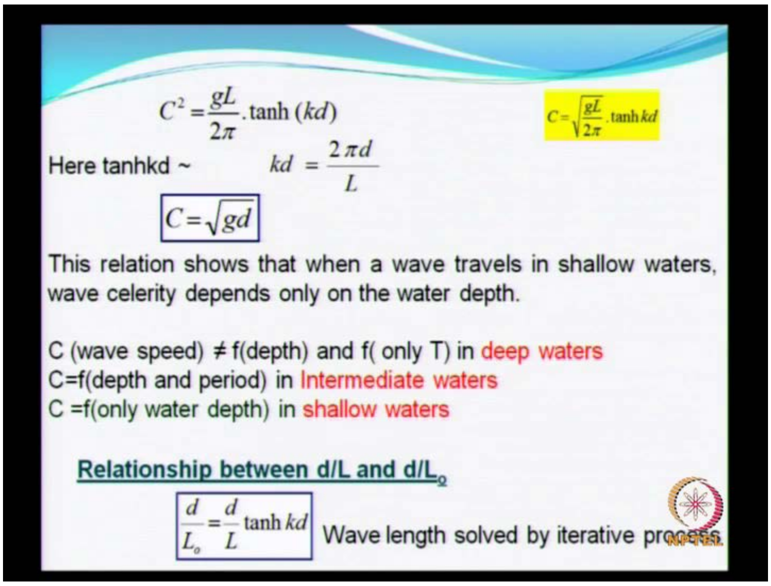

So, and we are using the same expression here, so tan h k d when d by L is less than 1 by 20 or 0.05 you will have k d equal to k d itself. So, hence if you, once you substitute here you see that the celerity in shallow water or the speed of the wave is going to be just square root of g into water depth.

So, now you see that the wave in shallow water when it is propagating, it is going to be a function of water depth that is all, it is not going to be a function of any other parameter. So, to summarize the behavior of celerity or the speed of the wave, the relationship that the relation shows that when a wave travels in shallows water that is what I was trying to say.

Now, to summarize you see that, the wave speed is just not a function of water depth it is just a function of wave period in deep water, in intermediate waters is going to be a function of both depth and water depth and their period, whereas in shallow waters is going to be just a function of water depth alone, is that clear.

So, if you start, so you will see that in case of intermediate water depth, the celerity is going to be the function of depth and period, that is I mean the wave length and if you start looking at the relation it is very easy to derive this expression for from the above expressions, that d by L naught is approximate is equal to d by L into tan h k d.

Now, this is iterative equation, so you have to solve this equation for the wave length by trial under a method, it is not a (( )) to get the wave length, so now, you see some slight difficulty is there in evaluating wave length. So, in overcome this difficulty you should not use the deep water wavelength, because it is very easy to calculate and then start using it for intermediate water depth conditions.

So, if you do, so you may have an error even up to about 50 percent it does depend on the water depth and other wave characters which you are dealing with, so this can be explained or this can be this will be seen later, with the help of a an example.

Now, you see that here, if you want to determine the wave length for the deep water straight away you can use this expression that is d by L naught you can calculate, because L naught is nothing, but 1.56 into d square.

So, in order to arrive at this value that is d by L from this kind, this using this equation we are equipped it what is called as wave tables, but these days you can easily, very easily calculate there are some wave calculators available on the net you can just simply go and you see wave calculator. So, you give the wave period water depth it will automatically give you the deep water wave length and also the wave length as well as the celerity and all other associated variable which we will be seeing later. So, in the case of wave tables, you have to initially calculate d by L naught and then from which using the wave table you can easily calculate d by L which will be seeing once again later.

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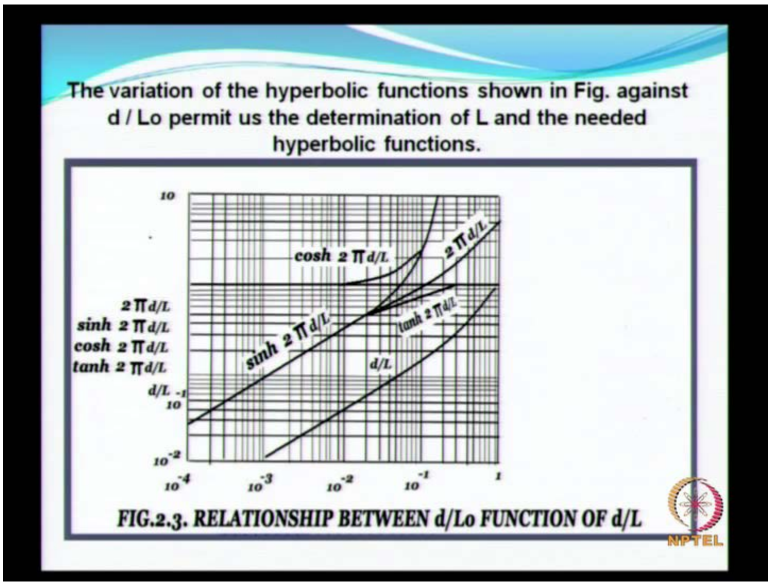

But this is a hyperbolic variation, this curve is quite handy if you want to arrive at the wave length and you also come across, so many hyperbolic function in understanding the physics of the waves, later you will see that the orbital velocity for example, you have already seen the variation of velocity potential varying you deal with the cos hyperbolic sine hyperbolic as well as tan hyperbolic.

So, all this function can easily be obtained from the single figure varying on the x axis you have the d by L naught variation, so d by L naught is quite easy to obtained once you have obtained to the d by L naught value from this picture, you can just use this a curve to get all the variables which you need, so from d by L you can straight away get your wave length, I hope it is clear now.

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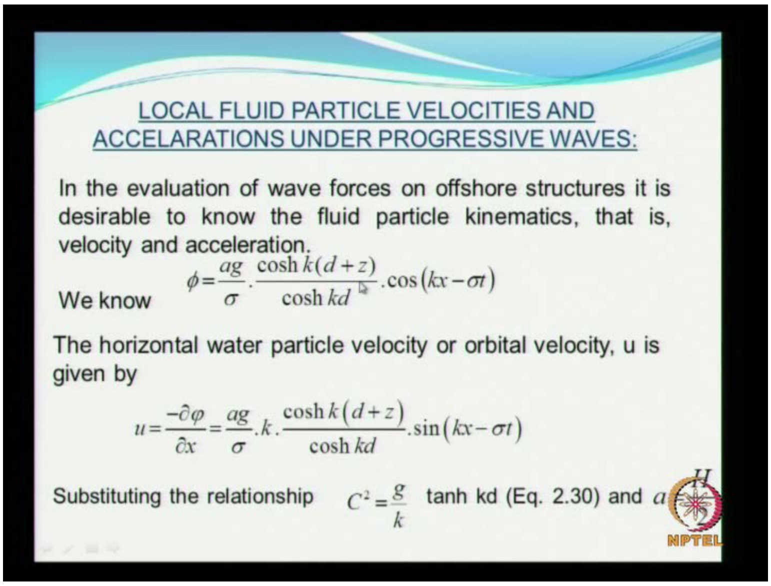

Now, let us understand, why did we derive the velocity potential and why are we now looking at fluid particles and acceleration, as I said earlier when there s a potential once you derive the velocity potential in a particular direction, you are going to get the velocity in that particular direction.

So, in the open ocean we have pile for example, now the waves are propagating, you have seen earlier that the velocity potential will be varying along the depth, when the velocity is velocity potential is varying along the depth. So, will be the horizontal velocity as well as we are since we are dealing with the two dimensional also the vertical velocity will be varying.

So, we are considering the three dimensional, that is the variation of the potential even in the other direction, the other transverse direction then the velocity in that direction also will be varying along the depth and this this variation is going to cause the force is acting on the structure. So, naturally you have to evaluate the variation of the velocity in order to find out the force variation along the structure, so I hope its clear now, why we are using the velocity potential and why are we deriving the expression for the particle velocity.

What we saw earlier is the speed of the wave, that is the speed with which the wave is moving as a whole what is the speed with which the wave is moving, that is celerity, but now what are we looking at, we are looking at the variation of particle velocity, that is again I am please recollect that we are talking about a oscillatory motion.

So, when it is oscillating is also will be oscillating and it will be varying along the depth, so we will get back to the expression, this is the expression which we have derived and when you differentiate it with respect to x, this is the expression your you will be having, but we have already derived the dispersion relationship; as stated earlier and we also know, that the a is nothing, but the amplitude which is half of the wave height.

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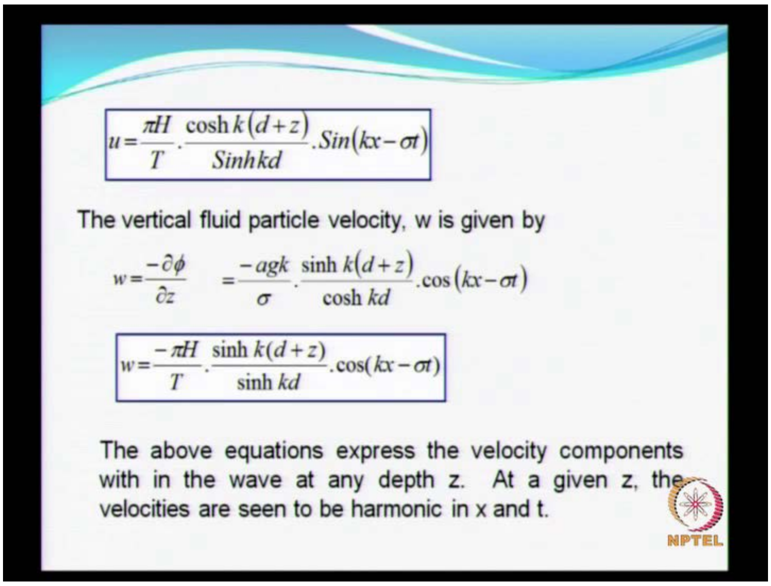

So, substituting in this expression all this you can easily derive the expression for the horizontal water particle velocity also refers to as inline velocity, which is going to be a function of wave height. So, it is going to be the function of wave height, wave period, and water depth, I am not using wave length, because once I said wave period it is automatically understood that wave length is included in that, because wave length is nothing but, a function of wave period.

So, now how does this velocity vary, this velocity the horizontal water particle velocity will be varying from z equal to 0 up to the seabed and how will the variation will look like, remember all theses expressions are for us sinusoidal curve; when you have sinusoidal wave moving under this we have already seen that the phase variation of velocity potential will be out of phase will be of cosine curve, but the horizontal water particle velocity will be sin curve, that is which you will be in phase with the evaluation, so you have sin theta here.

Now, z will be carrying from 0 to minus d up to the seabed, so what does this mean, this means that the velocity particle velocity will be a sine curve variation, with maximum amplitude maximum amplitude when z equals to 0, that is at the sea water level z equal to 0 that is the point where you will have maximum arbitral velocity. And the magnitude of which will keep on decreasing as you go towards it seabed and at each elevation it will be a the phase variation, will be a sine variation the same thing holds good in the case of a vertical velocity.

So, now vertical velocity will be out of phase with horizontal velocity and the theta and remember there is a negative also which will come in which will come in to exist, so again the wave phase can be obtained here, the phase variation can be obtained here (Refer slide time: 18:08).

So, at each elevation at each elevation you can obtain u max at each that is on the assumption that k x minus sigma T is 90, when k x minus sigma T is 90 that is the theta if you assume it as 90, that sine 90 is equal to 1. So, you will have the maximum value maximum value of u at a crest of the wave, because we are talking about sinusoidal wave and crest is theta equal to 90, because when you have the wave.

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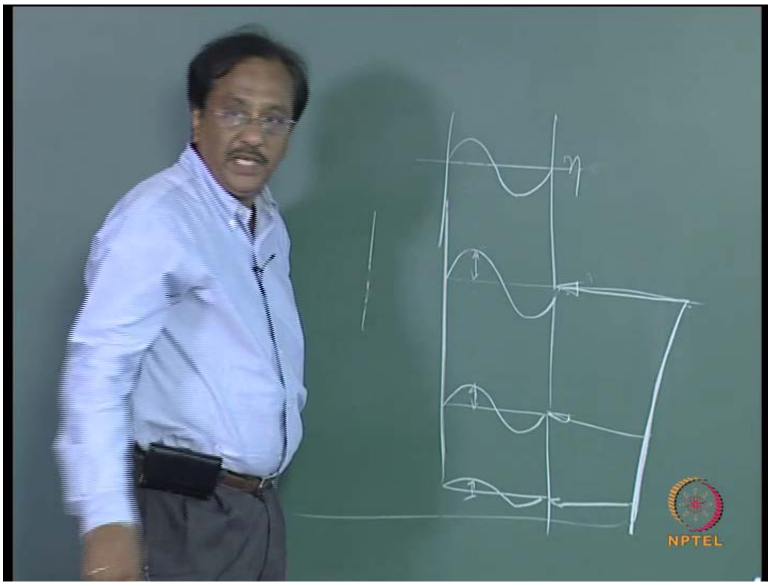

So, this is the variation of eta, so this will be u at somewhere close to the free surface and then, you see that the wave the magnitude of the horizontal, vertical velocity will be reducing as we go towards the, and each elevation it will be a sin curve. Now I am talking the magnitude if I plot, that how does the variation can look like, the variation will be like this, because this will be the maximum that I am potting only the magnitude here and then this will be at this, so you see that at this variation will be nothing but, a cos hyperbolic variation.

So, usually there will be you will be asked how does the particle velocity, vary under the a progressive way, you need to say that it is a hyperbolic variation is that clear; so now, you see that when you have a structure here, the distribution of the velocity is very important in order to evaluate the forces on the structure, so now similarly for w that is the vertical velocity.

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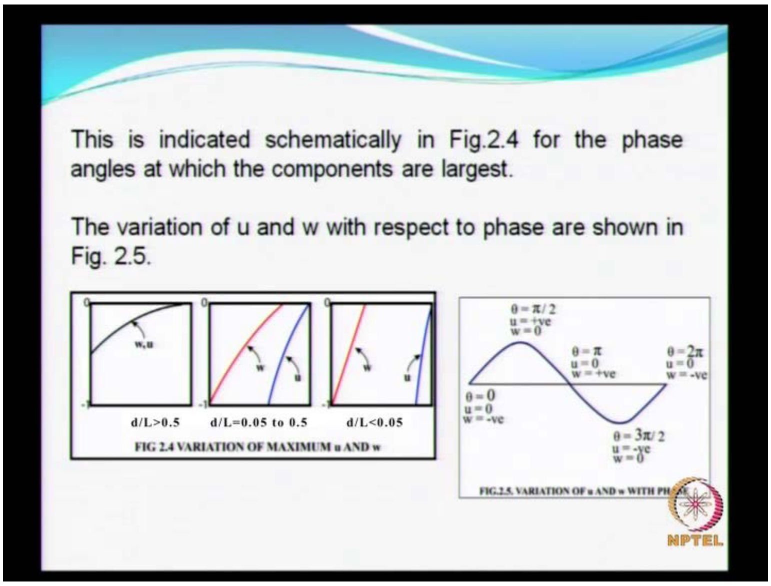

So, this is what I have explained here, so now, we look at the variation phase variation which has just now stated in this plot, how each one will vary this is the sine curve and you will be 0 at theta equal to 0 and w will be negative look at the expression and then whereas, here this will be maximum crest u, at that particular point u, w will be 0 and etcetera for the other phase.

As I said that, what I drew on the board the same thing now, I am representing here for u and w considering the 3 water depth condition which we have already discussed, that is in the case of deep waters, you will have u and w will be almost same will be same then here you see the variation of u this is the maximum velocity horizontal velocity at each of the elevation and similarly the maximum horizontal vertical maximum vertical velocity at each elevation.

So, this is actually for shallow water condition that is d by L equal to 0.05, so you see that the horizontal velocity much higher. So, what does this also employ if you have pipeline here, will it experience it any wave force it will be very negligible, but if you have a pipeline in the shallow waters, the magnitude all the force in the horizontal direction is expected to be almost close to that, what can be close to the free surface, so such information is easily obtained from this

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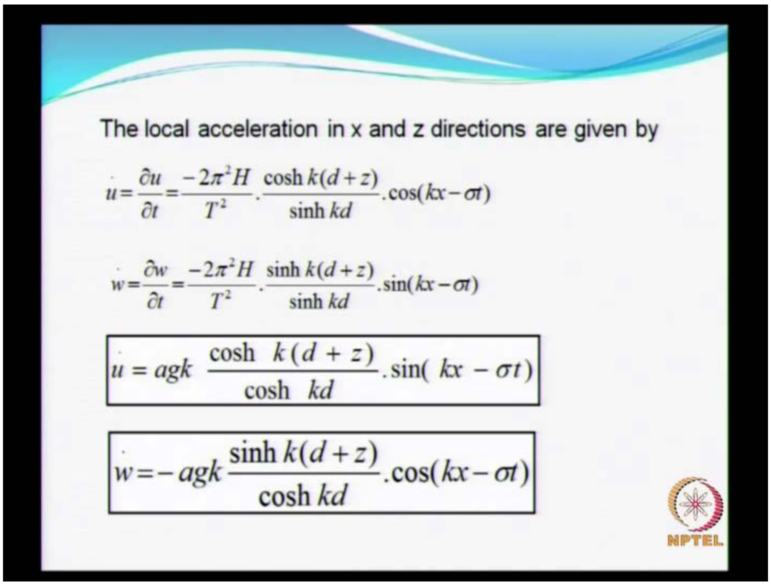

So now, we proceed further in order to to obtain the acceleration, the wave’s waves are an unsteady flow it is an unsteady flow, so the fluid particles the fluid properties vary with respect to time. So, when wave is acting on a pile or a structure, the force a sinusoidal the pile will be due to both particle velocities as well as due to accelerations which we will see later.

So, in order to obtained the total force, we need to have the particle acceleration also which are derived and these are simple derivations which you can straight away obtain and these in terms of h and this in terms of amplitude, only thing is the difference is just we have used k also here, substituted is that clear.

So, we have covered the velocities and particles velocities and accelerations, if I am floating on water surface and in the deeper water, then I will be moving from my mean positioning in circular orbits as like this and in case, I am in the shallow water conditions then my position from my mean position, my movement will be in elliptical orbits is that clear. So, this is very important, how the motions are water particles take place under different water depth conditions and that is what we are going to examine with this.

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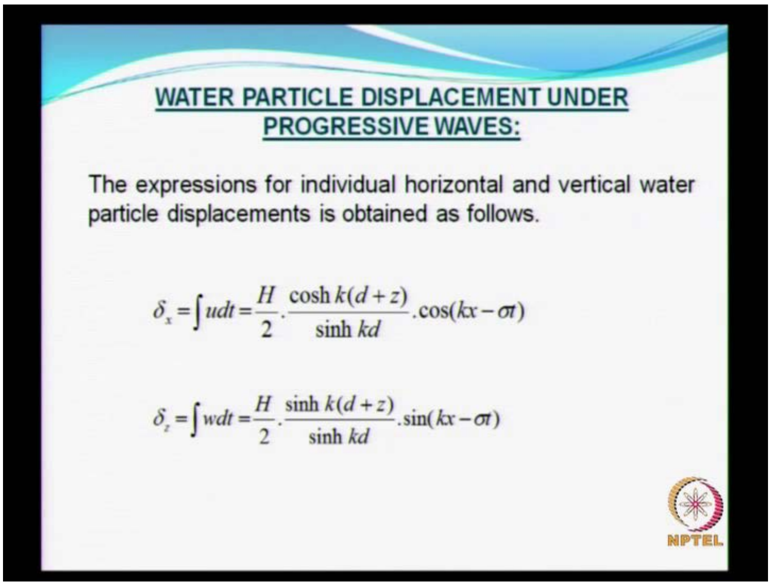

So, since the expressions for the individual I mean the horizontal and vertical particles, so this can be obtain from integrating the velocities, horizontal and vertical velocity, so you integrate. So, you substitute you look at the earlier expressions and then you land up with an expression for delta x, that is the displacement in the direction of wave propagation that is the direction.

Similarly, a vertical displacement in this direction is going to be like this, so from the earlier expression, from this expression, let me call this by an alphabet and this by another alphabet has done here.

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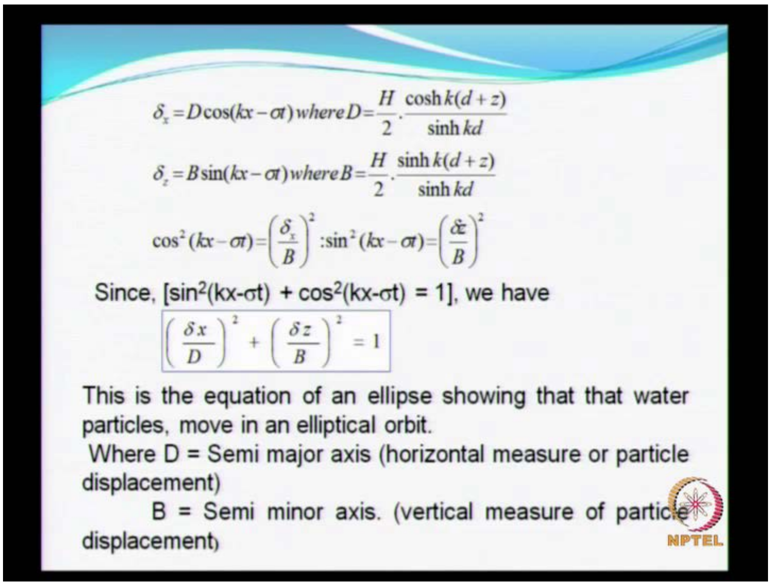

So, I have delta x as D into cos of k x minus sigma t, where k is where D is as shown here and similarly the vertical displacement is given by this where B is from the earlier a slide you get all this information. Now, I just take through this, take this sine square and k square and when I add them that it is equal to which 1, which all of us know from which we get an expression as you can see, which is, nothing but, the equation of an ellipse.

So, this is the equation of an ellipse, showing that the water particles moved in a elliptical orbit, where D is semi major axis horizontal horizontal that is horizontal measure of particle displacement whereas, your B is semi minor axis. So, now as we have examined the shallow, the celerity behavior of celerity in shallow and deep waters, we will go through that process for different water depth conditions.

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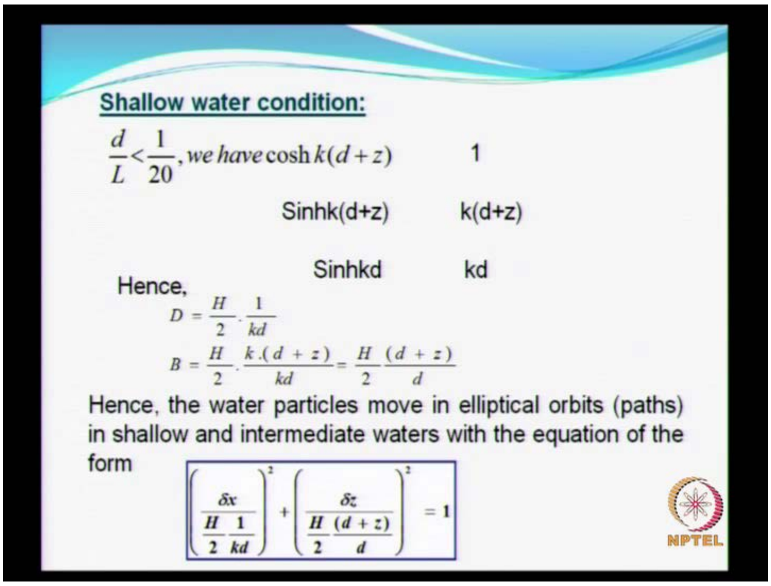

So, in the case of a shallow water condition, cos hyperbolic argument will become 1 where as sin hyperbolic will become the argument itself, so the argument here is k into d plus z in to sin hyperbolic you have two two argument, so you see B and D. So, you have to consider these two expressions, so using this, these limitations you will get d equal to this much and b equal to h by 2 into d plus z divided by d.

Now you substitute in this I will get expressions as shown here this is how, so you see the size of can be semi major axis and the size of the semi minor these two are different proving that the particles in shallow waters as well as in intermediate waters will be moving as elliptical orbits. So, if you physically stand there, you will be undergoing as a an elliptical orbit provided you are in intermediate of shallow waters, but in the deeper waters, in deep waters D is given by this expression as you have seen earlier.

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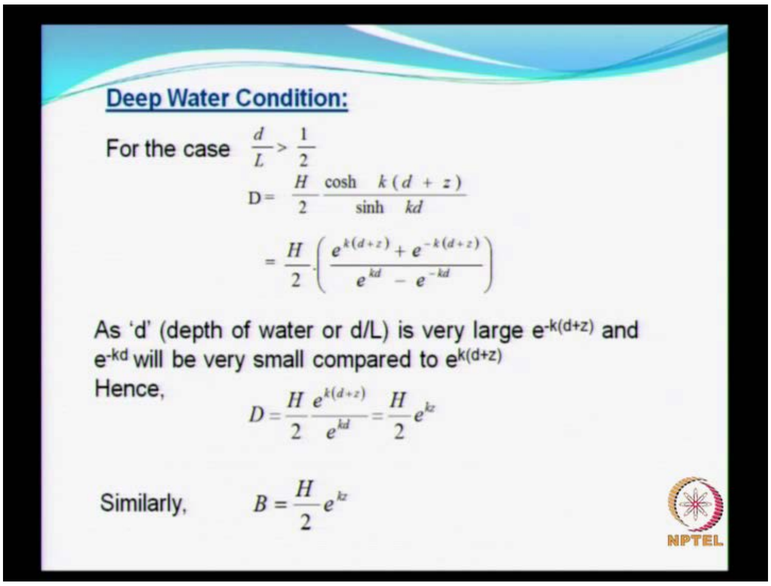

So, which can be written as in the form of exponential and when d is a very large the negative exponential e to the power minus k into d plus z and e to the power minus k d will be very small.

So, you can just simply write this expression as given here, which is nothing but, this much that is h into h by 2 into e to the power k z the same thing will hold good for the for b the other direction of displacement. So, you see that both the directions of displacement, when the way it is in deeper water deep water particles will be in circular orbits whereas in shallow waters it start moving in elliptical orbits.

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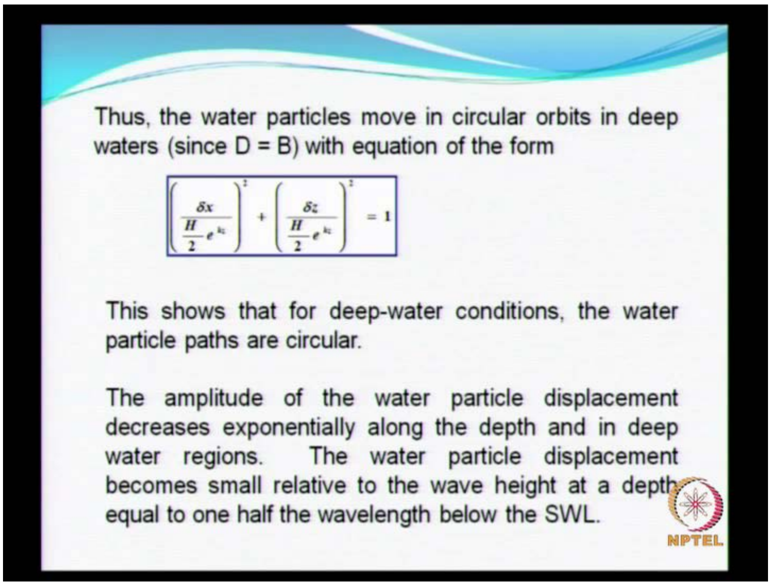

So, this is what I have written here, the amplitude of the water particle displacement decreases exponentially along the depth, because it is an exponential variation, so the d k or the reduction in the displacement for a wave which is in deeper water it will be more drastic. And in fact, at a distance of L naught by 2 L naught is nothing, but 1.6 in to T square divided by 2 at that up to that location only you will have some amount of displacement, beyond that the displacement will be negligible particular in deeper waters; in deep water the displacement will be almost zero negligible, at a distance of L naught by 2 below still water line.

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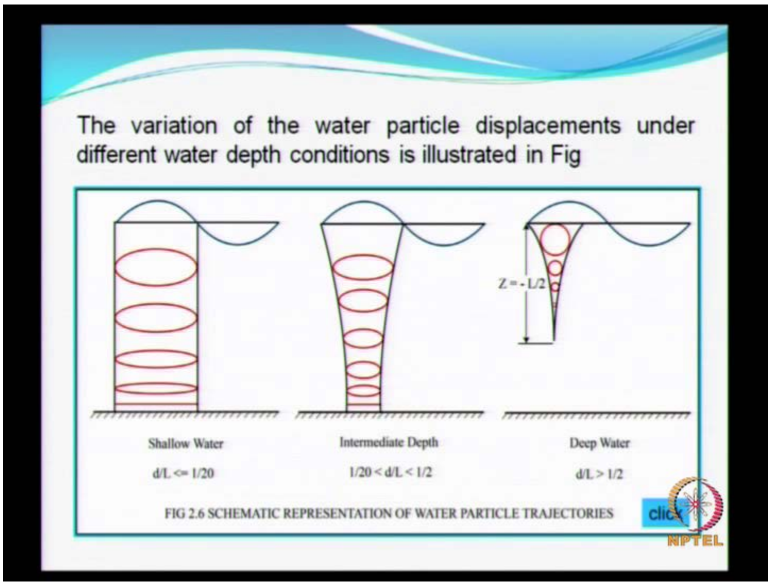

And this what pictorially represented as shown here, so in shallow waters you see that the displacement will be felt, the horizontal displacement will be felt up to the seabed, the vertical displacement will be reducing as you go below the (( )).

So, you see that if you have a pipeline here and the horizontal force will be almost same as that near the, now in the case of intermediate water the horizontal displacement also will be reducing as you go towards the seabed, in both these water depth conditions the particles orbit will be particle will be moving in elliptical orbits whereas, in deep waters as I have explained earlier at a distance of L naught by 2 this will be almost negligible.

Now, let us understand, how this can happen can we have some kind of a animation, the university of deliver under the leadership of professor Tony Dalrymple who has also brought out a nice book on wave mechanics, he has he has created animations to clearly visualize the effects of variation of the particle displacements, etcetera.

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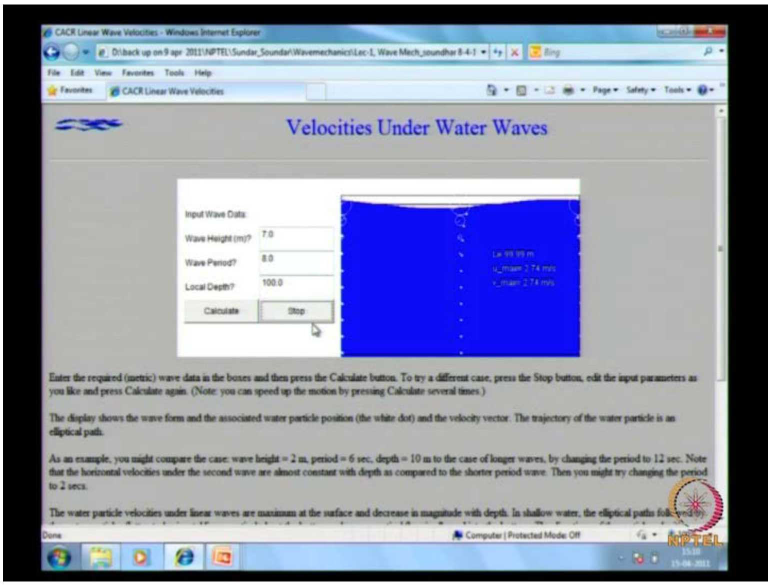

So, let us now look at this animation, initially I am trying to generate deep water conditions by assuming a wave height of 7 meters, wave period of 8 seconds, propagating in a wave in a water depth of 100 meters. So, let us see what happens in this case see, you look at the orbits here, near the free surface you see, a large orbit that is the displacement is quite large near the free surface and this and is all in circular orbit, because your wave length in this case is close to around 100 meters and the water depth is hundred meters.

So, d by L relative water depth is close to around 1 which is greater than 0.5, so deep water condition, so size of a orbit can continue to reduce as you go down towards the seabed and it is almost negligible you see here. So, this is how now particles will be moving under the wave, when it is propagating in deep waters, so now, let us change depth parameters in such a way, let us try to see simulate wave intermediate water depth conditions.

So, I am assuming wave height of 5 meters and I am just changing the water depth to 30 meters, so now, you see that, earlier case there are not much of disturbance here, now you see more or less it is circular, but it is tending towards shallower; if you change the water depth if you reduce the water depth or increase the wave length, what will happen your d by L will reduce, thereby you can have instead of circular orbit the waves would be moving in elliptical orbits.

So, let us examine for a wave height of about, I am since I am going to reduce the water depth drastically, I am going to reduce the wave height also, because you cannot have a huge wave height, you cannot sustain in a shallow very shallow waters the sustainability of wave height will be looking at when we are talking about a the wave deformation and this one I am changing it to 2 meters.

So, now you look at the variation, you see that the waves are propagating in elliptical orbits and the effect of this waves are felt up to the seabed, now one more interesting thing we can observe now, what we will see in deep waters; in deep waters we assume the for 100 meters water water depth, we saw that the orbits were only near the free surface and after certain distance it reduce.

Now, let us us try this example, let us assume that, the water depth is 4000 meters, let me have the wave period about 2 hours, I have said a wave can have only a wave period of up to 30 seconds, but now I am increasing the wave period to 2 hours, say approximately 2 hours that is 120 into 60, so 7200 seconds am I right? What can we see now, so we are talking about water depth of 4000 meters, now first initially let us restrict ourselves to you shall say ten seconds wave normal under normal circumstance.

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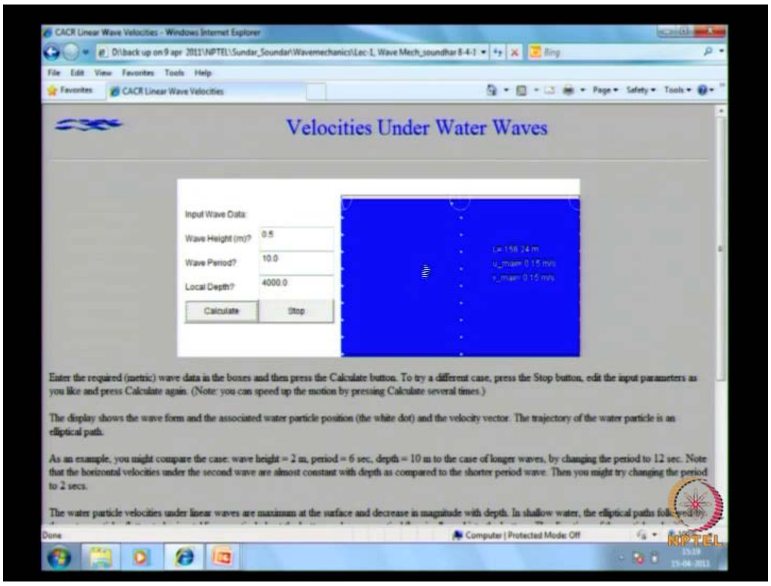

So, you see that in this case the water depth is a 4000 meters and a wave height of 0.5 meter, you see that the displacement of the water surface is very small and the displacement of the orbits is only near the free surface. Now I change this to 7200 seconds, that is equal to 2 hours, now you see what is happening in 4000 meters that is 4 kilometers you see that a effect of a long period wave is felt even up to the at the seabed.

What is this, what we have simulated here is nothing but, a tsunami, tsunami can have wave period ranging up to hours and that is what we have exactly simulated; now I hope you have understood, one of the difference between a wave and a tsunami. So, I am sure that this exercise as a given as a very clear idea about about the behavior of the waves in different water depth conditions.

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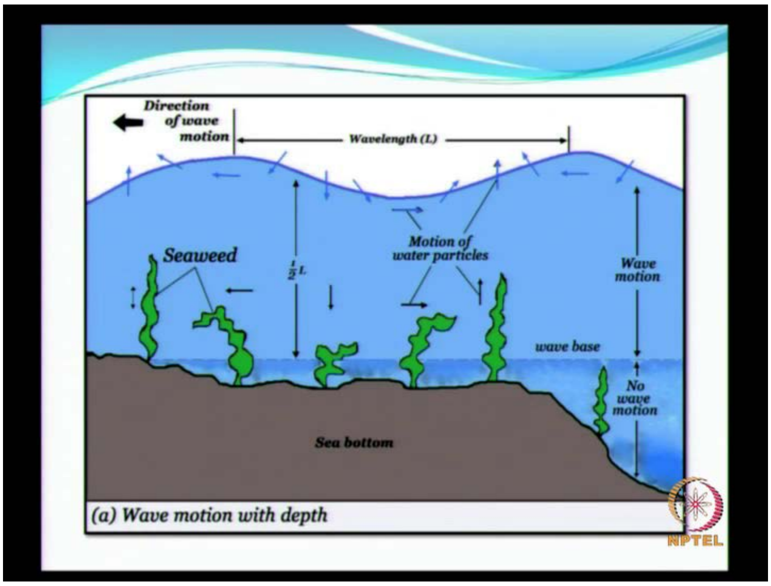

If you have a sea grass whatever it is, now this is how the water particles would be the looking like the displacement of or the motion of individual particles, when a wave is moving here, so you have the motion in this direction and here, so and when under the tough the direction will be in this, it will be focusing in this and around this time it will be vertical.

So, all this things I have already explained in terms of the phase variation, but this picture shows you how leaf can undergo the kind of motions when a wave is propagating over it.

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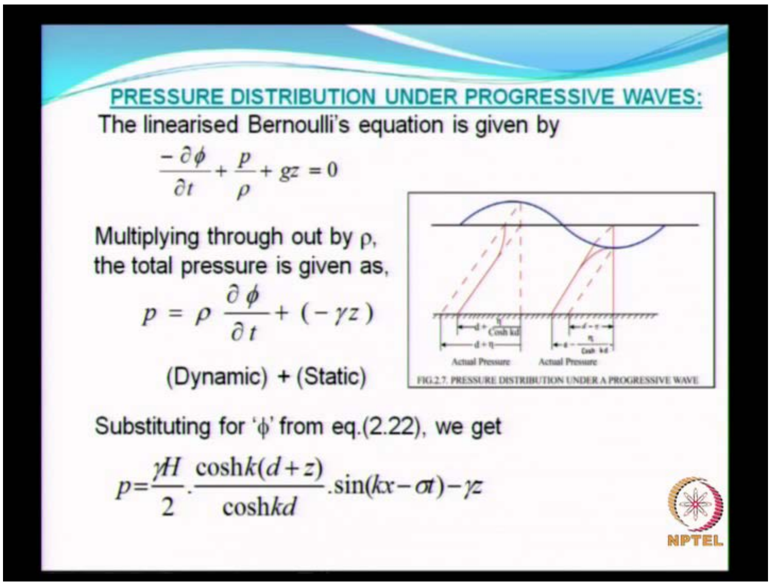

Now, we look at the pressure distribution and a progressive wave, so as I said earlier we have been, we are dealing with the linear theory and we have linearised the Bernoulli equation, which will be used for defining the pressure distribution under the waves. So, here is the linearised Bernoulli equation and the pressure distribution is shown here in this picture, now we multiply it, multiply the Bernoulli equation by rho and then to get the total pressure and now this will have two components, one is the dynamic components and the other one is the static pressure z component.

So, we already seen the expression for the velocity potential, which once you substitute will get an expression for the pressure as shown here, this would mean that, the pressure will be varying along the water depth and there are it is very important to have in mind.

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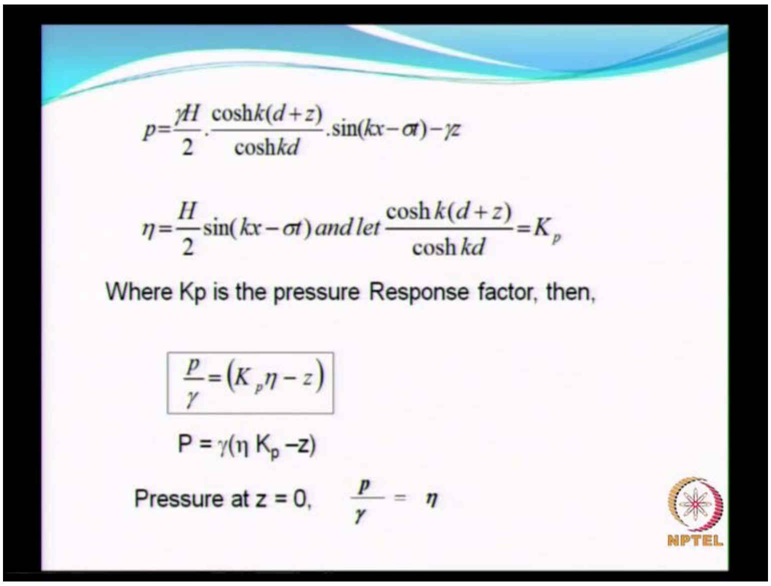

How this dynamic pressure component varies and how the total pressure vary, for this purpose, we again look at the expression for pressure, which is rewritten in this slide. And we know that eta is x by 2 into sine of a k x minus sigma T and this hyperbolic function is represented as or is defined as pressure response factor and it is it is represented as K of p.

So, hence the definition of the pressure head is given by this equation K p that pressure head equal to K p into eta minus z or pressure you just take the gamma into the equation and this will be the pressure in density. Now, we will consider the pressure variation at different variation, within the flowed medium, so at z equal to 0 at the still water line, using this expression this will be the basic equation, which a you will be using for defining how the pressure are going to be vary at z equal to 0, your pressure will be equal to P by gamma.

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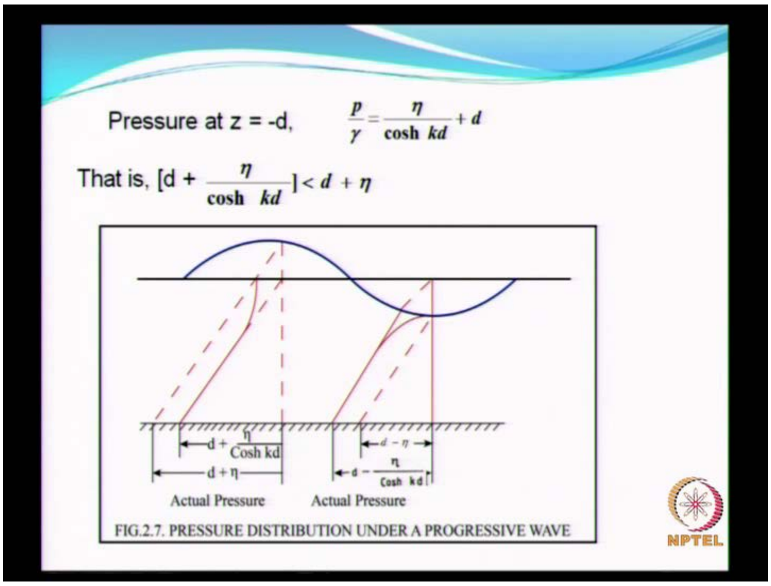

And next is at the seabed z takes the value of minus d and hence p by gamma will become d plus eta by cos of k d, so how will the static pressure head look at the below the crest below the crest it will be d plus eta, water depth plus that elevation about the water I mean mean water level or still water line. So, you will have d plus eta, but the actual pressure will be as indicated here, when the crest is crossing which will be less than the hydrostatic pressure, because cos k d, because of the variation of cos k d, that is cos k d is always greater than 1.

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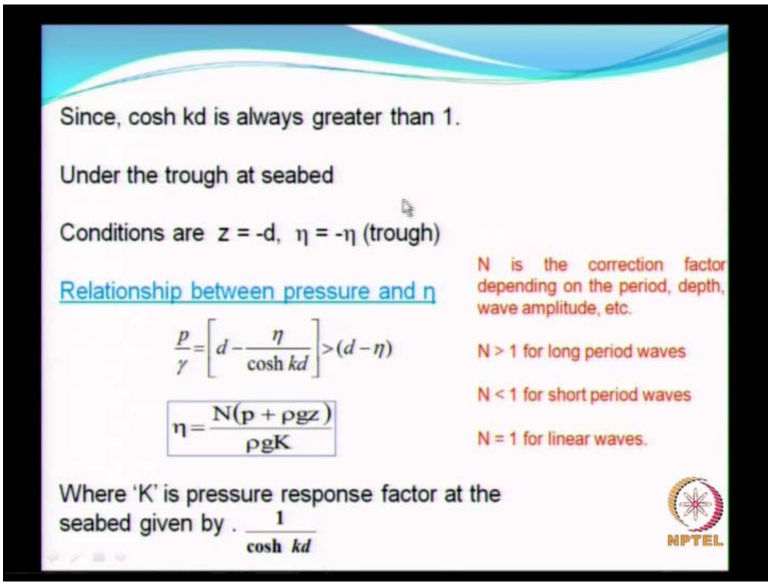

Now, under that trough if you look at, you have to use the same expression with z is equal to minus d and eta taking the value of a minus eta, you land up with this expression and in which case this actual pressure will be greater than the hydrostatic pressure and the tough crust.

So, this is the basic difference when a crust is passing by and when a trough is passing by, so finally, you have to you have an expression relating variation of eta and pressure dynamic pressure, this is why I said that this the one of the basic principle, main principle, why it is being used for the tsunami pre warning system are the pressure sensor is used for obtaining or measuring the wave climate within in a given size for measuring the wave climate.

So here, you see that the k can is just the pressure response factor are the seabed therefore, it is both are related pressure and n is nothing, but correction factor adopted and n takes a value slightly greater than 1, that is for long period waves, how big the values should be depends on how long the waves are, and a similarly for the short period in which case n will be less than 1 and in the case of linear waves we use the n is equal to 1. So, thus we have seen the dynamic pressures under a progressive waves, the dynamic pressure in fact it will be reducing from the still water level, as you go towards the seabed and it is going to be a cos hyperbolic variation.

So, I hope pressure distribution and a wave are clear and you get more information about the calculation of the pressures and other how once a pressure is given how you determine the wave valuation, vice versa in the some of the examples which we will work out later.